The Standard Model
Part 3

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Part 3 outline

- Symmetries in SM
  - – symmetries, groups and conservation laws
  - – rotation symmetry $O(3)$, symmetries of isospin $SU(2)_I$, favor $SU(3)_F$
- Discreet symmetries
  - – space reflection P, charge conjugation C
  - – combined CP symmetry
  - – time inversion T, CPT theorem, electric dipole moment
- More about quarks and color:
  - – evidence of 3 colors of quarks
  - – confinement hypothesis
- Quantum loops in QCD:
  - – renormalization, “running” coupling, asymptotic freedom

Symmetries in Particle Physics

What precisely is a symmetry? It is an operation you can perform on a system that leaves it invariant (or it carries it into a configuration indistinguishable from the original one).

Symmetries, Groups, and Conservation Laws.

In 1917 the dynamical implications of symmetry were understood. In that year, Emmy Noether published famous theorem relating symmetries and conservation laws: "Every symmetry of nature yields a conservation law; conversely, every conservation law reflects an underlying symmetry."

A few examples

Symmetry	$\Leftrightarrow$	Conservation law
Translation in time	$\Leftrightarrow$	energy E
Translation in space	$\Leftrightarrow$	3-momentum $\vec{P}$
Rotation of 3-dimensional space	$\Leftrightarrow$	angular momentum $\vec{J}$
Gauge transformations	$\Leftrightarrow$	electric charge, baryon number, etc. Q, B, ...

In mathematics, the **Group Theory** may be regarded as the systematic study of symmetries. The symmetry operations R_i can be combined in a group.

Symmetry in physics and groups

What groups do we know?
– Abelian, non-abelian, finite, infinite
– Continuous: elements depend on one or more continuous parameters, e.g. translation in space, time, rotation, gauge transformations
– Discrete: elements are numbered by index which takes integer values, 1,2,3, ...

In physics we are mostly interested in continuous groups of matrices.

Group	Matrices	Symmetry in physics
$U(n)$	$n \times n$ unitary: $U^\dagger U = UU^\dagger = 1$	$n = 1$: electric charge, baryon or lepton numbers
$SU(n)$	$n \times n$ unitary and $det(U) = 1$	for $n = 2$: isospin, $n = 3$: flavor and color in QCD
$O(n)$	$n \times n$ orthogonal: $O^T O = OO^T = 1$	
$SO(n)$	$n \times n$ orthogonal, and $det(O) = 1$	$n = 3$: rotations 3-dim. space

"U" means unitary, "S" means special, "O" means orthogonal. Also we have $U^\dagger = U^{T*}$, and T means transposition $U^T_{ij} = U_{ji}$. Lorentz transformations are also described by the continuous matrices.

Flavor symmetry $SU(2)_I$ and isospin

There was observation of W. Heisenberg in 1932: the neutron is almost identical to the proton. If we forget about the electric charge of the proton, then they would be identical and strong interaction is equal.

We can introduce isospin operator $\vec{I}$, similarly to the spin $\vec{S}$ and then the proton has isospin=1/2 and z-projection equal to +1/2, while the neutron has z-projection equal to -1/2. Therefore

$$p = |1/2, +1/2\rangle, \qquad n = |1/2, -1/2\rangle$$

So that they form the **isospin doublet** with the isospin $I = 1/2$.

The strong interactions are invariant with respect to rotation in the internal space, like all interactions are invariant with respect to rotations in our ordinary space.

According to Nother's theorem, $\vec{I}$ is conserved in the strong interaction, like angular momentum $\vec{J}$ is conserved for all interactions due to the rotational symmetry.

Isospin symmetry and quark model

All strongly interacting particles, hadrons, are classified according isospin multiplets. The electric charge of a particle is related to z-projection of the isospin via Gell-Mann–Nishijima relation:

$$Q = I_3 + \frac{1}{2}(A + S)$$

where A is the baryon number, and S is strangeness.

In the quark model we assume that the quarks u and d form isospin doublet, and s quark is isospin singlet,

$$u = |1/2, +1/2\rangle, \qquad d = |1/2, -1/2\rangle, \qquad s = |0, 0\rangle$$

The baryon number for any quark is $A_q = 1/3$ (for antiquark $A_{\bar{q}} = -1/3$), while strangeness of u, d is zero, and s has $S = -1$. Then formula of Gell-Mann–Nishijima is satisfied.

Finally, we should mention that

- For the strong interaction: the isospin I and all its components $\vec{I}$ are conserved.
- For the electromagnetic interaction: the component I_3 is conserved, however, the total isospin I is not conserved (for example, $\pi^0 \to \gamma\gamma$).
- For the weak interaction: both isospin I and I_3 are not conserved (for example, $\Lambda \to p + \pi^-$).

Flavor symmetry $SU(3)_F$

IIt was suggested later (M. Gell-Mann) to regard 8 baryons p, n, Λ, $\Sigma^{\pm,0}$, $\Xi^{+,0}$ as a supermultiplet, and this means that they belong in some representation of an enlarged symmetry group, in which the $SU(2)_I$ group of isospin is a subgroup

$$SU(2)_I \subset SU(3)_F$$

A problem here was related to the fact that the proton and neutron belong to the **fundamental representation** of the group $SU(2)_I$, i.e. there are exactly 2 states, but what particles form the fundamental representation of the group $SU(3)_F$?

