The Standard Model.
Part 2

Alexander Korchin

National Science Center “Kharkiv Institute of Physics and Technology”, Ukraine
V.N. Karazin Kharkiv National University, 61022 Kharkiv, Ukraine
Institute of Nuclear Physics Polish Academy of Sciences, PL-31342 Krakow, Poland

Trans-European School of High Energy Physics,
Bezmiechowa Gorna, Bieszczady mountains, Poland,
July 14-22, 2023

Standard Model, Part 2. Outline

- Spontaneous symmetry breaking
  - – Goldstone theorem
  - – Superconductor and hidden gauge symmetry
  - – Mechanism of Higgs in electroweak theory
  - – Masses of gauge bosons
  - – Fermion masses
- CP violation and matter-antimatter asymmetry in the Universe
  - – Flavour mixing and CKM matrix: CP violation
  - – CP violation and matter-antimatter asymmetry in the Universe

Spontaneous Symmetry Breaking (SSB)

Up to now we

- derived **charged- and neutral-current interactions** of the type needed to describe weak decays,
- incorporated **electromagnetic interactions**,
- got additional **self-interactions of the $W^{\pm}$ and Z bosons**,
- obtained a well-defined and consistent **renormalizable Lagrangian** (guaranteed by the local gauge symmetry).

However, the gauge bosons are still massless,

$$M_{W^{\pm}} = 0, \qquad M_Z = 0, \qquad m_{\gamma} = 0,$$

which is fine for the photon but not satisfactory for the heavy $W^{\pm}$ and Z.

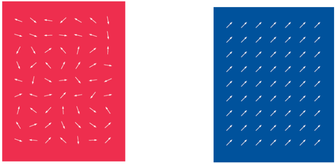
Dilemma: in order to generate masses, we need to break the gauge symmetry; however, a symmetric Lagrangian is needed to preserve renormalizability. Therefore, we need to find a way of getting non-symmetric results from symmetric (invariant) Lagrangian.

Spontaneous Symmetry Breaking (SSB)

Let us consider a physical system described by Lagrangian, which

1. is invariant under certain transformations,
2. has a degenerate set of states with minimal energy,
3. if one of those states is arbitrarily selected as the ground state of the system, then the symmetry is said to be spontaneously broken.

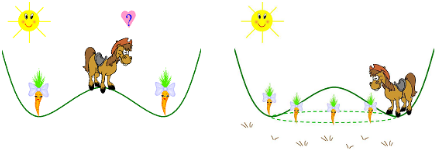
A well-known physical example is provided by a ferromagnet: although the Hamiltonian is invariant under rotations in 3-dim space, the ground state has the spins aligned into some arbitrary direction.



In a Quantum Field Theory (QFT), the ground state is the vacuum; thus SSB mechanism appears when there is a symmetric Lagrangian, but a non-symmetric vacuum.

Goldstone theorem

The horse illustrating in a very simple way the phenomenon of SSB [taken from review of A. Pich]



The existence of flat directions connecting the degenerate states of minimal energy is a general property of breaking the continuous symmetries.

In a QFT it implies the existence of massless degrees of freedom which are called Goldstone bosons.
The fact that there are massless excitations associated with the SSB mechanism is a completely general result, known as the Goldstone theorem.

Goldstone theorem

In Nature there are two ways how symmetry of Lagrangian is realized:

- Wigner-Weyl (conventional) - ground state (vacuum) is symmetrical
- Nambu-Goldstone (more complex) - ground state is not symmetrical. This is called “spontaneous symmetry breaking” (SSB), or “hidden symmetry”.

The Goldstone theorem (mathematical formulation):

if Lagrangian is invariant under a continuous symmetry group G, but the vacuum is invariant under a subgroup $H \subset G$, then there must exist as many massless spin-0 particles (Goldstone bosons) as broken group generators, i.e., generators of G which do not belong to H.

A simple model Lagrangian

To illustrate SSB and Goldstone theorem we need a simple model of Lagrangian.

Consider a complex scalar field $\phi(x) = \phi_1(x) + i\phi_2(x)$

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi)\,, \qquad V(\phi) = \mu^2 \phi^\dagger \phi + h \left(\phi^\dagger \phi\right)^2 .$$

$\mathcal{L}$ is clearly invariant under global $U(1)$ phase transformations

$$\phi(x) \longrightarrow \phi'(x) \equiv \exp\{i\theta\}\,\phi(x)\,.$$

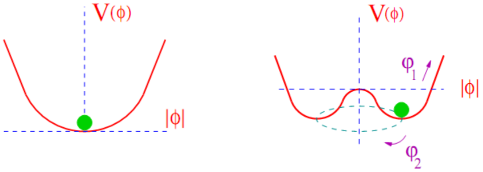
The parameter $h > 0$.
For the quadratic piece $\mu^2\phi^\dagger\phi$ there are two possibilities:

1. $\boldsymbol{\mu^2 > 0}$: The potential has only the trivial minimum at $\phi = 0$. It describes a massive scalar particle with mass μ and quartic interaction $h\left(\phi^\dagger\phi\right)^2$.
2. $\boldsymbol{\mu^2 < 0}$: The minimum of potential energy is obtained for the field configurations satisfying

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}} > 0\,, \qquad V(\phi_0) = -\frac{h}{4} v^4 .$$

“Mexican hat” potential

In order to have a ground state the potential should be bounded from below



The first possibility with $\mu^2 > 0$ is just the usual situation with a single ground state with $\phi_0 = 0$.
The other case with $\mu^2 < 0$ is more interesting. Owing to the $U(1)$ phase-invariance, there is an infinite number of degenerate states of minimum energy,

$$\phi_0(x) = |\phi_0| \exp\{i\theta\} = \frac{v}{\sqrt{2}} \exp\{i\theta\},$$

or $\phi_1 = \frac{v}{\sqrt{2}} \cos\theta$ and $\phi_2 = \frac{v}{\sqrt{2}} \sin\theta$.
By choosing a particular solution, for example $\theta = 0$, as the ground state $\phi(x)_0 = v/\sqrt{2}$, the symmetry gets spontaneously broken.

