Selected topics from QCD: Diffractive processes

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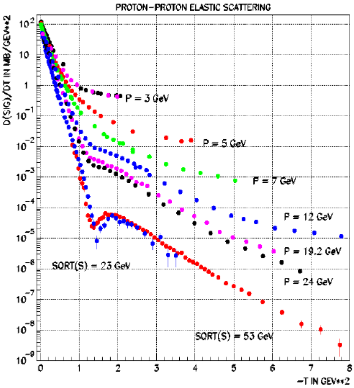
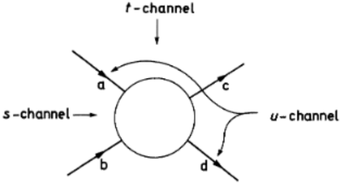
Elastic proton-proton scattering





- Elastic hadron-hadron scattering at high energies reminds of Fraunhofer diffraction.
- Scattering amplitude for black disc of size R, $q = \sqrt{-t}$:

$$f(q) = ikR^2 \frac{J_1(qR)}{qR}, \quad k = \frac{1}{2}\sqrt{s - 4m^2},$$

- Optical theorem: $4\pi/k\Im m f(0) = \sigma_{\rm tot} = 2\pi R^2$
- For the black disc: $\sigma_{\rm el} = 0.5\,\sigma_{\rm tot} = \pi R^2 = \sigma_{\rm inel}$.

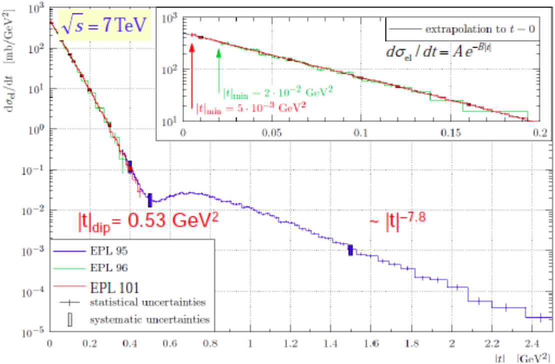
Mandelstam variables

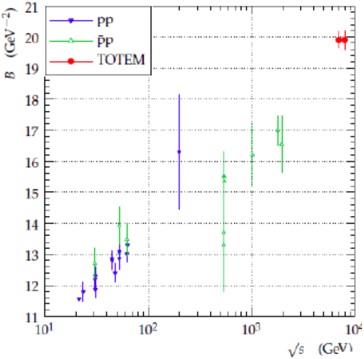
$$s = (p_a + p_b)^2 = E_{\rm cm}^2, \quad \cos\theta = 1 + \frac{2t}{s - 4m^2}$$

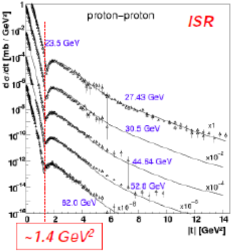
$$s + t + u = 4m^2 .$$

- small $|t|$ means scattering under small angles.
High energy limit: $-t/s \ll 1$.

Elastic proton-proton scattering

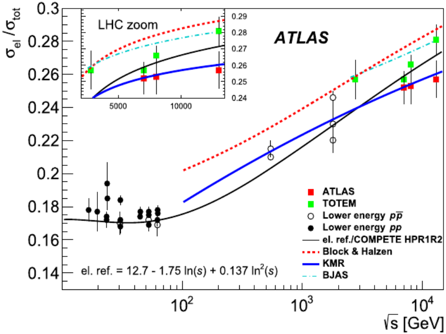




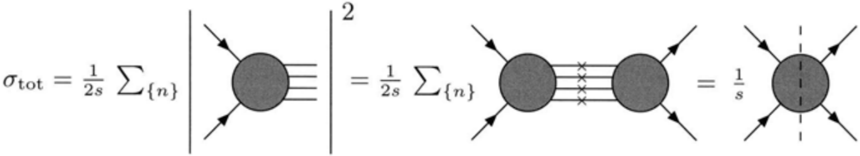


- sharp peak in the forward direction, well described by exponential $d\sigma/dt \propto \exp[-B|t|]$, B is the **elastic slope**.
- shrinkage of the diffraction cone: elastic slope rises with energy.
- dip moves to smaller $|t|$.

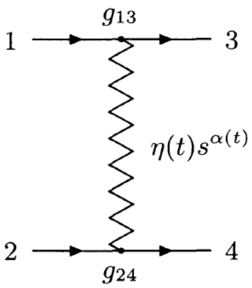
Elastic proton-proton scattering



- elastic/total ratio continues to rise.
- Indicative of a "blacker" interaction region. (Recall elastic/total = 0.5 for the black disc).
- elastic scattering is a substantial fraction of the total cross section.
- elastic scattering is a consequence of **unitarity**. (Optical theorem).