The answer was given in the quark model: **the 3 quarks** u, d, s **form the basis of the fundamental representation of** $SU(3)_F$**.**

The octets of baryons with $J^P = 1/2^+$ and mesons with 0^- constitute 8-dimensional representations of $SU(3)_F$, and baryon decuplet with $3/2^+$ – 10-dimensional representation.

Flavor symmetries $SU(4)$, $SU(5)$, ...

When the charm quark c was discovered, the symmetry was promoted to $SU(4)$, and with discovering the bottom quark, to $SU(5)$. However, this extension was not successful.

If one checks the masses of all particles, then for the $SU(2)_I$ isomultiplet:

$$\frac{m_n - m_p}{m_n} \approx 0.001, \quad \frac{m_{\pi^\pm} - m_{\pi^0}}{m_{\pi^0}} \approx 0.003,$$

and we see that accuracy of the isospin symmetry is $\sim 0.1\%$.

However, if we compare masses of particles in the supermultiplet of $SU(3)_F$

$$\frac{m_\Sigma - m_N}{m_\Sigma} \approx 0.2,$$

then the accuracy of $SU(3)_F$ is only $\sim 20\%$.

The situation for $SU(4)$ is worse, and for $SU(5)$ and $SU(6)$ is completely absurd. There is no such symmetries at al.

Quark masses and flavor symmetries

Why are the isospin $SU(2)_I$ a good symmetry, flavor symmetry $SU(3)_F$ approximate, and $SU(4)$, $SU(5)$ and $SU(6)$ symmetries so bad?

Quark	Bare mass (MeV)	"Effective" (constituent) mass in hadrons (MeV)
u	≈ 2	336
d	≈ 5	340
s	≈ 95	486
c	1300	1550
b	4200	4730
t	173 GeV	173 GeV

We see that the effective mass of the u and d quarks are very similar and are about 350 MeV, so that

$$m_u^{eff} \approx m_d^{eff} \approx 330 \text{ MeV},$$

so that in hadrons they are effectively equal, and this explains the isospin symmetry. The effective mass of the strange quark m_s^{eff} is larger than m_u^{eff}, m_d^{eff}, and this causes 20-30 % violation of $SU(3)_F$.
The masses of heavy quarks c, b, t are so large and different from the light quark masses, that the higher symmetries are not the symmetries at all.

Quark masses and flavor symmetries

This explanation raises two questions.

- Why is effective mass of the light quark larger than bare mass by about 300-350 MeV?
- Why do the bare masses of the quarks have these values?

Of course in the SM the bare quark masses m_q come from interaction with the Higgs scalar field. However as we know, the values of masses are not explained and are just free parameters. Only in theories beyond the SM there is a hope to explain origin and values of the quark masses.

The effective quark masses come from the interaction of the bare quark with other quarks and gluons inside medium of the hadrons – this should be explained and quantified in QCD.

Space reflection (parity)

In this famous experiment of Wu in 1957, the radioactive nucleus of cobalt 60 was polarized in the magnetic field of solenoid at the temperature $T = 0.01$ K. This nucleus beta-decays like

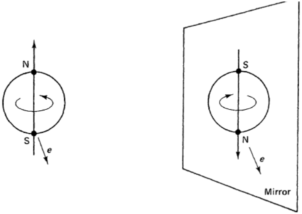
$$^{60}_{27}\vec{Co}_{\,(J=5)} \to e^- + \bar{\nu}_e + ^{60}_{28} Ni^{\star}_{(J=4)}$$

The electrons come out in this decay mostly in the directions opposite to the spin of the nucleus ^{60}Co. So, if we describe the angular distribution of electrons, it will look like this

$$W(\theta) \sim 1 - \alpha\, \vec{n}\, \vec{S}$$

with some coefficient $\alpha > 0$, $\vec{n}$ describes direction of the electron and $\vec{S}$ is the spin of the nucleus. What was so unexpected in this experiment?

Discrete symmetries in particle physics



The mirror reflects the axis OY, so that the coordinates transform: $x,\ y,\ z \to x,\ -y,\ z$. The spin is the axial vector which behaves similarly to orbital momentum $\vec{L} = \vec{r} \times \vec{p}$. Therefore $S_z \sim x\,p_y - y\,p_x$ changes sign if $y \to -y$. This means that

$$S_z \to -S_z, \quad but\ momentum\ of\ electron\ \ \vec{n} = (n_x, n_y, n_z) \to (n_x, -n_y, n_z)$$

The distribution in the mirror looks like

$$W(\theta) \sim 1 + \alpha\,\vec{n}\,\vec{S}$$

which is NOT observed in Nature, where the electrons always come in the opposite direction to the spin of *Co*!

Discrete symmetries in particle physics

This effect occurs in every weak-interaction process, and it means violation of parity symmetry.
In general one defines space reflection operation, or space parity P:

$$\hat{P}: \; x, y, z \Longrightarrow -x, -y, -z$$

which is directly related to the mirror reflection.

The parity violation is a signature of the weak interactions. It is most dramatically revealed in the behavior of neutrino, and in particular with neutrino **helicity**.
In general, for any particle one defines helicity

$$\lambda = \vec{p}\,\vec{s}/(|\vec{p}|s) = m_s/s, \qquad m_s = -s, -s+1, \ldots, s-1, s$$

For fermions with spin $1/2$ (electron, muon, quarks, protons, ...), $m_s = -1/2, +1/2$ and

$$\lambda = +1 \quad \mathrm{right-handed} \quad \mathit{and} \quad \lambda = -1 \quad \mathrm{left-handed}$$

In general, for any massive particle, the helicity is not a conserved quantum number – it can change under appropriate Lorentz transformation.
However, for a massless particle the helicity is conserved.

