Selected topics from Quantum Chromodynamics Part II: Electron scattering and hadron structure: Form factors and parton distributions

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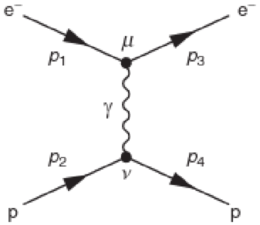
Photon exchange Feynman diagram



- four-momentum transfer $q = p_1 - p_3 = p_4 - p_2$. The four-momentum transfer squared is negative: $q^2 = -Q^2$. In the LAB frame, where the proton target is initially at rest, we have :

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

  here $\theta =$ scattering angle of electron, and E, E' energies of the electron before and after scattering.
- The Feynman amplitude for single-photon exchange takes the following form:

$$i\mathcal{M} = 4\pi\alpha_{em} \langle e^-, p_3 | j_\mu | e^-, p_1 \rangle \frac{-g^{\mu\nu}}{q^2} \langle p, p_4 | j_\nu | p, p_2 \rangle .$$

- Recall, that the probability amplitude for photon emission/absorption for a particle a is proportional to

$$\langle a, p' | j_\mu | a, p \rangle \varepsilon^{\mu *}(\lambda) .$$

Electron-nucleon (proton/neutron) scattering

- To calculate the electron-scattering cross section, we need to know the electromagnetic current for the target.
- for example for a **scalar** particle of charge Ze, the current has the simple form:

$$\langle p'|j_\mu|p\rangle = Ze(p+p')_\mu \, F(Q^2)\,,\; q = p' - p\,,\; Q^2 = -q^2\,.$$

- For the proton, the current also depends on its **polarization states**. There are **two independent form factors**:

$$\langle p', s'|j_\mu|p, s\rangle = e\,\bar{u}(p', s')\left(\gamma_\mu\, F_1(Q^2) + \frac{i}{2m}\sigma_{\mu\nu}q^\nu\, F_2(Q^2)\right)u(p,s)\,,\; \sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\mu\gamma_\nu)\,.$$

- **Dirac form factors** $F_1(Q^2), F_2(Q^2)$ are related to the more easily interpreted **Sachs**-form factors (or **electric** and **magnetic** formfactors). They read

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \tau F_2(Q^2)\,,\; \tau = \frac{Q^2}{4m_p^2} \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2)\,. \end{aligned}$$

- at $Q^2 = 0$, for the proton and neutron:

$$G_E^p(0) = F_1^p(0) = 1\,,\; G_E^n(0) = F_1^n(0) = 0\,.$$

- the magnetic form factors are normalized to the relevant magnetic moment:

$$G_M^p(0) = 1 + F_2^p(0) = +2.79\,,\; G_M^n(0) = F_2^n(0) = -1.91\,.$$

Differential cross section, measuring the two form factors

- Elastic $ep \to ep$ cross section depends on the charge and magnetic moment distributions. The **Rosenbluth formula** for the differential cross section reads:

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{Mott}} \left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \tan^2\frac{\theta}{2} \right), \quad \tau = \frac{Q^2}{4m_p^2} \, .$$

- Mott cross section is a reference cross section for a pointlike proton, neglecting proton spin.
- In the LAB frame:

$$Q^2 = 4EE' \sin^2\frac{\theta}{2}$$

  here $\theta=$ scattering angle of electron, and E, E' energies of the electron before and after scattering.
- In order to determine the formfactors at fixed values of Q^2, we must measure the differential cross section for various angles, i.e for different **beam energies**.
- In the Breit-Frame the Sachs form factors become related to Fourier transforms of charge- and magnetization densities:

$$\begin{aligned} G_E(Q^2) \to G_E(\vec{q}^2) &= \int d^3\vec{r} e^{i\vec{q}\vec{r}} \rho(\vec{r}) \\ G_M(Q^2) \to G_M(\vec{q}^2) &= \int d^3\vec{r} e^{i\vec{q}\vec{r}} \mu(\vec{r}) \, . \end{aligned}$$