Particle vs Reggeon exchange



- An amplitude for the exchange of a particle of spin J has the high energy limit:

$$A_J(s,t) \propto \frac{s^J}{t-m^2} \Rightarrow \frac{d\sigma}{dt} \propto s^{2(J-1)}$$

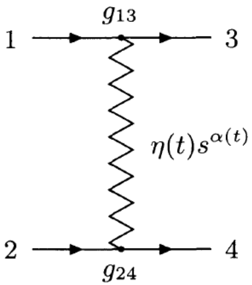
- this amplitude is useful only, if the pole is close to the physical region of the scattering process.
- In the high energy limit (or Regge limit), a coherent exchange of many particles of varying spin contributes (the so-called Regge trajectory). In effect, the spin becomes "running" with t: $J \to \alpha(t)$.

Reggeon exchange amplitude

$$A^{\pm}(s,t) = g_{13}(t) g_{24}(t) \left(\frac{s}{s_0}\right)^{\alpha(t)} \eta^{\pm}(\alpha(t))\,.$$

- Reggeons have a new quantum number: **signature** (it tells us about $s \leftrightarrow u$ crossing)
- $$\eta^{\pm}(\alpha(t)) = -\frac{e^{-i\pi\alpha(t)} \pm 1}{\sin(\pi\alpha(t))} \Rightarrow \eta^{+} = i - \cot\frac{\pi\alpha(t)}{2},\ \eta^{-} = i + \tan\frac{\pi\alpha(t)}{2}$$

Reggeon exchange



Reggeon exchange amplitude

$$A^{\pm}(s,t) = g_{13}(t)g_{24}(t)\left(\frac{s}{s_0}\right)^{\alpha(t)} \eta^{\pm}(\alpha(t))\,.$$

Three salient features of Reggeon exchange:

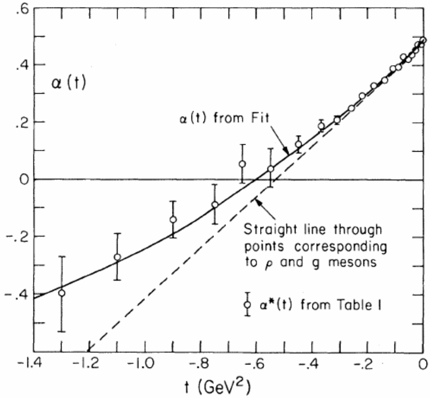
1. factorization of the "couplings" (residue function) and "propagator". Typically $g(t) \propto \exp(-B|t|/2)$
2. a t–dependent **phase** encoded in signature factor
3. the **Regge trajectory** $\alpha(t)$ determines energy dependence.

If we expand $\alpha(t) = \alpha(0) + \alpha'\,t$, we can write

$$A^{\pm}(s,t) = \left(\frac{s}{s_0}\right)^{\alpha(0)} \eta^{\pm}(\alpha(t)) \exp\left[-\frac{1}{2}\left(B_{13} + B_{24} + 2\alpha' \log(\frac{s}{s_0})\right)|t|\right],$$

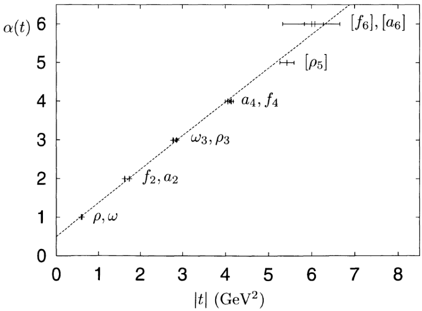
- The **intercept** $\alpha(0)$ controls the energy dependence.
- The **Regge slope** α' controls the **shrinkage of the forward cone**, which is a **prediction** of Regge pole exchange.

Regge trajectory of the ρ meson for $t < 0$



- Regge trajectory of the ρ meson from the charge exchange reaction $\pi^- p \to \pi^0 n$. (Fermilab 1976)
- Beam energy varies from 20 GeV to 200 GeV, the cross section drops over a factor ~ 15 over this energy range.
- deviations from linearity at larger $-t$.

Regge trajectories



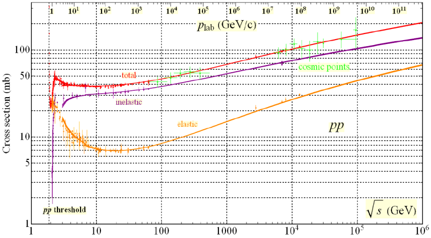
$$
\begin{aligned}
f_2 &: \quad P = +1,\ C = +1,\ G = +1,\ I = 0,\ \xi = +1\ , \\
\rho &: \quad P = -1,\ C = -1,\ G = +1,\ I = 1,\ \xi = -1\ , \\
\omega &: \quad P = -1,\ C = -1,\ G = -1,\ I = 0,\ \xi = -1\ , \\
a_2 &: \quad P = +1,\ C = +1,\ G = -1,\ I = 1,\ \xi = +1\ .
\end{aligned}
$$

