APPENDIX I

The Standard Model

in the

fermion sector

CKM matrix and CP Violation.
The Unitarity Triangle

Flavour Physics in the *Standard Model* (SM) in the quark sector:

10 free parameters

~ half of the *Standard Model*

6 quarks masses

4 CKM parameters

In the Standard Model, charged weak interactions among quarks are codified in a 3 X 3 unitarity matrix : the **CKM Matrix**.

The existence of this matrix conveys the fact that the quarks which participate to weak processes are a linear combination of mass eigenstates

The fermion sector is poorly constrained by SM + Higgs Mechanism

mass hierarchy and CKM parameters

The Standard Model is based on the following gauge symmetry

$$SU(2)_L \times U(1)_Y$$

Weak Isospin (symbol L because only the LEFT states are involved)

Weak Hypercharge : (LEFT and RIGHT states)

			I	I_3	Q	Y
Leptons	doublet L	ν_e	½	½	0	-1
		e_L^-	½	-½	-1	-1
	singlet R	e_R^-	0	0	-1	-2
quarks	doublet L	u_L	½	½	2/3	1/3
		d_L	½	-½	-1/3	1/3
	singlet R	u_R	0	0	2/3	4/3
	singlet R	d_R	0	0	-1/3	-2/3

Idem for the other families

Short digression on the mass

$$E^2 = \vec{p}^{\,2} + m^2 \quad \rightarrow \quad \partial^\mu \partial_\mu + m^2 \phi = 0 \quad \leftrightarrow \quad L = \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 = 0$$

$$(i\gamma^\mu \partial_\mu - m) = 0 \quad \leftrightarrow \quad L = i\overline{\psi}\gamma_\mu \partial^\mu \psi - m\overline{\psi}\psi$$

$$m\overline{\psi}\psi = m\overline{\psi}(P_L + P_R)\psi = m\overline{\psi}(P_L P_L + P_R P_R)\psi =$$

$$= m[(\overline{\psi}P_L)(P_L\psi) + (\overline{\psi}P_R)(P_R\psi)]\psi \qquad = m\left(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R\right)$$

The mass should appear in a LEFT-RIGHT coupling

ψ_R : SU(2) singlet

ψ_L : SU(2) doublet

The mass terms are not gauge invariant under

$SU(2)_L \times U(1)_Y$

ψ_R (I=0,Y=-2) leptoni$_R$

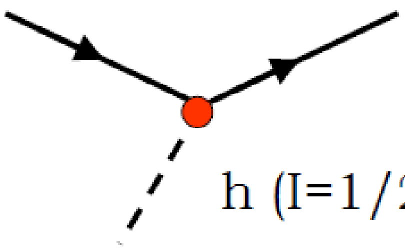
(I=0,Y=-2/3) quark d_R

(I=0,Y=4/3) quark u_R

ψ_L (I=1,Y=-1) leptoni$_L$

(I=1,Y=1/3) quark d_L

(I=1,Y=1/3) quark u_L



h (I=1/2,Y=1)

Adding a doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad I = \frac{1}{2} \quad Y = 1$$

Yukawa interaction : $\overline{\psi}_L \phi \psi_R$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$g_e(\overline{\psi}_L \phi \psi_R + \phi^+ \overline{\psi}_R \psi_L)$$

(le deuxieme terme est l'hermitien conjuge du premier)

After SSB

$$\frac{g_e v}{\sqrt{2}}(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L) + \frac{g_e}{\sqrt{2}}(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L)H$$

$$m_e = \frac{g_e v}{\sqrt{2}}$$

v/sqrt(2) ~natural mass (g~1)

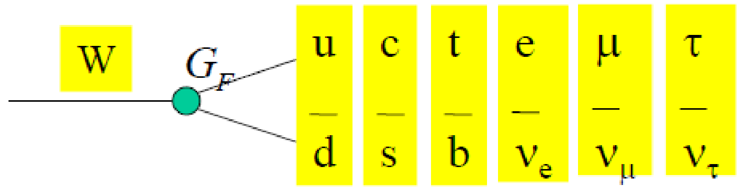
$$g_e = \frac{\sqrt{2} m_e}{v}$$

$$m_e \overline{e}e + \frac{m_e}{v} \overline{e}eH$$

$$\frac{g_e}{\sqrt{2}} = \frac{m_e}{v}$$ couplage Hee

$$L_W = \frac{g}{2}\overline{Q}^{Int.}_{L_i}\gamma^\mu\sigma^a Q^{Int.}_{L_i} W^a_\mu \qquad a = 1,2,3 \qquad Q^{Int.}_{L_i} = \begin{pmatrix} u_{L_i} \\ d_{L_i} \end{pmatrix} \quad L^{Int.}_{L_i} = \begin{pmatrix} \nu_{L_i} \\ l_{L_i} \end{pmatrix}$$

$$\overline{Q}^{Int.}_{L_i} Q^{Int.}_{L_i} = \overline{Q}^{Int.}_{L_i} 1_{ij} Q^{Int.}_{L_j} \quad \text{universality of gauge interactions}$$



The SM quantum numbers are I_3 and Y

→ The gauge interactions are

Flavour blind

In this basis the Yukawa interactions has the following form :

$$L_Y = Y^d_{ij}\overline{Q}^{Int.}_{L_i}\phi d^{Int.}_{R_j} + Y^u_{ij}\overline{Q}^{Int.}_{L_i}\phi u^{Int.}_{R_j} + Y^l_{ij}\overline{L}^{Int.}_{L_i}\phi l^{Int.}_{R_j}$$

$$SSB^* \rightarrow <\phi^0> = v/\sqrt{2}; \mathrm{Re}(\phi^0) \rightarrow (v + H^0)/\sqrt{2}$$

With: $\tilde{\phi} = i\sigma_2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^*$

To be manifestly invariant under SU(2)

Y_{ij} complex

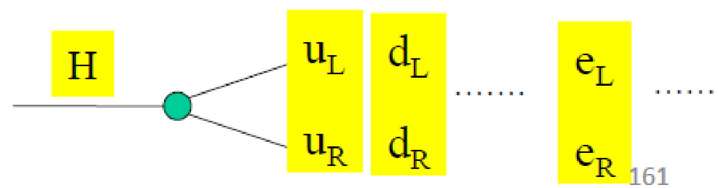
Two matrices are needed to give a mass term to the **u-type** and **d-type quarks**

....

$$L_M = M^d_{ij}\overline{d}^{Int.}_{L_j} d^{Int.}_{R_j} + M^u_{ij}\overline{u}^{Int.}_{L_j} u^{Int.}_{R_j} + M^l_{ij}\overline{l}^{Int.}_{L_j} l^{Int.}_{R_j}$$

We made the choice of having the Mass Interaction diagonal

where $M^f = (v/\sqrt{2})Y^f$



* SSB=Spontaneous Symmetry Breaking

To have mass matrices diagonal and real, we have defined: $M^{f}_{(diag)} = V_L^{f} M^{f} V_R^{f}$

The mass eigenstates are:

$$d_{L_i} = (V_L^d)_{ij}\, d_{L_j}^{Int.} \quad ; \quad d_{R_i} = (V_R^d)_{ij}\, d_{R_j}^{Int.}$$

$$u_{L_i} = (V_L^u)_{ij}\, u_{L_j}^{Int.} \quad ; \quad u_{R_i} = (V_R^u)_{ij}\, u_{R_j}^{Int.}$$

$$l_{L_i} = (V_L^d)_{ij}\, l_{L_j}^{Int.} \quad ; \quad l_{R_i} = (V_R^d)_{ij}\, l_{R_j}^{Int.}$$

$$\nu_{L_i} = (V_L^l)_{ij}\, \nu_{L_j}^{Int.}$$

ν_{L_i} arbitrary (assuming ν massless)

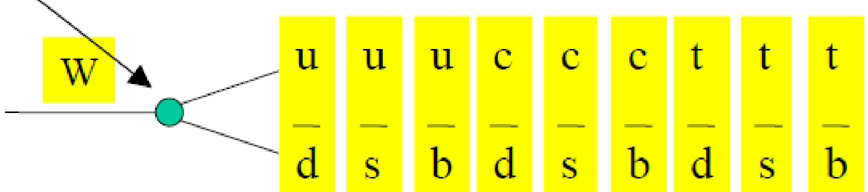
In this basis the Lagrangian for the gauge interaction is:

$$L_W = \frac{g}{2} \bar{u}_{L_i} \gamma^{\mu} (V_L^u V_L^{d\dagger}) d_{L_j} W_{\mu}^{a} + h.c.$$

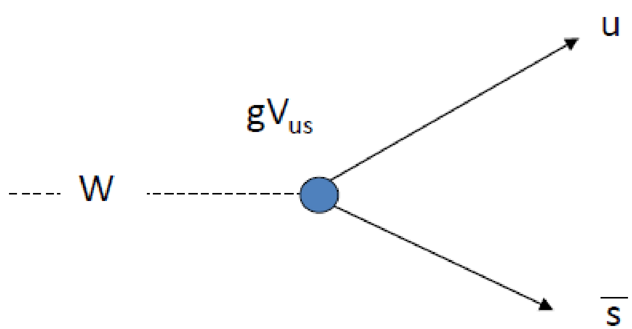
The coupling is not anymore universal

$$V(CKM) = (V_L^u V_L^{d\dagger})$$

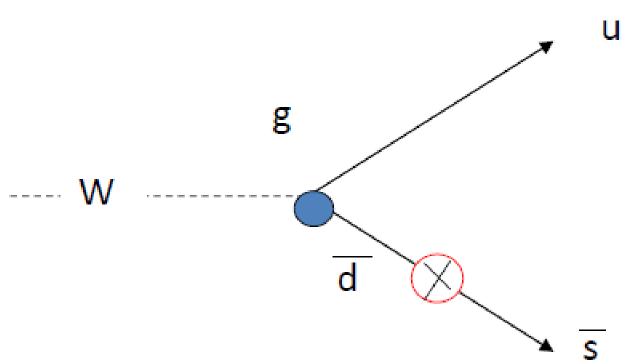
Unitary matrix



Two different way of seeing the charged interactions among quarks

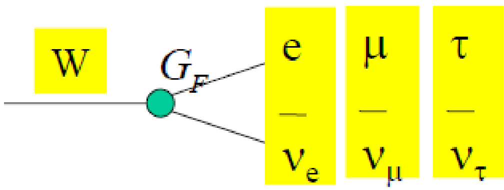


In the basis where :
the masses are real
and diagonal



In the basis where :
charged interactions are just
between members of the same family
and CKM is diagonal

If a similar procedure is applied to the lepton sector



Since the neutrino are (were) massless the matrix which change the basis from int-> mass is in principle arbitary
We can always choose $V_L^\nu = V_L^l$

Now the neutrino have a mass, it exists a similar matrix in the lepton sector with mixing a CP violation

For the Z^0

Facultatif

$$L_W = \frac{g}{2}\bar{Q}^{Int.}_{L_i}\gamma^\mu\sigma^a Q^{Int.}_{L_i} W^a_\mu \qquad a = 1,2,3$$

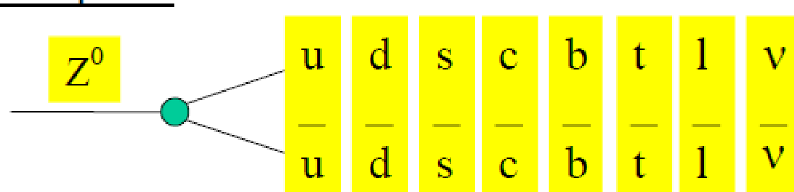
$$-L_B = g'\left[\frac{1}{6}\bar{Q}^{Int.}_{L_i}\gamma^\mu 1_{ij} Q^{Int.}_{L_j} + \frac{2}{3}\bar{u}^{Int.}_{R_i}\gamma^\mu 1_{ij} u^{Int.}_{R_j} - \frac{1}{3}\bar{d}^{Int.}_{R_i}\gamma^\mu 1_{ij} d^{Int.}_{R_j}\right]B_\mu$$

for the Z^0 $\quad Z^\mu = \cos\vartheta_W W_3^\mu - \sin\vartheta_W B^\mu \;;\; \tan\vartheta_W = g'/g$

in the mass basis (example for d_L)

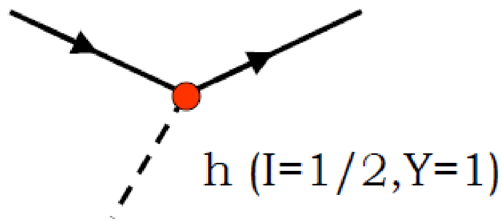
$$-L_Z = \frac{g}{\cos\vartheta_W}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\vartheta_W\right)\bar{d}_{L_i}\gamma^\mu (V^\dagger_{dL} V_{dL}) d_{L_i} Z_\mu = \frac{g}{\cos\vartheta_W}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\vartheta_W\right)\bar{d}_{L_i}\gamma^\mu d_{L_i} Z_\mu$$

The neutral currents stay universal, in the mass basis :
<u>we do not need extra parameters for their complete description</u>

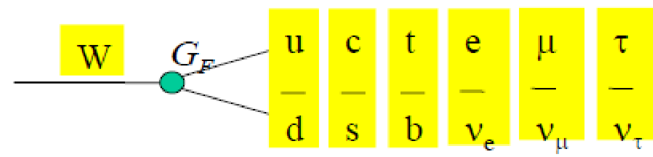


SUMMARY

The mass is a LEFT-RIGHT coupling and has to respect the gauge invariance $SU(2)_L \times U(1)_Y$



$$\overline{\psi}_L \phi \psi_R \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad I=\frac{1}{2} \quad Y=1$$



$$M^D = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} \qquad M^U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}$$

9+9 Complex parameters

$$M^{D,U}_{DIAG} = V_L^{D,U} M^{D,U} (V_R^{D,U})^+$$

$$M^D_{DIAG} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \qquad M^U_{DIAG} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$V(CKM) = V_L^U (V_L^D)^+ = \begin{pmatrix} \text{4 parameters} \\ \lambda, A, \rho, \eta \end{pmatrix}$$

$$L_M = M^d_{ij} \overline{d}^{Int.}_{L_j} d^{Int.}_{R_j} + M^u_{ij} \overline{u}^{Int.}_{L_j} u^{Int.}_{R_j} + M^l_{ij} \overline{l}^{Int.}_{L_j} l^{Int.}_{R_j}$$

To have mass matrices diagonal and real, we have defined: $M^f(diag) = V_L^f M^f V_R^f$

The mass eigenstates are:

$$d_{L_i} = (V^d_L)_{ij} d^{Int.}_{L_j} \quad ; \quad d_{R_i} = (V^d_R)_{ij} d^{Int.}_{R_j}$$

The Lagrangian for the gauge interaction is:

$$L_W = \frac{g}{2} \overline{u}_{L_i} \gamma^\mu (V^u_L V^{d\dagger}_L) d_{L_j} W^a_\mu + h.c.$$