Massless and massive bosons

The excitations over this ground state should describe the particles in this model. Let us parameterize them

$$\phi(x) = \phi(x)_0 + \phi_{exc}(x) \equiv \frac{1}{\sqrt{2}} \left[v + \varphi_1(x) + i\,\varphi_2(x) \right] ,$$

where φ_1 and φ_2 are real scalar fields, and the potential is then (recall that $\mu^2 < 0$)

$$V(\phi) = V(\phi_0) - \mu^2 \varphi_1^2 + h\,v\,\varphi_1 \left(\varphi_1^2 + \varphi_2^2 \right) + \frac{h}{4} \left(\varphi_1^2 + \varphi_2^2 \right)^2$$

and the Lagrangian becomes

$$\begin{aligned} \mathcal{L} \quad = \quad & \frac{1}{2}\partial_\mu \varphi_2 \partial^\mu \varphi_2 + \frac{1}{2}\partial_\mu \varphi_1 \partial^\mu \varphi_1 - \frac{1}{2} m_{\varphi_1}^2 \varphi_1^2 \\ & - h\,v\,\varphi_1 \left(\varphi_1^2 + \varphi_2^2 \right) - \frac{h}{4} \left(\varphi_1^2 + \varphi_2^2 \right)^2 . \end{aligned}$$

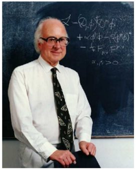
A massless field φ_2 describes excitations around a flat direction in the potential, i.e., into states with the same energy as the chosen ground state. Those excitations do not cost any energy, and correspond to a massless state – Goldstone boson.

The massive field φ_1 with the mass $m_{\varphi_1} = \sqrt{2|\mu^2|}$ is a prototype of the Higgs boson.

Mechanism of Higgs – generation of masses

J. J. Sakurai Prize for Theoretical Particle Physics

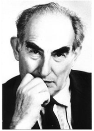
2010: (L to R) Kibble, Guralnik, Hagen, Englert, Brout, Higgs

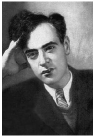


"For elucidation of the properties of spontaneous symmetry breaking in four-dimensional relativistic gauge theory and of the mechanism for the consistent generation of vector boson masses."

Hidden gauge symmetry in superconductor

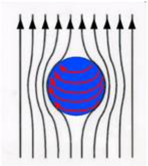
Superconductor provides an example of physical system in which the photon acquires a mass inside a medium, as a consequence of a symmetry-reducing phase transition.





Ginzburg-Landau phenomenological description of superconductor (1950):
there two types of charge carries
(i) normal, or resistive, and
(ii) superconducting, or resistanceless.

- Electric current without resistance;
- magnetic fields are expelled from superconductor – this is called Meissner effect.



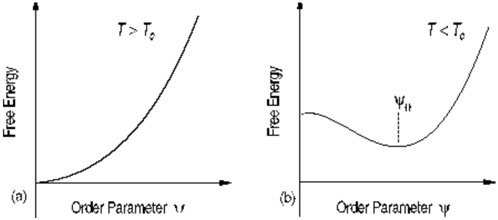
Breaking of symmetry in superconductor

The free energy (analogue of Hamiltonian)

$$G_{\rm super}(0) = G_{\rm normal}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4 \quad ,$$

where $\beta > 0$, $|\psi|^2$ is called order parameter – density of superconducting charge carriers, and

(a) $\alpha > 0$ if $T > T_c$ (minimum energy at $|\psi_0|^2$ corresponds to a resistive flow with no superconducting carriers),
(b) $\alpha < 0$ if $T < T_c$ (free energy is minimized when $|\psi_0|^2 = -\alpha/(2\beta) \neq 0$).



Photon acquires a mass inside superconductor

In an applied magnetic field $\vec{H}$ the free energy is

$$G_{\rm super}(\vec{H}) = G_{\rm super}(0) + \frac{\vec{H}^2}{8\pi} + \frac{1}{2m^\star}|-i\hbar\nabla\psi - \frac{e^\star}{c}\vec{A}\psi|^2 \ ,$$

$e^\star = -2|e|$. Here you probably recognize covariant derivative.
The equation of motion for the photon (for slowly varying field $\vec{H} \approx 0$ and $\psi \approx \psi_0$):

$$\nabla^2\vec{A} - \frac{4\pi e^\star}{m^\star c^2}\,|\psi_0|^2\,\vec{A} = 0 \ ,$$

A massive vector field satisfies the Klein-Gordon equation

$$\Box\, A^\mu + m^2\, A^\mu = 0$$

For $A^0 = 0$ and $\partial\vec{A}/\partial t \approx 0$ it leads to the conclusion that:
inside superconductor the photon acquires a mass

$$m_\gamma^{eff} = \sqrt{\frac{4\pi|e^\star|}{m^\star c^2}}\,|\psi_0| \neq 0.$$

Therefore the magnetic field in the medium fades away on the distance $\lambda_L = \hbar/m_\gamma^{eff}$ – this is the origin of **Meissner effect**. Typically $\lambda_L \sim 10^{-5} - 10^{-6}$ cm.