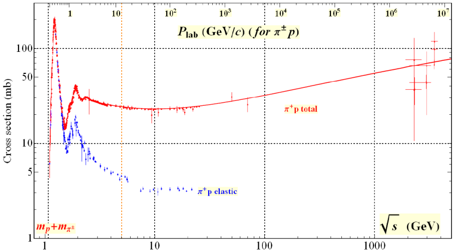
- As the Reggeon amplitude comes from an analytic continutation of t-channel amplitudes it has well defined quantum numbers, except of spin! We can assign Parity P, charge parity C, Isospin I, I_3 and G parity.
- In the **scattering region**, $t < 0$, and the Regge trajectory controls the phase, shrinkage, and energy dependence of the amplitude.
- There is a highly nontrivial link between these properties and the **spectrum** of the strongly interacting theory. Namely at $t > 0$, where we can interpret $t = M^2$, the Regge trajectories **pass through the spins of resonances with the appropriate quantum numbers**.

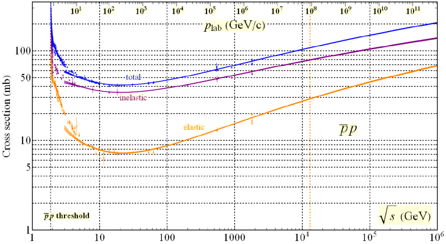
$$\alpha(M^2) = J$$

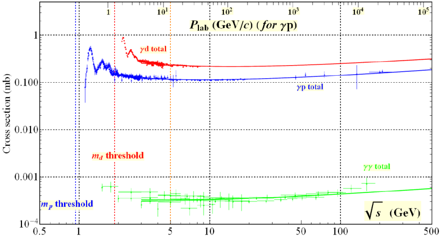
- Regge trajectories with $\alpha(0) = 0.5$ are called **secondary trajectories**. In practice there occurs a degeneracy of the f_2, ρ, ω, a_2 trajectories.
- **the pion** is a still subleading Regge trajectory with $\alpha(0) = 0$.

Total cross sections









The leading Regge trajectory – the Pomeron

$$I\!P: \quad P=+1,\ C=+1,\ G=+1,\ I=0,\ \xi=+1\,.$$

- By the optical theorem, we have for the total cross section:

$$\sigma_{\rm tot} \propto \frac{1}{s}\,\Im m\, A(s,0) \propto s^{\alpha(0)-1}$$

- The **secondary Regge trajectory** would give a total cross section **decreasing with energy:**

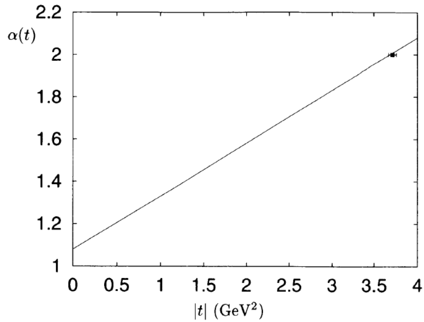
$$\sigma_{\rm tot} \propto 1/\sqrt{s}.$$

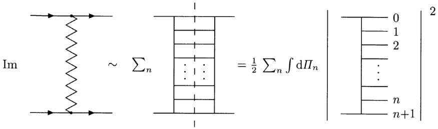
- Data suggest the presence of another trajectory, introduced by Chew and Frautschi (1961) and Gribov (1961). It was named the **Pomeron** by M. Gell-Mann after I.Ya.Pomeranchuk.
- In 1960's theorists favoured asymptotically constant cross sections, and the original Pomeron trajectory was supposed to have $\alpha(0)=1$.
- In practice, data are well described by a "supercritical" Pomeron with

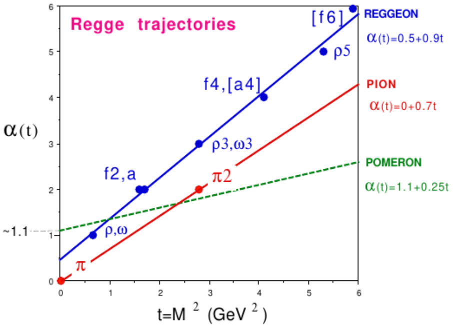
$$\alpha(t) = 1.08 + 0.25\,\mathrm{GeV}^{-2}\,t$$

- The Pomeron exchange amplitude is **predominantly imaginary**!.