Parity violation and helicity

As for neutrino's, until 1950-1960, it had been considered that 50% neutrino are left-handed, and 50% are right-handed. It came later as a surprise that **all neutrino are left-handed with $\lambda_\nu = -1$, and all antineutrino are right-handed with $\lambda_{\bar{\nu}} = +1$.**

(a) Right-handed (b) Left-handed

Parity transformation applied to left-handed neutrino gives right-handed neutrino which does not exist

$$\hat{P}\,|\nu, \lambda = -1\rangle = |\nu, \lambda = +1\rangle$$

If we recall the structure of the weak interactions, we can understand this: only left-handed neutrino's $\nu_L = \frac{1}{2}(1 - \gamma_5)\nu$ participate in the charged and neutral weak interactions. In fact, violation of parity in weak interactions is maximally possible.

Of course, in the strong and electromagnetic interactions parity is strictly conserved.

Parity

Group of parity transformations
Apply the parity transformation two times: $\hat{P} \cdot \hat{P} = \hat{P}^2 = 1$, so the group consists of two elements: $G = \{\hat{P}, 1\}$, such that $\hat{P}^{-1} = \hat{P}$.
Since $\hat{P}$ is an operator in quantum theory, it should have eigenstates and eigenvalues:

$$\hat{P}|n\rangle = \varepsilon|n\rangle, \quad \hat{P}\hat{P}|n\rangle = \varepsilon^2|n\rangle = |n\rangle$$

and we find that

$$\varepsilon^2 = 1, \quad and \ \ \varepsilon = \pm 1$$

Particles are eigenstates of parity operator and are classified by their parity, that is they have the parity +1, or -1 (with exception of Z, $W^{\pm}$, $\nu, \bar{\nu}$). According to Quantum Field Theory:

parity of antifermion = – parity of fermion : e^- and e^+, q and $\bar{q}$, p and $\bar{p}$, etc.
parity of antiboson = parity of boson: π^- and π^+, K^+ and K^-, etc.

By convention, the parity of particles is +1, parity of antiparticles is -1 (in principle, it could have been vise verse).

Charge conjugation symmetry

In particle physics, the charge conjugation transformation (or $\hat{C}$ parity)

$$\hat{C}|a\rangle \equiv |\bar{a}\rangle$$

for arbitrary particle $a = e, \mu, \ldots, q, p, n, \Lambda, \ldots$.
All quantum numbers of particles, such as : $Q, B, L_e, L_\mu, L_\tau, S, C, B, T$ have to be changed to the opposite. A few examples:

Quantum number	proton	antiproton	electron	positron
Charge Q	$+e$	$-e$	$-e$	$+e$
Baryon number B	+1	-1	-	-
Lepton number L	-	-	+1	-1
Strangeness S	0	0	-	-
Magnetic moment μ	$2.79\frac{e\hbar}{2m_pc}$	$-2.79\frac{e\hbar}{2m_pc}$	$-\frac{e\hbar}{2m_ec}$	$\frac{e\hbar}{2m_ec}$
Spin	$\frac{1}{2}\hbar$	$\frac{1}{2}\hbar$	$\frac{1}{2}\hbar$	$\frac{1}{2}\hbar$

Group property
As for the parity, charge conjugation transformations form a group of two elements: $G = \{\hat{C}, 1\}$. Apparently, $\hat{C}^2 = 1$ and $\hat{C}^{-1} = \hat{C}$.

Charge conjugation symmetry

If there are eigenstates of $\hat{C}$, then it should be

$$\hat{C}|a\rangle = \xi|a\rangle, \quad \textit{then} \;\; \hat{C}^2|a\rangle = \xi^2|a\rangle = |a\rangle$$

therefore $\xi = \pm 1$ and

$$\hat{C}|a\rangle = \pm|a\rangle, \quad \textit{but} \;\; \hat{C}|a\rangle = |\bar{a}\rangle$$

It follows that

$$|\bar{a}\rangle = \pm|a\rangle$$

which is only possible, if all quantum numbers of particle *a* are equal to zero. There are only several particles which have this property:

$$\gamma,\ \pi^0,\ \eta,\ \eta',\ \rho^0,\ \omega,\ \phi,\ J/\psi,\ \Upsilon, \ldots$$

These particles have all quantum numbers equal to zero. The charge parity *C* is conserved in the strong and electromagnetic interactions. However, it is violated in the weak interactions.

