The Standard Model.
Part 4

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July 14-22, 2023

Standard Model, Part 4. Outline

- More about QCD and strong interaction
  – flavor symmetries: vector and isospin symmetry in QCD
  – chiral symmetry breaking, quark condensate
  – brief summary for QCD
- Higgs boson
  – Production of the Higgs boson at the LHC
  – Decay modes and decay rate
- Quantum loops in the SM
- Is the SM the final theory of particle physics?
- Conclusion and outlook

Flavor symmetries of QCD

Recall Lagrangian of QCD:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}\sum_{a=1}^{8} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_{f=1}^{N_f} \bar{q}_f^{\alpha}\,(i\gamma^\mu\partial_\mu - m_f)\,q_f^{\alpha}$$

$$+g_s \sum_{a=1}^{8}\sum_{f=1}^{N_f} G_a^{\mu}\,\bar{q}_f^{\alpha}\gamma_\mu \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q_f^{\beta}, \qquad (\alpha,\,\beta = 1,2,3)$$

What can one say about symmetries of this Lagrangian and conserved currents?

- Global phase symmetry $U(1)_V$:

$$q_f \to e^{i\theta} q_f, \qquad \theta = \text{const for all quarks}$$

$\Longrightarrow$ conservation of baryon number $\frac{dB}{dt} = 0$

Define for quarks: $B_q = 1/3$ and $B_{\bar{q}} = -1/3$, then for hadrons $B_p = 1$, $B_{\bar{p}} = -1$, $B_\pi = 0, \ldots$

Conclusion: in any process the baryon number is conserved, i.e.

$$B_{\text{initial}} = B_{\text{final}}$$

Equivalently, $N_{\text{quarks}} - N_{\text{antiquarks}}$ is conserved.

Flavor symmetries of QCD

- Global phase symmetry $U(1)$ of each quark flavor $f = u,\ d,\ s, \ldots$:

$$q_f \to e^{i\theta_f} q_f, \qquad \theta_f = \mathrm{const}$$

$\Longrightarrow$ quark flavor is conserved in the strong interaction, i.e.

$$q_f \to q_f + G,\ \ q_f + \bar{q}_f \to G, \ldots$$

Moreover, strong interactions do not depend on flavor f, e.g. all vertices are characterized by one coupling constant:

$$g_{\bar{q}_f q_f G} = g_s \qquad \text{for any flavor}$$

Vector symmetries of QCD

- If masses of quarks are equal, i.e. $m_f = m$, then there is an additional global vector symmetry $SU(N_f)_V$:

$$q_f \to U_{ff'} q_{f'}$$

where U is $N_f \times N_f$ matrix (unitary $UU^\dagger = U^\dagger U$) with parameters θ_i

$$U = \exp(i\lambda^i \theta^i/2), \qquad i = 1, \ldots, N_f^2 - 1$$

1. $N_f = 2$, u, d quarks only: ⇒ isospin symmetry $SU(2)_I$ – this is rather good symmetry, accuracy $\sim 1\%$,
2. $N_f = 3$, u, d, s quarks: ⇒ "Eight-fold way" symmetry $SU(3)_F$ – not so good, accuracy 20-30% ,
3. for $N_f = 4, 5, 6$: not a symmetry at all because of $m_{c,b,t} \gg m_{u,d,s}$

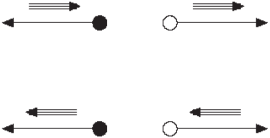
Chiral symmetry of QCD

If quark masses are zero, $m_f = 0$, then a new symmetry arises

$$\bar{q}\,\gamma^\mu\,q = \bar{q}\,(\Pi_R + \Pi_L)\,\gamma^\mu\,(\Pi_R + \Pi_L)\,q = \bar{q}_R\,\gamma^\mu\,q_R + \bar{q}_L\,\gamma^\mu\,q_L,$$
$$q_R = \Pi_R\,q, \quad q_L = \Pi_L\,q, \qquad \bar{q}_R = \bar{q}\,\Pi_L, \quad \bar{q}_L = \bar{q}\,\Pi_R$$

with

$$\Pi_R = \frac{1+\gamma_5}{2}, \qquad \Pi_L = \frac{1-\gamma_5}{2}$$ right and left projectors field



$$\bar{q}_R\gamma^\mu q_R$$

$$\bar{q}_L\gamma^\mu q_L$$

Then the QCD Lagrangian becomes

$$\mathcal{L}_{QCD} \Rightarrow \mathcal{L}^0_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} + i\bar{q}_L\gamma^\mu D_\mu q_L + i\bar{q}_R\gamma^\mu D_\mu q_R$$