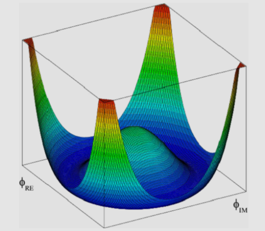
It is interesting to mention, that this effect was independently discovered experimentally by a physicist from Kharkov - Lev Shubnikov

Superconductor gives a hint how a symmetry-hiding phase can lead to a massive gauge boson!

And we need masses for $W^{\pm}$ and Z bosons, but photon has to stay massless.

Introduce an $SU(2)$ doublet of complex scalar fields

$$\phi(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix} = \begin{pmatrix} \mathrm{Re}\,\phi^{(+)}(x) + \mathrm{i}\,\mathrm{Im}\,\phi^{(+)}(x) \\ \mathrm{Re}\,\phi^{(0)}(x) + \mathrm{i}\,\mathrm{Im}\,\phi^{(0)}(x) \end{pmatrix}$$



The charges $Q_{\phi^{(+)}} = +1$, $Q_{\phi^{(0)}} = 0$ and weak isospin projections $T_{3\,\phi^{(+)}} = +1/2$, $T_{3\,\phi^{(0)}} = -1/2$.

Mechanism of Higgs

Take the scalar Lagrangian of the Goldstone model, and introduce there instead of usual derivative ∂^μ the **covariant derivative** D^μ:

$$\mathcal{L}_S = (D_\mu \phi)^\dagger D^\mu \phi - \mu^2 \phi^\dagger \phi - h \left(\phi^\dagger \phi\right)^2 \qquad (h > 0\,,\ \mu^2 < 0)\,,$$

$$D_\mu \phi = \left(\partial_\mu + i\, g\, \frac{\sigma^i}{2}\, W^i_\mu + i\, g'\, y_\phi\, B_\mu\right) \phi, \quad \text{with} \quad y_\phi = Q_\phi - T_3 = \frac{1}{2}$$

It is invariant under local $SU(2)_L \otimes U(1)_Y$ transformations (similarly to QED and QCD) with appropriate transformation of the gauge fields W^μ and B^μ.

For $\mu^2 < 0$ there is a infinite set of degenerate states which minimize the potential energy, satisfying

$$\left|\langle 0|\phi^{(0)}|0\rangle\right| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}\,,$$

where v is called the vacuum expectation value (VEV) of the neutral scalar.

Since the electric charge is a conserved quantity, only the neutral scalar field $\phi^{(0)}$ can acquire VEV (because the vacuum is neutral).

Mechanism of Higgs

Once we choose a particular ground state, the $SU(2)_L \otimes U(1)_Y$ symmetry gets spontaneously broken. However, the electromagnetic subgroup of QED $U(1)_{\rm QED}$ should remain a true symmetry of the vacuum, i.e.

$$SU(2)_L \otimes U(1)_Y \;\Rightarrow\; U(1)_{\rm QED}.$$

According to the Goldstone theorem three massless scalar states should appear. These are unphysical particles.
Let us parametrize four fields of scalar doublet in the general form

$$\phi(x) \;=\; \exp\left\{ i\,\frac{\sigma_i}{2}\,\theta^i(x) \right\} \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + H(x) \end{array} \right),$$

with four real fields $\theta^1(x)$, $\theta^2(x)$, $\theta^3(x)$ and $H(x)$.
The crucial point is that the local $SU(2)_L$ invariance allows us to rotate away any dependence on $\theta^i(x)$. These three fields are precisely the would-be massless Goldstone bosons associated with the SSB mechanism.

The condition $\theta^i(x) = 0$ is called the physical (unitary) gauge in which we get rid of unphysical massless excitations and

$$\phi(x) \;=\; \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + H(x) \end{array} \right),$$

Masses of gauge bosons

The covariant derivative couples the scalar doublet to the gauge bosons. The kinetic piece of the scalar Lagrangian is:

$$(D_\mu\phi)^\dagger D^\mu\phi \quad \xrightarrow{\theta^i=0} \quad \frac{1}{2}\,\partial_\mu H \partial^\mu H + (v+H)^2 \left(\frac{g^2}{4}\, W_\mu^\dagger W^\mu + \frac{g^2}{8\cos^2\theta_W}\, Z_\mu Z^\mu \right) .$$

The VEV of the neutral scalar generates a quadratic term for $W^\pm$ and Z, i.e., these gauge bosons acquire masses:

$$M_Z \cos\theta_W = M_W = \frac{1}{2}\, v\, g\, .$$

This is a very important relation between masses of gauge bosons and VEV.

In addition, the interaction of the Higgs field with the gauge bosons is induced

$$\mathcal{L}_{HWW,\,HZZ} = \left(\frac{1}{v}\, H + \frac{1}{2v^2}\, H^2 \right) \left(2M_W^2\, W_\mu^\dagger W^\mu + M_Z^2\, Z_\mu Z^\mu \right)$$

Number of degrees of freedom

Therefore, we have found a way of giving masses to the intermediate carriers of the weak force – $W^{\pm}$ and Z, but not the γ, because $U(1)_{\rm QED}$ is an unbroken symmetry.