The easiest way to see this is to look at the properties of neutrino and antineutrino. We know that ν are only left-handed, but the charge conjugation changes neutrino to antineutrino without changing helicity,

$$\hat{C}\,|\nu;\lambda = -1\rangle = |\bar{\nu};\lambda = -1\rangle, \qquad \textit{but such antineutrino do not exist}$$

Therefore ***C* parity is violated in the weak interactions similarly to *P* parity.**

Discrete symmetries: time inversion

The operation $t \to -t$ is denoted by $\hat{T}$ and is called time inversion.
All physical operators have definite properties with respect to the time inversion. For example, in electrodynamics from the law

$$\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c}\,\vec{v} \times \vec{B}$$

the following properties follow

$$t \to -t: \qquad \vec{p} \to -\vec{p}, \;\; \vec{E} \to \vec{E}, \;\; \vec{v} = \frac{d\vec{r}}{dt} \to -\vec{v}, \;\; \vec{B} \to -\vec{B}$$

In addition one can see that

$$t \to -t: \qquad \vec{L} = \vec{r} \times \vec{p} \to -\vec{L}, \;\; \vec{S} \to -\vec{S}, etc.$$

The strong and electromagnetic interactions are invariant under the time inversion.
Time reversion invariance leads to the so-called principle of detailed balance

$$\frac{d\sigma_{i\to f}}{d\Omega_f}\,\frac{p_i}{p_f}\,(2s_1+1)(2s_2+1) = \frac{d\sigma_{f\to i}}{d\Omega_i}\,\frac{p_f}{p_i}\,(2s_3+1)(2s_4+1)$$

The relation between cross sections of direct and reverse processes was tested many times and was confirmed.

CP symmetry

As we saw earlier, using an example of neutrino, the parity P and charge conjugation C are violated in the week interactions, i.e.

$$\hat{P}|\nu, \lambda = -1\rangle = |\nu, \lambda = +1|\nu, \lambda = +1\rangle, \qquad \textit{does not exist}$$

$$\hat{C}|\nu, \lambda = -1\rangle = |\bar{\nu}, \lambda = -1\rangle, \qquad \textit{also does not exist}$$

However, let us apply both transformations

$$\hat{C}\hat{P}|\nu, \lambda = -1\rangle = \hat{C}|\nu, \lambda = +1\rangle = |\bar{\nu}, \lambda = +1\rangle$$

and we see that such antineutrino exists and is observed.
Then the symmetry $\hat{C}\hat{P} = \hat{P}\hat{C}$ seems to be satisfied in the weak interactions (of course, also in the strong and electromagnetic interactions).

This symmetry, as a generalized mirror symmetry, leads to unexpected and beautiful quantum-mechanic phenomenon – oscillation of neutral kaons (M. Gell-Mann and A. Pais, 1955).

Much more on oscillation of kaon K^0, and other mesons you will learn from lectures at this School by Achille Stocchi and Marie-Helene Schune.

CP symmetry violation

In 1964 Christenson, Cronin, Fitch and Turlay performed experiment with the neutral kaons, which unexpectedly demonstrated violation of the *CP* symmetry.

Here I will no go into details of this phenomenon, it is a separate and very interesting topic which will be covered in other lectures at this School.

In fact, we partly addressed *CP* violation, and at least theoretically understand that in the SM this effect arises due to complex phase in the quark-mixing CKM matrix. This phenomenon has many implications which are studied at the LHC by the LHCb collaboration.

Also, as we already discussed, *CP* violation plays important role in the problem of matter-antimatter asymmetry in the Universe.

CPT theorem

One of the fundamental results of the Quantum Field Theory is the famous *CPT* theorem (J. Schwinger, G. Luders, W. Pauli). It is based on general assumptions of Lorentz invariance, quantum mechanics, Bose-Einstein and Fermi-Dirac statistics, the theorem states that all interactions are invariant with respect to the combined operation

$$\hat{C}\hat{P}\hat{T} = \hat{P}\hat{C}\hat{T} = \hat{T}\hat{C}\hat{P} = \hat{T}\hat{P}\hat{C} = \dots$$

for any order of operators. It is just impossible to formulate a consistent theory in which *CPT* would be violated.

What are the consequences of the *CPT* theorem? It follows, that for any particle A there is antiparticle $\bar{A}$

$$M_{\bar{A}} = M_A, \quad \tau_{\bar{A}} = \tau_A, \quad Q_{\bar{A}}| = |Q_A|$$

At present there are no indications of violation of *CPT* symmetry in the properties of particles and antiparticles. Therefore, we take it to be the exact symmetry.

- For the strong and EM interactions, C, P and T are separately exact symmetries, and the *CPT* as well.
- However the weak interaction violates C, P and CP symmetries, then it follows, that if CPT is exact symmetry, and CP is violated, then T is violated !

Thus the time inversion symmetry T is expected to be violated by the weak interaction.

CPT theorem and time-inversion violation

How to test time-inversion symmetry in the weak processes?

As it turns out, time reversal is much harder to test than *P* or *C*. First of all, we know that many particles are eigenstates of *P*, and some particles, γ, π^0, ρ^0, $J/\psi, \ldots$, are eigenstates of *C*, however there are no particles which are eigenstates of *T*. Indeed, particle can be identical to the mirror image (*P* symmetry), or to its antiparticle (*C* symmetry if all quantum numbers are zero), but neither particle can be identical to itself going backward in time. So we cannot check conservation of *T* simply by multiplying numbers, as one can do for *P* and *C*.

The most direct test would be to take a particular reaction (say, $a + b \to c + d$), and run it in reverse ($c + d \to a + b$), and then test the principle of detailed balance.

Unfortunately, the inverse processes are very hard to perform for the weak interaction. For example, consider the direct and inverse processes

$$(1)\ \Lambda \to p + \pi^-, \quad \textit{and} \quad (2)\ p + \pi^- \to \Lambda$$

However, the inverse reaction (2) is not possible to see, because of the strong interaction processes, like $p + \pi^- \to p + \pi^- + \pi^0$, $p + \pi^- \to n + \pi^0$, etc., which are much more probable.