The charge radius

- an effective size, the **charge radius** can be related to the slope of the charge formfactor at small momentum transfers.
- recall the definition in terms of the charge distribution

$$\begin{aligned} G_E(\vec{q}^2) &= \int d^3\vec{r}\rho(\vec{r})e^{i\vec{q}\cdot\vec{r}} = \int d^3\vec{r}\rho(\vec{r})\left(1 + \sum_{n=1}^{\infty} i^n \frac{(\vec{q}\cdot\vec{r})^n}{n!}\right) \\ &= 1 - \frac{1}{6}\vec{q}^2\, 4\pi \int_0^\infty r^2 dr\, r^2\rho(\vec{r}) \equiv 1 - \frac{1}{6}\vec{q}^2\langle r^2\rangle + \ldots \end{aligned}$$

- Here we define the **charge radius** as $r_{ch} = \sqrt{\langle r^2\rangle}$, with

$$\langle r^2\rangle = \int d^3\vec{r}\, r^2\rho(r) \qquad (1)$$

- or, starting from the directly measured formfactor

$$\langle r^2\rangle = -6\frac{dG_E(\vec{q}^2)}{d\vec{q}^2}\bigg|_{\vec{q}^2=0} .$$

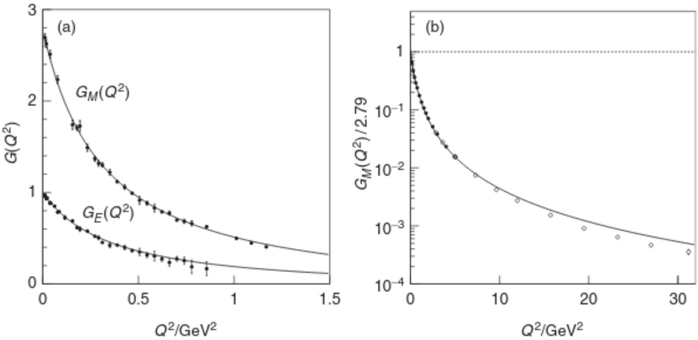
- if the charge is concentrated in the origin $\rho(\vec{r}) = \delta^3(\vec{r})$ ("pointlike particle"), the form factor is just constant: $G_E(\vec{q}^2) = 1$!

Dipole parametrization

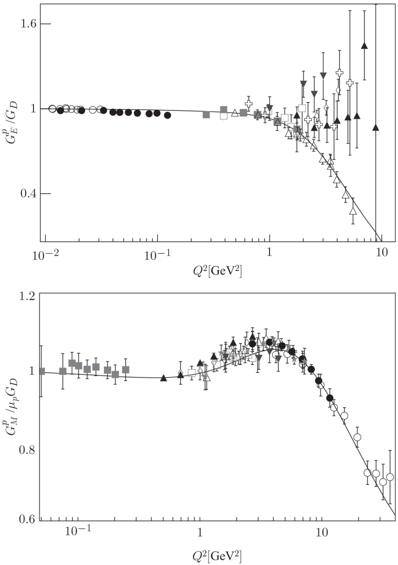
- Traditionally proton electric and magnetic and neutron magnetic form factors have been fitted by a so-called "dipole" functional shape. Older data at small and intermediate Q^2 suggest, that they are all proportional to the same function

$$G_E^p(Q^2) = G_M^p(Q^2)/\mu_p = G_M^n(Q^2)/\mu_n = G_D(Q^2) = \frac{1}{(1+\frac{Q^2}{\Lambda^2})^2}\,,\ \Lambda^2 = 0.71\,\mathrm{GeV}^2$$

- after Fourier transform it corresponds to an charge distribution with an exponential tail and rms-radius $r_{\mathrm{ch}} \sim 0.8\,\mathrm{fm}$.