Pattern	U	D	$\lvert V_{us}\rvert$ $(Exp.\ 0.22)$	$\lvert V_{ub}\rvert$ $(Exp.\ 0.0036)$	$\lvert V_{cb}\rvert$ $(Exp.\ 0.040)$	
1 M_7, M_3	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17, 0.28)	$\sqrt{\frac{m_d m_u}{m_b m_c}}$ 0.0023	$\sqrt{\frac{m_d}{m_b}}$ 0.040	No (V_{ub})
2 M_8, M_3	$\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_u}{m_c}}\left[\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}\right]$ (0.0011, 0.0058)	$\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}$ (0.022, 0.10)	No (V_{ub}, V_{cb})
3 M_6, M_3	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_d}{m_b}}$ 0.040	OK
4 M_3, M_7	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_u^2}{m_c m_t}}$ 0.00021	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{ub}, V_{cb})
5 M_2, M_7	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s} \pm \frac{m_u}{m_c}}$ (0.22,0.23)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{cb})

Pattern	U	D	$\lvert V_{us}\rvert$ $(Exp.\ 0.22)$	$\lvert V_{ub}\rvert$ $(Exp.\ 0.0036)$	$\lvert V_{cb}\rvert$ $(Exp.\ 0.040)$	
1 M_1, M_7	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{cb})
2 M_2, M_3	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_d}{m_b}} \pm \sqrt{\frac{m_u}{m_t}}$ (0.036,0.043)	OK
3 M_2, M_4	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_d m_s}{2m_b^2}} \pm \sqrt{\frac{m_u}{m_t}}$ (0.0013,0.0085)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{cb})
4 M_3, M_4	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$\sqrt{\frac{m_d m_s}{2m_b^2}} \pm \sqrt{\frac{m_u^2}{m_c m_t}}$ (0.0047, 0.0051)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	No (V_{ub}, V_{cb})
5 M_4, M_3	$\begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_t}}$ (0.22,0.23)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_d}{m_b}}$ 0.040	OK
6 M_5, M_3	$\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{2m_u}{m_t}}$ (0.22,0.23)	$\sqrt{\frac{m_u}{m_t}}$ 0.0036	$\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}$ (0.022,0.10)	? (V_{cb})
7 M_6, M_1	$\begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}}$ 0.22	$\sqrt{\frac{m_u}{m_t}} \pm 2\sqrt{\frac{m_d^{\ 2}}{m_s m_b}}$ (0.014,0.021)	$\sqrt{\frac{m_d}{m_b}}$ 0.040	No (V_{ub})
8 M_7, M_1	$\begin{pmatrix} 0 & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$2\sqrt{\frac{m_d^{\ 2}}{m_s m_b}} \pm \sqrt{\frac{m_d m_u}{m_b m_c}}$ (0.015,0.020)	$\sqrt{\frac{m_d}{m_b}}$ 0.040	No (V_{ub})
9 M_8, M_1	$\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$	$\begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}$	$\sqrt{\frac{m_d}{m_s}} \pm \sqrt{\frac{m_u}{m_c}}$ (0.17,0.28)	$2\sqrt{\frac{m_d^{\ 2}}{m_s m_b}} \pm \sqrt{\frac{m_d m_u}{m_b m_c}}$ (0.015,0.020)	$\sqrt{\frac{m_c}{m_t}} \pm \sqrt{\frac{m_d}{m_b}}$ (0.022,0.10)	No (V_{ub}, V_{cb})

The matrix $(V_{uL}V_{dL}^\dagger)$ is the mixing matrix for 2 quark generations. It is a 2×2 unitary matrix. As such, it generally contains 4 parameters, of which one can be chosen as a real angle, θ_C, and 3 are phases:

$$(V_{uL}V_{dL}^\dagger) = \begin{pmatrix} \cos\theta_C\ e^{i\alpha} & \sin\theta_C\ e^{i\beta} \\ -\sin\theta_C\ e^{i\gamma} & \cos\theta_C\ e^{i(-\alpha+\beta+\gamma)} \end{pmatrix}. \tag{4.11}$$

By the transformation

$$(V_{uL}V_{dL}^\dagger) \to V = P_u(V_{uL}V_{dL}^\dagger)P_d^*, \tag{4.12}$$

with

$$P_u = \begin{pmatrix} e^{-i\alpha} & \\ & e^{-i\gamma} \end{pmatrix}, \quad P_d = \begin{pmatrix} 1 & \\ & e^{i(-\alpha+\beta)} \end{pmatrix}, \tag{4.13}$$

we eliminate the three phases from the mixing matrix. (We redefine the mass eigenstates $u_{L,R} \to P_u u_{L,R}$ and $d_{L,R} \to P_d d_{L,R}$, so that the mass matrices remain unchanged. In particular, they remain real.) Notice that there are three independent phase differences between the elements of P_u and those of P_d, and three phases in $(V_{uL}V_{dL}^\dagger)$. Consequently, there are no physically meaningful phases in V, and hence no CP violation:*

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}. \tag{4.14}$$

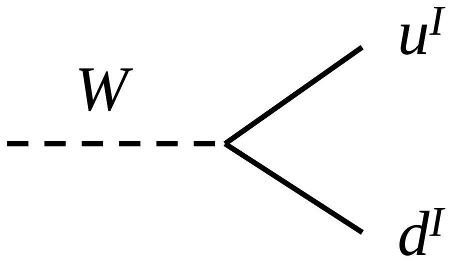
For two generations, V is called the Cabibbo matrix [1]. If $\sin\theta_C$ of (4.14) is different from zero, then the $W^\pm$ interactions mediate generation-changing currents.

Recap

$$L_{SM} = L_{Kinetic} + L_{Higgs} + L_{Yukawa}$$

$$-L_{Yuk} = Y_{ij}^{d}\,(\overline{u_L^I}, \overline{d_L^I})_i \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} d_{Rj}^I + \ldots$$

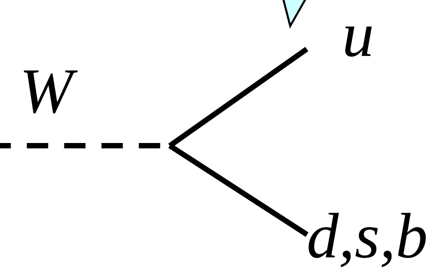
$$L_{Kinetic} = \frac{g}{\sqrt{2}} \overline{u_{Li}^I} \gamma^\mu W_\mu^- d_{Li}^I + \frac{g}{\sqrt{2}} \overline{d_{Li}^I} \gamma^\mu W_\mu^+ u_{Li}^I + \ldots$$



Diagonalize Yukawa matrix Y_{ij}

- Mass terms
- Quarks rotate
- Off diagonal terms in charged current couplings

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} \rightarrow V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$$-L_{Mass} = (\bar{d}, \bar{s}, \bar{b})_L \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + (\bar{u}, \bar{c}, \bar{t})_L \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \ldots$$

$$L_{CKM} = \frac{g}{\sqrt{2}} \overline{u_i} \gamma^\mu W_\mu^- V_{ij} \left(1 - \gamma^5\right) d_j + \frac{g}{\sqrt{2}} \overline{d_j} \gamma^\mu W_\mu^+ V_{ij}^* \left(1 - \gamma^5\right) u_i + \ldots$$

$$L_{SM} = L_{CKM} + L_{Higgs} + L_{Mass}$$

M(diag) is unchanged if $V'^{f}_{L} = P^{f} V^{f}_{L}$; $V'^{f}_{R} = P^{f} V^{f}_{R}$ $\quad V(CKM) = P^{u} V(CKM') P^{*d}$

P^f = phase matrix

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} = \begin{pmatrix} e^{-i\varphi_1} & 0 \\ 0 & e^{-i\varphi_2} \end{pmatrix} \begin{pmatrix} V'_{11} & V'_{12} \\ V'_{21} & V'_{22} \end{pmatrix} \begin{pmatrix} e^{-i\chi_1} & 0 \\ 0 & e^{-i\chi_2} \end{pmatrix} = \begin{pmatrix} V'_{11} e^{-i(\varphi_1-\chi_1)} & V'_{12} e^{-i(\varphi_1-\chi_2)} \\ V'_{21} e^{-i(\varphi_2-\chi_1)} & V'_{22} e^{-i(\varphi_2-\chi_2)} \end{pmatrix}$$

$u \rightarrow u e^{i\phi_{11}}$ $\qquad V_{11} e^{i\phi_{11}} e^{-i(\varphi_1-\chi_1)}$

Redifine the quark field

I choose $\varphi_1 - \chi_1$ such than V_{11} real

I choose $\varphi_1 - \chi_2$ such than V_{12} real

I choose $\varphi_2 - \chi_1$ such than V_{21} real

BUT: $(\varphi_2 - \chi_2) = (\varphi_2 - \chi_1) + (\varphi_1 - \chi_2) - (\varphi_1 - \chi_1)$

I cannot play the same game with all four fields
but only with 3 over 4

(2n-1) irreducible phases

APPENDIX III

JARSLOG DISCRIMINANT

UT area and condition for CP violation (formal)

The standard representation of the CKM matrix is:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \qquad \begin{aligned} c_{ij} &\equiv \cos\theta_{ij} \\ s_{ij} &\equiv \sin\theta_{ij} \end{aligned}$$

However, many representations are possible. What are the invariants under re-phasing?

- Simplest: $U_{\alpha i} = |V_{\alpha i}|^2$ is independent of quark re-phasing
- Next simplest: Quartets: $Q_{\alpha i \beta j} = V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*$ with $\alpha \neq \beta$ and $i \neq j$
  - "Each quark phase appears with and without *"
- $V^\dagger V = 1$: Unitarity triangle: $V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0$
  - Multiply the equation by $V_{us}^* V_{cs}$ and take the imaginary part:
  - $Im\,(V_{us}^* V_{cs} V_{ud} V_{cd}^*) = -\,Im\,(V_{us}^* V_{cs} V_{ub} V_{cb}^*)$
  - $J = Im\, Q_{udcs} = -\,Im\, Q_{ubcs}$
  - The imaginary part of each Quartet combination is the same (up to a sign)
  - In fact it is equal to 2x the surface of the unitarity triangle

    Area = ½ $|V_{cd}||V_{cb}|$ h ; h=$|V_{ud}||V_{ub}|\sin\arg(-V_{ud}V_{cb}V_{ub}^*V_{cb}^*)|$

    $=1/2\ |Im(V_{ud}V_{cb}V_{ub}^*V_{cb}^*)|)|$
- $Im[V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*] = J \sum \varepsilon_{\alpha\beta\gamma}\, \varepsilon_{ijk}$ where J is the universal Jarlskog invariant
- Amount of CP Violation is proportional to J

The Amount of CP Violation

Using Standard Parametrization of CKM:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

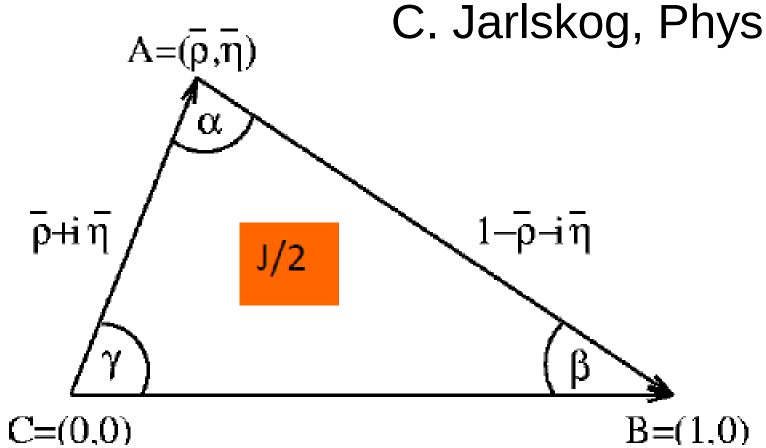
$$c_{ij} \equiv \cos\theta_{ij}$$
$$s_{ij} \equiv \sin\theta_{ij}$$

$$J \equiv c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta = (3.0 \pm 0.3) \times 10^{-5} = \lambda^6 A^2 \eta$$

(eg.: $J=Im(V_{us}\, V_{cb}\, V_{ub}^{*}\, V_{cs}^{*})$)

(The maximal value J might have = 1/(6√3) ~ 0.1)

C. Jarlskog, Phys. Rev. Lett.55, 1039 (1985)



More details

CP Violation at the Lagrangian level

$$L_W = \frac{g}{2}\overline{Q}^{Int.}_{L_i}\gamma^\mu\sigma^a Q^{Int.}_{L_i} W^a_\mu \qquad a=1,2,3 \qquad Q^{Int.}_{L_i} = \begin{pmatrix} u_{L_i} \\ d_{L_i} \end{pmatrix} \quad L^{Int.}_{L_i} = \begin{pmatrix} \nu_{L_i} \\ l_{L_i} \end{pmatrix}$$

$$L_M = M^d_{ij}\overline{d}^{Int.}_{L_j} d^{Int.}_{R_j} + M^u_{ij}\overline{u}^{Int.}_{L_j} u^{Int.}_{R_j} + M^l_{ij}\overline{l}^{Int.}_{L_j} l^{Int.}_{R_j} \qquad \text{where} \quad M^f = (v/\sqrt{2})Y^f$$

Accept that (or verify) the most general CP transformation which leave the lagrangian invariant is

$$d^{Int.}_L -> W_L C d^{Int.*}_L \quad ; \quad d^{Int.}_R -> W^d_R C d^{Int.*}_R$$

$$u^{Int.}_L -> W_L C u^{Int.*}_L \quad ; \quad u^{Int.}_R -> W^u_R C u^{Int.*}_R$$

$$(C = i\gamma^2\gamma^0 \qquad W_L, W^u_R, W^d_R \quad \text{unitarity matrices})$$

In order to have L_M to be invariant under CP, the M matrices should satisfy the following relations :

$$W_L^\dagger M_u W^u_R = M^*_u \qquad W_L^\dagger H_u W_L = H^*_u \qquad \text{where} \quad H_u = M_u M_u^\dagger \text{ and } \quad W^u_R = M_u^\dagger W_L$$

$$W_L^\dagger M_d W^d_R = M^*_d \qquad W_L^\dagger H_d W_L = H^*_d \qquad \text{where} \quad H_d = M_d M_d^\dagger \text{ and } \quad W^d_R = M_d^\dagger W_L$$

in this form, these conditions are of little use. A way of doing is :

$$W_L^\dagger H_u H_d W_L = H_u^T H_d^T$$

$$W_L^\dagger H_d H_u W_L = H_d^T H_u^T$$

- The existence of charged current contrains u_L,d_L to trasform in the same way under CP while the absence of right charged current allow u_R,d_R to tranform differentely under CP

