PART III

30 min… more advanced

- **How well we know the CKM matrix ?**

- **what do we learn from this knowledge ?**

The quarks c b and t were discovered in an indirect way by using **rare decays**, and more precisely **FCNC**, such as

$K^0 \rightarrow \mu\mu$, $K_L \rightarrow \pi\pi$ and **B oscillations**

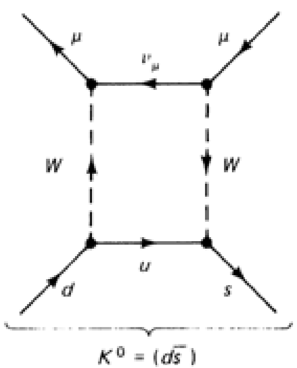
1. ~1970 charm quark from FCNC and GIM-mechanism $K^0 \rightarrow \mu\mu$
2. ~1973 3rd generation from CP violation in kaon (ε_K) KM-mechanism
3. ~1990 heavy top from B oscillations Δm_B

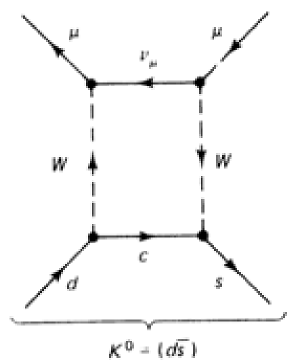
The Quantum path

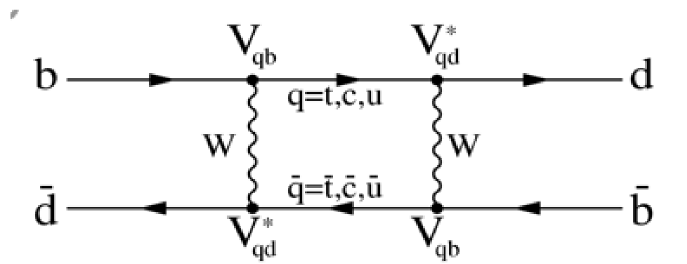
The indirect searches

look for "New Physics"

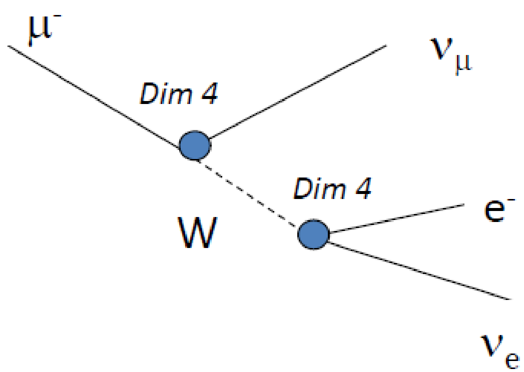
through virtual effects from new particles in loop corrections







SM



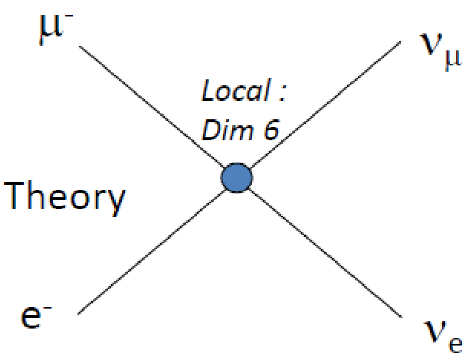
$$L_{CC} = \frac{g}{\sqrt{2}}\left(J^-_\mu W^\mu_- + J^+_\mu W^\mu_+\right)$$

$$M = \left(\frac{g}{\sqrt{2}}\bar{u}_{\nu_\mu}\gamma^\mu \frac{1}{2}(1-\gamma_5)u_\mu\right)\frac{1}{M_W^2 - q^2}\left(\frac{g}{\sqrt{2}}\bar{u}_e\gamma_\mu \frac{1}{2}(1-\gamma_5)u_{\nu_e}\right)$$

if $q^2 << M_W^2$ (case of beta decay)

$$M = \frac{g^2}{8M_W^2}\left(\bar{u}_{\nu_\mu}\gamma^\mu(1-\gamma_5)u_\mu\right)\left(\bar{u}_e\gamma_\mu(1-\gamma_5)u_{\nu_e}\right)$$

Fermi Effective Theory



$$M = \frac{G_F}{\sqrt{2}}\left(\bar{u}_{\nu_\mu}\gamma^\mu(1-\gamma_5)u_\mu\right)\left(\bar{u}_e\gamma_\mu(1-\gamma_5)u_{\nu_e}\right)$$

Experimentally from muon decay $G_F = 1.16\times 10^{-5}\,GeV^{-2}$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

The effective coupling constant GF is expressed as the SM « fundamental » coupling constant g divided by the mass of the propagator M_W squared (consequence of Dim=6 operators [4legs])

In this specific case we know

→ from SM $e = g\sin(\theta_W)$

→ Experimentally M_W~80 GeV

The weak interaction is not weak because of *g<<e* but because of the large value for the W mass

Effective Flavour Theory to New Flavour Physics :
A game of scale and coupling

$$\mathscr{L}_{eff} = \mathscr{L}_{SM} + \sum_{k=1} \left(\sum_i C_i^k \, Q_i^{(k+4)} \right) / \Lambda^k$$

New operators which are of dimension >4, in principle the theory is not Renormalizable...as Fermi theory was not..!
[You can show that in B physics the new operators have dimension 6]

NP flavour effects are governed by two players

→ the value of the new phyiscs scale Λ

→ the effective flavour-violating coupling C's

In explicit models

Λ ~ mass of virtual particle (Fermi theory : M_W)

C ~loop coupling × flavour coupling (SM/MFV $\alpha_w \times$ CKM)

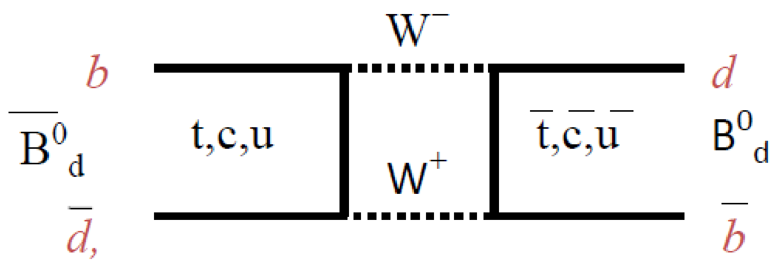
Example for B oscillations (FCNC-$\Delta B=2$)

FCNC porcesses are ideal place to look for NP effects because they are suppressed in SM

Precise measurements are needed.