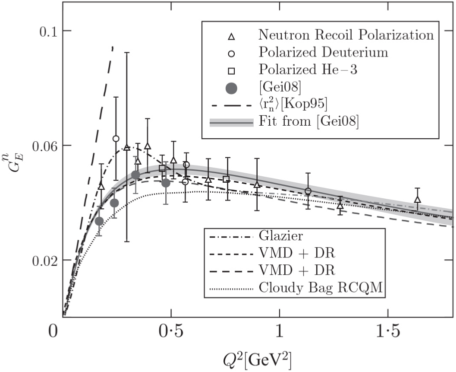


Deviation from the dipole parametrization



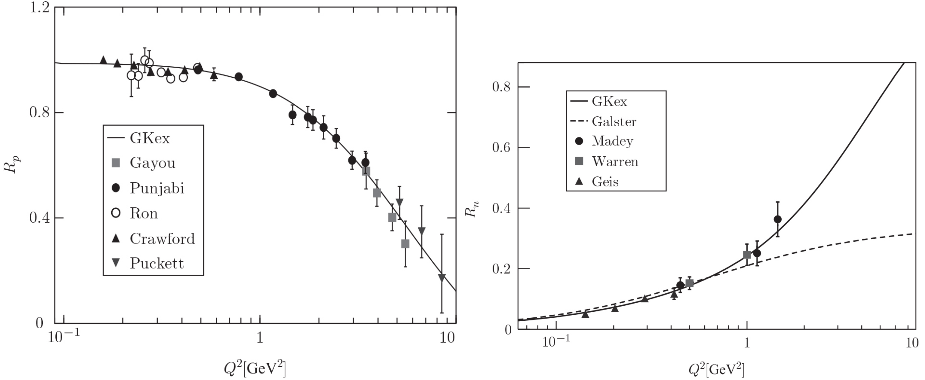
- Data from Jefferson lab show a clear deviation from the dipole parametrization. Above $Q^2 \gtrsim 1\,\mathrm{GeV}^2$ the electric FF falls off substantially faster.

Neutron electric form factor



- Neutron electric form factor exctracted from polarized scattering from light nuclei (D, ^{3}He).
- slope at $Q^2 = 0$ has been measured from neutron scattering off atomic electrons. Notice that $\langle r_n^2 \rangle < 0$.

Electric/magnetic form factor ratios

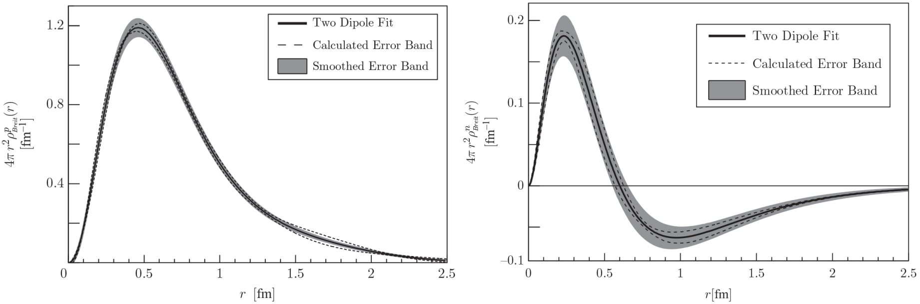


- Ratios

$$R = \frac{\mu G_E(Q^2)}{G_M(Q^2)} .$$

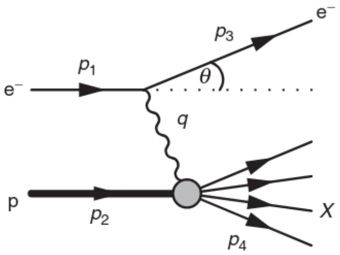
- solid lines are results of a VDM + pQCD model as proposed by Gari and Krümpelmann.
- significant difference of G_E and G_M at large Q^2 for the proton. Deviation from the common dipole form.

Breit frame charge distributions for proton and neutron



- charge density of neutron integrates to zero, as it must be. It has a positive charge core, and a negative tail. The latter may be suggestive of $p\pi^-$ fluctuations in the neutron wave function.

Kinematics for inclusive electron proton scattering



- **Inclusive** electron scattering: measure only the scattered electron. From the scattered electron (lepton) kinematics determine the four-momentum transfer.
- for the **elastic** $e^- p \to e^- p$ process, we had only the dependence on the scattering angle. Now, we sum over all hadronic final states X. The invariant mass of the recoiling hadronic system X is not fixed. Conventionally we denote $W^2 \equiv p_4^2 = M_X^2$.