More details

Substracting these two equations

$$W_L^\dagger [H_u H_d] W_L = -[H_u H_d]^T$$

If one evaluates the traces of both sides, they vanish identically and no constraints is obtained. In order to obtain no trivial contrain, we have to multiply the previous equation a odd number of times :

$$W_L^\dagger [H_u H_d]^r W_L = -\{[H_u H_d]^r\}^T \qquad (r \text{ odd})$$

Taking the traces one obtain :

$$Tr[H_u H_d]^r = 0$$

For n=1, and n=2 the previous equations are automatically satified for harbitrary hermitian H matrices (it is the same as the counting of the physical phase of the CKM matrix). For n=3 or larger the previous eq. provides non trivial contraints on the H matrix. It can be shown that for n=3 it implies

$$Tr[H_u H_d]^3 = 6\Delta_{21}\Delta_{31}\Delta_{32} \,\mathrm{Im}\, Q$$
$$\Delta_{21} = (m_s^2 - m_d^2)\times(m_c^2 - m_u^2)$$
$$\Delta_{31} = (m_b^2 - m_d^2)\times(m_t^2 - m_u^2)$$
$$\Delta_{32} = (m_b^2 - m_s^2)\times(m_t^2 - m_c^2)$$

CP violation vanish in the limit where any two quarks of the same charge become degenerate. But it does not necessarily vanish in the limit where one quark is massless (m_u=0) or even in the chiral limit (m_u=m_d=0)

CP violation vanish if the triangle has area equal to 0

CP Violation in the Standard Model

Requirements for CP violation

$$\left(m_t^2 - m_c^2\right)\left(m_t^2 - m_u^2\right)\left(m_c^2 - m_u^2\right)$$
$$\times\left(m_b^2 - m_s^2\right)\left(m_b^2 - m_d^2\right)\left(m_s^2 - m_d^2\right)\times J_{CP} \neq 0$$

where

$$J_{CP} = \left|\mathrm{Im}\left\{V_{i\alpha}V_{j\beta}V_{i\beta}^{*}V_{j\alpha}^{*}\right\}\right|\left(i \neq j, \alpha \neq \beta\right)$$

Jarlskog determinant

Using above parameterizations

$$J_{CP} = s_{12}s_{13}s_{23}c_{12}c_{23}c_{13}\sin\delta = \lambda^6 A^2 \eta = O\left(10^{-5}\right)$$

→ CP violation is small in the Standard Model

APPENDIX III

Experimental techniques

for B Physics

Introduction to mixing and CP phenomena

Pairs of self-conjugate mesons that can be transformed to each other via flavour changing weak interaction transitions are:

$$|K^0\rangle = |\bar{s}d\rangle \qquad |D^0\rangle = |c\bar{u}\rangle \qquad |B_d^0\rangle = |\bar{b}d\rangle \qquad |B_s^0\rangle = |\bar{b}s\rangle$$

They are **flavour eigenstates** with definite quark content $|B^0\rangle, \ |\overline{B}^0\rangle$

- useful to understand particle production and decay

Apart from the flavour eigenstates there are **mass eigenstates**: $|B_L\rangle, \ |B_H\rangle$

- eigenstates of the Hamiltonian
- states of definite mass and lifetime

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle$$

$|B_L\rangle, \ |B_H\rangle$: mass eigenstates

$|B^0\rangle, \ |\overline{B}^0\rangle$: flavour eigenstates

Since flavour eigenstates are not mass eigenstates, the flavour eigenstates are mixed with one another as they propagate through space and time

$|B^0(t)\rangle$ ($|\bar{B}^0(t)\rangle$) : the flavour state of a B meson that was a B^0 ($\bar{B}^0$) at $t=0$.

Schrödinger equation governs time evolution of the B^0-$\bar{B}^0$ System:

$$i\frac{d}{dt}\begin{pmatrix}|B^0(t)\rangle\\|\bar{B}^0(t)\rangle\end{pmatrix}=\underbrace{\left(M-\tfrac{i}{2}\Gamma\right)}\begin{pmatrix}|B^0(t)\rangle\\|\bar{B}^0(t)\rangle\end{pmatrix}$$

T conservation ➡ $|H_{21}| = |H_{12}|$

CP conservation ➡ $|H_{21}| = |H_{12}|$, $H_{11} = H_{22}$

CPT conservation ➡ $H_{11} = H_{22}$

=> ***H*** (effective Hamiltonian)

$$H|B_L^0\rangle = (M_L - i/2\Gamma_L)|B_L^0\rangle$$
$$H|B_H^0\rangle = (M_H - i/2\Gamma_H)|B_H^0\rangle$$

eigenvalues

Mass states are eigenvectors of ***H***

$$\Delta m_B \equiv M_H - M_L \approx 2|M_{12}|$$
$$\Delta\Gamma_B \equiv \Gamma_H - \Gamma_L = 2\mathrm{Re}\left(M_{12}\Gamma_{12}^*\right)/|M_{12}|$$
$$m_B \equiv \frac{M_H + M_L}{2}$$
$$\Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}$$

$$\frac{q}{p} \equiv -\sqrt{\frac{H_{21}}{H_{12}}} = \frac{\Delta m_B + i\Delta\Gamma_B/2}{2M_{12} - i\Gamma_{12}}$$

The time evolution of the mass eigenstates is governed by their eigenvalues :

$$|B_{H,L}(t)\rangle = e^{-i\left(M_{H,L} - i\frac{\Gamma_{H,L}}{2}\right)t}|B_{H,L}(t=0)\rangle$$

+

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$
$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

Time evolution

Time evolution of the physical states $\left|B^0(t)\right\rangle$ $(\left|\bar{B}^0(t)\right\rangle)$

$$\left|B^0(t)\right\rangle = g_+(t)\left|B^0\right\rangle + \frac{q}{p} g_-(t)\left|\bar{B}^0\right\rangle \quad g_+(t) = e^{-i\left(m_B - i\frac{\Gamma_H}{2}\right)t}\left[\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta m t}{2} - i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta m t}{2}\right]$$

$$\left|\bar{B}^0(t)\right\rangle = \frac{p}{q} g_-(t)\left|B^0\right\rangle + g_+(t)\left|\bar{B}^0\right\rangle \quad g_-(t) = e^{-i\left(m_B - i\frac{\Gamma_H}{2}\right)t}\left[-\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta m t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta m t}{2}\right]$$

More general formulae

$$\Delta m_B \equiv M_H - M_L$$
$$\Delta\Gamma_B \equiv \Gamma_H - \Gamma_L$$
$$m_B \equiv \frac{M_H + M_L}{2}$$
$$\Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}$$
$$\frac{q}{p} = \frac{\Delta m + i\Delta\Gamma/2}{2M_{12} - i\Gamma_{12}}$$

When ΔΓ is small they simplify to :

$$\left|B^0(t)\right\rangle = e^{-im_B t} e^{-\Gamma_B t/2}\left(\cos\frac{\Delta m_B t}{2}\left|B^0\right\rangle + i\frac{q}{p}\sin\frac{\Delta m_B t}{2}\left|\bar{B}^0\right\rangle\right)$$

$$\left|\bar{B}^0(t)\right\rangle = e^{-im_B t} e^{-\Gamma_B t/2}\left(\cos\frac{\Delta m_B t}{2}\left|\bar{B}^0\right\rangle + i\frac{p}{q}\sin\frac{\Delta m_B t}{2}\left|B^0\right\rangle\right)$$

Probability to observe in the state *f* a B^0 produced at time t=0:

$$P\left(B^0(0) \to f\right) = \left|\left\langle f\right|H\left|B^0(t)\right\rangle\right|^2$$

Probability to observe in the state $\bar{f}$ a B^0 produced at time t=0:

$$P\left(\bar{B}^0(0) \to f\right) = \left|\left\langle f\right|H\left|\bar{B}^0(t)\right\rangle\right|^2$$

The two master formulae (having however neglected $\Delta\Gamma$:

$$P\left(B^0(0)\to f\right)=\frac{e^{-\Gamma t}}{2}\left\{(1+\cos\Delta m t)\left|\langle f|H|B^0\rangle\right|^2+(1-\cos\Delta m t)\left|\frac{q}{p}\right|^2\left|\langle f|H|\overline{B}^0\rangle\right|^2\right.$$
$$\left.-2\sin\Delta m t\times\mathrm{Im}\left(\frac{q}{p}\langle f|H|\overline{B}^0\rangle\langle f|H|B^0\rangle^*\right)\right\}$$

$$P\left(\overline{B}^0(0)\to f\right)=\frac{e^{-\Gamma t}}{2}\left\{(1+\cos\Delta m t)\left|\langle f|H|\overline{B}^0\rangle\right|^2+(1-\cos\Delta m t)\left|\frac{p}{q}\right|^2\left|\langle f|H|B^0\rangle\right|^2\right.$$
$$\left.-2\sin\Delta m t\times\mathrm{Im}\left(\frac{p}{q}\times\langle f|H|B^0\rangle\langle f|H|\overline{B}^0\rangle^*\right)\right\}$$

Considering only the mixing :

Starting from a B^0

$$\left|\langle B^0 | H | B^0(t)\rangle\right|^2 = \frac{e^{-\Gamma t}}{2}(1+\cos\Delta m t)$$

CP violation is neglected : q/p=1

Starting from a $\overline{B}^0$

$$\left|\langle \overline{B}^0 | H | B^0(t)\rangle\right|^2 = \frac{e^{-\Gamma t}}{2}(1-\cos\Delta m t)$$

If one does not neglect $\Delta\wp$ (useful for charm or B_s) the previous formulae become

$$\frac{e^{-\Gamma t}}{4}(\underbrace{e^{\frac{\Delta\Gamma}{2}t}+e^{-\frac{\Delta\Gamma}{2}t}}_{\cosh\left(\frac{\Delta\Gamma}{2}t\right)} \pm 2\cos\Delta m t)$$

So that one finds for the time dependent mixing asymmetry:

$$A_{\text{mix}}(t) \equiv \frac{N(\text{unmixed})-N(\text{mixed})}{N(\text{unmixed})+N(\text{mixed})}(t) = \frac{\cos(\Delta m t)}{\cosh(\Delta\Gamma t/2)}$$

Mixed : $\overline{B}^0 \rightarrow B^0$ or $B^0 \rightarrow \overline{B}^0$

UnMixed : $B^0 \rightarrow B^0$ or $\overline{B}^0 \rightarrow \overline{B}^0$

$$\cosh(\Delta\Gamma t/2) \rightarrow 1 \text{ when } \Delta\Gamma \rightarrow 0$$

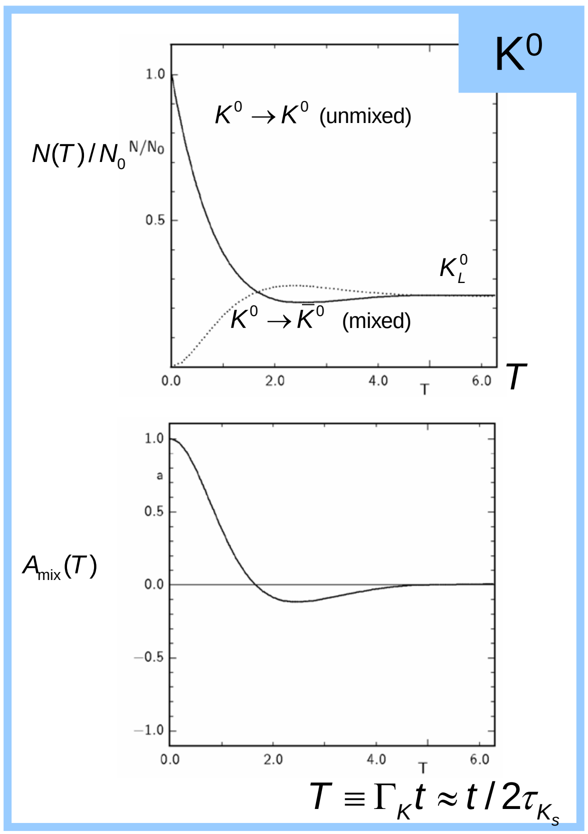
$$\cos \Delta m t = \cos\left(\frac{\Delta m}{\Gamma}\right)\left(\frac{t}{\tau}\right) \quad ; \quad x \equiv \left(\frac{\Delta m}{\Gamma}\right) \quad y \equiv \left(\frac{\Delta\Gamma}{2\Gamma}\right)$$

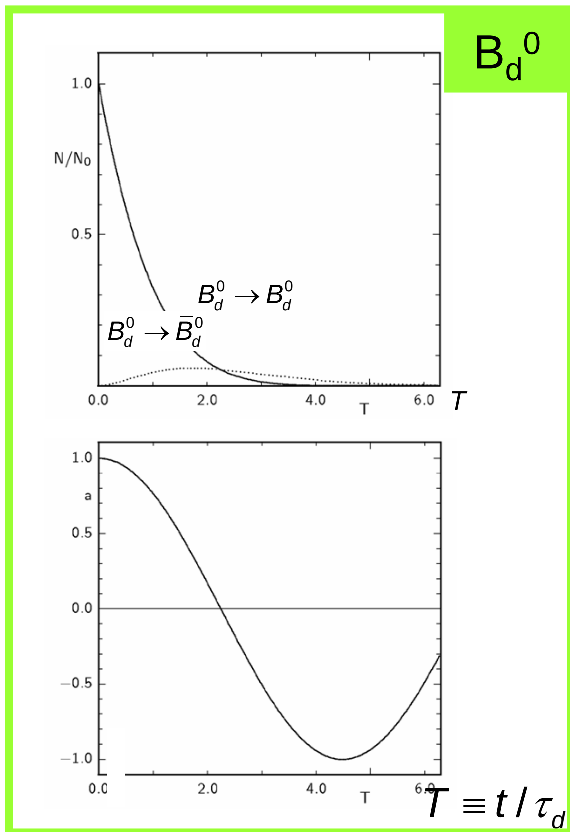
x : the mixing frequency in unit of lifetime

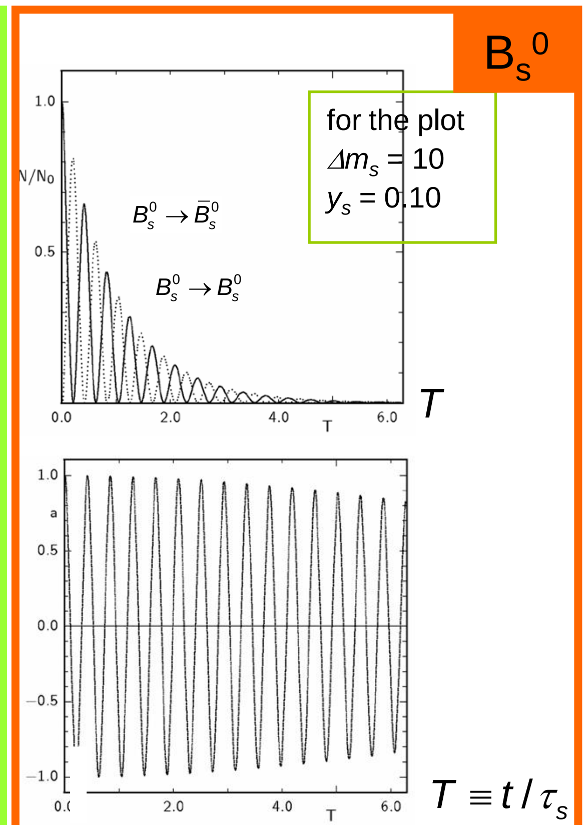
x>>1 rapid oscillation
x<<1 slow oscillation

	x=Δm/Γ	y=ΔΓ/Γ
K^0	~1	~1
D^0	10^{-3}-10^{-5}	10^{-3}-10^{-5}
B_d^0	~0.75	~few%
B_s^0	~25	(10-15)%