Chiral symmetry of QCD

There are independent global transformations of right and left quarks:

$$\begin{aligned} (q_L)_f &\rightarrow e^{i\theta_L} U^{(L)}_{ff'} (q_L)_{f'} \\ (q_R)_f &\rightarrow e^{i\theta_R} U^{(R)}_{ff'} (q_R)_{f'} \end{aligned}$$

This is called chiral group:
$SU(N_f)_L \times U(1)_L \times SU(N_f)_R \times U(1)_R \sim SU(N_f)_V \times U(1)_V \times SU(N_f)_A \times U(1)_A$

1. $U(1)_V = e^{i\theta_V}$ is global phase $\Longrightarrow$ baryon number conservation
2. $U(1)_A = e^{i\theta_A \gamma_5}$ looks like symmetry though it is not, because of quantum corrections – is called "axial anomaly".
3. Global $SU(N_f)_L \times SU(N_f)_R \sim SU(N_f)_V \times SU(N_f)_A$ is called chiral symmetry.

   Note that vector $SU(N_f)_V$ is a subgroup of chiral group (as a special case with $U^{(L)} = U^{(R)}$).

Chiral symmetry

Since m_u, m_d, m_s are indeed small, $m_u \approx 2.2$ MeV, $m_d \approx 4.7$ MeV, $m_s \approx 93.4$ MeV, chiral symmetry seems to be a good symmetry.

However, if this symmetry existed this would result in parity doublet particles, but in the hadron spectra such particles are not observed.

What does that mean?

Recall the Goldstone-Nambu mechanism of SSB in the EW theory.
In QCD, the chiral symmetry $SU(3)_L \times SU(3)_R$ is spontaneously broken to the more familiar vector symmetry, flavor symmetry $SU(3)_{V=L+R}$.

The group $SU(3)_L \times SU(3)_R$ has 8+8 group generators, and after symmetry breaking only 8 remain, therefore according to Goldstone theorem 16-8=8 Goldstone bosons should appear.

This mechanism in QCD reminds symmetry breaking in EW theory, where crucial role is played by the scalar Higgs doublet ϕ and its nonzero vacuum expectation value:

$$\langle \phi_0 \rangle = v/\sqrt{2} \approx 174 \text{ GeV} \qquad \Rightarrow \text{ Higgs mechanism : gives masses to quarks and leptons}$$

However there is no scalar field in QCD. What plays its role?

Quark condensate

Quarks are the only fields carrying flavor and they are responsible for breaking the symmetry via **quark condensate**:

$$v = \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = \langle 0|\bar{s}s|0\rangle \approx (-240 \pm 10\ \mathrm{MeV})^3$$

(recall the Cooper pairs in superconductor.)
This is called dynamical breaking of chiral symmetry.

Note that $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{u}_L u_R + \bar{u}_R u_L|0\rangle$

so that QCD vacuum mixes left quarks with right quarks, and
light quark with mass $m_q \sim$ few MeV, moving through the vacuum, gets an effective mass inside hadrons:

$$m_{eff} \sim\ m_q + \Lambda_{QCD} \sim 300\ \mathrm{MeV}$$

It is not surprising that QCD scale parameter Λ_{QCD} and condensate value are related

$$\Lambda_{QCD} \sim (-\langle 0|\bar{q}q|0\rangle)^{1/3} = 200 - 300\ \mathrm{MeV}$$

Goldstone bosons

Do we see 8 Goldstone bosons of the broken chiral symmetry?
Yes, these are light pseudoscalar mesons with $J^P = 0^-$:

$$\underbrace{\pi^+,\ \pi^0,\ \pi^-}_{I=1},\ \underbrace{K^+,\ K^0,\ \bar{K}^0,\ K^-}_{I=1/2},\ \underbrace{\eta_8}_{I=0}$$

Let us combine these mesons in the 3 × 3 matrix

$$\Phi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta_8/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6} \end{pmatrix}$$

There exists the so-called effective field theory for these Goldstone bosons called Chiral Perturbation Theory χPT ($SU(2)$ and $SU(3)$).