CPT theorem and electric dipole moment

Another possibility would be reactions with neutrino, like

$$(1)\ \nu_\mu + e^- \to \mu^- + \nu_e, \quad \textit{and} \quad (2)\ \mu^- + \nu_e \to \nu_\mu + e^-$$

These are the pure weak processes without contamination due to the strong or electromagnetic effects. However, it is extremely difficult to do accurate measurements on neutrinos, and here we are looking for a very small effect.

Electric dipole moment

The practical way to look for violation of T is accurate measurements of quantities that should be precisely zero if T is a perfect symmetry. An example is a static electric dipole moment (EDM) of an elementary particle. Recall interaction of a system with external electric field $\vec{E}$

$$\mathcal{H} = -\vec{d}\,\vec{E}, \qquad \textit{where} \qquad \vec{d} = \sum_i e_i\,\vec{r}_i$$

CPT theorem and electric dipole moment

If we have a particle, say the neutron, what is the direction of $\vec{d}$?

For any particle at rest, and with nonzero spin s, the only direction available is the direction of the spin vector $\vec{s}$. Therefore we assume that $\vec{d} = A\,\vec{s}$ with some constant A. Let us check symmetries of this equation under $\hat{P}$ and $\hat{T}$ transformations:

$$\hat{P}: \; \vec{d} = A\,\vec{s} \implies -\vec{d} = A\,\vec{s}$$

so if P is the exact symmetry, then $\vec{d} = -\vec{d}$, or $\vec{d} = 0$. Thus P should not be the symmetry in order to have $\vec{d} \neq 0$.

Now check properties under $\hat{T}$ transformation:

$$\hat{T}: \; \vec{d} = A\,\vec{s} \implies \vec{d} = -A\,\vec{s}$$

and again we see that for the exact T symmetry EDM would be zero.

The conclusion is: to have a nonzero EDM both P and T should be violated. As we know, the parity is violated by the weak interactions, but now one can test if the time-inversion symmetry is violated as well.

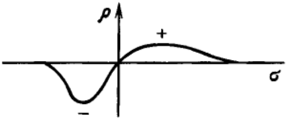
EDM of the neutron

Let us see what we can say for the neutron. The overall charge is zero,

$$Q = \int \rho(\vec{r})\, d^3 r = 4\pi \int_0^\infty \rho(r)\, r^2\, dr = 0$$

If the distribution $\rho(\vec{r}) = \rho(r)$, i.e. spherically symmetric, the EDM will be exactly zero. To have a nonzero $\vec{d}$ there should be some shift of the positive charge and the negative charge:

$$\vec{d} = \sum_i e_i \vec{r}_i = |Q|(\vec{R}_+ - \vec{R}_-) = |Q|\vec{L}_{eff} \sim \vec{s},$$



This difference of the centers of positive and negative charges $L_{eff} = |\vec{R}_+ - \vec{R}_-|$ should be extremely small: (i) this is effect of the weak interaction, and (ii) it is related to a very small effect of the *CP* violation which is $\sim 10^{-3}$.

Electric dipole moment

Let us make an approximate estimate of the EDM.

$$d \approx e\, L_{eff} \times\ (CP\, violation\, parameter) \approx e\, L_{eff} \times 10^{-3}$$

Based on the dimensional arguments

$$L_{eff} \sim G_F\, m_p \approx 10^{-19}\, cm \ll R_n \approx 10^{-13}\, cm$$

where the Fermi constant is $G_F = 1.166 \times 10^{-5}\ GeV^{-2} \approx 10^{-5}/m_p^2$.

And then we can find EDM of the neutron

$$d \sim 10^{-22}\, |e| \cdot cm \qquad (theoretical\, estimate)$$

(the units $e \cdot cm$ are typical for the EDM).

Actual theoretical estimates of the EDM vary from $10^{-22}\, e \cdot cm$ to $10^{-34}\, e \cdot cm$.

Measurements of EDM

Here I will show experimental constraints on EDM of several particles.

- Neutron. Experiments started in 1968, they continue today. The most recent value cited by the PDG 2022 is

$$d_n < 0.18 \times 10^{-25}\, e \cdot cm$$

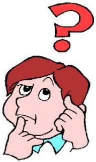
- Electron: $d_e < 0.11 \times 10^{-28}\, e \cdot cm$
- Muon: $d_\mu < 1.8 \times 10^{-19}\, e \cdot cm$
- Tauon (τ lepton):

$$\mathrm{Re} d_\tau = (-0.22,\ +0.45) \times 10^{-16}\, e \cdot cm, \quad \mathrm{Im} d_\tau = (-0.25,\ +0.008) \times 10^{-16}\, e \cdot cm$$

- Proton: $d_p < 0.021 \times 10^{-23}\, e \cdot cm$
- Λ hyperon: $d_\Lambda < 1.5 \times 10^{-16}\, e \cdot cm$

Values for EDM place strong constraints on the scale of *CP*-violation that extensions to the SM allow. Some theories beyond the SM are inconsistent with the current limits on EDM and have been ruled out. Current generations of experiments are designed to be sensitive, for example, to the range of EDM predicted by SUSY.

Time for questions



1. Why do we say that reflection in the mirror $\hat{O}_y$ (of the axis OY) is equivalent to the space reflection $\hat{P}$?