Different behaviors for the neutral mesons :







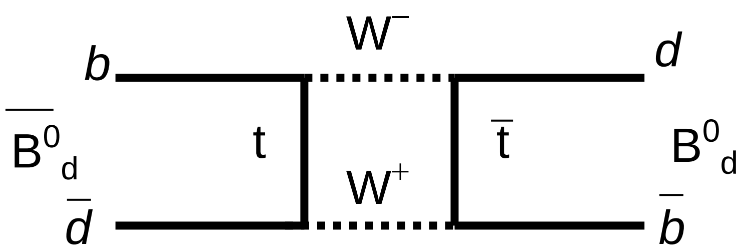
$$\cos \Delta m t = \cos\left(\frac{\Delta m}{\Gamma}\right)\left(\frac{t}{\tau}\right) \quad ; \quad x \equiv \left(\frac{\Delta m}{\Gamma}\right)$$

x is a number the mixing frequency in unit of lifetime

x>>1 rapid oscillation
x<<1 slow oscillation

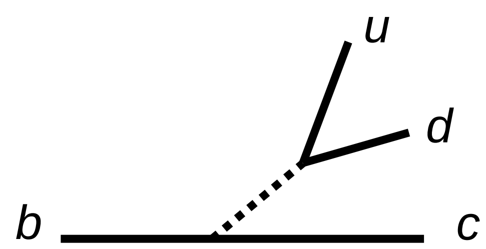
We also define $y \equiv \left(\frac{\Delta\Gamma}{2\Gamma}\right)$

B_d



$$f(m_t)[V^*_{td}V_{tb}]^2 \sim m_t^2\lambda^6$$

$f(m_t)[V^*_{cd}V_{cb}]^2 \sim m_c^2\lambda^6$ totally negligible



$$[V^*_{ud}V_{cb}]^2 \sim \lambda^4$$

$$\Delta m_d / \Gamma_d \sim m_t^2\lambda^2 \sim \text{large}$$

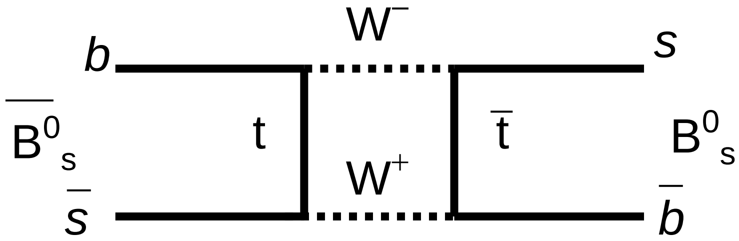
$$\Delta m_d \sim 0.50\, ps^{-1}$$

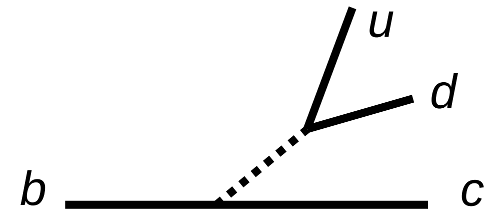
$$1/\Gamma_d \sim 1.50\, ps$$

$$x = \Delta m_d / \Gamma_d \sim 0.75$$

Slow oscillations

B_s





$$f(m_t)[V_{ts}^* V_{tb}]^2 \sim f(m_t)\lambda^4$$

$$[V_{ud}^* V_{cb}]^2 \sim \lambda^4$$

$\Delta m_s / \Gamma_s \sim f(m_t) \sim$ very large $x_s >> 1$

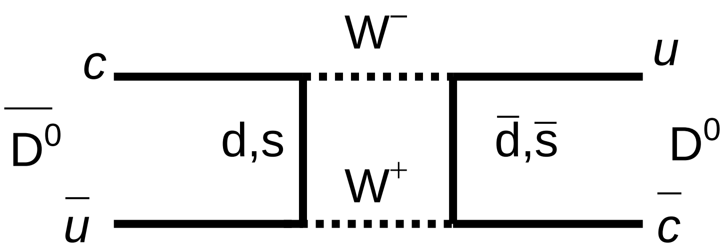
$\Delta m_s \sim 17\, ps^{-1}$

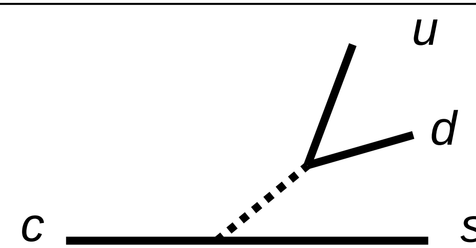
$1/\Gamma_s \sim 1.50\, ps$

$x = \Delta m_s / \Gamma_s \sim 25$

Rapid oscillations

D^0





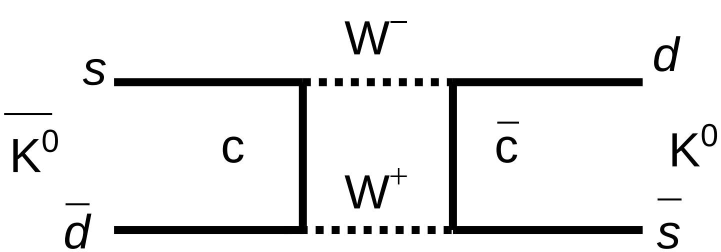
$x << 1$

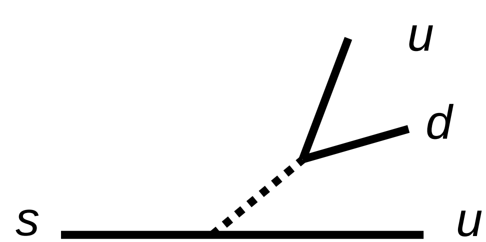
$x \sim 10^{-3} - 10^{-5}$

$$f(m_s)[V_{cs}^* V_{us}]^2 \sim f(m_s)\lambda^2$$

$$[V_{ud}^* V_{cs}]^2 \sim 1$$

K^0





$x \sim 1$

$$f(m_c)[V_{cd}^* V_{cs}]^2 \sim f(m_c)\lambda^2$$

$$[V_{ud}^* V_{us}]^2 \sim \lambda^2$$

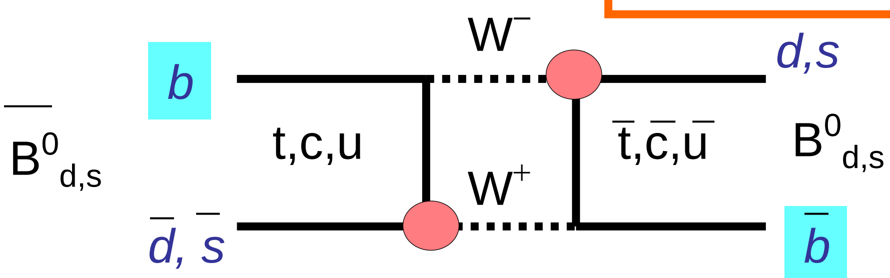
More…

The probability that the meson B^0 produced (by strong interaction) at *t = 0* transforms (weak interaction) into $\overline{B^0}$ (or stays as a B^0) at time *t* is given by :

$$P_{B_q^0 \to B_q^0 (\overline{B_q^0})} = \frac{1}{2} e^{-t/\tau_q} (1 \pm \cos \Delta m_q t)$$

Δm_q can be seen as **an oscillation frequency : 1 ps^{-1} = 6.58 10^{-4} eV**

$$\Delta m_B = 2 |M_{12}|$$



In SM : ΔF=2 process

GIM mechanism (Rate ~ $m_1^2 - m_2^2$)

Dominated by ***t*** *exchange*

Rate LARGE

V_{td} V_{ts}

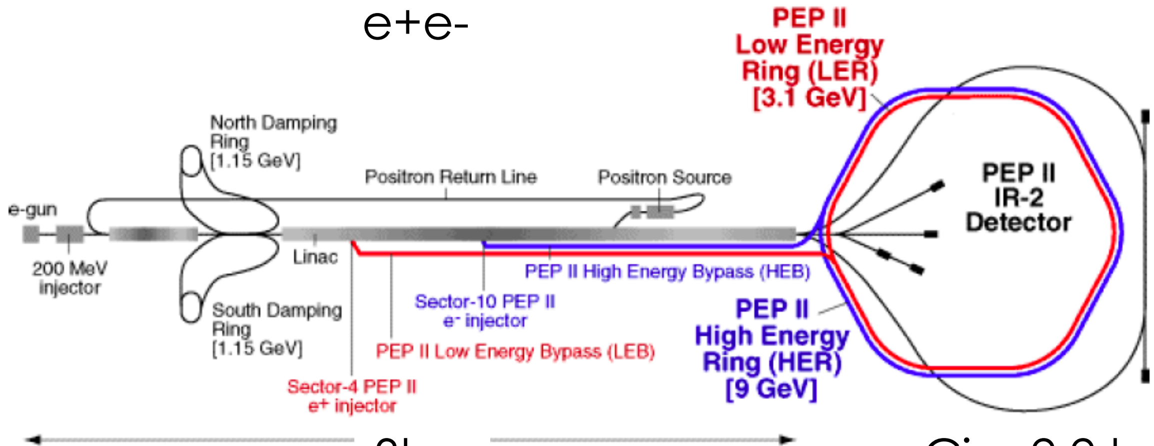
Allow to access fundamental parameters of the Standard Model

(Super) B-factories and LHC

e+e-

(Super) B-factories :
E_{CM} = 10.58 GeV

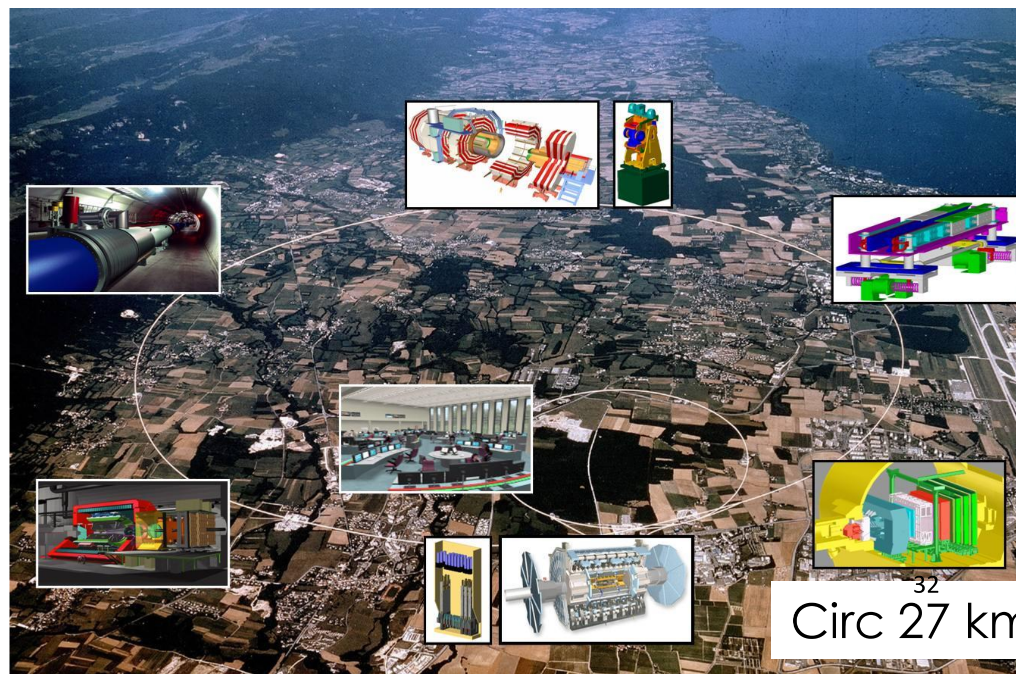
$L = 3\ 10^{33}$ cm^{-2} s^{-1}
…10^{36} cm^{-2} s^{-1} (Super B)



Circ 2.2 km

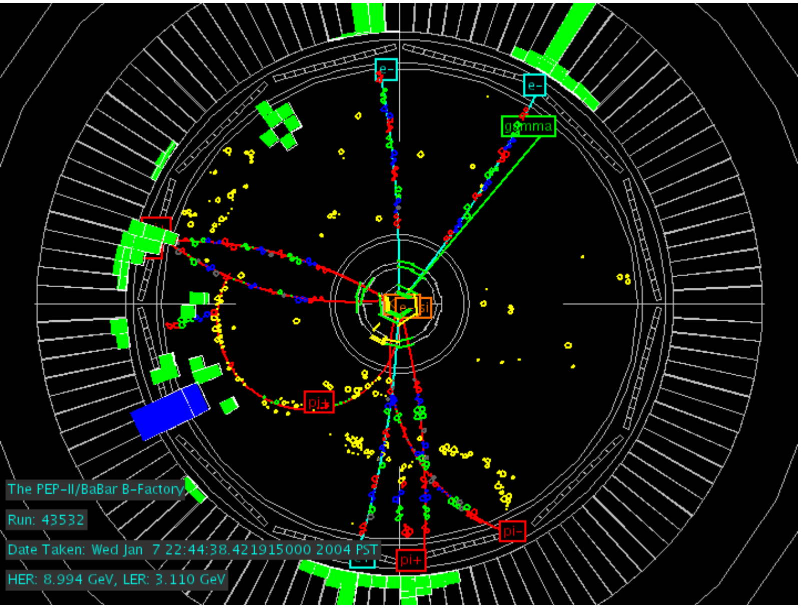
pp

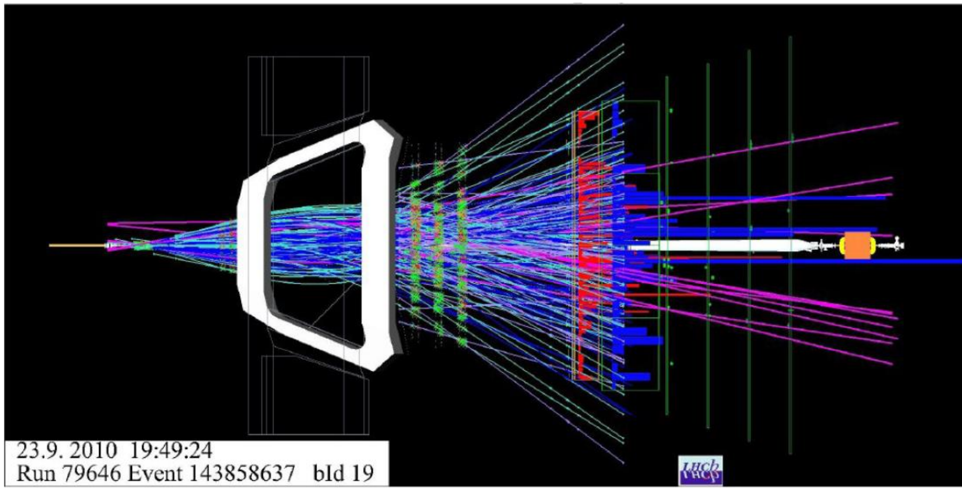
LHC: E_{CM} = 7, 8 TeV , (later 14 TeV)
$4\ 10^{32}$ cm^{-2} s^{-1} (design was $2\ 10^{32}$)
… 10^{33} cm^{-2} s^{-1} (upgrade)