Pomeron trajectory

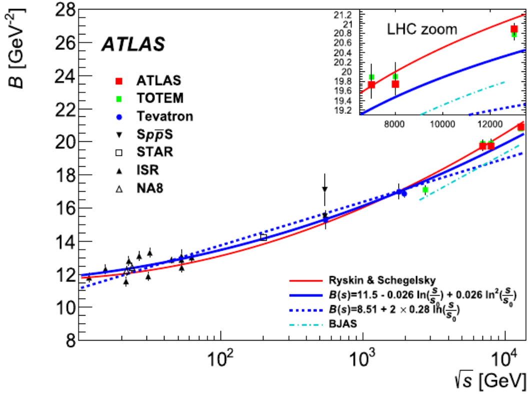


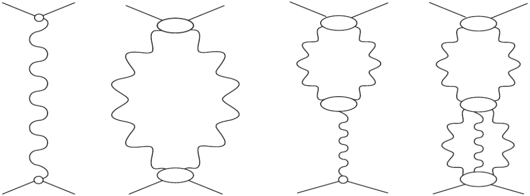




- In QCD, spin-1 gluon exchange naturally induces flat cross sections. As the Pomeron has to be **color singlet**, it is often modelled as a two-gluon exchange.
- Somewhat speculatively, the phenomenological Pomeron trajectory appears to include a $J = 2$ tensor glueball candidate. Some class of models treat the Pomeron as the Regge trajectory of such spin-2,4 etc. exchanges.

Limitations of the Regge pole picture

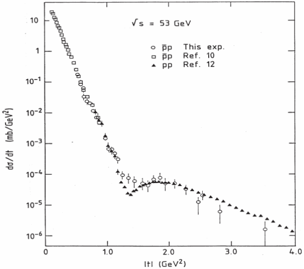


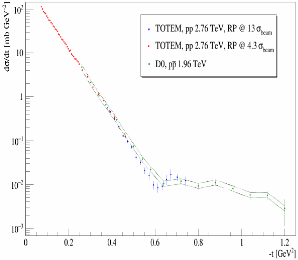


- Elastic slope dependence on energy deviates from the log(s) behaviour predicted for the Regge pole.
- The simple pole picture must be replaced by a Reggeon Field Theory. It takes into account the unitarity effect of diffractive channels on the total cross section/elastic amplitude.
- unfortunately there is no clearcut systematic expansion i.e. on the order of multipomeron vertices, but generally succesful phenomenological models include multipomeron effects in one way or another.

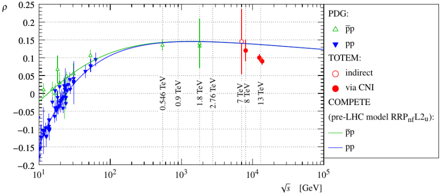
pp vs. $p\bar{p}$ – the Odderon

$$pp = \mathbb{P} + f_2 - \rho + a_2 - \omega, \qquad p\bar{p} = \mathbb{P} + f_2 + \rho + a_2 + \omega$$





- direct measurement of Odderon would require comparison of pp and $p\bar{p}$ at the same energy.
- Tevatron: $p\bar{p}$ at 1.96 TeV.
  LHC: pp at 2.76 TeV.

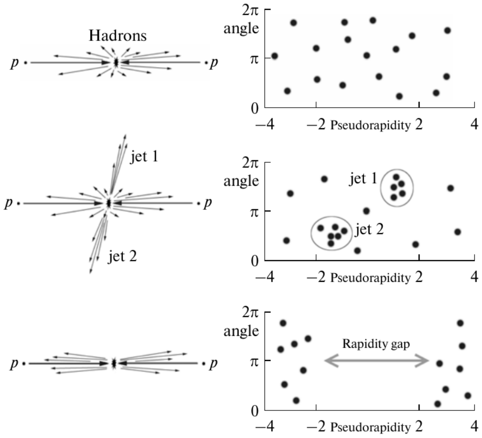


$$\rho(s) = \Re e A(s,0)/\Im m A(s,0)$$

- a nonvanishing difference between proton and antiproton scatering at high energies requires a new Regge trajectory - the **Odderon**.
- the Odderon has $C = -1$, **odd signature**, and the amplitude is predominantly real.
- in QCD can be thought of as **three gluon exchange** in totaly symmetric color singlet d_{abc}.

Topologies of inelastic interactions

- inelastic processes on the "cylinder phase space" of azimuthal angle ϕ and (pseudo-)rapidity.
- The typical **inelastic event** populates the cylinder phase space rather uniformly.
- Events with large rapidity gaps share many features with elastic scattering. In particular they also feature the forward cone. We call these events **diffractive** or diffractive dissociation of the beam particle(s).
- Notice, that the dominant diffractive channel is single diffraction, where only one of the colliding particles dissociate.