Effects goes $1/\Lambda^2$

SM

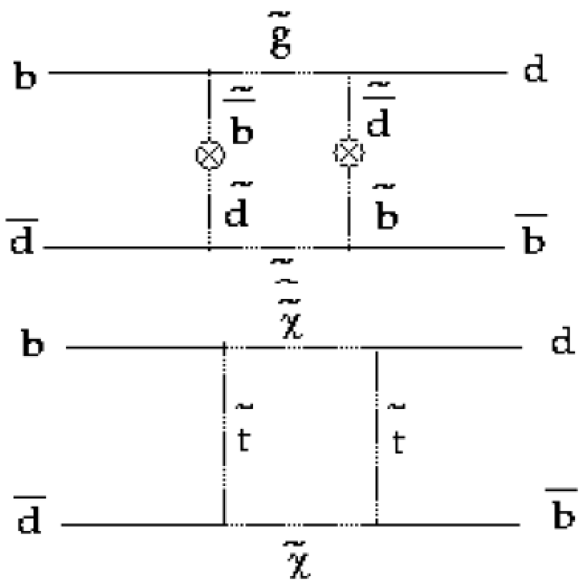


$$f(m)\ X\ \frac{\left|V_{tb}^{*}V_{tq}\right|^{2}}{M_W^2}$$

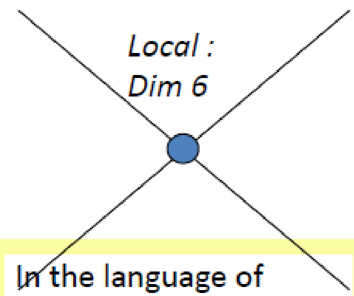
GIM mechanism

When I squared I have the m_t^2 at the numerator and M_W^4 at denominator. IMPORTANT at the end the dimension is $1/E^2$

BSM



$$\frac{\left|\delta_{bq}\right|^{2}}{\Lambda_{eff}^{2}}$$



In the language of The EFT, the operators have dimension 6 → $1/\Lambda^2$

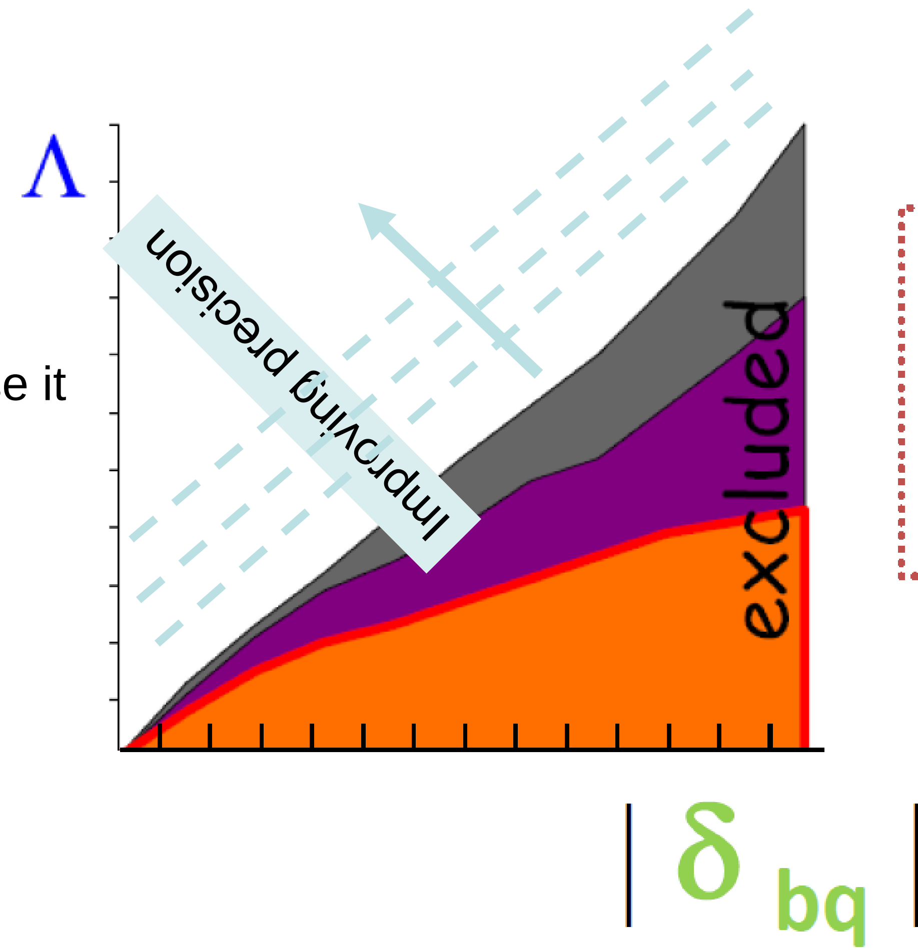
The measurements (in this case Δm_d)
are modified wrt the predictions of the SM by the presence of BSM particles.
modifications are important if couplings are larger and/or NP masses are lighter

Pictorially

Flavour Physics

It is a game of couplings and scales

This is the basic region because it is important to have precise measurements



$$\frac{|\delta_{bq}|}{\Lambda_{eff}^2}$$

The Unitarity Triangle

The CKM is unitary

$$V V^{\dagger} = V^{\dagger} V = 1$$

The non-diagonal elements of the matrix products correspond to 6 triangle equations

$$V_{ud}^{*} V_{us} + V_{cd}^{*} V_{cs} + V_{td}^{*} V_{ts} = 0 \qquad \lambda \ \lambda \ \lambda^{5}$$

$$V_{ub}^{*} V_{ud} + V_{cb}^{*} V_{cd} + V_{tb}^{*} V_{td} = 0 \qquad \lambda^{3} \ \lambda^{3} \ \lambda^{3}$$

$$V_{us}^{*} V_{ub} + V_{cs}^{*} V_{cb} + V_{ts}^{*} V_{tb} = 0 \qquad \lambda^{4} \ \lambda^{2} \ \lambda^{2}$$

$$V_{ud}^{*} V_{td} + V_{us}^{*} V_{ts} + V_{ub}^{*} V_{tb} = 0 \qquad \lambda^{3} \ \lambda^{3} \ \lambda^{3}$$

$$V_{td}^{*} V_{cd} + V_{ts}^{*} V_{cs} + V_{tb}^{*} V_{cb} = 0 \qquad \lambda^{4} \ \lambda^{2} \ \lambda^{2}$$

$$V_{ud}^{*} V_{cd} + V_{us}^{*} V_{cs} + V_{ub}^{*} V_{cb} = 0 \qquad \lambda \ \lambda \ \lambda^{5}$$