DIS variables

$$
\begin{aligned}
\nu &= \frac{P\cdot q}{m_N} = \frac{W^2+Q^2-m_N^2}{2m_N} \\
x &= \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2m_N\nu} = \frac{Q^2}{Q^2+W^2-m_N^2} \\
y &= \frac{P\cdot q}{P\cdot l} = \frac{W^2+Q^2-m_N^2}{s-m_N^2} \,.
\end{aligned}
$$

DIS kinematic variables

DIS variables

$$\begin{aligned}
\nu &= \frac{P \cdot q}{m_N} = \frac{W^2 + Q^2 - m_N^2}{2m_N} \\
x &= \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2m_N \nu} = \frac{Q^2}{Q^2 + W^2 - m_N^2} \\
y &= \frac{P \cdot q}{P \cdot l} = \frac{W^2 + Q^2 - m_N^2}{s - m_N^2} .
\end{aligned}$$

- any of the pairs (Q^2, x), (Q^2, y), (x, y), (ν, x), (Q^2, ν), or Q^2, W^2 can be used to analyze the cross section.
- ν has the meaning of the **photon energy in the target rest frame**.
- y is the **fraction of the incoming electrons's energy carried by the photon**, $y = \nu/E$ (sometimes called "inelasticity"). By definition $0 \leq y \leq 1$.
- x is the so-called **Bjorken-variable**, also by its definition $0 \leq x \leq 1$.
- We conventionally speak of **Deep Inelastic Scattering** (DIS), when $\nu \gg m_N$, and $Q^2 \gg m_N^2$. In this regime we can mostly neglect the proton mass compared to the other large scales.
- Notice, that for elastic scattering, where $X = p$, we have $W = m_N$, and therefore $x = 1$!

Inclusive $ep \to eX$ cross section

- Let's rewrite the **elastic** $ep \to ep$ cross section as

$$\frac{d\sigma}{[dx]dQ^2} = [\delta(1-x)]\frac{4\pi\alpha_{em}^2}{Q^4}\left\{\left(1-y-\frac{m_N^2 y^2}{Q^2}\right)\frac{G_E^2(Q^2)+\tau G_M(Q^2)}{1+\tau}+\frac{1}{2}y^2 G_M^2(Q^2)\right\}$$

- The inclusive cross section for $ep \to eX$ can be written in an analogous way:

$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha_{em}^2}{Q^4}\left\{\left(1-y-\frac{m_N^2 y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x}+y^2 F_1(x,Q^2)\right\}$$

- Instead of two form factors which depend only on Q^2 we have two **structure functions**. Our structure functions depend on two independent variables x, Q^2.
- This notation F_2, F_1 and the fact that one of them comes with a factor $1/x$, is purely historical accident. We shall see later that in fact $2xF_1(x,Q^2)$ and $F_2(x,Q^2) - 2xF_1(x,Q^2)$ have a straightforward physical meaning.
- for comparison: scattering off **pointlike** particle:

$$\frac{d\sigma}{[dx]dQ^2} = [\delta(1-x)]\frac{4\pi\alpha_{em}^2}{Q^4}\left(1-y+\frac{1}{2}y^2\right)$$

Inclusive electron proton scattering

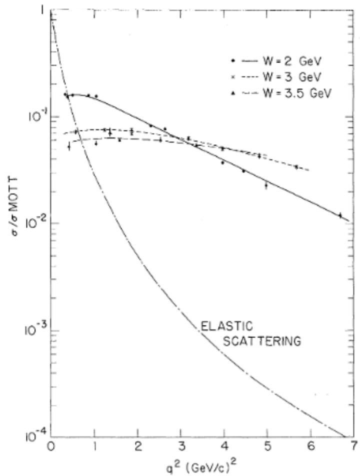
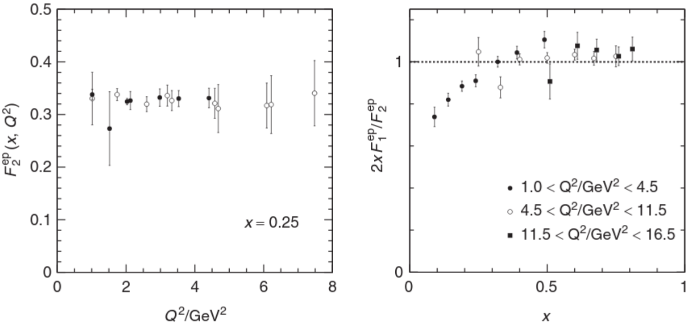


FIG. 1. $(d^2\sigma/d\Omega dE')/\sigma_{\text{Mott}}$, in GeV^{-1}, vs q^2 for $W = 2$, 3, and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e–p scattering divided by σ_{Mott}, $(d\sigma/d\Omega)/\sigma_{\text{Mott}}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.