It is instructive to count the number of degrees of freedom (d.o.f.) in this model, to make sure that we have not introduced or lost d.o.f.

Before SSB, we had 3 massless W^+, W^- and Z bosons, i.e., $3 \times 2 = 6$ d.o.f. (two polarizations of each massless spin-1 field), and four real scalar fields θ^i, H. Therefore the number of d.o.f. is

$$3 \times 2 + 4 = 10.$$

After SSB, 3 Goldstone modes θ^i are "eaten" by the weak gauge bosons $W^{\pm}$, Z, which become massive and, therefore, acquire one additional longitudinal polarization. Then we have $3 \times 3 = 9$ d.o.f. in the gauge sector, plus the remaining one scalar Higgs boson H, or in total

$$3 \times 3 + 1 = 10.$$

The total number of d.o.f. in this model remains the same.

Electro-weak phenomenology

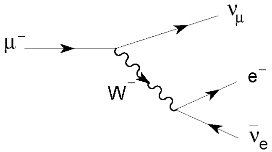
The SM predicts that $M_Z > M_W$ in agreement with the measured masses:

$$M_Z = 91.1876 \pm 0.0021\,\mathrm{GeV}\,, \qquad M_W = 80.377 \pm 0.012\,\mathrm{GeV}\,.$$

From these experimental numbers, one obtains the electroweak mixing angle

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223\,.$$

Independent estimate of $\sin^2\theta_W$ can be obtained from the decay $\mu^- \to e^-\bar{\nu}_e\,\nu_\mu$, in which the measured muon lifetime is $\tau_\mu = (2.197019 \pm 0.000021)\cdot 10^{-6}$ s.



The W propagator “shrinks” to a point because of small value of momentum transfer $q^2 = (p_\mu - p_{\nu_\mu})^2 = (p_e + p_{\nu_e})^2 \lesssim m_\mu^2 \ll M_W^2$.

$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4\sqrt{2}\,G_F$$

where G_F is the Fermi constant for the weak interaction.

A few predictions

One obtains for the lifetime

$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\,\pi^3} f(m_e^2/m_\mu^2)\,(1+\delta_{\rm RC})\,, \quad f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \log x\,,$$

where $\delta_{\rm RC}$ is radiative corrections to $O(\alpha^2)$. Then

$$G_F = 1.1663788(6) \cdot 10^{-5}\,{\rm GeV}^{-2}\,.$$

The Fermi coupling gives a direct determination of the scalar VEV

$$v = \left(\sqrt{2}\,G_F\right)^{-1/2} = 246\,{\rm GeV}$$

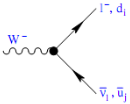
which determines the electroweak energy scale.

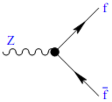
There is additional relation, which we discussed earlier, $g = e/\sin\theta_W$. Using the measured fine-structure constant α_{em}, and M_W, G_F, allows one to find weak-mixing angle

$$\sin^2\theta_W = 0.215\,.$$

Small difference between the two numbers for $\sin^2\theta_W$, 0.215 and 0.223, can be understood in terms of higher-order quantum corrections (which we did not include).

W and Z decay modes





At tree level (=no loops), the W and Z partial widths,

$$\Gamma\left(W^- \to \bar{\nu}_l l^-\right) = \frac{G_F M_W^3}{6\pi\sqrt{2}}\,, \qquad \Gamma\left(W^- \to \bar{u}_i d_j\right) = N_C\,|\mathbf{V}_{ij}|^2\,\frac{G_F M_W^3}{6\pi\sqrt{2}}$$

$$\Gamma\left(Z \to \bar{f} f\right) = \frac{G_F M_Z^3}{6\pi\sqrt{2}}\left(|v_f|^2 + |a_f|^2\right) \times N_f$$

with $N_f = 1$ for leptons and $N_f = N_C = 3$ for quarks.

- For W the widths are equal for all leptonic decay modes (up to small mass corrections);
- The quark modes the mixing factor $\mathbf{V}_{ij}$ relating weak and mass eigenstates, $d'_i = \mathbf{V}_{ij}\, d_j$ (will be discussed later);
- The Z partial widths are different for each decay mode, i.e. depends on v_f and a_f (its couplings depend on the fermion charge).

W and Z decay modes

Summing over all possible final fermion pairs, one predicts the total widths

$$\Gamma_W = 2.09\,\mathrm{GeV}\,, \quad \Gamma_Z = 2.48\,\mathrm{GeV} \qquad \text{vs. experiment}$$
$$\Gamma_W = (2.085 \pm 0.042)\,\mathrm{GeV}\,, \quad \Gamma_Z = (2.4952 \pm 0.0023)\,\mathrm{GeV}$$

The universality of the W and Z couplings implies

$$\mathrm{Br}(W^- \to \bar{\nu}_l\, l^-) = \frac{1}{3 + 2 \times N_C} = 11.1\%\,, \qquad \Gamma(Z \to l^+ l^-) = 84.85\ \mathrm{MeV}$$

These are in good agreement with the measured leptonic widths confirming the universality of the W and Z leptonic couplings.

leptonic	e	μ	τ	average
$Br(W^- \to l^- \bar{\nu}_l)$, %	10.71 ± 0.16	10.63 ± 0.15	11.38 ± 0.21	10.86 ± 0.09
$\Gamma(Z \to l^+ l^-)$, MeV	83.92 ± 0.12	83.99 ± 0.18	84.08 ± 0.22	83.984 ± 0.086