Remember that :

$$\begin{pmatrix} 1-\lambda^2/2-\lambda^4/8 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{A^2\lambda^5}{2}(1-2\rho)-i\,A^2\lambda^5\eta & 1-\lambda^2/2-\lambda^4(\frac{1}{8}+\frac{A^2}{2}) & A\lambda^2 \\ A\lambda^3(1-(1-\lambda^2/2)(\rho+i\eta)) & -A\lambda^2(1-\lambda^2/2)(1+\lambda^2(\rho+i\eta)) & 1-\frac{A^2\lambda^4}{2} \end{pmatrix}$$

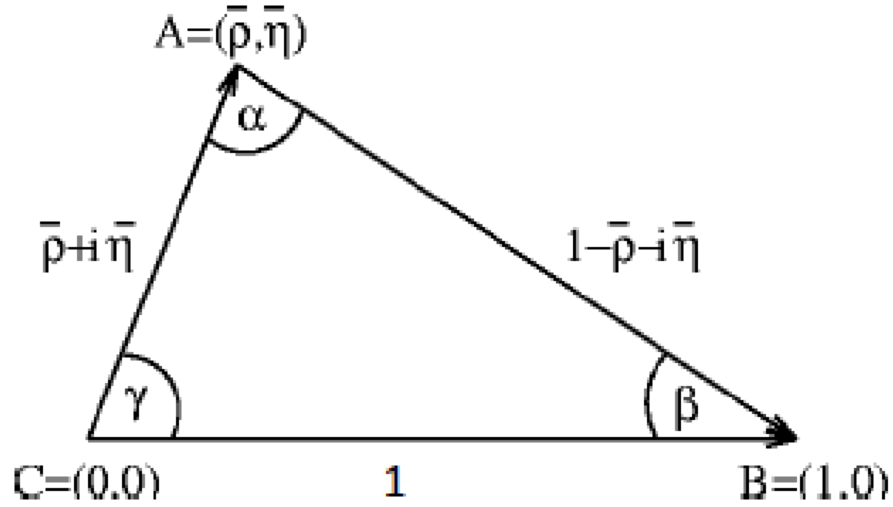
$$\overline{\rho}(\overline{\eta}) = (1-\lambda^2/2)\rho(\eta)$$

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

$$V_{ud} V_{ub}^* = A\lambda^3(\bar{\rho} + i\bar{\eta})$$

$$V_{cd} V_{cb}^* = -A\lambda^3$$

$$V_{td} V_{tb}^* = A\lambda^3(1 - \bar{\rho} - i\bar{\eta})$$



$$\overline{AB} = \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| \sim \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right|$$

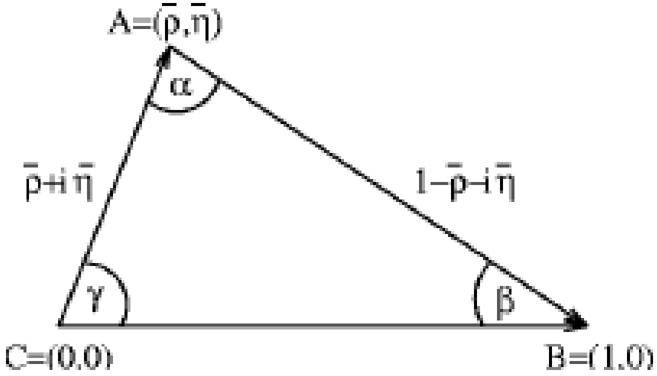
$$\overline{AC} = \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

Each of the angles of the unitarity triangle is the relative phase of two adjacent sides (a part for possible extra π and minus sign)

$$\beta = \arg\left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right) = \mathrm{atan}\left(\frac{\bar\eta}{(1-\bar\rho)}\right)$$

$$\gamma = \arg\left(\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \mathrm{atan}\left(\frac{\bar\eta}{\bar\rho}\right)$$

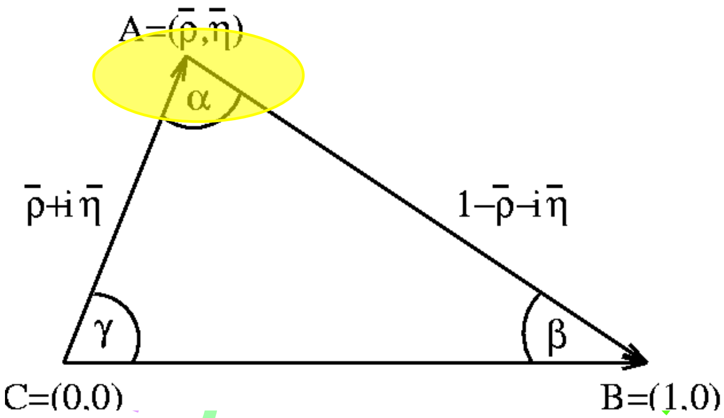
$$\alpha + \beta + \gamma = \pi$$



The reason of making the arg of the ratio of two legs is simple

$$x = |x|e^{i\vartheta}; y = |y|e^{i\chi} \qquad x/y = (|x|/|y|)e^{i(\vartheta-\chi)}$$

$\rightarrow \quad \arg(x/y) = (\vartheta - \chi)$ So the relative phase



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$$\alpha + \beta + \gamma = \pi$$

$$\overline{AB} = \left|\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{(1-\bar{\rho})^2+\bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| \sim \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right|$$

$$\overline{AC} = \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right| = \sqrt{\bar{\rho}^2+\bar{\eta}^2} = \left(1-\frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