   Hint: parity reflection means transformation $(x, y, z) \to (-x, -y, -z)$*, and reflection in the mirror means* $(x, y, z) \to (x, -y, z)$*.*

2. Main decay channel of π^0 meson ($J^P = 0^-$) is $\pi^0 \to \gamma + \gamma$. Is the decay $\pi^0 \to \gamma + \gamma + \gamma$ possible? Please justify your answer.

3. The neutron mass is 939.56 MeV and the proton mass is 938.27 MeV.

   What effect(s) contribute(s) to this mass difference?

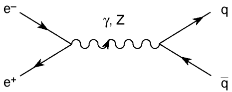
   If the proton would be heavier than the neutron, what would happen?

More about QCD. Evidence of color

Confinement hypothesis: quarks and gluons are confined in color–singlet, or colorless, bound states.

A direct test of the quark color is obtained from the ratio

$$R_{e^+e^-} \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



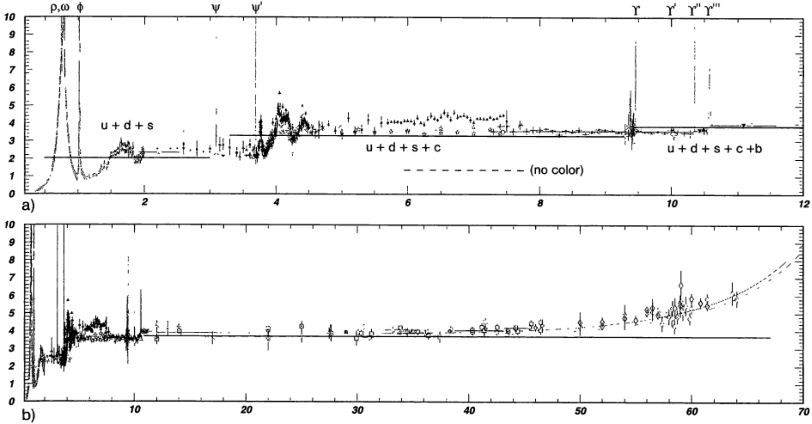
Quarks are confined, therefore *they have to hadronize with probability equal to one*. Further, at energies below Z-boson,

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}, \qquad \sigma(e^+e^- \to q\bar{q}_f) = N_C \frac{4\pi\alpha^2 Q_f^2}{3s}\theta(\sqrt{s} - 2m_f)$$

and in the lowest order

$$R_{e^+e^-} \approx N_C \sum_{f=1}^{N_f} Q_f^2\, \theta(\sqrt{s} - 2m_f) = \begin{cases} \frac{2}{3}N_C = 2\,, & (N_f = 3\ :\ u, d, s) \\ \frac{10}{9}N_C = \frac{10}{3}\,, & (N_f = 4\ :\ u, d, s, c) \\ \frac{11}{9}N_C = \frac{11}{3}\,, & (N_f = 5\ :\ u, d, s, c, b) \end{cases} .$$

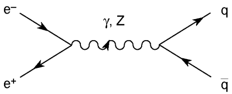
Measurement of $R_{e^+e^-}$



Quark-gluon interaction and hadronization

Suppose we measure the **inclusive** cross section

$$\sigma(e^+e^- \to \mathrm{hadrons}) = \sigma(e^+e^- \to q\bar{q} + q\bar{q}G + q\bar{q}GG + \ldots \to hadrons)$$



Quarks and gluons are created at very short distances,

$$x_0 \sim 1/\sqrt{s} \sim 10^{-3}\ \mathrm{fm} \qquad (where\ \mathrm{fm} = 10^{-13}\ \mathrm{cm})$$

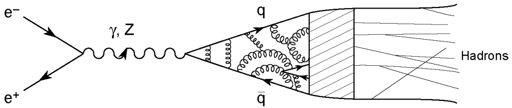
at energies $\sqrt{s} \sim 200$ GeV (and much smaller at the LHC energy).

After that quarks and gluons radiate additional “soft” gluons with smaller energies, lose energy, and at much larger distances

$$x_1 \sim 1/\Lambda_{QCD} \sim 1\ \mathrm{fm} \gg x_0$$

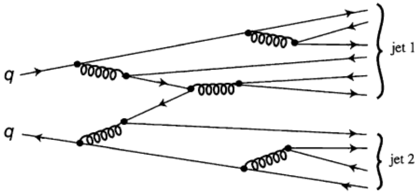
interaction becomes very strong with coupling $\alpha_s(\Lambda_{QCD}) \sim 1$ ($\Lambda_{QCD} = 200 - 300\ \mathrm{MeV}$ is the energy scale of QCD).

Quark-gluon interaction and hadronization

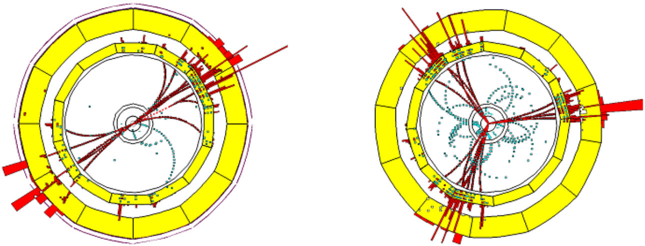


Finally quarks and gluons transform into hadrons with probability equal to one because of the confinement. This is called hadronization.

The final hadrons appear in the form of jets. In case of 2 jets, they move in the directions of original q and $\bar{q}$.