You will learn more about this very interesting direction of the low-energy QCD from the lectures of Wolfgang Schafer at this School.

Summary for QCD

QCD correctly describes hadronic world. Its predictions have remarkable success explaining a wide range of phenomena.

Three main properties of the strong interaction:

- **Asymptotic freedom**, which is due to gluonic self-interactions – coupling α_s becomes smaller at short distances, or large momentum transfers. The running of α_s has been experimentally tested at different energy scales.
- **Confinement of quarks and gluons** into color–singlet hadrons, which happens at low energies, where α_s increases. A rigorous proof is still lacking, and dynamical details of “hadronization” of quarks to hadrons are not very well understood.
- **Dynamical chiral symmetry breaking**, which is is due to a non-zero condensate of $q\bar{q}$ pairs in vacuum. However, details of this phenomena are not completely clear.

Problem: the so-called strong *CP* problem of QCD.
Why does QCD not violate space inversion and time inversion symmetries, like the weak interaction?

Time for questions

- The W boson decays to leptons and quarks (the top quark is excluded as being too heavy). The experimental decay width of W is 2.085 GeV.

  Can you find with minimal calculations the width of the decay

$$W^- \to \mu^- + \bar{\nu}_\mu \ ?$$

Hint: you can neglect the masses of leptons and quarks compared with the W-boson mass.

Time for questions

- With neutrino oscillations, the individual lepton numbers L_e, L_μ, L_τ are no longer conserved. This means that muon decay $\mu^- \to e^- + \gamma$ becomes possible, in principle.

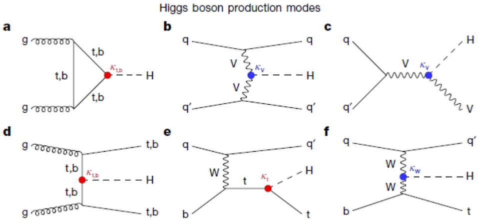
  (i) Draw a Feynman diagram for this process.

  Hint: neutrino oscillation can be represented by a blob ν_μ—●—ν_e.

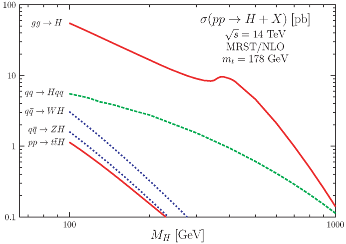
  (ii) Do you think that the process $\mu^- \to e^- + \gamma$ can physically occur?

  Hint: neutrino oscillations occur over large distances L_{osc} proportional to their energy, for example, at $E_\nu \sim 10$ MeV the oscillation distance is $L_{osc} \sim 25$ m.

Production mechanisms of the Higgs boson

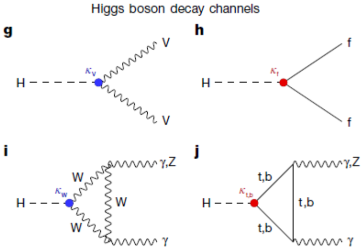


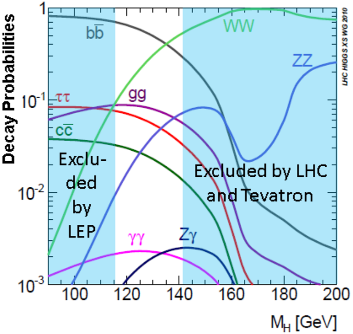
Cross sections for the principal reactions studied at the LHC:



The largest cross section occurs in the reaction $p^{\pm}p \to H +$ anything, which proceeds by gluon fusion $g + g \to H$ through heavy-quark loops.