Z decay to neutrinos

Another interesting quantity is the Z decay width into "invisible" modes, i.e. to the channels not detected (neutrino's)

$$\frac{\Gamma_{\rm inv}}{\Gamma_l} \equiv \frac{N_\nu \times \Gamma(Z \to \bar{\nu}\,\nu)}{\Gamma_l} = \frac{2\,N_\nu}{(1-4\,\sin^2\theta_W)^2+1} = 5.865$$

On the other hand, from the total width and all measured channels one finds "invisible" decay width

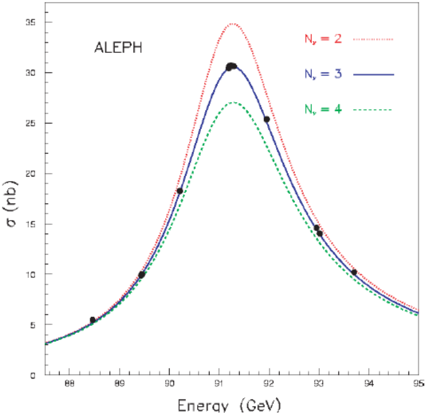
$$\Gamma_{\rm inv} = \Gamma_Z - \Gamma_l - \Gamma(Z \to {\rm hadrons}) = 499.0 \pm 1.5\,{\rm M}eV$$

and comparison with the measured value

$$\frac{\Gamma_{\rm inv}}{\Gamma_l} = \frac{499.0 \pm 1.5\,{\rm M}eV}{83.984 \pm 0.086\,{\rm M}eV} = 5.941 \pm 0.016,$$

provides experimental evidence for existence of three different light neutrinos.

Number of neutrinos



In fact, the combined analysis of ALEPH, DELPHI, L3, OPAL at LEP collaborations gives the number of neutrino's (with the masses less than $M_Z/2$):

$$N_\nu = 2.985 \pm 0.009 \approx 3$$

Fermion masses

We know that fermionic mass term $\mathcal{L}_m = -m\overline{\psi}\psi = -m\left(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L\right)$ is not allowed (it breaks the gauge symmetry – left- and right-handed fields transform differently under $SU(2)_L \otimes U(1)_Y$).
However, we introduced an additional scalar doublet, and can write gauge-invariant Yukawa-type fermion-scalar coupling (for simplicity for one family):

$$\begin{aligned}\mathcal{L}_Y \;=\; & -c_1\,(\bar{u},\bar{d})_L\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} d_R - c_2\,(\bar{u},\bar{d})_L\begin{pmatrix}\phi^{(0)*}\\ -\phi^{(-)}\end{pmatrix} u_R \\ & - c_3\,(\bar{\nu}_e,\bar{e})_L\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} e_R - c_4\,(\bar{\nu}_e,\bar{e})_L\begin{pmatrix}\phi^{(0)*}\\ -\phi^{(-)}\end{pmatrix} \nu_{eR} + \text{h.c.}\,,\end{aligned}$$

where the 2nd term involves the $\mathcal{C}$-conjugate scalar field $\phi^c \equiv i\,\sigma_2\,\phi^*$.

	Q (charge)	T_3 (weak-isospin projection)	y (hypercharge)
$\phi^{(+)}$	1	1/2	1/2
$\phi^{(0)}$	0	-1/2	1/2
$\phi^{(0)*}$	0	1/2	-1/2
$\phi^{(-)}$	-1	-1/2	-1/2

Fermion masses

After SSB, this Lagrangian takes the simpler form

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}}(v+H)\left\{c_1\,\bar{d}d + c_2\,\bar{u}u + c_3\,\bar{e}e + c_4\,\bar{\nu}\nu\right\} .$$

Therefore, the SSB mechanism also generates fermion masses !

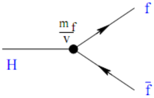
$$m_d = c_1\frac{v}{\sqrt{2}}\,, \qquad m_u = c_2\frac{v}{\sqrt{2}}\,, \qquad m_e = c_3\frac{v}{\sqrt{2}}\,, \qquad m_\nu = c_4\frac{v}{\sqrt{2}}$$

and the overall scale of masses is set by VEV: $v/\sqrt{2} \approx 174$ GeV.

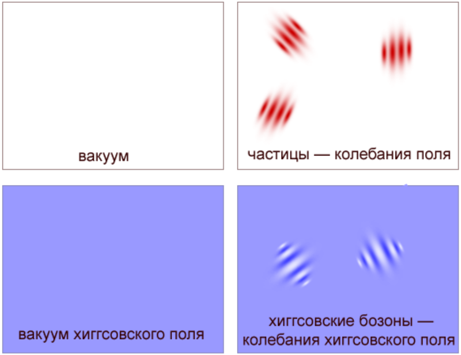
Unfortunately we do not know parameters c_i, and thus the values of the fermion masses are arbitrary.

However, all Yukawa couplings to the Higgs field are fixed in terms of the masses:

$$\begin{aligned}\mathcal{L}_Y = & -\left(m_d\,\bar{d}d + m_u\,\bar{u}u + m_e\,\bar{e}e + m_\nu\,\bar{\nu}\nu\right) \\ & -\frac{H}{v}\left(m_d\,\bar{d}d + m_u\,\bar{u}u + m_e\,\bar{e}e + m_\nu\,\bar{\nu}\nu\right)\end{aligned}$$



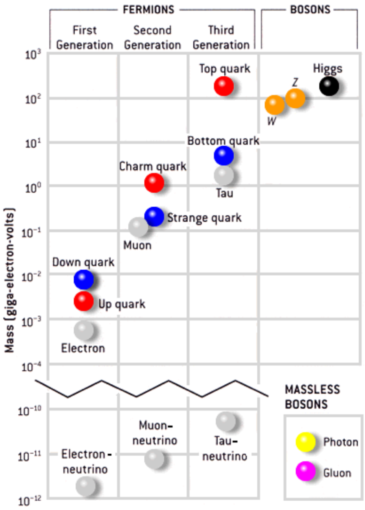
How does the vacuum in the SM look like?