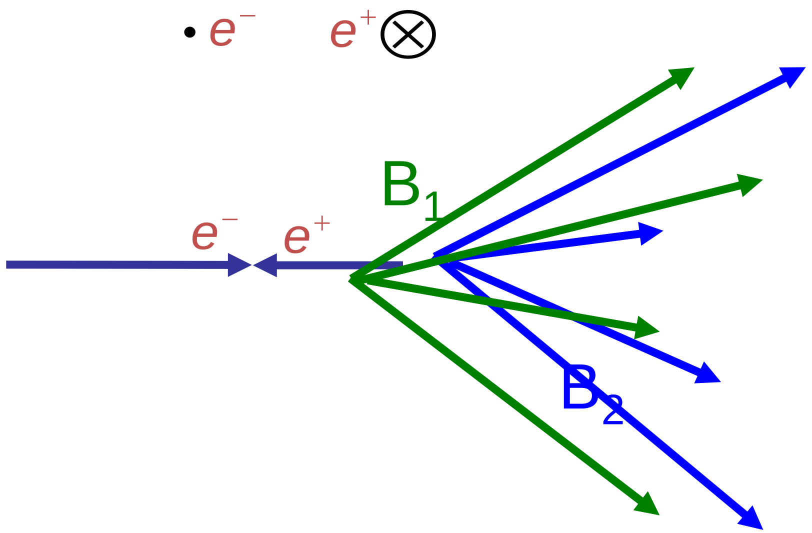


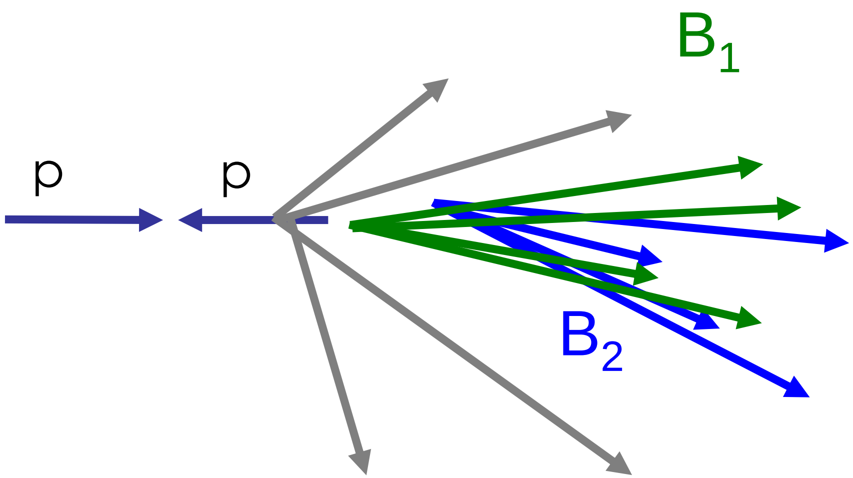
Circ 27 km

sketch of an event at B-factory and at LHCb



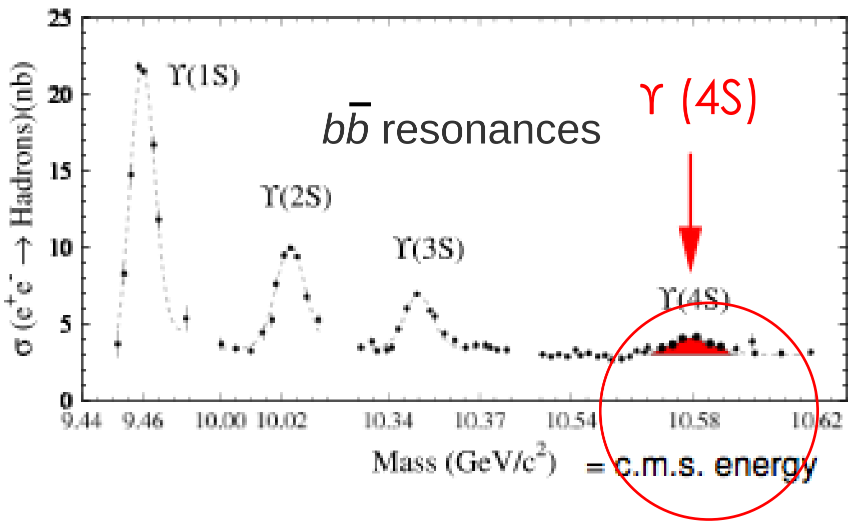






(Super) B-factories

ϒ(1S), ϒ(2S) and ϒ (3S) : not enough mass to decay into BB pair



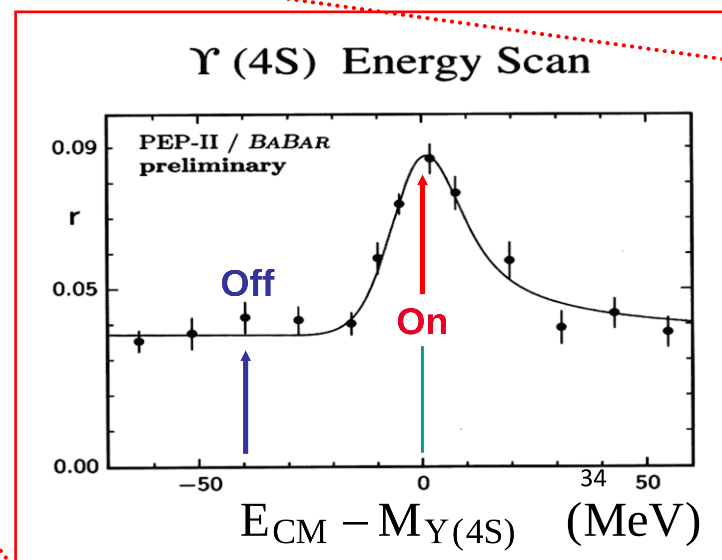
$e^+ e^- \to \Upsilon(4S) \to B\bar{B}$ at $\sqrt{s}$ = 10.58 GeV

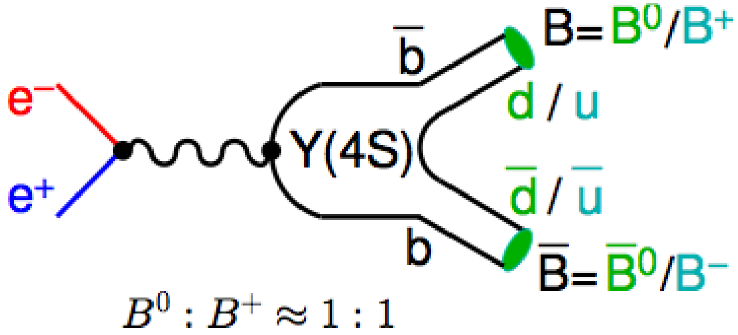
Production of coherent $\bar{B}B$ pairs with a cross section of 1.1 nb   (over a continuum of ~3 nb)

Hadronic cross sections at $\sqrt{s} = 10.58$ GeV:

h	σ[nb]
b	1.05
c	1.3
d,s	0.3
u	1.4

$\Upsilon(4S) \to B^+B^-$, B^0B^0
to approx. 50% each





M(ϒ(4S))=10.58 Gev

$M(B^+, B^0)$ = 5.28 GeV

$M(B_s)$ = 5.37 GeV > M(ϒ(4S))/2

only (B^+, B^0) are produced

(B^+, B^0) are produced nearly at rest in the ϒ(4S)

A $B^0\,\bar{B}^0$ or B^+B^- coherent pair in the L=1 state is produced

$B^0 \overline{B^0}$ or B^+B^- coherent L=1 pairs are produced nearly at rest in the $\Upsilon(4S)$

$$a_{f_{CP}}(t) = \frac{\mathrm{Prob}(B^0(t) \to f_{CP}) - \mathrm{Prob}(\overline{B^0}(t) \to f_{CP})}{\mathrm{Prob}(\overline{B^0}(t) \to f_{CP}) + \mathrm{Prob}(B^0(t) \to f_{CP})} =$$

$$= C_f \cos \Delta m_d t + S_f \sin \Delta m_d t$$

$$= \pm \sin 2\beta \sin \Delta m_d t \quad \text{for } J/\psi, K^0$$

$t = t(B_1) - t(B_2)$

The decay of the first B *starts* the clock $t(B_1)$

The decay of the other B *stops* the clock $t(B_2)$

Time integrated measurement :

$$\int_{-\infty}^{+\infty} \sin \Delta m_d t \, dt = 0 \ !!$$

t can be >0 or <0 ….

One should measure t in order to probe CP violation

It was not the case for the observation of B mixing performed at an previous $\Upsilon(4S)$ collider because :

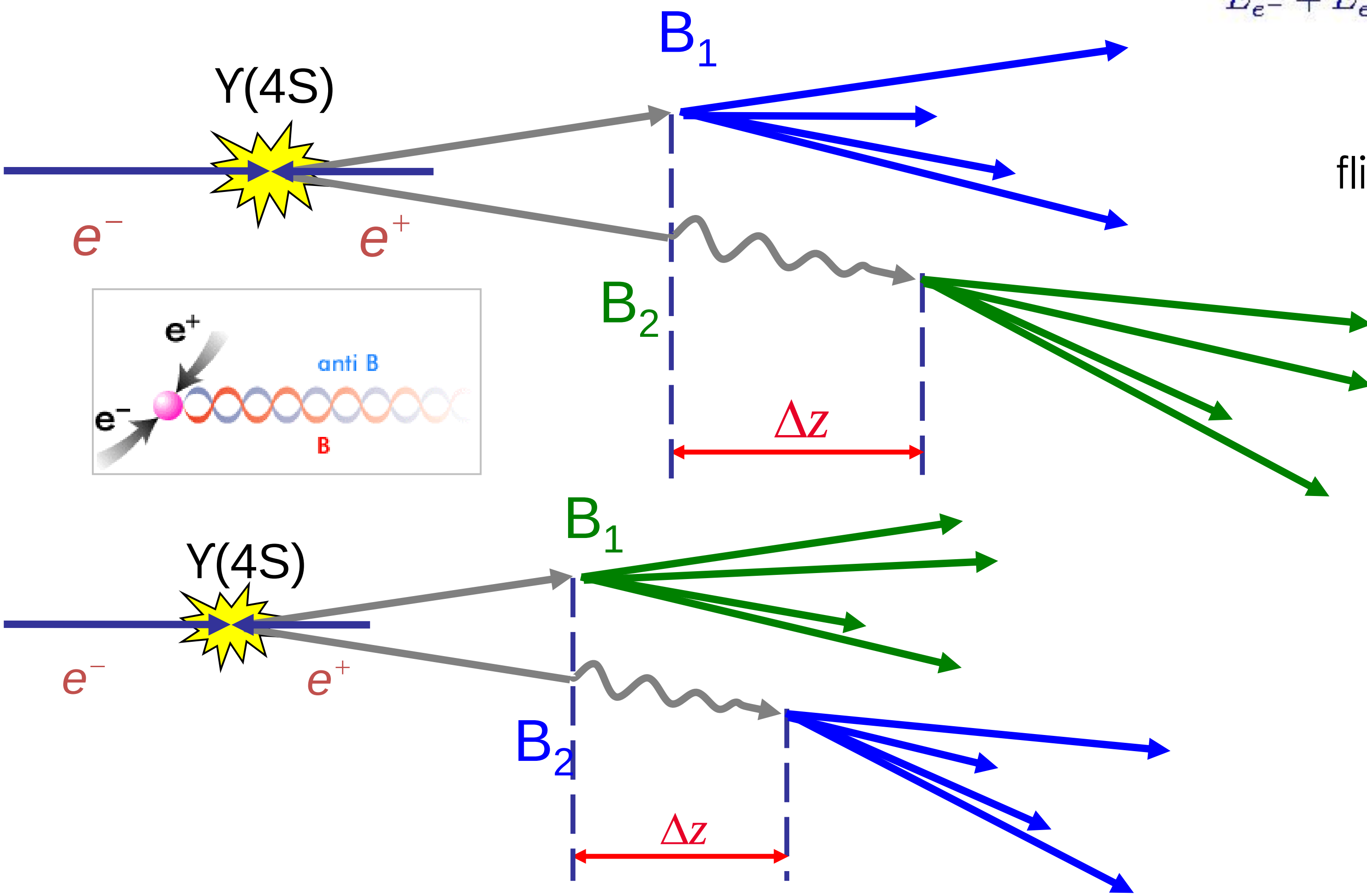
$$a_{mixing}(t) = \cos \Delta m_d t$$

In the $\Upsilon(4S)$ rest frame p(B) ~ 300 MeV : $\beta\gamma = .3/5.28 = 0.06$ flight ~ 30μm
Boost the $\Upsilon(4S)$!