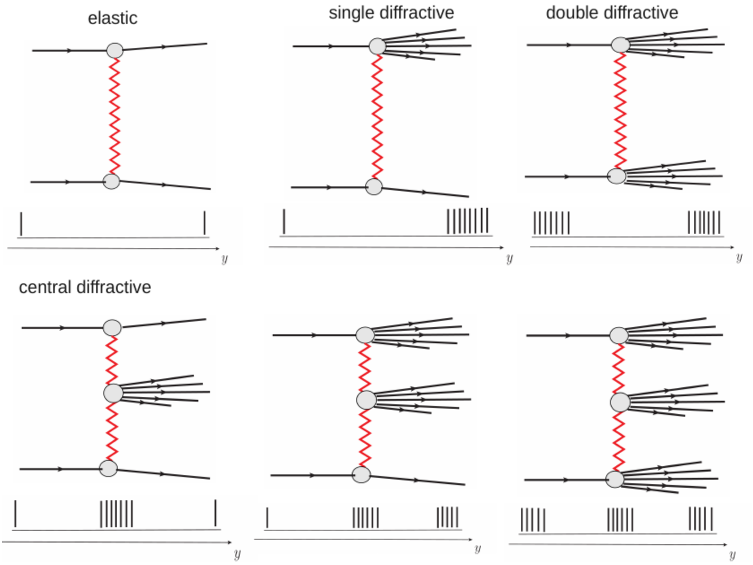


Rapidity y, pseudorapidity η

$$y = \frac{1}{2}\log\left(\frac{E+p_z}{E-p_z}\right), \quad \eta = \frac{1}{2}\log\left(\frac{|\vec{p}|+p_z}{|\vec{p}|-p_z}\right) = -\log\tan(\theta/2),$$

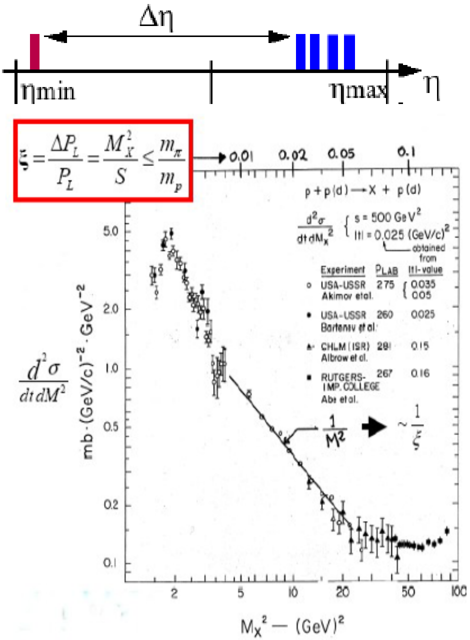
$$(E, p_x, p_y, p_z) = (\sqrt{p_\perp^2 + m^2}\cosh y, p_\perp\cos\phi, p_\perp\sin\phi, \sqrt{p_\perp^2 + m^2}\sinh y)$$

Diffractive event topologies



- This is a simplified picture. In fact also here we expect multipomeron effects which are related to so-called **gap survival probabilities.**

Diffractive dissociation



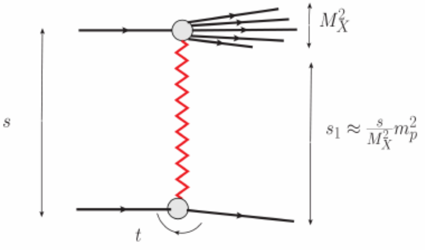
$$\xi = \frac{\Delta P_L}{P_L} = \frac{M_X^2}{S} \leq \frac{m_\pi}{m_p}$$

$$\frac{d^2\sigma}{dt\,dM^2}$$

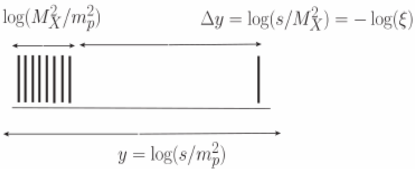
$$\frac{1}{M^2} \rightarrow \sim \frac{1}{\xi}$$

- Above $p_{\rm lab} \gtrsim 5GeV$ diffractive transitions become important. Proton loses small fraction $\xi = M_X^2/s$ of its momentum, **large rapidity gap** between outgoing proton & diffractive system.
- Shares many properties with elastic scattering: sharp forward peak, dominance of vacuum quantum number exchange, imaginary amplitude ...
- Another optical analogy: different absorption strength for different components of the beam's wavefunction imply the presence of diffraction dissociation (Glauber, Akhiezer & Sitenko, Landau & Pomeranchuk)

Inelastic diffraction: kinematics & t-channel exchanges



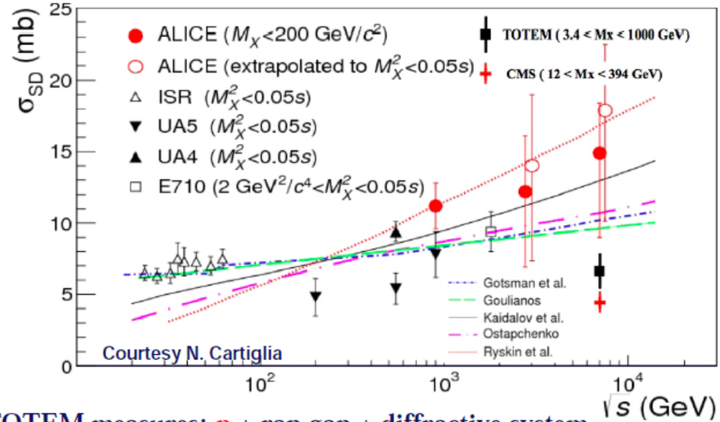
$$\frac{\Delta p_L}{p_L} \equiv \xi \sim \frac{M_X^2}{s} \ll 1$$