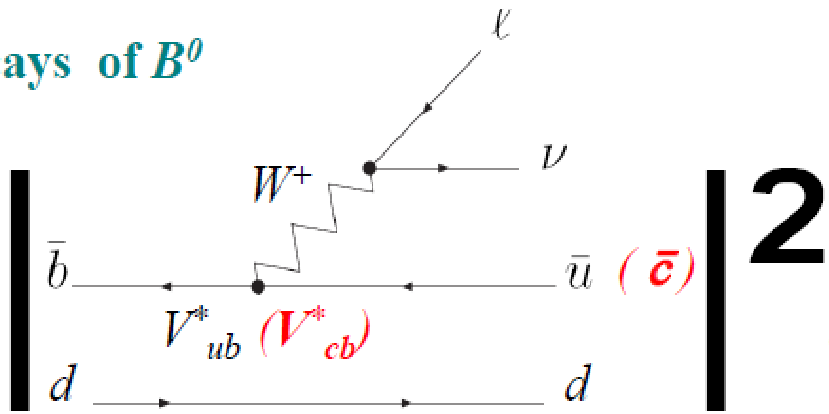
$$\beta = \arg\left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right) = \operatorname{atan}\left(\frac{\bar{\eta}}{(1-\bar{\rho})}\right)$$

$$\gamma = \arg\left(\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \operatorname{atan}\left(\frac{\bar{\eta}}{\bar{\rho}}\right)$$

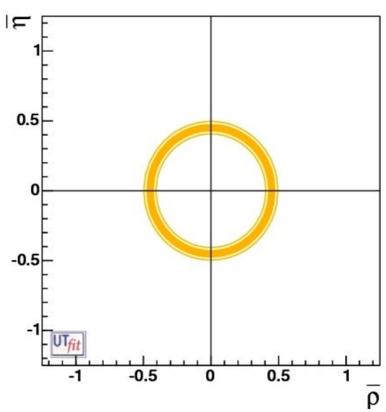
More explicitely with two examples :

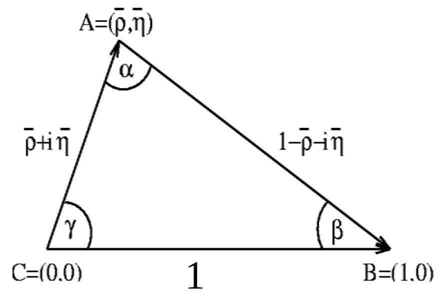
- **Rates of semileptonic decays of B^0**

Provide information on V_{ub} (V_{cb})



$\propto |V_{ub}|^2$ ($|V_{cb}|^2$)



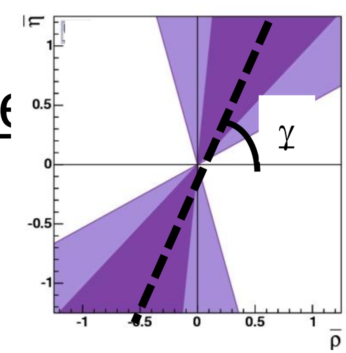


$$\overline{AC} = \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

Circle around (0,0) in the $\bar{\rho}-\bar{\eta}$ plane

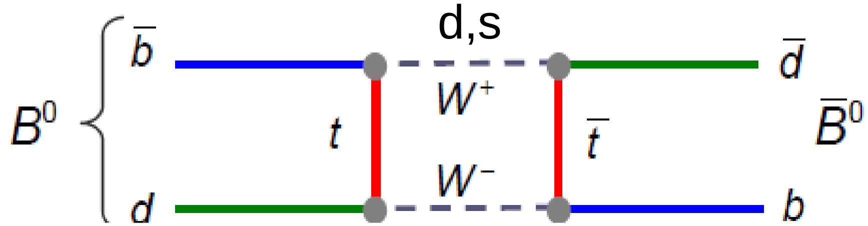
If we can access to the imaginary part of the ampliutude involving V_{ub} → access to γ angle

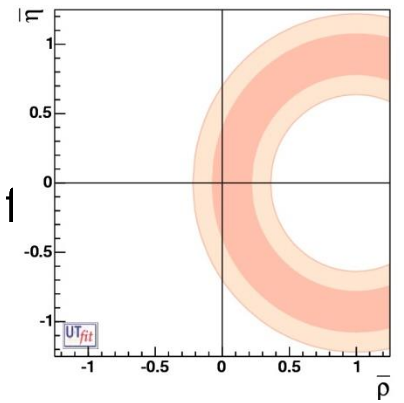
$$\gamma = \arg\left(\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \operatorname{atan}\left(\frac{\bar{\eta}}{\bar{\rho}}\right)$$



$B^0 \leftrightarrow \overline{B}^0$ Oscillations

$\sim (V_{td} V^*_{tb})^2$





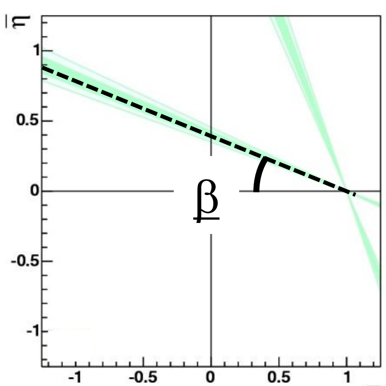
Oscillation frequency of $B_d \sim |V_{td}|^2$

$$\overline{AB} = \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| \sim \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right|$$