KEKB – Belle – Japan.
8 vs 3.5 GeV. =0.425

PEPII – BaBar – US.
9 vs 3.1 GeV. =0.56.

$$p_{cm} = p_{\Upsilon(4S)} = (E_{e^-} + E_{e^+}, (E_{e^-} - E_{e^+})\hat{\mathbf{z}})$$
$$E_{cm} = \sqrt{4E_{e^-}E_{e^+}} = M(\Upsilon(4S))$$
$$\beta = \frac{E_{e^-} - E_{e^+}}{E_{e^-} + E_{e^+}} \qquad \beta\gamma = \frac{E_{e^-} - E_{e^+}}{2E_{e^-}E_{e^+}}$$



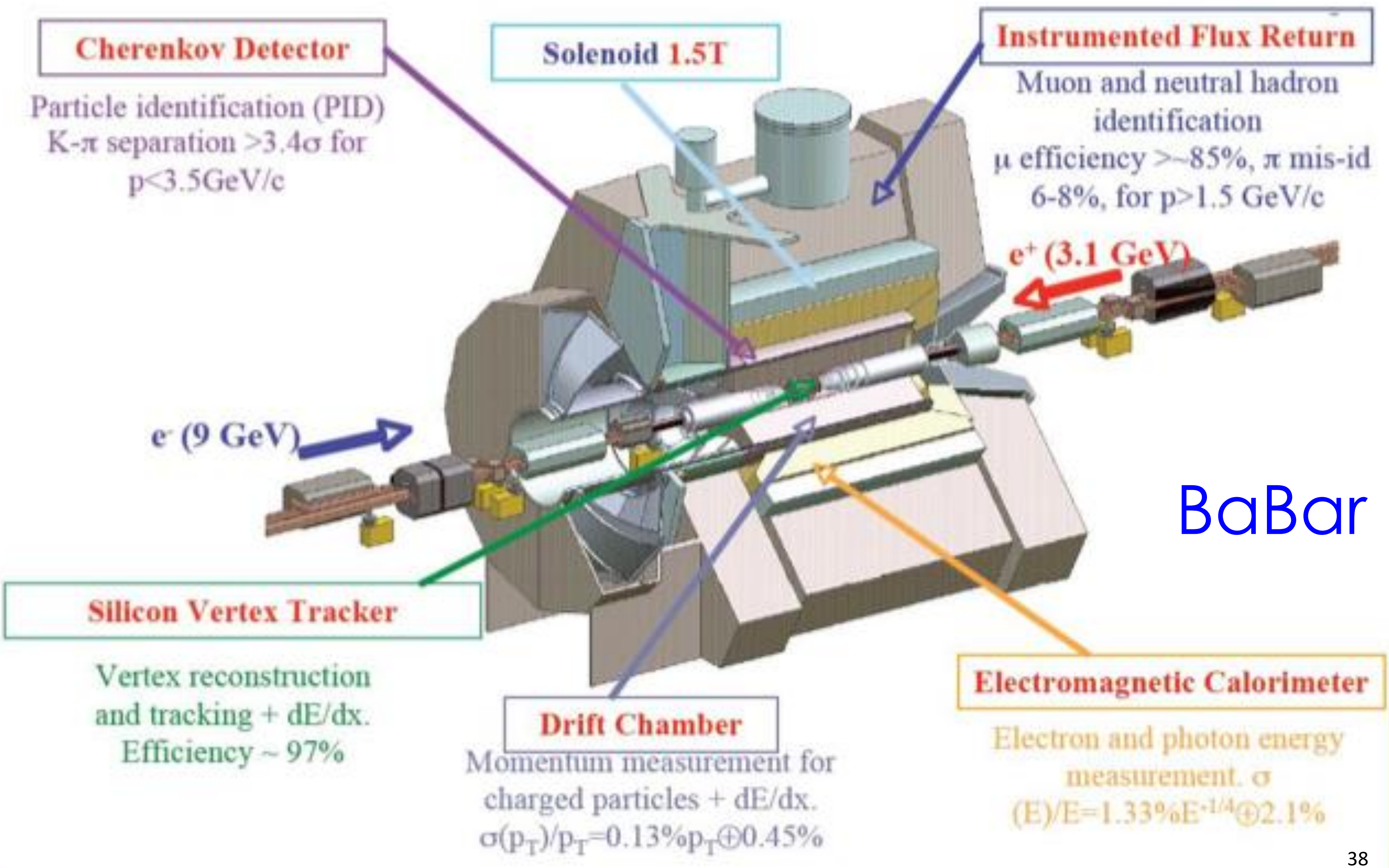
cτ (B)= 450μm

flight Δz ~ 250 μm

or

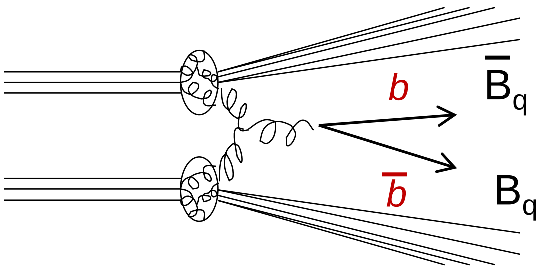
- By measuring Δz, we can follow time dependent effects in B decays.
- distance scale is much smaller than in the kaon decay exp. that first discovered CP

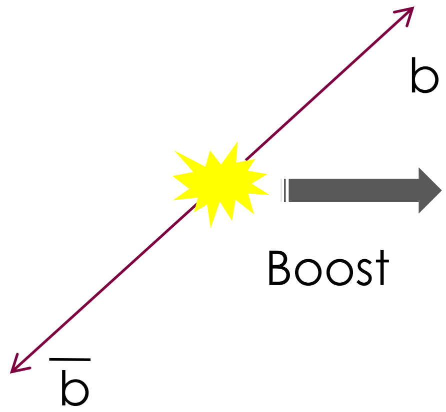
Slightly asymmetric detector

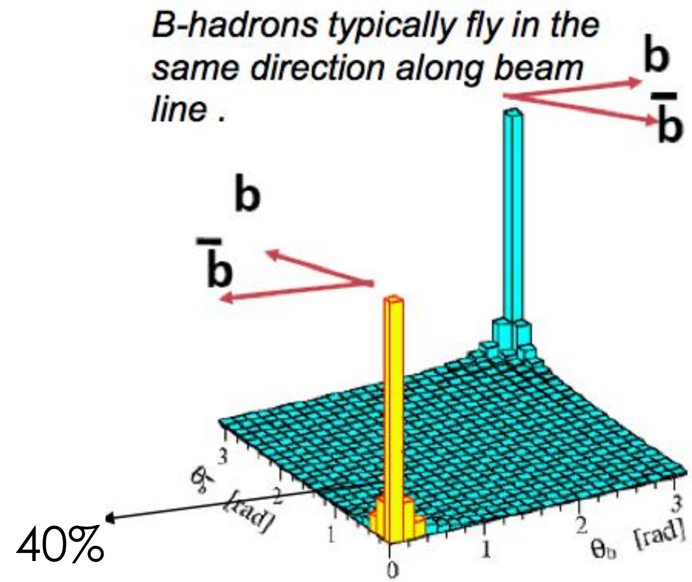


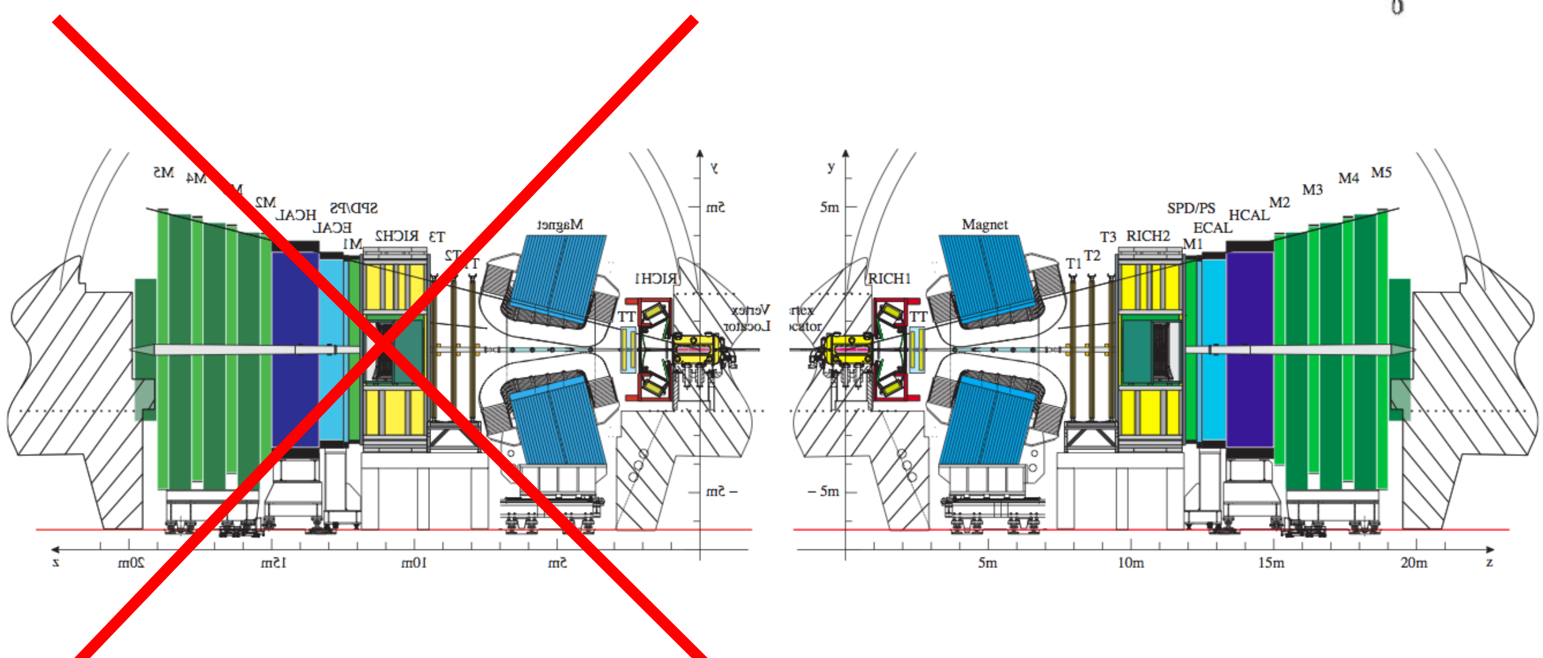
LHC

The 2 b-quarks are produced in the same direction along the beam axis







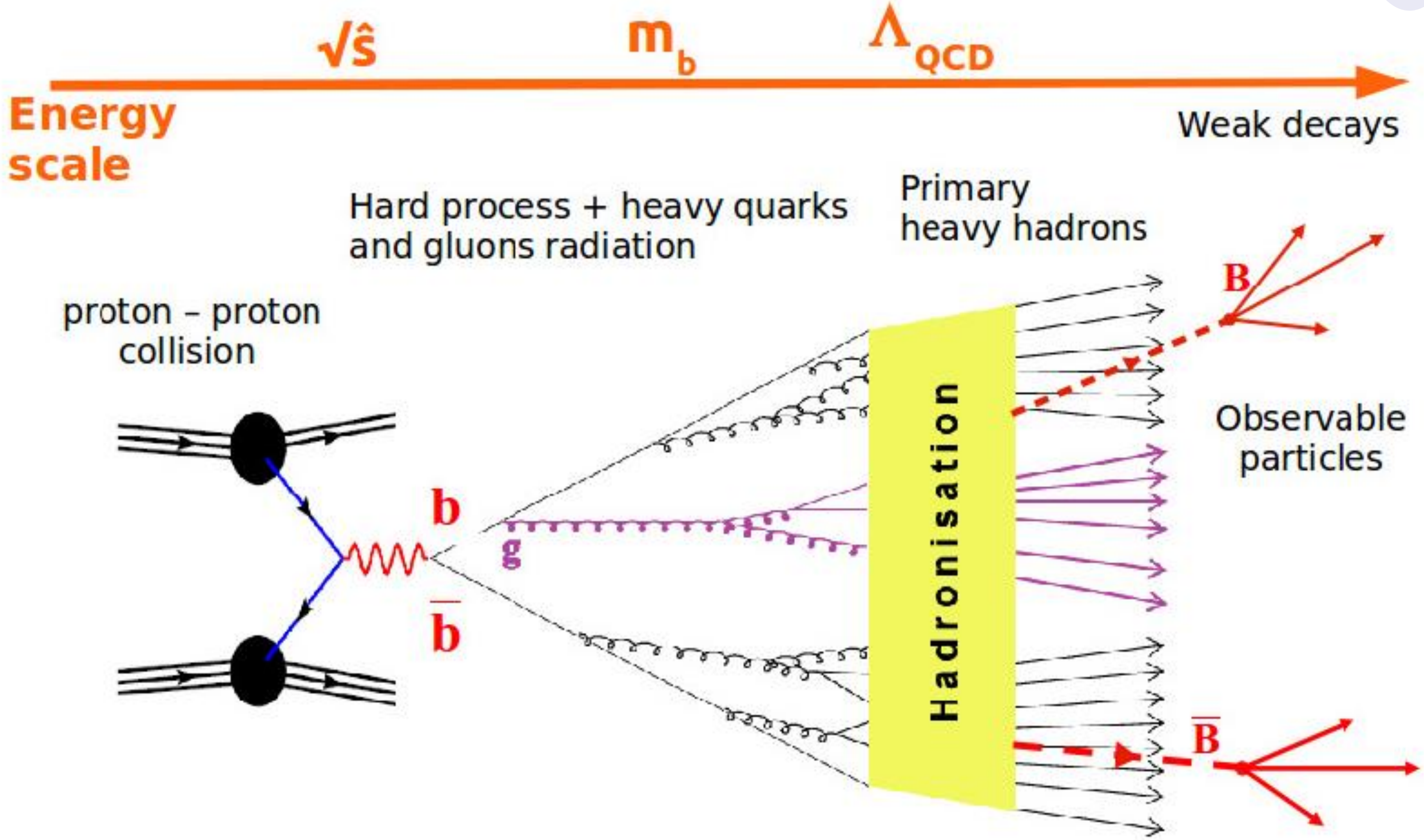


Energy in the CM 8 TeV
B energy ~ 100 GeV

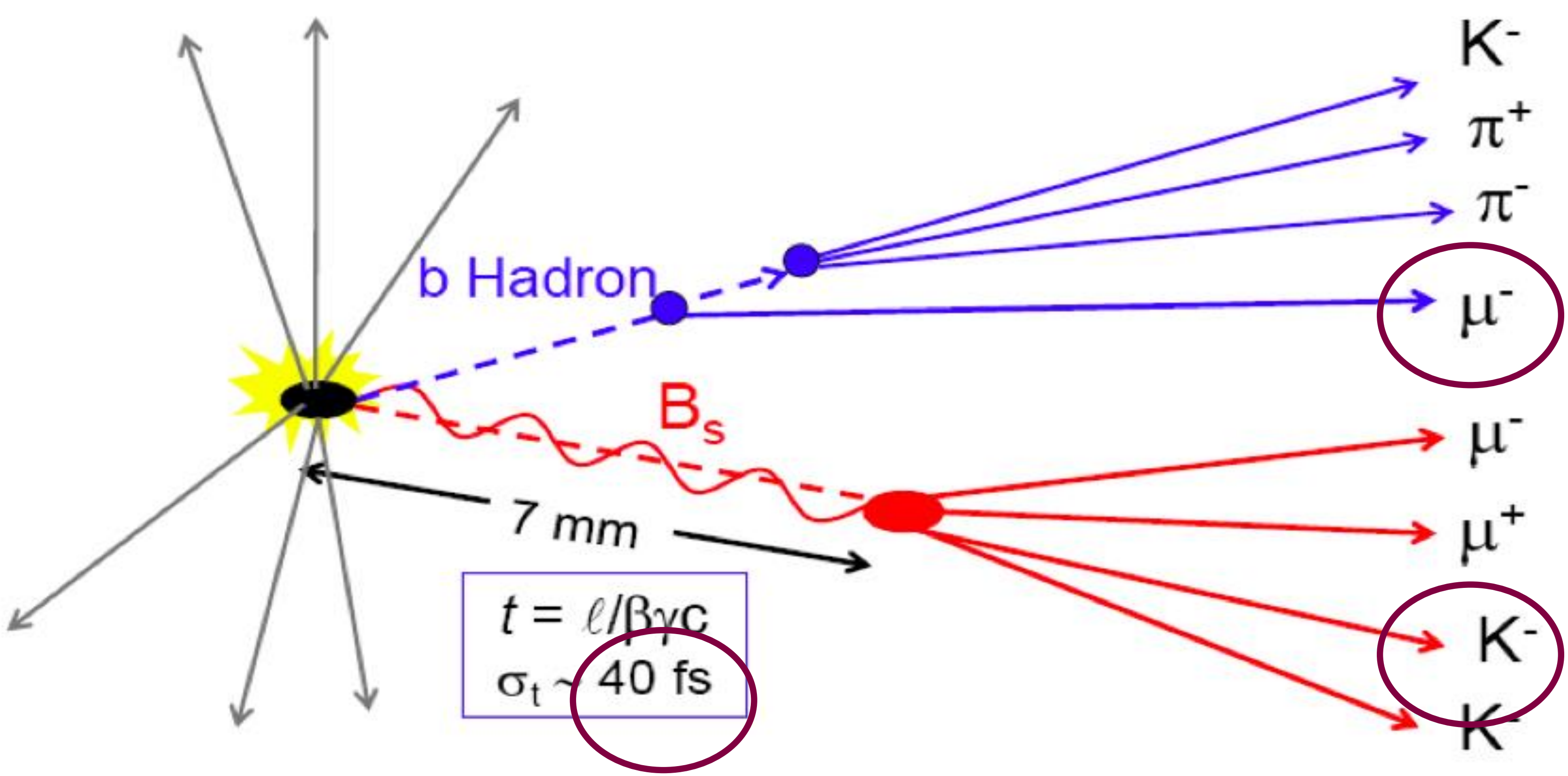
$\bar{b}$
q=u,d,s,c

$\bar{b}$
q_1
q_2

all types of b-hadrons can be produced :