$$A(s_1, M_X^2, t) \propto \left(\frac{s_1}{m_p^2}\right)^{\alpha(t)} = \left(\frac{s}{M_X^2}\right)^{\alpha(t)} = \left(\frac{1}{\xi}\right)^{\alpha(t)} \Rightarrow \sigma \propto \exp[2(\alpha(0)-1)\cdot \Delta y]$$

- **To bridge a large gap, say $\Delta y \gtrsim 3$,** we need $\alpha(0) \geq 1$. Pomeron (C=+1), Odderon (?) (C $= -1$).
- Exchange of secondary Reggeons $\alpha(0) \sim 0.5$ for $\rho, \omega, f_2 a_1$, $\alpha(0) = 0$ for pions, **dies out exponentially at large gap size!**.
- NB: **Photons** ($J = 1$, $C = -1$) also qualify! In elastic scattering, the interference between Coulomb and strong amplitude gives access to the **phase** of the strong amplitude. The interference is concentrated at very small $|t|$.

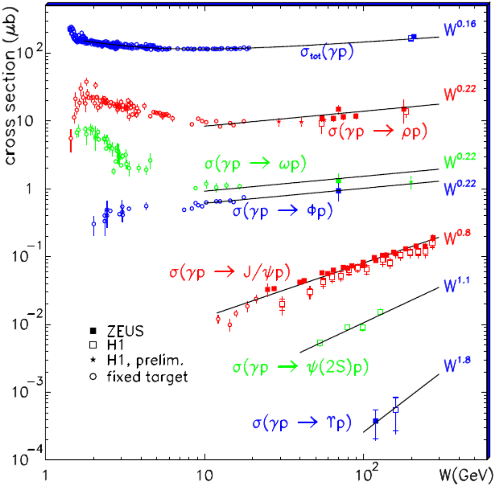
Diffractive dissociation

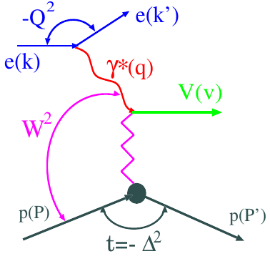


TOTEM measures: p + rap gap + diffractive system,
ALICE & CMS: "rap gap + diffractive system"

- Note: $M_X^2 < 0.05 \cdot s \Rightarrow$ gap size $\Delta y > 3$.
- Single diffractive cross section is an important observable for Regge models of soft inelastic interactions.

Vector meson photoproduction, soft to hard diffraction





- **Process dependent Pomeron!?** Steep energy dependence for for heavy quark mesons. Some authors postulate separate soft and hard Pomerons.

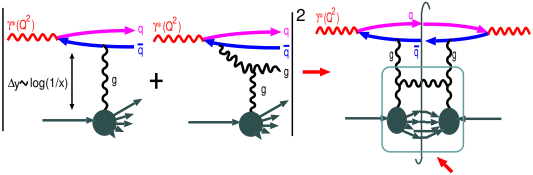
Color dipoles as diffraction scattering eigenstates

- An approach that allows the simultaneous inclusion of inelastic and diffractive channels is the **color dipole approach**. In the limit of large photon energy ω, a coherence length $l_c = 2\omega/M_{q\bar{q}}$ becomes much larger then the size of the target.
- We describe the photoabsorption process photon splits into a $q\bar{q}$ pair a long distance upstream the target.

$$\sigma(\gamma^* p) = \int dz d^2\boldsymbol{r} |\psi_{q\bar{q}}(z, \boldsymbol{r})|^2 \sigma(x, \boldsymbol{r}) = \langle \hat{\sigma} \rangle$$

total photoabsorption cross section

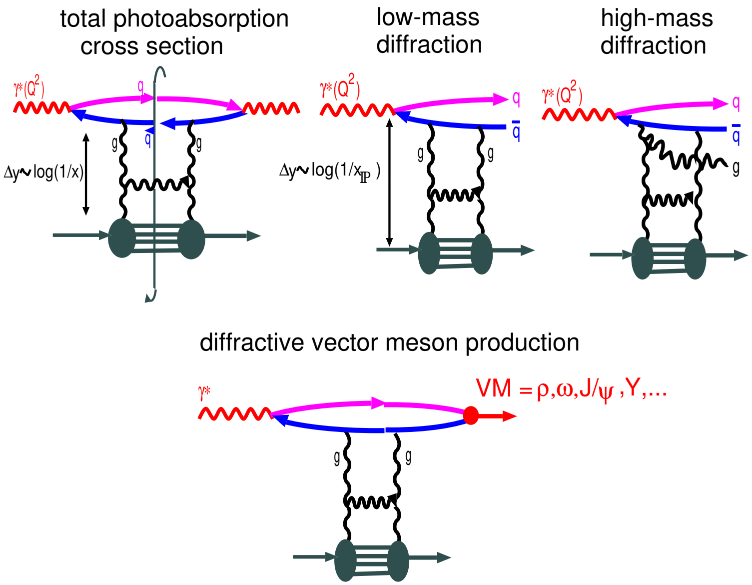
sum over final states → imaginary part of forward amplitude