Circle around (1,0) in the $\bar{\rho}-\bar{\eta}$ plane

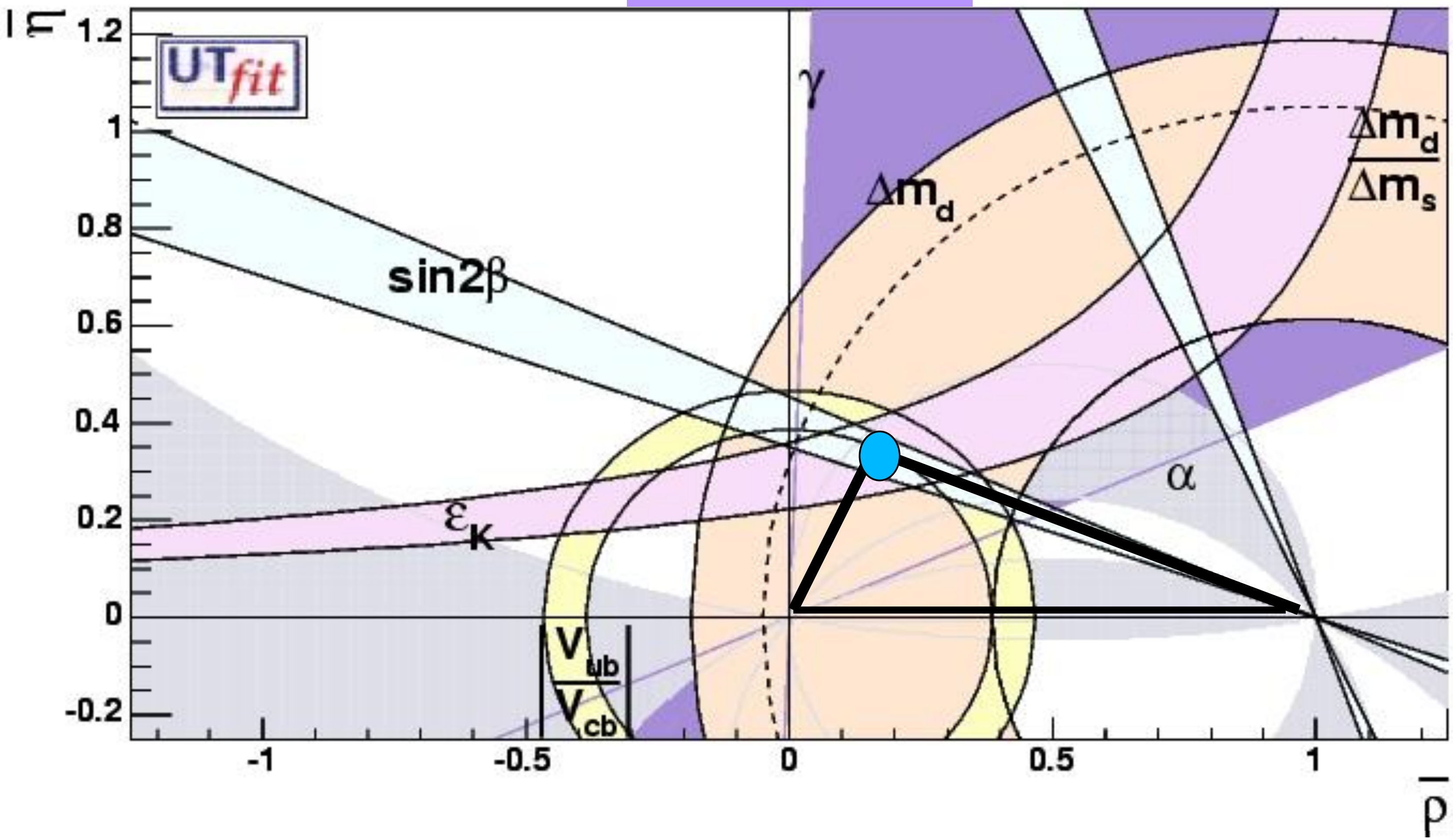
<u>If we can access to the imaginary part of the ampliutude involving V_{td} → access to β angle</u>

$$\beta = \arg\left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}\right) = \operatorname{atan}\left(\frac{\bar{\eta}}{(1-\bar{\rho})}\right)$$



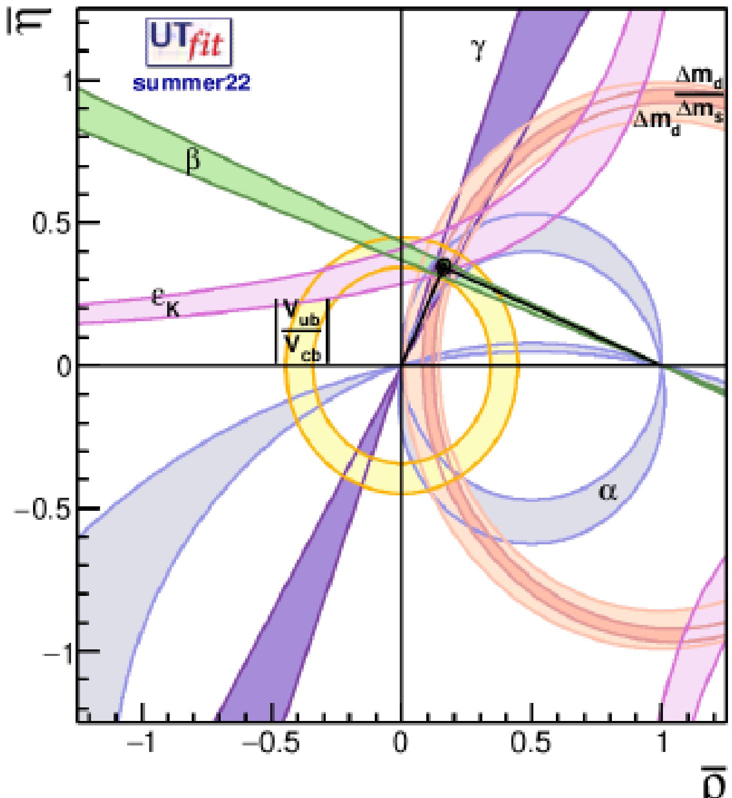
An example on how to fit the UT parameters and fit new physics

B→DK : ϕ_3/γ



Global Fit within the SM

All the constraints
Look compatibles !