Incoherent $B\bar{B}$ production : a B^0 and a B- for example



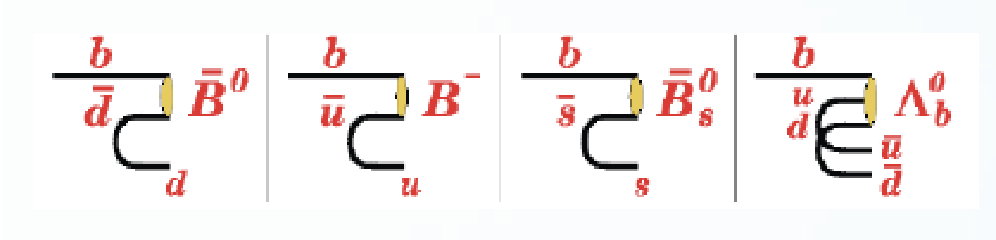
lifetime of a B : 1500 fs

Drives the detector design :

- ability to reconstruct the B vertex and to measure its decay time
- K/π discrimination
- μ identification

All this is similar to (super)B-Factories, but with different kinematic ranges

What is not similar to (super)-B-Factories :



All type of b-hadrons are produced at the LHC

Probability that a b quark hadronize a into a $B_{u,d,s}$ meson or a Λ_b baryon.

Important input for BR measurements since most of the measurements are done relative to another well known BR (B-Factories)

Cross sections at 14 TeV:

Total	100 mb
Inelastic	80 mb
$c\bar{c}$	3.5 mb
$b\bar{b}$	500 μb

x 160

A trigger is needed to:
- reject the light flavours (u,d,s)
- keep only the interesting events

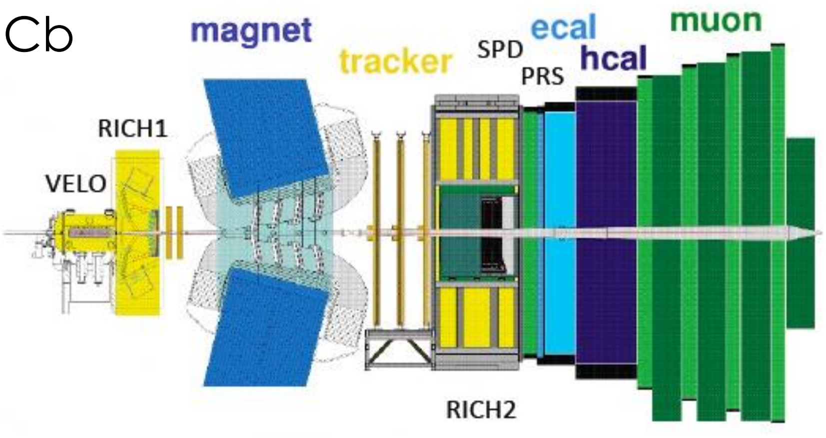
In 1 every 200 collisions a b-bbar pair is produced bb production cross section is huge : 290 mb ….

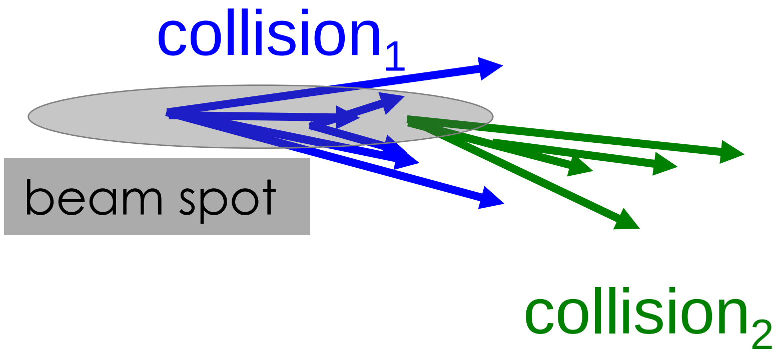
but the inelastic cross section is about 300 times larger

L limited to 4 10^{32} cm^{-2} s^{-1} to stay with a limited number of primary vertices

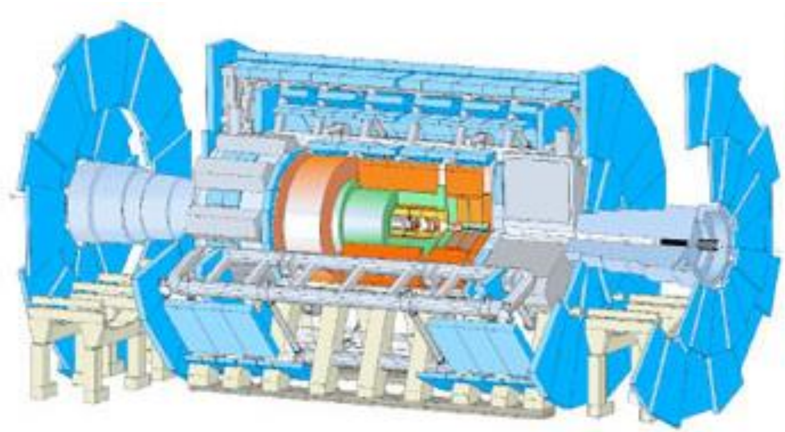
LHCb cannot deal with 30-40 interactions as ATLAS/CMS :