- at large Q^2 "flat" dependence of the cross section ratio to the Mott cross section.
- the cross section rather resembles the scattering off an almost pointlike constituent. That cannot be the proton itself– we are talking about inelastic collisions!
- Apparently we observe structure probed by a photon at distance scales $\sim 1/Q^2$.
- $\Rightarrow$ parton model, (Bjorken, Feynman).
- SLAC experiments from 1960's. Nobel prize 1990 (Friedman, Kendall, Taylor).

Inclusive electron proton scattering



- the early DIS experiments suggested that the structure function F_2 is only a function of Bjorken-x and does not depend on Q^2 (**Bjorken scaling**).
- there was also reasonable support for the identity $2xF_1 = F_2$ (**Callan-Gross relation**).
- these results strongly suggested the existence of "pointlike" charged constituents in the proton and gave credibility to the parton model.

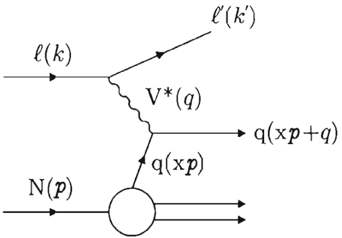
DIS: electron-quark scattering

- Let's adopt the idea of Bjorken-scaling and the Callan-Gross relation. Then:

$$\frac{d\sigma(ep \to eX)}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{Q^4}\left(1 - y + \frac{1}{2}y^2\right)\frac{F_2(x)}{x}$$

- With the ansatz ($f =$ flavor)

$$F_2(x) = \sum_f e_f^2 \, x \, p_f(x)$$



- Meaning of Bjorken-x: Let quark before the scattering carry momentum ξP, then from the on-shell condition of final state quark $(\xi P + q)^2 = m_q^2 = 0 \to \xi = Q^2/(2P \cdot q) = x$!
- we obtain the cross section

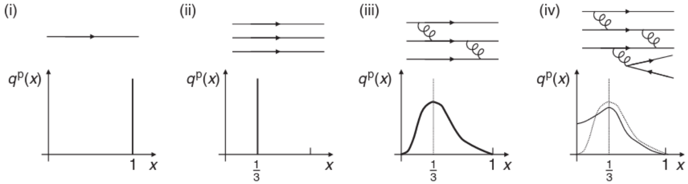
$$\frac{d\sigma(ep \to eX)}{dx dQ^2} = \sum_f \frac{d\sigma(eq_f \to eq_f)}{dQ^2} \cdot p_f(x)$$

- Here $p_f(x)$ is the **parton density** or **parton distribution function (pdf)** .
- naturally, we should identify $p_f(x) = q_f(x) + \bar{q}_f(x)$.
- Here $p_f(x)\,dx$ is the probability to find a parton of flavor x carrying a fraction of the proton's momentum in the interval $x, x + dx$.
- the **Callan-Gross relation** can be directly related to the fact that **quarks have spin 1/2**.

Parton distributions, flavor decomposition

- In the parton model we assume that we can neglect interactions between partons during the time of the interaction (the "hard process"). The interactions in the initial state (which "bind" the proton) and in the final state (which convert the quarks/partons into final state hadrons) take place at large spacetime distances – of order of the proton mass. These space and time scales are much larger than the size of the quark or the time of interaction.
- Hence, quark distributions (pdf's) cannot be calculated in perturbation theory. They carry information on the nonperturbative structure of the proton. Relations to correlation functions of QCD quark fields on along the light-cone can be derived using a very involved machinery of so-called "factorization theorems".
- For the proton we expect **valence quarks** (uud) and in addition $q\bar{q}$-pairs (the **sea quarks**). We expect sea quarks at smaller values of x (recall the Bremsstrahlung spectrum dx/x).
- For proton and neutron we would expand

$$\frac{1}{x}F_2^p = \frac{4}{9}(u_p(x) + \bar{u}_p(x)) + \frac{1}{9}(d_p(x) + \bar{d}_p(x)) + \frac{1}{9}(s_p(x) + \bar{s}_p(x)) + \dots$$
$$\frac{1}{x}F_2^n = \frac{4}{9}(u_n(x) + \bar{u}_n(x)) + \frac{1}{9}(d_n(x) + \bar{d}_n(x)) + \frac{1}{9}(s_n(x) + \bar{s}_n(x)) + \dots$$