Decays of the Higgs boson


Higgs boson decay channels



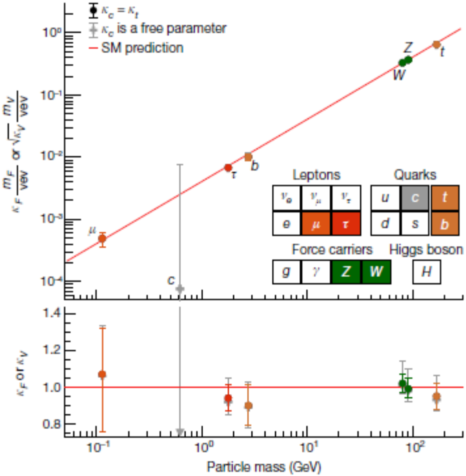
Main decay channels of the Higgs boson and corresponding branching fractions. Of course, in the decays $H \to ZZ$ and $H \to W^+W^-$, both vector bosons cannot be on-mass-shell because of $M_H < 2M_W,\, 2M_Z$.

The CMS and ATLAS data give $M_H \approx 125\ \mathrm{GeV}$, and relatively small decay rate [PDG 2022]:

$$\Gamma_H = 3.2^{+2.8}_{-2.2}\ \mathrm{MeV}, \quad \text{with the lifetime } \tau = 1/\Gamma_H \approx 2 \times 10^{-22}\ s$$

Coupling of the Higgs to fermions and bosons

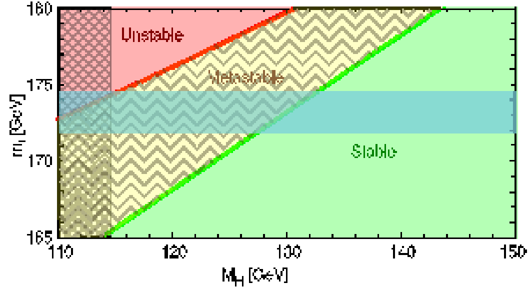
As we saw the coupling of the Higgs boson to all particles is proportional to masses of these particles (surprisingly this is similar to gravitation).
Let us see what is known up to now from experiments at the LHC.



Do we live in a stable vacuum?

There is an interesting question related to the Higgs boson mass: does the EW vacuum (ground state) correspond to the absolute minimum of the Higgs potential, or it is a *false* (unstable) state that has survived quantum fluctuations until now?

The condition of stability is that the mean time to tunnel from our vacuum to a deeper vacuum, $T_{\rm tun}$, exceeds the age of the Universe $T_{\rm Universe} \approx 13.7 \times 10^9$ years.



Present values of M_H and m_t suggest that we do not live in the unstable vacuum but rather in acceptably long-lived state with $T_{\rm tun} \gg T_{\rm Universe}$.
This is a comforting conclusion, although it can be a model dependent result based on a renormalization-group-improved one-loop calculation of the tunneling probability at zero temperature [G. Isidori].

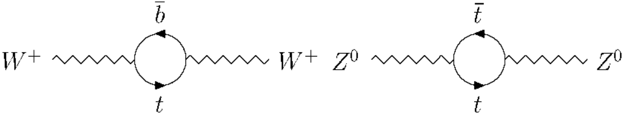
Quantum loops in the SM: masses of gauge bosons

Quantum corrections

Since we deal with the quantum field theory, various quantum corrections arise.

- 1st example

Before the top quark was discovered in 1995, quantum corrections had given indications that t quark would be much more massive than the other quarks. The self-energy corrections to M_W and M_Z arise from different quark loops:



$t\bar{b}$ – for M_W, and $t\bar{t}$, $b\bar{b}$ – for M_Z. This changes the tree-level relation between the W- and Z-boson masses to

$$\frac{M_W}{M_Z \cos\theta_{\rm W}} \approx \left(1 + \frac{3G_{\rm F}(m_t^2 - m_b^2)}{8\pi^2\sqrt{2}}\right)^{1/2} \approx 1 + 0.0047$$

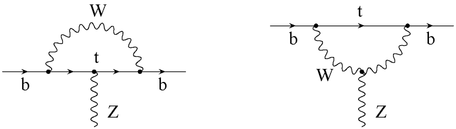
There is a strong dependence on the top-quark mass.

In general, there are observables which are sensitive to virtual effects !

Quantum loops in the SM: decay $Z \to b\bar{b}$

- 2nd example

The $Z \to \bar{b}\, b$ decay width turns out to be sensitive to the top quark mass.



The CKM factor $|\boldsymbol{V}_{tb}| \approx 1$ is big, and for the $Z \to \bar{b}b$ vertex there is no suppression. The induced correction, for $m_t \approx 173$ GeV, amounts to a 1.6% reduction of the decay width $\Gamma(Z \to \bar{b} + b)$.