Coherent picture of FCNC and CPV processes in SM

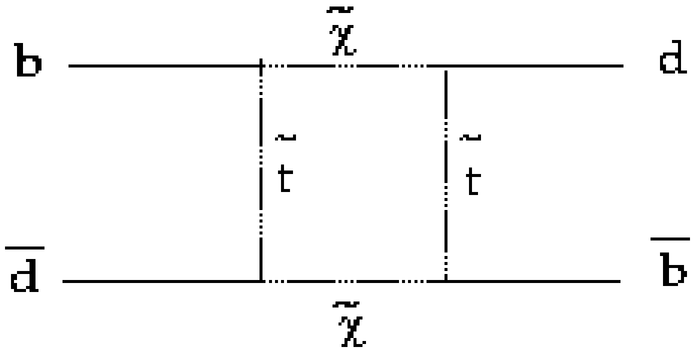
CKM matrix is the dominant source of flavour mixing and CP violation

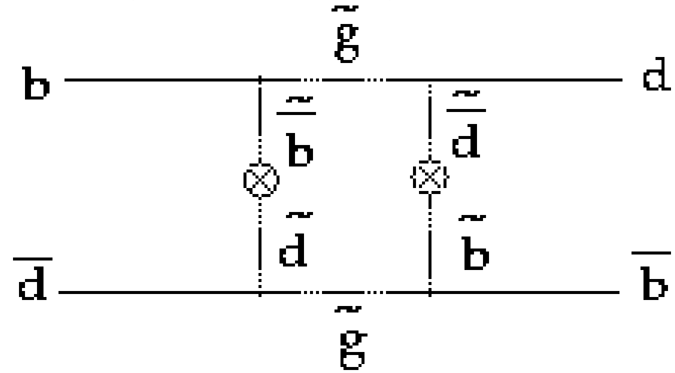
Out of these measurement there a general agreement that we have limited the contributions of New Physics amplitudes (A_{NP}) wrt to SM ones (A_{SM}) at the the level of

$$R = \frac{A_{NP}}{A_{SM}} < 20\%$$

What does it imply ?

Simplify the discussion with one example : B oscillations (FCNC-ΔB=2) :





$$\left|\frac{Q^{NP}_{\Delta B=2}}{Q^{SM}_{\Delta B=2}}\right| \leq r \;\rightarrow\; \frac{|\delta_{bq}|}{\Lambda_{eff}} \leq \sqrt{r}\,\frac{|V^{*}_{tb}V_{tq}|}{M_W}$$

r upper limit of the relative NP contribution

δ_{bd} NP physics coupling

Λ_{eff} NP scale (masses of new particles)

Minimal Flavour Violation

≡ no new sources of flavour and CP violation NP contributions governed by SM Yukawa couplings.

$$\delta_{q'd} \approx V^{*}_{tq'}V_{td}$$

(couplings small as CKM elements)

$\Lambda_{eff} \sim 0.08/\sqrt{r}$ TeV

If couplings ~ 1

$\delta_{bq} \sim 1$ $\Lambda_{eff} \sim 10/\sqrt{r}$ TeV

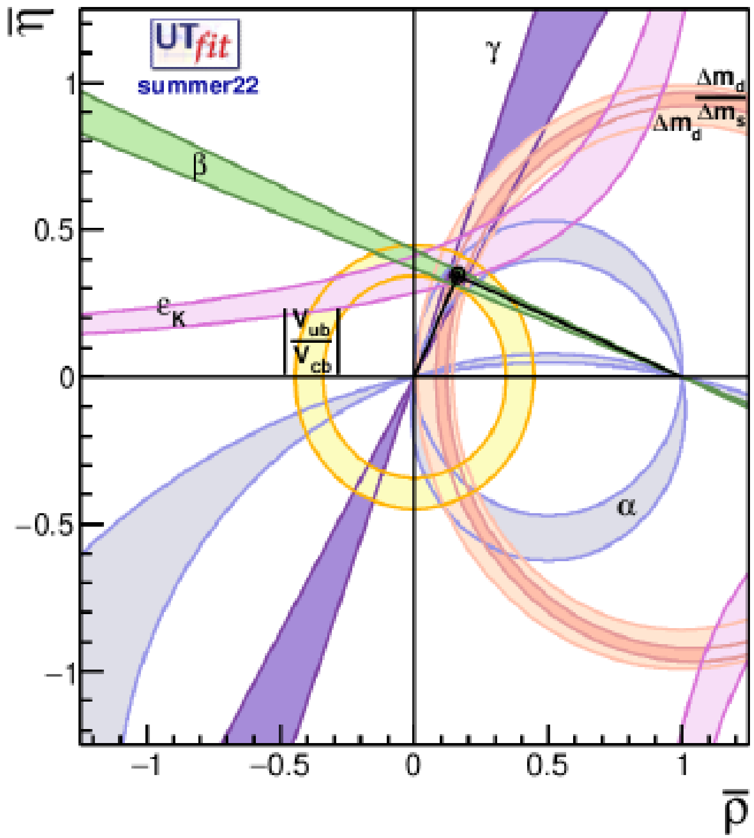
$\delta_{bs} \sim 1$ $\Lambda_{eff} \sim 2/\sqrt{r}$ TeV

Coupling δ	r=20% today
Order 1	$\Lambda_{eff} \sim 20$ TeV
MFV	$\Lambda_{eff} \sim 180$ GeV

r<0.2 → Λ_{eff} > 180 GeV . Particles below 180 GeV circulating in the loop would have given visible effects within the present level of precision

The test of the SM (in fermion sector)

1990-now → a huge number of precise measurements



Discovery : absence of New Particles up to the ~2×Electroweak Scale !