Valence and sea decomposition

- **Isospin symmetry** suggests $u_p(x) = d_n(x) \equiv u(x)$ and $d_p(x) = u_n(x) \equiv d(x)$. Furthermore $s_p(x) = s_n(x) \equiv s(x)$.
- Motivated by the simple valence quark picture, we decompose quark distributions into their **valence** and **sea** components.
- $u(x) = u_V(x) + u_S(x)$. The antiquark densities have only a sea quark component $\bar{u}(x) = u_S(x)$, so that $u_V(x) = u(x) - \bar{u}(x)$.
- Valence distributions are normalized as

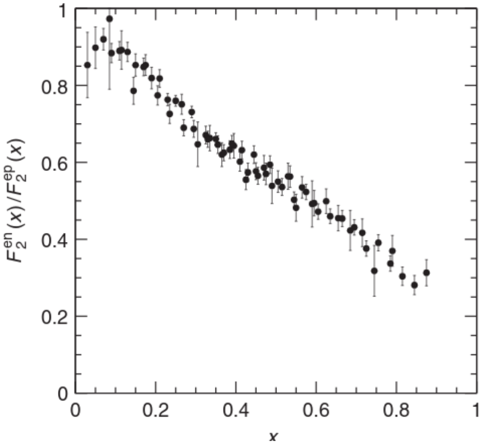
$$\int_0^1 dx\, u_V(x) = 2\,, \int_0^1 dx\, d_V(x) = 1\,.$$

- at **small** x we expect sea quarks to dominate.
- one often assumes, that the sea quark distribution is **flavour symmetric**, e.g. $\bar{u}(x) = \bar{d}(x)$. This is not borne out by many fits. In fact a nonperturbative sea from meson-baryon fluctuations is expected to exist. For simplicity let us though assume, that $\bar{u}(x) = \bar{d}(x) = S(x)$.
- Then, for the ratio of neutron to proton structure functions we obtain:

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

- at small x: $\frac{F_2^n(x)}{F_2^p(x)} \to 1$.

Neutron to proton ratio



$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

- a naive assumption $u_V(x) \sim 2d_V(x)$ would yield $\frac{F_2^n(x)}{F_2^p(x)} \to 2/3$ which appears to be in conflict with data. The ratio rather approaches $\to 1/4$ which suggests $d_V(x) \ll u_V(x)$ at large x

Behaviour of parton distributions and the momentum sum rule

- rough x-dependence of quark valence and sea distributions

$$
\begin{aligned}
xq_V(x) &\sim (1-x)^3 \text{ for } x \to 1 \\
xq_V(x) &\sim x^{0.5} \text{ for } x \to 0 \\
xq_S(x) &\sim (1-x)^7 \text{ for } x \to 1 \\
xq_S(x) &\sim x^{-0.2} \text{ for } x \to 0
\end{aligned}
$$

- it makes no sense to ask for the **total number pf partons in the proton**:

$$\int_0^1 dx\, q(x) \to \infty$$

The integral diverges at the lower boundary! Sea quark distribution is not integrable.

- it makes sense to ask for the momentum fraction carried by a parton:

$$\int_0^1 dx\, x\, q(x) \equiv \langle x \rangle_q = \text{finite} < 1$$

- one finds roughly

$$\langle x \rangle_{u_V} \sim 0.27\,,\ \langle x \rangle_{d_V} \sim 0.11,\ \langle x \rangle_{\text{sea}} \sim 0.17$$

- quarks and antiquarks carry only about $\sim 55\%$ of the proton's momentum.
- to save the momentum sum, the proton must contain neutral partons - **gluon distribution**.

Structure functions, hadronic tensor, etc.

- The amplitude for the deep inelastic ep scattering has the familiar form

$$\mathcal{M} = \frac{4\pi\alpha_{em}}{Q^2} j_\mu g^{\mu\nu} \langle X|J_\nu|p\rangle \,.$$

- We cannot parametrize the hadronic current $\langle X|J_\nu|p\rangle$ in any convenient way, but after squaring there appears the so-called **hadronic tensor**:

$$W_{\mu\nu} \propto \sum_X \int d\Phi_X \, \langle p|J_\mu|X\rangle\langle X|J_\nu|p\rangle$$