Saying loosely, the Universe is filled with constant scalar field $\phi^{(0)} = v/\sqrt{2} = 174$ GeV. The particles (quarks, leptons, gauge bosons) "catch" on this field and become massive. However, the photon, neutrinos, gluons do not catch on and stay massless.

Question: where does the mass of the Higgs boson come from in this description ?

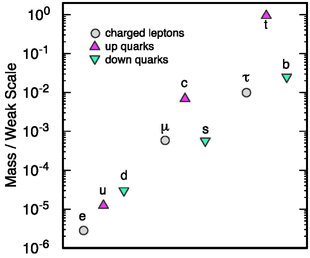
Origin of fermion masses is a mystery in the SM



Masses vary over 5 orders of magnitude! If we include neutrinos, then it is even 14 orders of magnitude.

Problems with masses in the SM

One can see that the Yukawa couplings $c_i = \sqrt{2} m_i / v$ range from $\approx 3 \times 10^{-6}$ for electron, to ≈ 1 for the top quark.



We conclude that in the SM there are two different ways of generating masses:

- masses of $W^{\pm}$, Z are directly related to mechanism of SSB and Higgs mechanism,
- masses of fermions depend on the Yukawa couplings c_i, which are undetermined and set the scale of fermion masses.

Clearly the fermion masses involve physics beyond the SM !

Quark flavor mixing

We learned that there are 6 different quark flavors u, d, s, c, b, t,
3 different charged leptons e, μ, τ and their corresponding neutrinos ν_e, ν_μ, ν_τ.
They are organized into 3 nearly identical families of quarks + leptons. Thus, we have 3 nearly identical copies of the same $SU(2)_L \otimes U(1)_Y$ structure.
This leads to the phenomenon of mixing quarks from different families. For example, for the **charged-current interactions** we earlier obtained for one family

$$\mathcal{L}_{\rm CC} = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \left[\bar{u}\gamma^\mu(1-\gamma_5)d + \bar{\nu}_e\gamma^\mu(1-\gamma_5)e \right] + \text{h.c.} \right\} .$$

Now generalizing this to 3 families and mixing we have

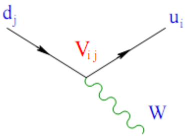
$$\mathcal{L}_{\rm CC} = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij}\, d_j + \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5)\, l \right] + \text{h.c.} \right\} ,$$

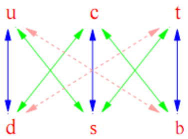
with

$$u_i = u,\ c,\ t, \qquad d_j = d,\ s,\ b\,.$$

The matrix **V** couples any ‘up-type’ quark with all ‘down-type’ quarks.

Unitary 3×3 matrix $\mathbf{V}_{ij}$, mixing any 'up-type' quark with all 'down-type' quarks is called the Cabibbo–Kobayashi–Maskawa (CKM) matrix.





This CKM matrix is very important – see below.

Despite the quark mixing, there is no mixing for the **neutral currents**. This leads to an important consequence – absence of the so-called flavor-changing neutral current vertices (called sometimes GIM mechanism).

Flavor mixing in the leptonic sector

Now what about the lepton mixing?

(i) If neutrinos are massless then any lepton mixing can be eliminated from the charged- and neutral current vertices.

(ii) If masses are not zero – and we know that they are not – then there indeed is leptonic mixing described by the 3×3 unitary matrix $\mathbf{V}_{L,\,ij}$ which is (almost) analogous to the quark mixing matrix.

You will know more about neutrino properties from lectures of Marie-Helene Schune.

CP violation and matter - antimatter asymmetry

After the Big Bang the matter and antimatter are created in equal amounts. Then how did it happen that we are surrounded by electrons, protons, and neutrons, with no positrons, antiprotons, or antineutrons?

- Of course, if a positron appears, in a short time it annihilates with an electron, i.e. $e^+e^- \to \gamma\gamma$. But this does not explain why only electrons are left over.
- Perhaps this is a local phenomenon: in our region of space only matter dominates, but in other part of the Universe there is a region of only antimatter. However, all astrophysical observations indicate that the known Universe is all matter (if there were an antimatter zone, the border would be source of very strong radiation of photons due to annihilation like $e^+e^- \to \gamma\gamma$, $p\bar{p} \to \gamma\gamma$, etc., and this has never been observed).
- Apparently some processes must have favored matter over antimatter during the evolution of Universe. What sort of mechanism may it be?

Sakharov's criteria for the matter-antimatter asymmetry

In 1968 Andrey Sakharov formulated the criteria for this asymmetry.