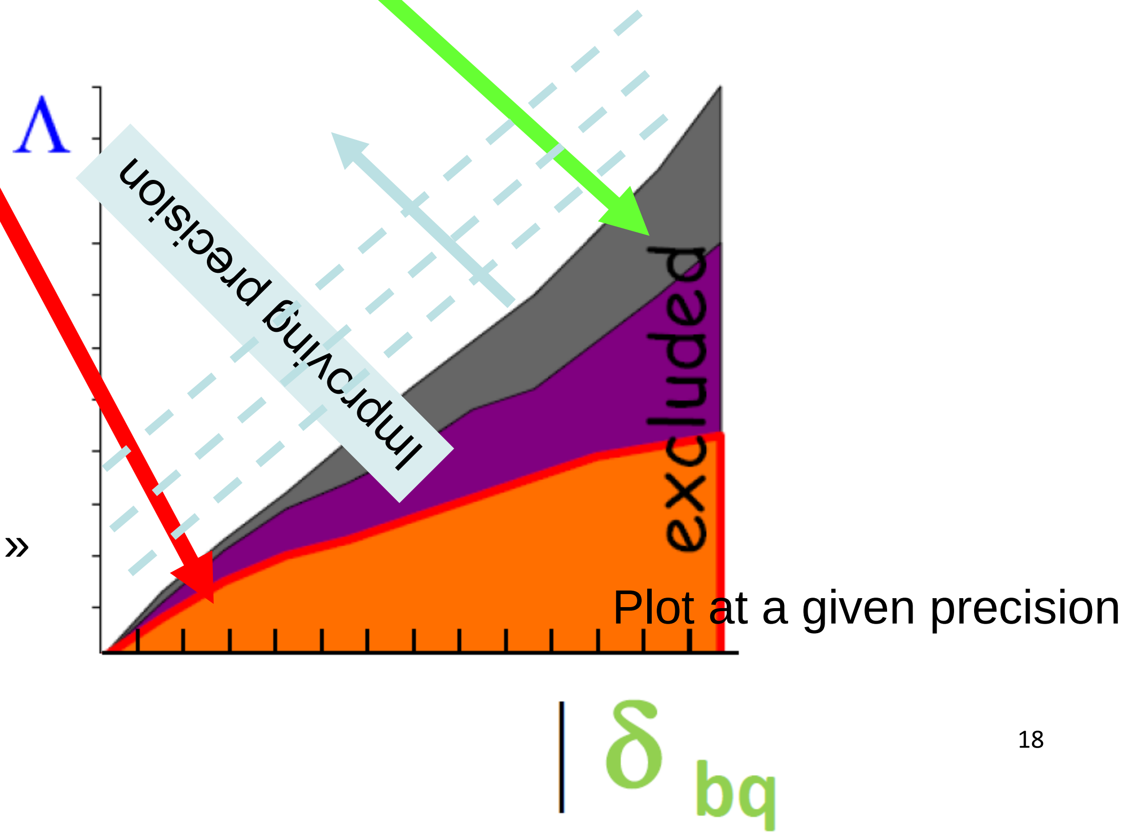
….and at much larger scale (>TeV) for some New physics model

1. ~1970 charm quark from FCNC and GIM-mechanism $K^0 \to \mu\mu$
2. ~1973 3rd generation from CP violation in kaon (ε_K) KM-mechanism
3. ~1990 heavy top from B oscillations Δm_B
4. >2010 success of the description of FCNC and CPV in SM

Coupling δ	r=20% today	r=10% tomorrow	r=1% after tomorrow
Order 1	Λ_{eff} ~ 20 TeV	Λ_{eff} ~ 30 TeV	Λ_{eff} ~ 100 TeV
MFV	Λ_{eff} ~ 180 GeV	Λ_{eff} ~ 250 GeV	Λ_{eff} ~ 800 GeV

FLAVOUR PHYSICS IS CENTRAL IN THE NP SEARCH

because ALLOW TO« DISCOVER » VERY HEAVY PARTICLE NOT ACCESSIBLE TO DIRECT SEARCH DEPENDING ON COUPLINGS

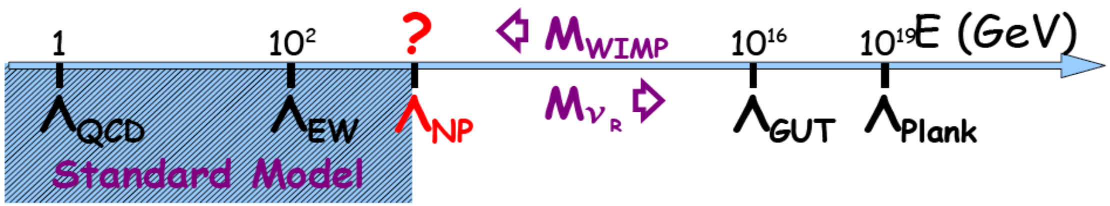


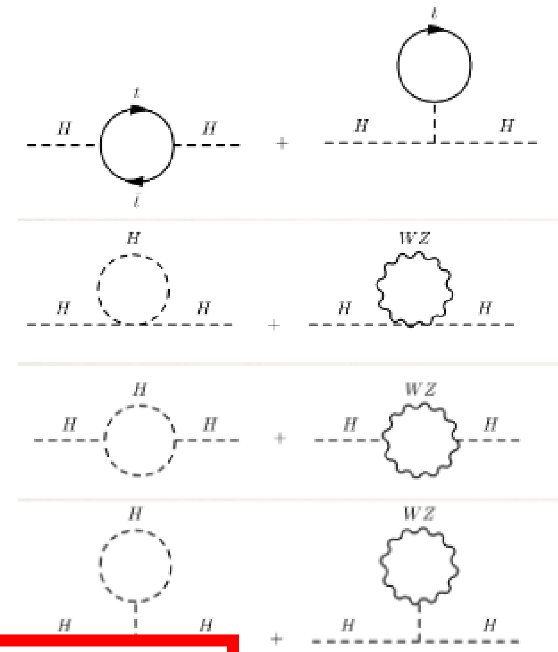
FLAVOUR PHYSICS IS AN EXPLORATORY PHYSICS !
IT ALLOWS TO EXPLORE SCALE FAR BEYOND THE REACH OF THE DIRECT SEARCH

SPAN FROM ELECTROWEAK SCALE TO SEVERAL TEV (up to 100TEV) !

Where do we expect the scale of New Physics??
This IS THE QUESTION !

The problem of particle physics today is : where is the NP scale $\Lambda \sim 0.5, 1\ldots10^{16}$ TeV





$$M_H^2(p^2) = M_H^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \ldots$$

$$m_H^2 \rightarrow m_{bare}^2 + \delta m_H^2$$

$$\delta M_H^2 = \frac{G_F \Lambda^2}{4\pi^2\sqrt{2}}(6\,M_W^2 + 3\,M_Z^2 + M_H^2 - 12\,m_t^2) \approx -\frac{3G_F}{\pi^2\sqrt{2}} m_t^2 \Lambda^2 \approx -0.075\,\Lambda^2.$$

The corrections to the **Higgs mass are quadratically divergent*** and Λ is the Energy cut-off to the divergent loop integrals. (*ΔM_H increseas depending of the choosed cut off Λ.)

hierarcy problem

Stabilizing the mass $M_H = 125$ GeV requires DELICATE BALANCE of TWO NUMBERS.