- one can show the following: firstly the hadronic tensor can be parametrized as

$$W_{\mu\nu} = \Big(-g_{\mu\nu} + \frac{q_\mu q\nu}{q^2}\Big) 2F_1(x,Q^2) + \frac{2}{P\cdot q}(P_\mu - \frac{P\cdot q}{q^2} q_\mu)(P_\nu - \frac{P\cdot q}{q^2} q_\nu) F_2(x,Q^2)$$

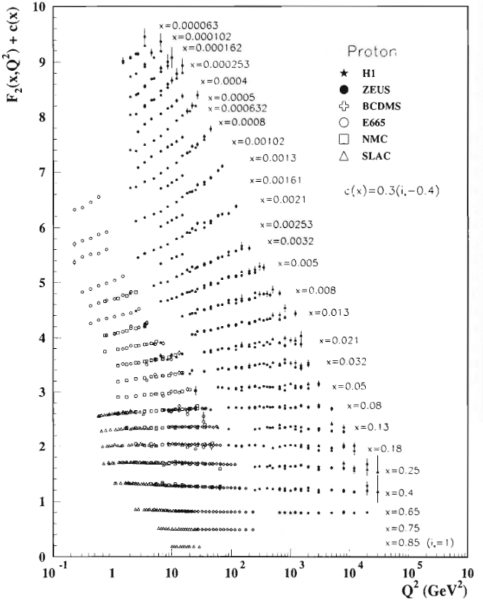
- secondly, $W_{\mu\nu}$ is the imaginary part of a forward Compton amplitude. Introducing virtual photon polarization vectors we can introduce virtual photoabsorption cross sections:

$$\sigma_\lambda^{\gamma^* p} \propto \varepsilon_\mu(\lambda)\varepsilon^*_\nu(\lambda) W_{\mu\nu} \,.$$

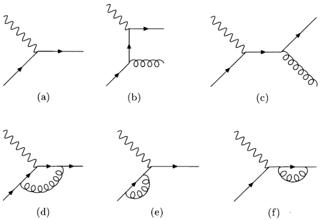
- structure functions $F_T = 2xF_1,\; F_L = F_2 - 2xF_1$

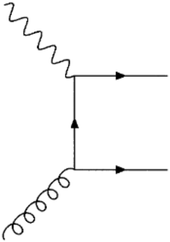
$$\sigma_T^{\gamma^* p} = \frac{4\pi^2\alpha_{em}}{Q^2} 2xF_1(x,Q^2)\,,\; \sigma_L^{\gamma^* p} = \frac{4\pi^2\alpha_{em}}{Q^2}(F_2(x,Q^2) - 2xF_1(x,Q^2))$$

Structure function $F_2(x, Q^2)$, violations of Bjorken scaling



QCD improved parton model





- in full QCD structure functions and parton densities **depend on** Q^2
- longitudinal structure function F_L is induced by the photon-gluon fusion and is a measure of the gluon distribution.
- Radiative corrections contain $\alpha_S \log Q^2$ enhancements from (near-)collinear emission of partons in the $q \to qg$, $g \to q\bar{q}$ and $g \to gg$ transitions.
- These "collinear logarithms" can be systematically summed up by an evolution equation. They explain the scaling violations of F_2 in a wide kinematic range.
- in the region of small-x, say $x \ll 10^{-3}$ summation of collinear logs is not enough and one has to also address $\alpha_S \log(1/x)$ corrections (Balitsky-Fadin-Kuraev-Lipatov).
- eventually, at very small-x a regime of very large parton (gluon) densities emerges, in which gluon-fusion corrections (or gluon saturation effects) lead to nonlinear evolution equations.

DGLAP evolution equations

- We cannot predict parton distributions in perturbative QCD, but we can predict their **dependence on** Q^2.
- Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations:

$$\frac{d}{d\log Q^2} q_{NS}(x,Q^2) = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1 \frac{dz}{z} P_{qq}(\frac{x}{z}) q_{NS}(z,Q^2)$$

$$\frac{d}{d\log Q^2}\begin{pmatrix}\Sigma(x,Q^2)\\ g(x,Q^2)\end{pmatrix} = \frac{\alpha_S(Q^2)}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix}P_{qq}(\frac{x}{z}) & 2n_f P_{qg}(\frac{x}{z})\\ P_{gq(\frac{x}{z})} & P_{gg}(\frac{x}{z})\end{pmatrix}\begin{pmatrix}\Sigma(z,Q^2)\\ g(z,Q^2)\end{pmatrix}$$