Two and three jet modes in e^+e^- annihilation

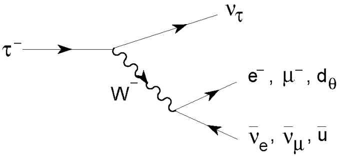


Annihilation of e^+e^- to hadrons

Left: final hadrons are collimated in 2 jets (LEP, CERN (DELPHI)).

Right: 3 jet event in $Z \to q\bar{q}G$ decay at LEP, CERN (DELPHI). This is evidence for the quark-gluon interaction, i.e. $q \to q + G$.

Tau-lepton decays



Tau-lepton decay modes:

$$\tau^- \to \nu_\tau e^- \bar{\nu}_e, \qquad \tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu, \qquad \tau^- \to \nu_\tau d_\theta \bar{u}$$

$d_\theta \equiv V_{ud} d + V_{us} s \approx d \, \cos\theta_C + s \, \sin\theta_C \approx 0.97 \, d + 0.22 \, s$.

Quark-lepton universality leads to branching ratios

$$B_{\tau\to l} \equiv \mathrm{Br}(\tau^- \to \nu_\tau l^- \bar{\nu}_l) \approx \frac{1}{2 + N_C} = \frac{1}{5} = 20\%$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \mathrm{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C = 3$$

Tau-lepton decays and color

Experimental averages of branching ratios [Particle Data Group]:

$$B_{\tau\to e} = (17.791 \pm 0.054)\%\,, \qquad B_{\tau\to\mu} = (17.333 \pm 0.054)\%\,,$$
$$R_\tau = (1 - B_{\tau\to e} - B_{\tau\to\mu})/B_{\tau\to e} = 3.647 \pm 0.014\,.$$

Of course we expect $N_C = 3$, and the deviation by $\sim$20% from the simple estimates is related to radiative corrections.

Many other observables, such as partial widths of Z and $W^\pm$ bosons, can be analyzed in a similar way to conclude that $N_C = 3$.

$SU(3)$ color symmetry, mesons, baryons, etc.

Among the Lie groups only $SU(3)$ is suitable group to describe gauge color symmetry of QCD. Using the language of the Group Theory:

quarks belong to triplet representation of $SU(3)_C$: $\underline{3}$, while antiquarks – to $\underline{3}^*$.
(quarks and antiquarks are different particles, therefore $\underline{3}^* \neq \underline{3}$).

One can build various combinations of quark and antiquark:

$$\begin{aligned}
q\bar{q}: &\quad \underline{3} \otimes \underline{3}^* = \underline{1} \oplus \underline{8} \\
qqq: &\quad \underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10} \\
qq: &\quad \underline{3} \otimes \underline{3} = \underline{3}^* \oplus \underline{6} \\
qqqq: &\quad \underline{3} \otimes \underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{3} \oplus \underline{3} \oplus \underline{3} \oplus \underline{6}^* \oplus \underline{6}^* \oplus \underline{15} \oplus \underline{15} \oplus \underline{15} \oplus \underline{15}^*
\end{aligned}$$

where $\underline{1}$ is color singlet (= colorless state), as required by the principle of confinement.

From $q\bar{q}$ we build the colorless mesons, from qqq – the baryons, and from $\bar{q}\bar{q}\bar{q}$ – antibaryons.

Di-quarks, four-quarks, glueballs, ...

Exotic combinations, such as di-quarks qq, $\bar{q}\bar{q}$ or four–quarks $qqqq$, $\bar{q}\bar{q}\bar{q}\bar{q}$ are not allowed, because they do not form color-singlet components.

However, two-quark + two-antiquark state (tetraquark) can exist

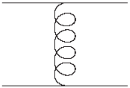
$$qq\bar{q}\bar{q}: \qquad \underline{3} \otimes \underline{3} \otimes \underline{3}^* \otimes \underline{3}^* = \underline{1} \oplus \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{8}^* \oplus \underline{10} \oplus \underline{10}^* \oplus \underline{35}$$

as well as two-gluon bound states (glueballs)

$$GG: \qquad \underline{8} \otimes \underline{8} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10} \oplus \underline{10}^* \oplus \underline{27}$$

and some other “exotic” particles, like $q\bar{q}G$ (hybrids), $q\bar{q}qqq$ (pentaquarks).

How does quark interact with quark (antiquark)?



Recall that $q\bar{q}$ and qq pairs can be in the color states:

$$q\bar{q}: \quad \underline{3} \otimes \underline{3}^* = \underline{1} \oplus \underline{8}, \qquad\qquad qq: \quad \underline{3} \otimes \underline{3} = \underline{3}^* \oplus \underline{6}$$

Potential for $q\bar{q}$ (like Coulomb for opposite electric charges), $(\alpha_s = g_s^2/(4\pi))$:

$$V_{q\bar{q}}(r) = -\frac{\alpha_s}{r} \times f_c, \qquad f_c = \begin{cases} +\frac{4}{3}, & \text{singlet } \underline{1}, \text{ attractive} \\ -\frac{1}{6}, & \text{octet } \underline{8}, \text{ repulsive} \end{cases}$$

Thus quark and antiquark attract each other in the singlet state. For qq (like Coulomb potential for similar electric charges):

$$V_{qq}(r) = +\frac{\alpha_s}{r} \times f'_c, \qquad f'_c = \begin{cases} -\frac{2}{3}, & \text{antitriplet } \underline{3}^*, \text{ attractive} \\ +\frac{1}{3}, & \text{sixtet } \underline{6}, \text{ repulsive} \end{cases}$$

How does quark interact with quark (antiquark)?