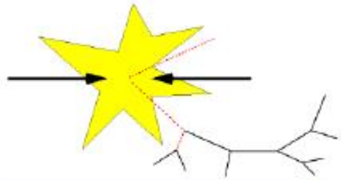
$\Lambda = 1$ TeV

$M_H^2(p^2) = (125\,\text{GeV})^2 = 1{,}56\,10^4\,\text{GeV}^2 = M_H^2(\Lambda^2) - 7.5\times10^4\,\text{GeV}^2$

$\Lambda = 10$ TeV . . . . $= M_H^2(\Lambda^2) - 7.5\,10^6\,\text{GeV}^2$

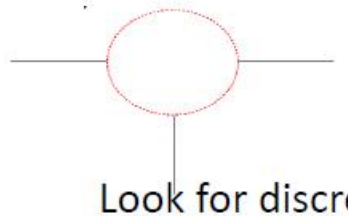
This delicate balance become more and more delicate as much as Λ increase ... up to the Planck scale

…Indeed historically we have always followed the two ways…



“Relativistic path”

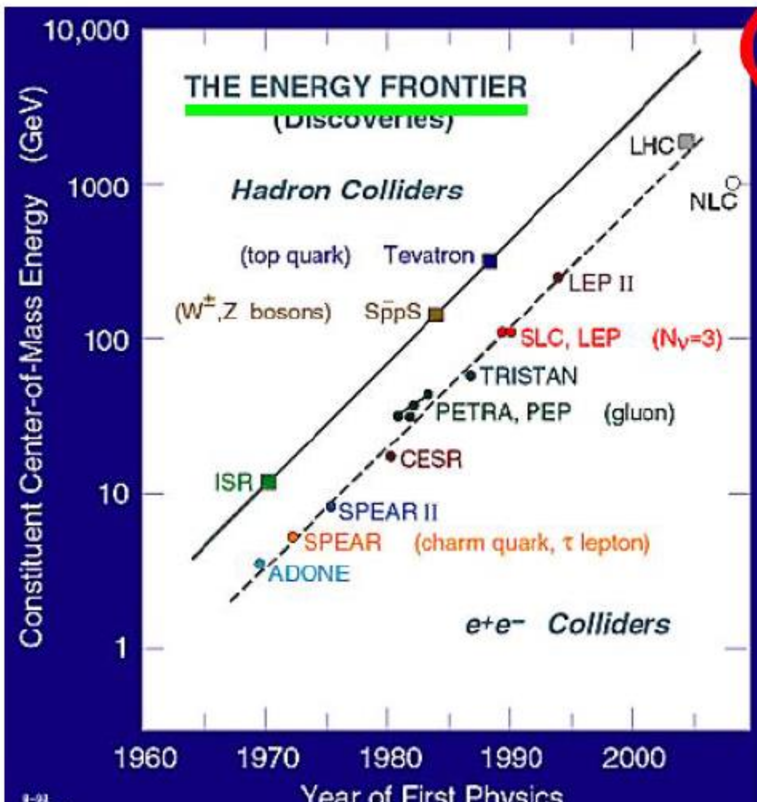
Crucial : Center-of-mass energy

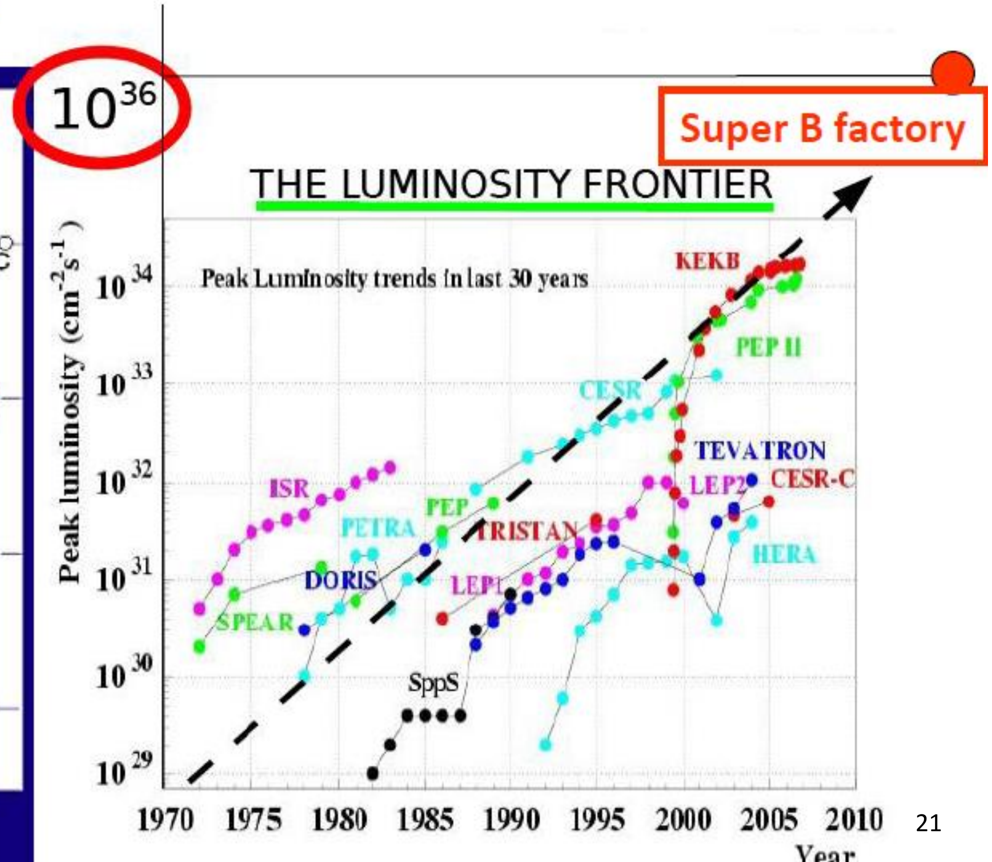


“Quantum path”

Crucial : Luminosity

Look for discrepancy wrt SM on many different measurements





дуже дякую

bardzo dziękuję

Muito obrigado

merci beaucoup

დიდი მადლობ

thank you very much

Grazie mille

شكرا جزيلا

Vielen Dank!

muchas gracias

बहुत - बहुत धन्यवाद

for your attention and your questions !!