gluon density

$$\sigma(x, \boldsymbol{r}) \quad = \quad \frac{4\pi\alpha_S}{N_c} \int \frac{d^2\boldsymbol{\kappa}}{\boldsymbol{\kappa}^4} \frac{\partial xg(x, \boldsymbol{\kappa}^2)}{\partial \log(\boldsymbol{\kappa}^2)} \left(1 - e^{i\boldsymbol{\kappa}\cdot\boldsymbol{r}}\right) \approx \boldsymbol{r}^2 \frac{\pi^2\alpha_S}{N_c} xg(x, \frac{1}{\boldsymbol{r}^2})$$

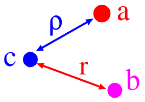
Color dipoles as diffraction scattering eigenstates



- On one footing, one can access total cross section, inclusive diffraction, as well as exclusive diffraction.

Color dipoles as diffraction scattering eigenstates

- Besides the dipole cross section also the cross section for the $q\bar{q}g$ state is important (a = g, b = q, c = $\bar{q}$):



$$\sigma_{q\bar{q}g}(\boldsymbol{r},\boldsymbol{\rho}) \quad = \quad \frac{N_c^2}{N_c^2-1}[\sigma(\boldsymbol{\rho})+\sigma(\boldsymbol{\rho}+\boldsymbol{r})] - \frac{1}{N_c^2-1}\sigma(\boldsymbol{r})\,.$$

Cross sections in the color dipole approach:

$$\sigma_{\rm tot}(\gamma^* p) \quad = \quad \int_0^1 dz \int d^2\boldsymbol{r} |\psi_{q\bar{q}}(z,\boldsymbol{r})|^2 \sigma(x,\boldsymbol{r})$$

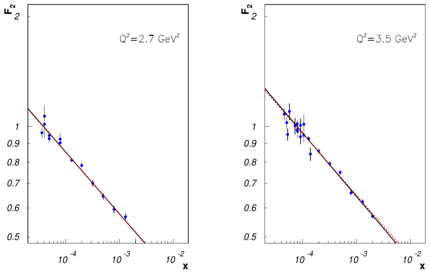
$$\frac{d\sigma(\gamma^* p \to q\bar{q}p)}{dt}\Big|_{t=0} \quad = \quad \frac{1}{16\pi}\int_0^1 dz \int d^2\boldsymbol{r} |\psi_{q\bar{q}}(z,\boldsymbol{r})|^2 \sigma^2(x,\boldsymbol{r})$$

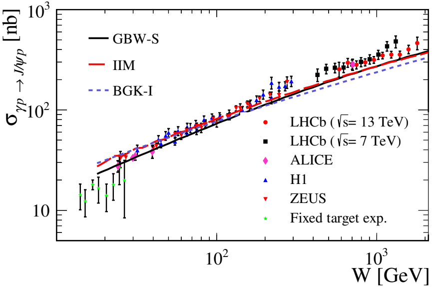
$$\frac{d\sigma(\gamma^* p \to q\bar{q}gp)}{dt}\Big|_{t=0} \quad = \quad \frac{1}{16\pi}\int_0^1 dz \frac{dz_g}{z_g} d^2\boldsymbol{r} d^2\boldsymbol{\rho} z_g |\psi_{q\bar{q}g}(z,z_g,\boldsymbol{r},\boldsymbol{\rho})|^2 [\sigma^2_{q\bar{q}g}(\boldsymbol{r},\boldsymbol{\rho}) - \sigma^2(x,\boldsymbol{r})]$$

$$\frac{d\sigma(\gamma^* p \to Vp)}{dt}\Big|_{t=0} \quad = \quad \frac{1}{16\pi}\left| \int_0^1 dz d^2\boldsymbol{r} \psi^*_V(z,\boldsymbol{r}) \psi_{q\bar{q}}(z,\boldsymbol{r}) \sigma(x,\boldsymbol{r}) \right|^2$$