What about interaction between 3 quarks? Situation is more complex, however we know that bound states exist – protons, neutrons, baryon resonances . . .

Recall now that in the color space

$$\begin{aligned} qqq: \qquad & \underline{3} \otimes \underline{3} \otimes \underline{3} = (\underline{3}^* \oplus \underline{6}) \otimes \underline{3} \\ = \quad & (\underline{3}^* \otimes \underline{3}) \oplus (\underline{6} \otimes \underline{3}) = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10} \end{aligned}$$

Then any pair of quarks, say # 1 and # 2, should be in $\underline{3}^*$ state to build color singlet $\underline{1}$ for three quarks, thus there is an attraction in any pair of quarks in the three-quark system.

Conclusion: one-gluon exchange interaction in $q\bar{q}$ and qq systems is favorable for binding $q\bar{q}$ in mesons and qqq in baryons.

This supports the hypothesis that physical particles are color singlets. However this cannot be a proof of confinement.

In QED the running coupling constant is

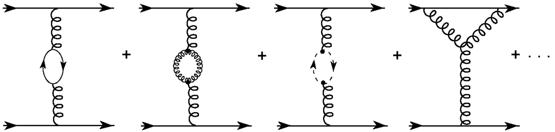
$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \frac{\beta_1 \alpha(Q_0^2)}{2\pi} \ln(Q^2/Q_0^2)}$$

with

$$\beta_1 = \frac{2}{3} > 0$$

This running coupling reflects effect of screening of electric charge in QED.

What happens in QCD? In addition to quark-gluon interaction now there is gluon self-interactions:



Asymptotic freedom in QCD

Calculating the quantum loops one finds the QCD running coupling $\alpha_s(Q^2) = g_s^2(Q^2)/(4\pi)$:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \frac{\beta_1 \alpha_s(Q_0^2)}{2\pi} \ln\left(Q^2/Q_0^2\right)}$$

with

$$\beta_1 = \frac{2\,N_f - 11\,N_C}{6} < 0, \quad \text{if} \quad N_C = 3, \qquad N_f \leq 16$$

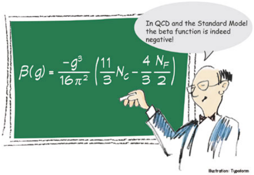
It decreases at large energies, or short distances [Gross, Wilczek, Politzer]:

$$\boxed{\lim_{Q^2\to\infty} \alpha_s(Q^2) = 0}$$

This is the property of asymptotic freedom; it is a consequence of "anti-screening" of strong charge in QCD.

Note that this result is obtained for the massless quarks, $m_q = 0$. There is no any energy scale in QCD; thus, there is no way to decide whether the energy of a given process is large or small.

Energy scale of QCD



Define energy Λ_{QCD} such that $\alpha_s(\Lambda^2_{QCD}) \to \infty$:

$$1 - \frac{\beta_1 \alpha_s(Q_0^2)}{2\pi} \ln\left(\Lambda^2_{QCD}/Q_0^2\right) = 0\,, \qquad \text{then}$$

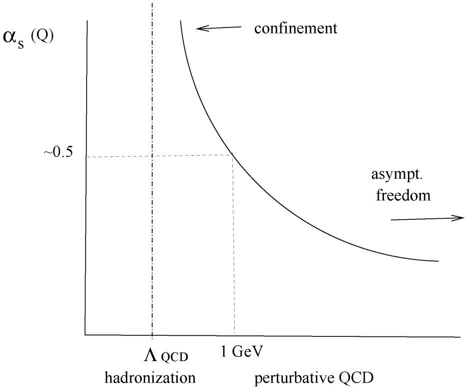
$$\alpha_s(Q^2) = \frac{2\pi}{-\beta_1 \ln\left(Q^2/\Lambda^2_{QCD}\right)}$$

where

$$\Lambda_{QCD} \approx 200 - 300\ \mathrm{MeV} \sim R^{-1}_{hadron} \sim 1\ \mathrm{fm}^{-1}\,.$$

Dimensionless parameter g_s is traded by energy scale Λ_{QCD} — this is called dimensional transmutation

Energy dependence of strong coupling



Measurements of strong coupling

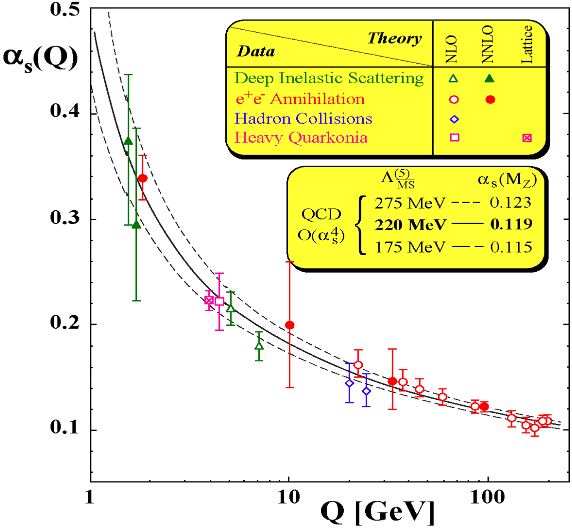


Figure: α_s measurements as function of energy

End of Part 3

Thank you for attention!