In principle, instead of $Z \to b\bar{b}$ we may have, e.g., $Z \to b\bar{s}$ or $Z \to s\bar{d}$ decays, so that the top quark also induces
flavor-changing neutral-current (FCNC) decays

$$Z \to \bar{d}_i + d_j \qquad \text{with} \quad i \neq j \quad \text{and} \quad Q_i = Q_j = -1/3|e|$$

which change the flavor of the "down" quarks $d_i \equiv (d, s, b)$. Therefore, such FCNC decays exist, but the amplitudes are suppressed by the small CKM mixing factors $|\boldsymbol{V}_{tj}\, \boldsymbol{V}^*_{ti}|^2 \ll 1$ and are very small.

Is SM the final theory of particle physics?

Incompleteness of the SM

- Too many free parameters
- Does not explain negative μ^2 required to break the EW symmetry
- Does not predict the masses of quarks and leptons and quark mixing parameters
- Does not explain origin of *CP* violation
- Neutrino nonzero masses and mixing require extension of the SM
- Higgs sector is unstable under radiative corrections –“hierarchy puzzle”
- Scalar field $\langle\phi\rangle_0$ is not compatible with observations in cosmology
- Dark matter and dark energy
- *CP* violation through the CKM matrix is too small to explain excess of matter over antimatter in the Universe
- Quantization of electric charge
- Absence of gravity
- . . .

Too many parameters - problem of identity

In total there are 26 parameters (compare with 2 parameters in QED: $\alpha_{\rm em},\ m_e$) :

$$\alpha_s,\ \alpha_{\rm em},\ \sin^2\theta_{\rm W},$$

parameters of the Higgs potential

$$\mu,\ \lambda,$$

6 quark masses and 4 parameters of CKM matrix

$$m_u, \cdots, m_t,\ \theta_{12},\ \theta_{23},\ \theta_{13},\ \delta,$$

6 masses of leptons and neutrinos and mixing parameters

$$m_e, \cdots, m_{\nu_\tau},\ \theta'_{12},\ \theta'_{23},\ \theta'_{13},\ \delta'$$

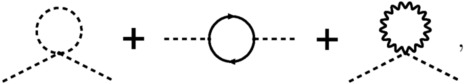
(plus 2 parameters $\alpha_1,\ \alpha_2$, if neutrinos are Majorana particles), plus QCD vacuum phase in the CP-violating strong Lagrangian.

From these 26 parameters 20 are related to physics of flavor!
Is it physics beyond the SM? Is the Higgs boson responsible for fermion masses, or only for $W^\pm$, Z masses?

Hierarchy problem and mass of the Higgs boson

Beyond the classical picture (“tree” level), the Higgs mass gets corrections from the loops:

$$M_H^2(p^2) = M_H^2(\Lambda^2) +$$



or more formally $$M_H^2(p^2) = M_H^2(\Lambda^2) + \mathcal{C} g^2 \int_{p^2}^{\Lambda^2} dk^2 + \ldots$$

The top quark gives the biggest contribution

$$\delta M_H^2 = \frac{G_\mathrm{F}\Lambda^2}{4\pi^2\sqrt{2}}\left(6\,M_W^2 + 3\,M_Z^2 + M_H^2 - 12\,m_t^2\right) \approx -\frac{3G_\mathrm{F}}{\pi^2\sqrt{2}} m_t^2\Lambda^2 \approx -0.075\,\Lambda^2 .$$

What should we choose for the upper energy Λ?
(i) Natural reference scale is the Planck mass,

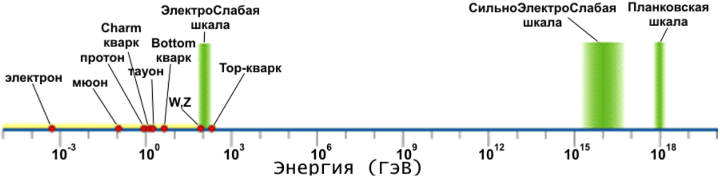
$$\Lambda \sim M_\mathrm{Planck} = (\hbar c/G_\mathrm{Newton})^{1/2} \approx 1.22 \times 10^{19}\,\mathrm{GeV}/c^2 = 2.17 \times 10^{-5}\,\mathrm{g},$$

where the gravitation becomes the dominant interaction.
It is actually a macroscopic mass, of the order of mass of a flea!