- Here

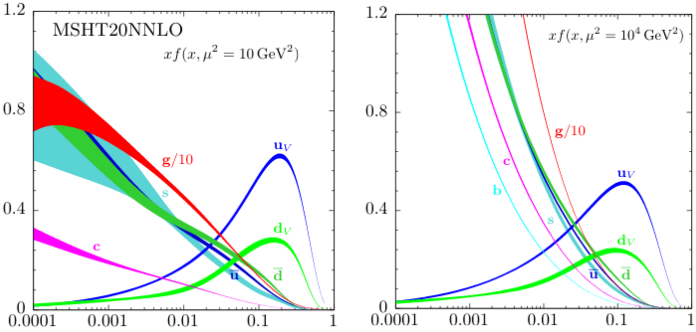
$$q_{NS}(x,Q^2) = q(x,Q^2) - \bar{q}(x,Q^2)\,,\ \Sigma(x,Q^2) = \sum_f [q_f(x,Q^2) + \bar{q}_f(x,Q^2)]$$

- We must specify a boundary condition at some starting scale Q_0, typically $Q_0 \sim$ a few GeV.
- typical ansatz of the form

$$xf_i(x,Q_0^2) = A_i x^{A_i}(1-x)^{B_i} P_i(x), \quad i = u, \bar{u}, , \bar{d}, s\bar{s}, g$$

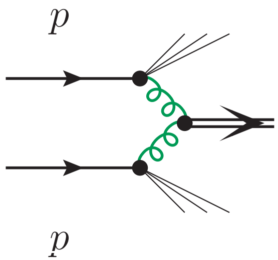
- This formalism is used to **fit** parton distributions at Q_0 to experimental data. Heavy quarks are often assumed to be fully generated by gluon splitting. (see however "intrinsic charm").

Parton distributions



- Parton distributions obtained by the Durham group for two scales.
- Note the flavour asymmetric sea!
- at small x the gluon distribution by far dominates.
- modern fits are performed at next-to-next-to leading order.

Applications to hard processes at hadron colliders



$\eta_{c,b}, \chi_{c,b}$

- factorization theorems ensure, that parton distributions are **universal** and can be used to predict cross sections of **hard processes** in hadronic collisions.
- examples: Drell-Yan (lepton pairs of large invariant mass), dijet or multijet production, quarkonium production, open haevy flavoured mesons, light mesons/hadrons at large p_T, Higgs bosons...

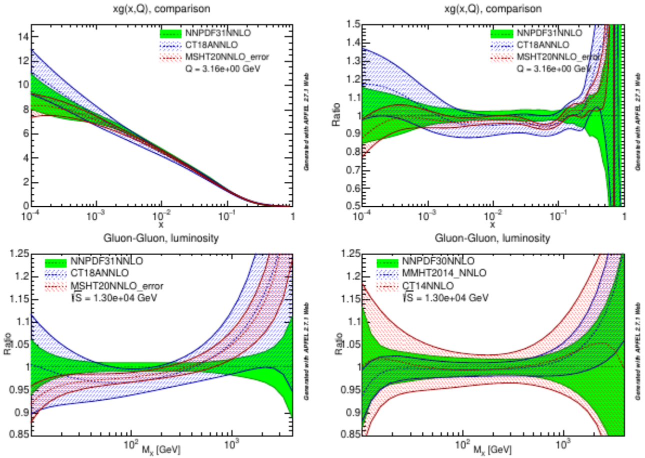
QCD factorization for hard production of final state f:

$$d\sigma(p_A, p_B, Q^2) = \sum_{a,b} \int dx_a dx_b f_{a/A}(x_a, \mu^2) f_{b/B}(x_b, \mu^2)\, d\sigma_{ab\to f}(\alpha_S(\mu^2), Q^2/\mu^2)$$

A useful quantity is the parton-parton luminosity

$$\mathcal{L}(M^2) = \int dx_a dx_b f_{a/A}(x_a, \mu^2) f_{b/B}(x_b, \mu^2)\, \delta(x_a x_b - M^2/s)$$

Uncertainties in parton distributions 2023



from A. Cooper-Sarkar, 2302.11788 [hep-ph], Proceedings of Cracow Ephipany 2023 Conference.

- pdf uncertainties are still an issue.