- Violation of conservation of baryon and lepton numbers, i.e. B and $L_e,\ L_\mu,\ L_\tau$ are not conserved. This requires models of New Physics beyond the SM (possibly in the grand unification theories).
- In the evolution of the Universe there was period of time far from equilibrium, so that the processes $i \to f$ and $f \to i$ proceed with different rates, $W(i \to f) \neq W(f \to i)$ (in the equilibrium there is no overall change of baryon or lepton numbers).
- The crucial condition is violation of the CP symmetry. It means that $W(i \to f) \neq W(\tilde{i} \to \tilde{f})$, so that the number of particles will be different from the number of antiparticles.
  Here, for any state $|i\rangle$ we define $|\tilde{i}\rangle = \hat{C}\hat{P}|i\rangle$. For example, if we have left-handed polarized electron, then

$$\hat{C}\hat{P}|e^-(\vec{p}, \lambda = -1)\rangle = |e^+(-\vec{p}, \lambda = +1)\rangle$$

CP violation via the CKM matrix

If we take, for example, decays $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$, which are related by application of $\hat{C}\hat{P}$, then experiment shows that

$$\Gamma(B^0 \to K^+\pi^-) > \Gamma(\bar{B}^0 \to K^-\pi^+) \;\; by \;\; 14\%$$

Why does it happen?

As mentioned before, there is the CKM mixing matrix for the 3 generations of quarks:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{0.1}$$

with the elements (in the so-called Wolfenstein parametrization)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

with $\lambda = 0.22535$.

CP violation via the CKM matrix

and all other parameters have been found from many measurements:

$$A = 0.811, \quad \bar{\rho} = (1 - \lambda^2/2)\rho = 0.131, \quad \bar{\eta} = (1 - \lambda^2/2)\eta = 0.349$$

Clearly, the dominant elements connect the quarks of the same generation:

$$u \leftrightarrow d, \quad c \leftrightarrow s, \quad t \leftrightarrow b, \quad V_{ud,\,cs,\,tb} \sim 1,$$

$$\begin{array}{lll} u \leftrightarrow s, & d \leftrightarrow c, & V_{us,\,dc} \sim 0.22, \\ c \leftrightarrow b, & s \leftrightarrow t, & V_{cb,\,st} \sim 0.05, \\ u \leftrightarrow b, & d \leftrightarrow t, & V_{ub,\,dt} \sim 0.01 \end{array}$$

However, the most important property of this matrix are the complex elements V_{ub} and V_{td} and the presence of a complex phase:

$$V_{ub} \approx 0.0035\, e^{-i\delta}, \qquad \delta = 1.20\, rad = 68.75^o$$

The other complex element is V_{td}, which in general has the form

$$V_{td} \approx 0.0094 - 0.0035\, e^{i\delta}$$

This complex phase δ is the only source of CP violation in the Standard Model.

CP violation and matter-antimatter asymmetry

Now, come back to the inequality $W(i \to f) \neq W(\tilde{i} \to \tilde{f})$ and try to answer why this happens.
Suppose that the processes $i \to f$ and $\tilde{i} \to \tilde{f}$ are described by the amplitudes:

$$\mathcal{M} = |\mathcal{M}| e^{i\phi} e^{i\delta}, \qquad \tilde{\mathcal{M}} = |\mathcal{M}| e^{i\phi} e^{-i\delta}$$

where ϕ is ordinary phase of any amplitude ("strong" phase), and δ is related to the CKM matrix ("weak" phase).
If we consider the decay width $\Gamma(i \to f) \sim |W(i \to f)|^2$, then

$$W(i \to f) \sim |\mathcal{M}|^2, \qquad W(\tilde{i} \to \tilde{f}) \sim |\tilde{\mathcal{M}}|^2 = |\mathcal{M}|^2$$

so that we will not see any effect of the *CP* violation in the decay width!

But suppose that there are 2 different mechanisms of the same process, that is

$$\mathcal{M} = |\mathcal{M}_1| e^{i\phi_1} e^{i\delta_1} + |\mathcal{M}_2| e^{i\phi_2} e^{i\delta_2}$$

and for the *CP* conjugated process $\tilde{i} \to \tilde{f}$ the amplitude is

$$\tilde{\mathcal{M}} = |\mathcal{M}_1| e^{i\phi_1} e^{-i\delta_1} + |\mathcal{M}_2| e^{i\phi_2} e^{-i\delta_2}$$

CP violation and matter-antimatter asymmetry

If we calculate the difference $|\mathcal{M}|^2 - |\tilde{\mathcal{M}}|^2$ we can see that now it is not zero:

$$|\mathcal{M}|^2 - |\tilde{\mathcal{M}}|^2 = -4|\mathcal{M}_1||\mathcal{M}_2|\sin(\phi_1 - \phi_2), \sin(\delta_1 - \delta_2) \neq 0$$

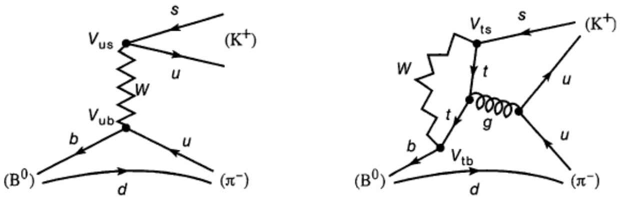
we need also $\phi_1 \neq \phi_2$ and $\delta_1 \neq \delta_2$ to demonstrate effect of the *CP* violation. Clearly we get the same effect if take 3 or more terms in the amplitude. Interference of amplitudes leads to the difference of probabilities:

$$W(i \to f) \neq W(\tilde{i} \to \tilde{f}), \quad where\ |\tilde{i}\rangle = \hat{C}\hat{P}|i\rangle,\ etc.$$

Consider example: decays $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$.
The 2nd decay is application of *CP* operation on the 1st decay,

$$\hat{C}\hat{P}:\ B^0 \to K^+\pi^- \Longrightarrow \bar{B}^0 \to K^-\pi^+$$

CP violation and matter-antimatter asymmetry



Two diagrams for the decay $B^0 \to K^+\pi^-$. The 2nd is called “penguin” diagram. For *CP* conjugated process $\bar{B}^0 \to K^-\pi^+$ there are also 2 diagrams, in which all lines of quarks are reversed.