Mass of the Higgs and hierarchy problem

(ii) Or, in a unified theory of the strong, weak, and electromagnetic interactions, a natural scale is the unification scale,

$$\Lambda \sim M_{GUT} \approx 10^{15} \text{ - } 10^{16}\,\mathrm{GeV}$$



(iii) Even if we assume that the SM is an effective theory at relatively low scale

$$\Lambda \sim 10\ \mathrm{TeV}\,,$$

then stabilizing the mass at $M_H = 125$ GeV requires “delicate” balance of two numbers of the order 10^7 GeV^2:

$$M_H^2(p^2) = (125\,\mathrm{GeV})^2 = 1.56 \cdot 10^4\,\mathrm{GeV}^2 = M_H^2(\Lambda^2) - 7.5 \cdot 10^6\,\mathrm{GeV}^2\,.$$

If we choose $\Lambda = M_{\mathrm{Planck}}$, then $m_H = 125$ GeV requires cancellation of two huge numbers $\sim 10^{37}$GeV^2, which is unbelievable.

Vacuum energy and cosmology

Problem of cosmological constant – why empty space is so nearly massless—is one of the great mysteries of cosmology.
At the VEV $\langle\phi\rangle_0 = v/\sqrt{2}$ of the Higgs field, the value of the Higgs potential is

$$V(\langle\phi^\dagger\phi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{M_H^2 v^2}{8} < 0.$$

If we take $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV and insert $M_H \approx 125$ GeV, we find contribution of the background field to a uniform vacuum energy density

$$\varrho_H \equiv \frac{M_H^2 v^2}{8} \gtrsim 10^8\,\mathrm{GeV}^4.$$

On the other hand, in the general relativity, this amounts to adding a cosmological constant Λ to Einstein's equation

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} - \Lambda g_{\mu\nu} &= 8\pi G_{\mathrm{Newton}}\, T_{\mu\nu}\,, \\ \text{where} \qquad \Lambda &= (8\pi G_{\mathrm{Newton}}/c^4)\varrho_H. \end{aligned}$$

Vacuum energy and cosmology

Observations of the accelerating expansion of the Universe allow for determination of Λ and tell us that the observed vacuum energy density must be extremely small:

$$\varrho_{\mathrm{vac}} \lesssim 10^{-46}\,\mathrm{GeV}^4 \approx (\text{a few meV})^4\ .$$

There is a puzzle:

Comparing $\varrho_H \gtrsim 10^8\,\mathrm{GeV}^4$ with $\varrho_{\mathrm{vac}} \lesssim 10^{-46}\,\mathrm{GeV}^4$ we see that the scalar field contribution is 54 orders of magnitude (!) larger than the upper bound inferred from the cosmology.

Obviously something essential is missing in our understanding of the vacuum energy due to the scalar field.

Conclusions and outlook

- The SM is a remarkable achievement.

  It accounts for a wide variety of experimental measurements, and has survived many tests as a consistent quantum field theory (see also other lectures at this TESchool).

  It meets the most important criteria for a good theory: we get more out than we put in, and it raises new and significant questions.

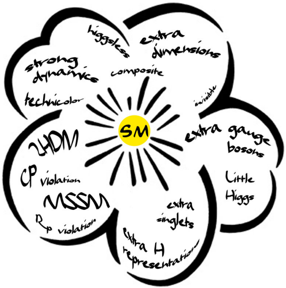
  Therefore we may regard the SM as a **Law of Nature.**

- Experiments at the LHC have been probing the symmetry breaking sector on the 10 TeV scale.

  Measurements at the LHC indicated that the Higgs boson mass is about 125 GeV. It is important to explore in detail all its properties at the LHC and future colliders.

Conclusions and outlook

- It is also clear that the SM is incomplete theory. And we hope to find signs of physics beyond the SM at the LHC. We need much more to learn . . . and new interesting physics is ahead.



In this search for new physics it is important to remember the words of

Richard P. Feynman:

It doesn't matter how beautiful your theory is, and it doesn't matter how smart you are. If it (theory) doesn't agree with experiment, it's wrong.

A few selected references

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