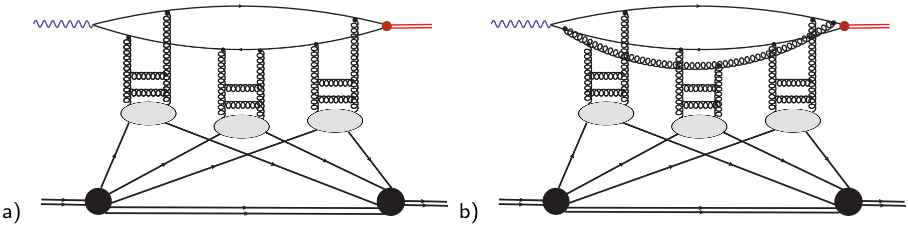
Dipole cross section

- We use dipole cross sections that have been fitted to precise HERA data on the proton structure function $F_2(x, Q^2)$. See for example the fit by Łuszczak and Kowalski.
- the dipole approach relates the **rise of** F_2 **at small** x with the **energy dependence of the VM photoproduction**.
- a good agreement with J/ψ diffractive data up to γp-cm energies of a few hundred GeV is obtained.





Diffractive photoproduction on nuclei



- rescattering of the $c\bar{c}$ dipole only can be obtained by a simple Glauber exponential

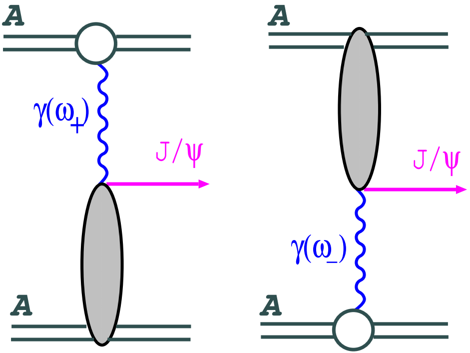
$$\Gamma_A(x, \boldsymbol{b}, \boldsymbol{r}) = 1 - S_A(x, \boldsymbol{b}, \boldsymbol{r}), \text{ with } S_A(x, \boldsymbol{b}, \boldsymbol{r}) = \exp\left[-\frac{1}{2}\sigma(x, \boldsymbol{r}) T_A(\boldsymbol{b}) \right].$$

- the rescattering of the $c\bar{c}$ pair corresponds to a resummation of higher-twist terms, **not** to a nuclear modification of the nuclear glue! The $c\bar{c}g$ yields a correction:

$$\Gamma_A(x, \boldsymbol{r}, \boldsymbol{b}) = \Gamma_A(x_A, \boldsymbol{r}, \boldsymbol{b}) + \log\left(\frac{x_A}{x}\right) \Delta\Gamma_A(x_A, \boldsymbol{r}, \boldsymbol{b})$$

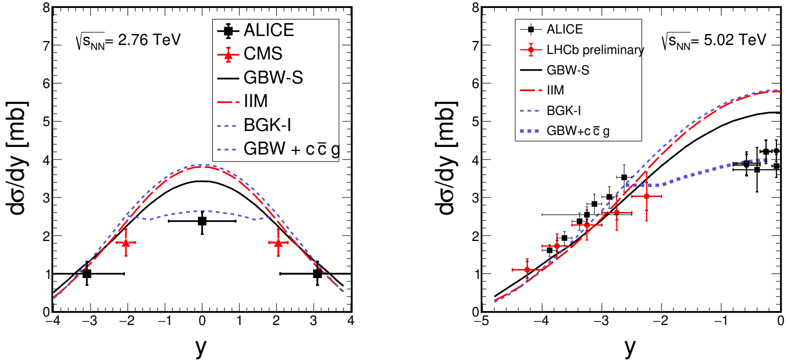
- the **nuclear glue** appears only, once the rescattering of $c\bar{c}g$ states are taken into account. But here the splitting amplitude $q \to qg$ in impact parameter space needs an infrared regularization (**gluon propagation radius** R_c). $x_A \sim 0.01$ also is a parameter of the model.

Diffractive photoproduction in ultraperipheral heavy ion collisions



- Heavy ions are accompanied by the large Weizsäcker-Williams flux of photons, which is enhanced by the charge squared of the ion Z^2.
- Each of the ions can be an emitter, so that there can appear subtle interference effects at very small P_T of the vector meson.
- At midrapidity both processes contribute with the same photon energy. In other cases there is a low-energy and high-energy contribution. The rather soft photon fluxes prefer the lower energy.

Ultraperipheral nuclear collisions



- Glauber-Gribov theory including only rescattering of the $c\bar{c}$ dipole works well in the forward region(large rapidities).
- In the central rapidity region inclusion of the $c\bar{c}g$ state introduces additional shadowing which is needed to describe the data. Strong dependence on R_c, a rather small $R_c \sim 0.21$ fm is preferred.
- Shadowing due to the $c\bar{c}g$ state can be (roughly) identified with gluon shadowing of the nuclear pdf. It depends on the infrared regulator. For J/ψ gluon shadowing is not a prediction of perturbation theory (hard scale $Q^2 \sim 2.25\,\mathrm{GeV}^2$). **Moderate gluon shadowing.**