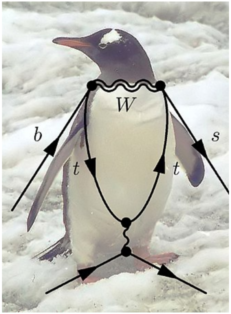
The vertex factor V_{ub} has the complex phase, $V_{ub} = |V_{ub}|e^{i\delta}$. This means that the vertex for the transition

$$u \to b + W^+ \quad \text{is complex, and has } e^{i\delta}$$

Now, recall weak charged interaction in the SM

$$\mathcal{L}_{weak} = \frac{g_w}{2\sqrt{2}}\left[V_{ub}\, W_\mu\, \bar{b}\,\gamma^\mu(1-\gamma_5)\,u + \left(V_{ub}\, W_\mu \bar{b}\,\gamma^\mu(1-\gamma_5)\,u\right)^\dagger\right]$$

“Penguin” diagram



Typical diagram contributing, for example, to $\bar{B}^0 \to K^- \pi^+$ process.

CP violation in $B^0 \to K^+\pi^-$ and $\bar{B}^0 \to K^-\pi^+$

which is equivalent to

$$\mathcal{L}_{weak} = \frac{g_w}{2\sqrt{2}}\left[V_{ub}\, W_\mu\, \bar{b}\,\gamma^\mu(1-\gamma_5)\,u + V_{ub}^*\, W_\mu^\dagger\, \bar{u}\,\gamma^\mu(1-\gamma_5)\,b\right]$$

This means that the transition

$$b \to u + W^- \quad \text{has complex phase} \;\; e^{-i\delta}$$

Then we see that

$$\mathcal{M}(i \to f) = M_1\, e^{i\delta} e^{i\phi_1} + M_2\, e^{i\phi_2}, \quad \mathcal{M}(\tilde{i} \to \tilde{f}) = M_1\, e^{-i\delta} e^{i\phi_1} + M_2\, e^{i\phi_2}$$

and $M_1,\ M_2$ are real numbers.

Then the difference of probabilities is

$$W(i \to f) - W(\tilde{i} \to \tilde{f}) \sim M_1\, M_2\, \sin(\delta)\, \sin(\phi_1 - \phi_2)$$

This can (possibly) explain the difference of the branching fractions:

$$\mathrm{Br}(B^0 \to K^+\pi^-) > \mathrm{Br}(\bar{B}^0 \to K^-\pi^+)$$

CP violation in the CKM matrix is far too small

This is fine, however, unfortunately the violation via the CKM matrix appears to be too small to account for the matter dominance in the Universe.
Much stronger CP violation is needed to explain the matter dominance. The matter-antimatter asymmetry problem remains the great mystery of cosmology.

There are various ideas on additional sources of CP violation, not via the CKM matrix. One model is called leptogenesis, and it is related to properties of neutrino.
We know now that neutrinos are massive and can oscillate from one flavor to other: $\nu_e \leftrightarrow \nu_\mu$, $\nu_\mu \leftrightarrow \nu_\tau$, $\nu_e \leftrightarrow \nu_\tau$. Let us only mention that for neutrino there can also be effect of CP violation. It would result in

$$W(\nu_e \to \nu_\mu) \neq W(\bar{\nu}_e \to \bar{\nu}_\mu)$$

Any asymmetry in the charged leptons and antileptons (electrons, muons, tau-leptons) can be of the same order of magnitude as the baryon asymmetry.

This idea has not been tested experimentally, but there are some new results, see, e.g. Probing Leptogenesis with the Cosmological Collider, Yanou Cui, Zhong-Zhi Xianyu, Physical Review Letters, 2022.

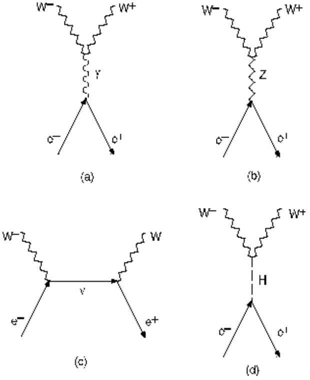
Time for questions

- How many different meson combinations can you make with 1, 2, 3, 4, 5, or 6 different quark flavors?
  What is the general formula for N_f flavors?
- How many baryon combinations can you make with 1, 2, 3, 4, 5, or 6 different quark flavors?
  What is the general formula for N_f flavors?

Hint: recall that meson is built of quark antiquark, $q_i\,\bar{q}_j$, and baryon is built of 3 quarks, $q_i\,q_j\,q_k$, where $i, j = 1, 2, \ldots, N_f$.

Time for questions

- The process $e^+ + e^- \to W^+ + W^-$ is described by the 4 diagrams.



Suppose we study the process $e^+ + e^- \to \gamma + W^+ + W^-$ with additional photon in the final state. How many Feynman diagrams in the lowest order (tree level = no loops) do we get?

End of Part 2.

Thank you for attention!