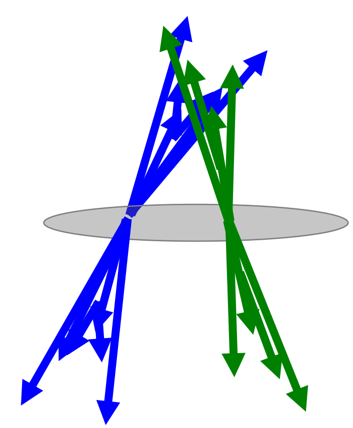
LHCb



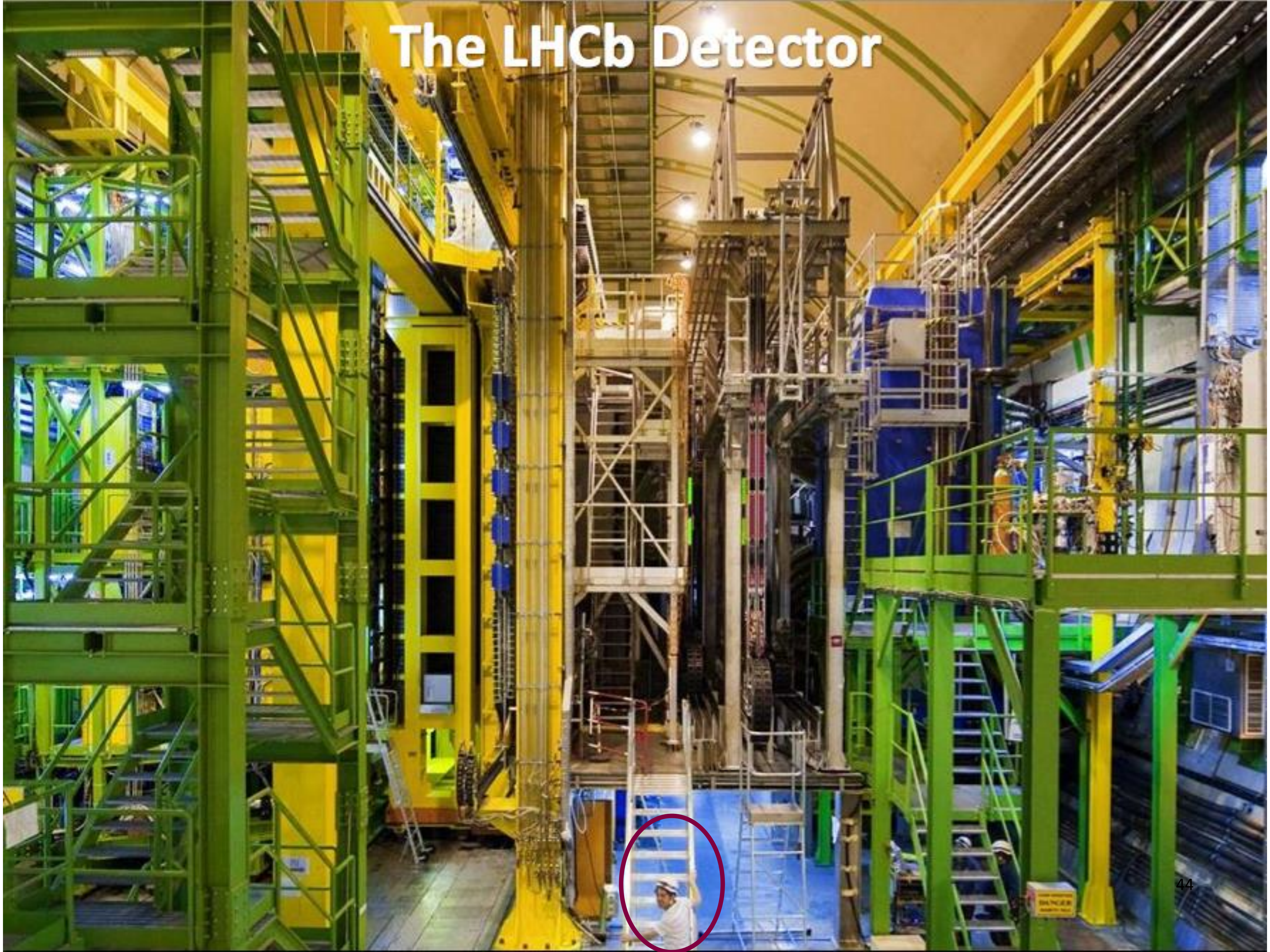


ATLAS/CMS

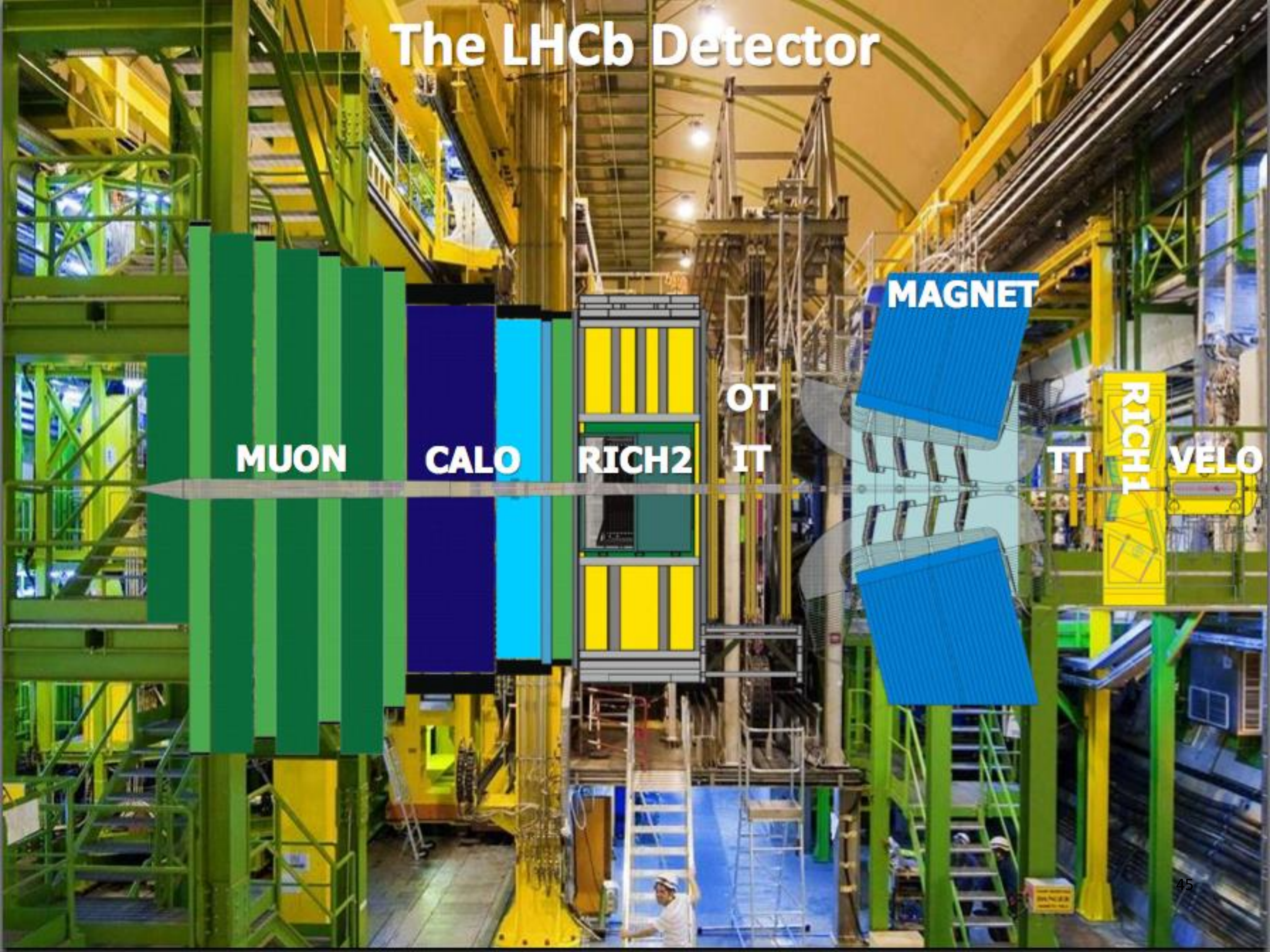




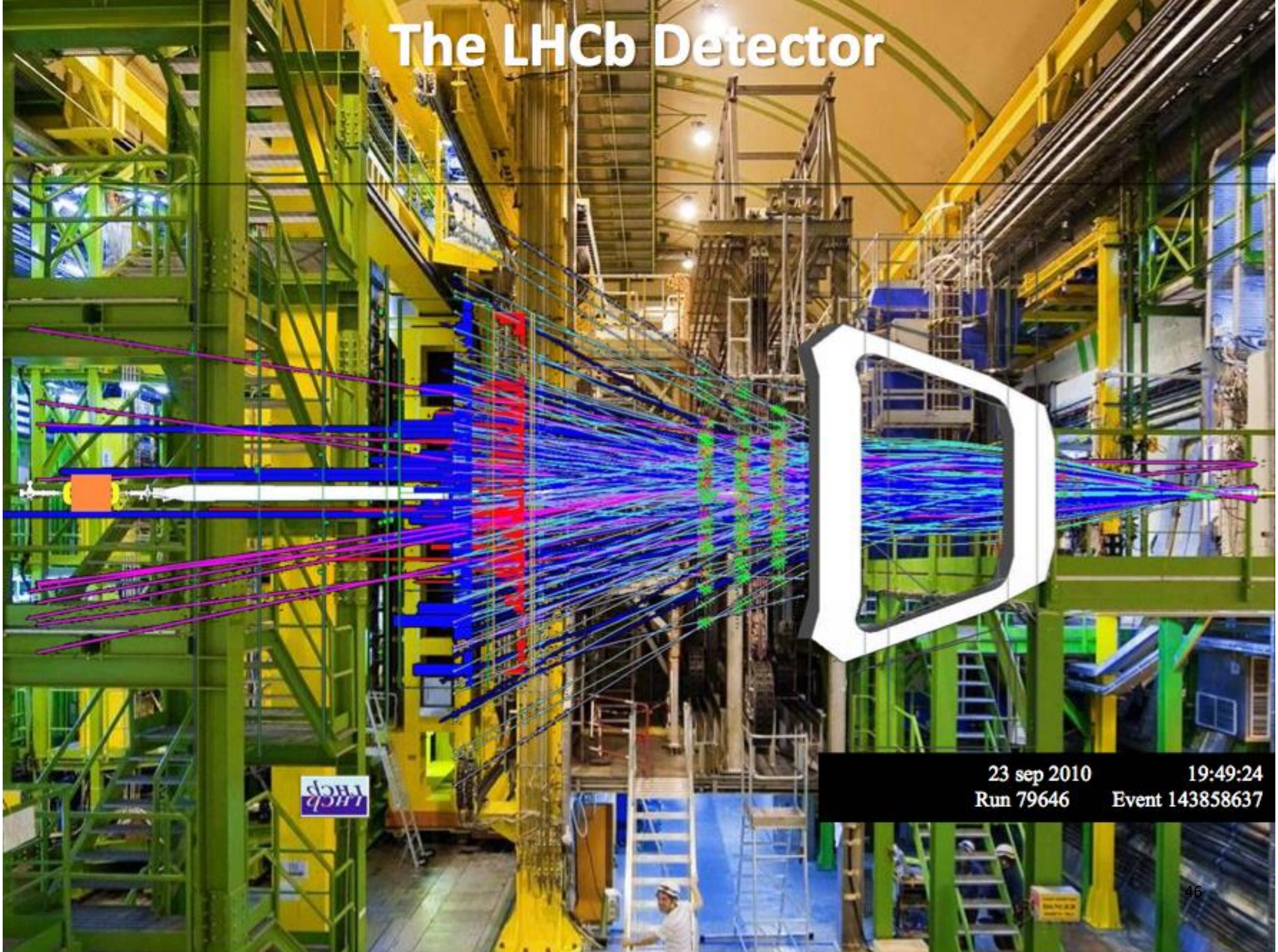
The LHCb Detector



The LHCb Detector

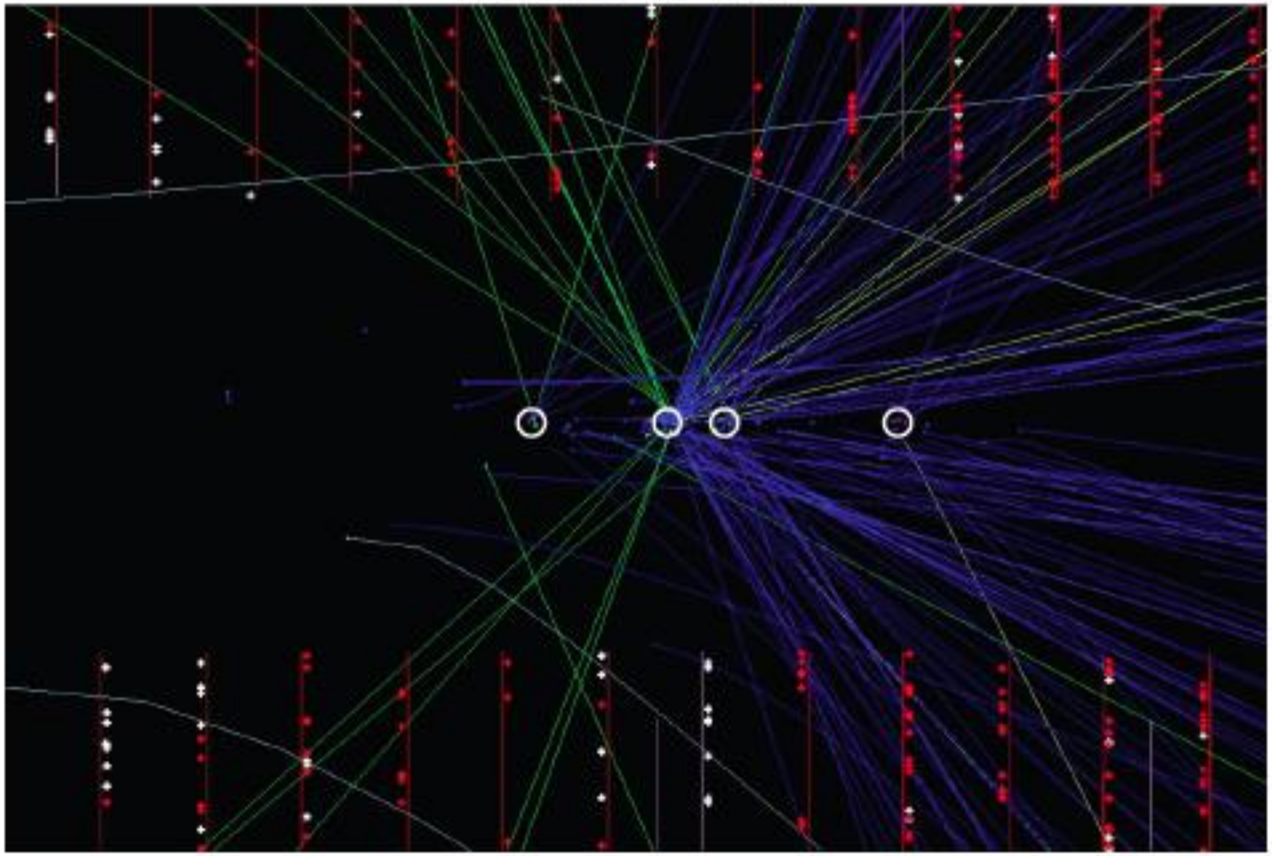


The LHCb Detector



Event at LHCb

VELO rz view



In order to record as much data as possible : “luminosity leveling”

$$\frac{dN}{dt} = L \times \sigma$$

$$L = \frac{kfN_1N_2}{4\pi s_x s_y}$$

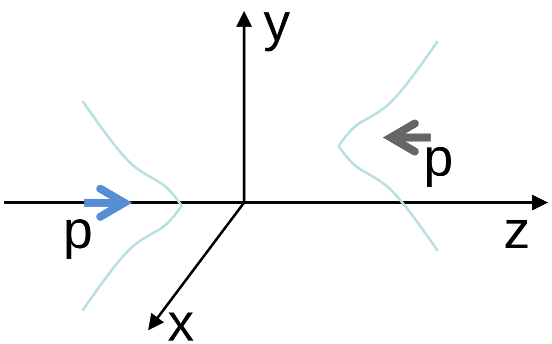
k bunches

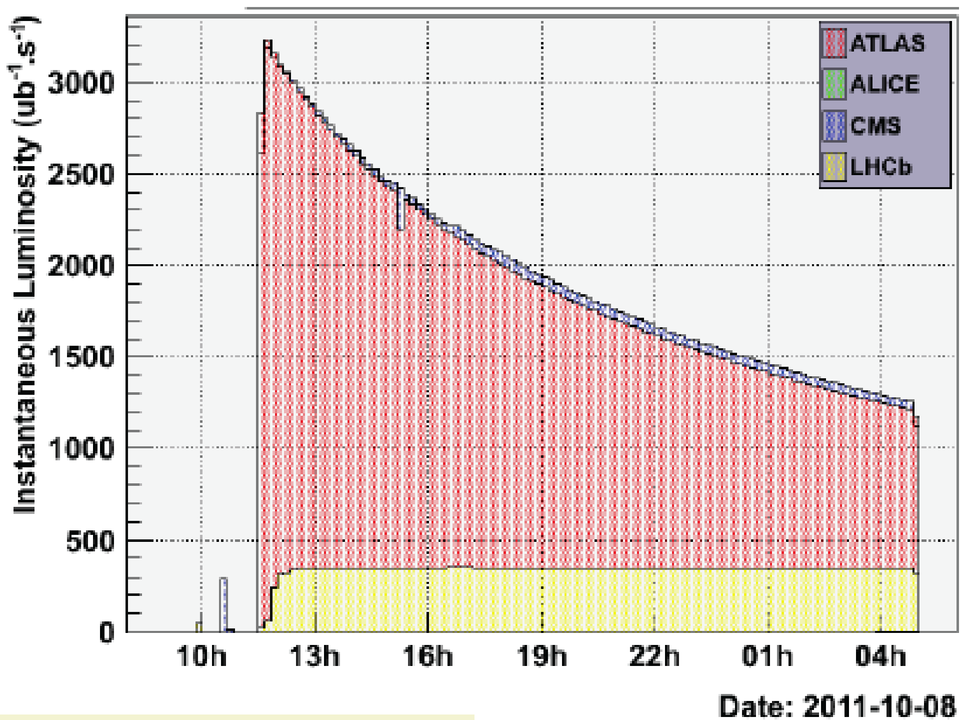
f frequency

N_1 : number of protons in a bunch

N_2 : number of protons in a bunch

$$\rho_{1/2}(x,y) = \frac{1}{2\pi s_x s_y} e^{-\frac{x^2}{2s_x^2}} e^{-\frac{y^2}{2s_y^2}}$$

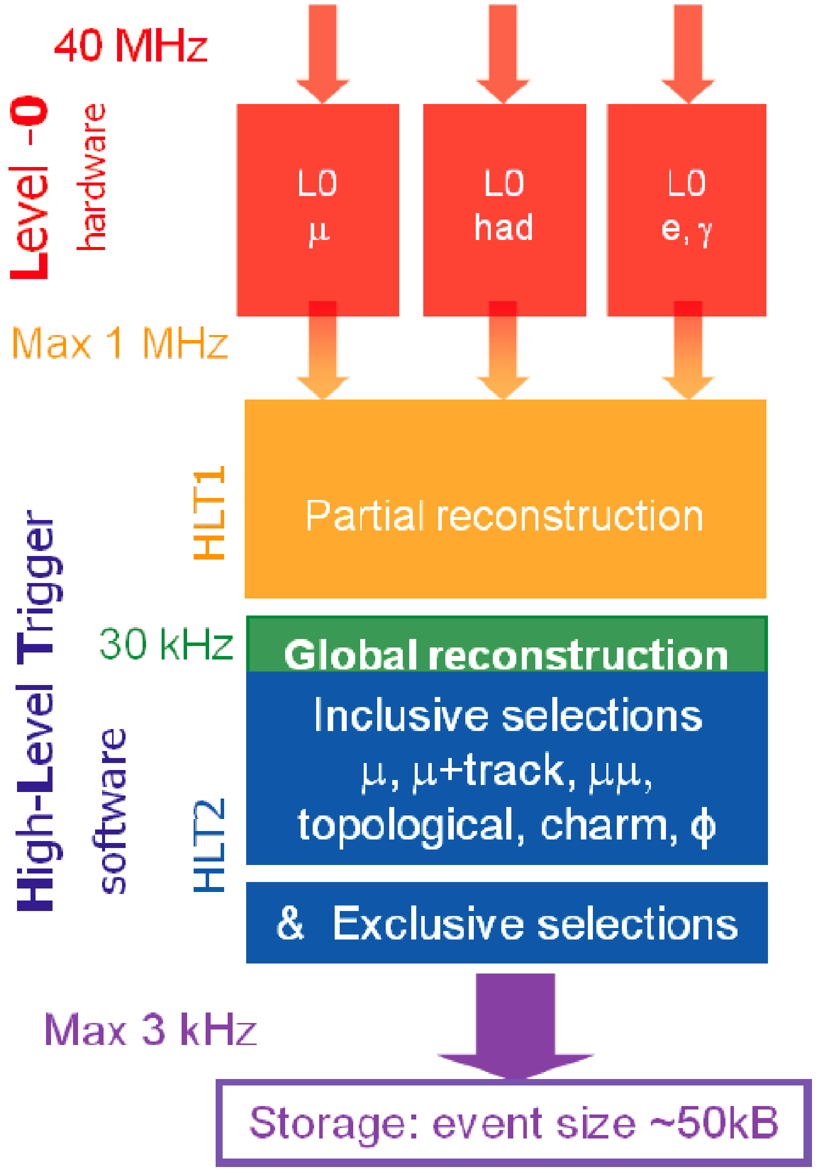


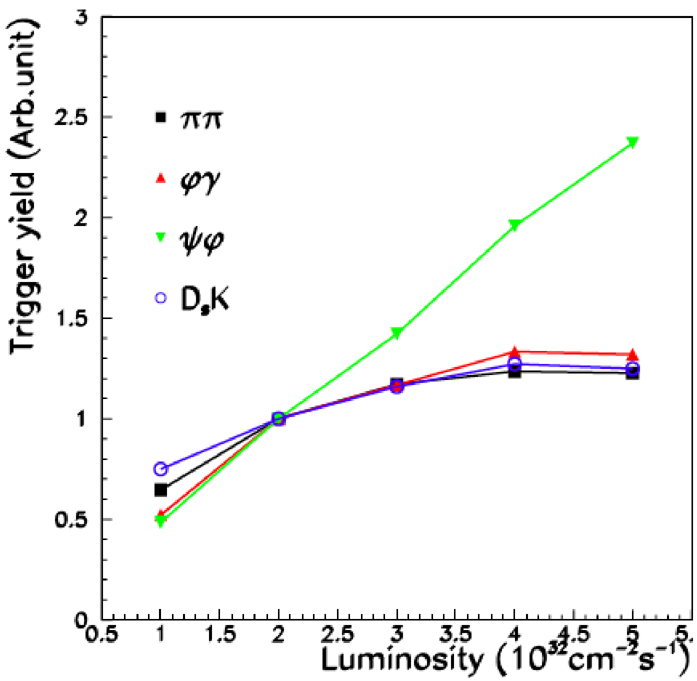


luminosity decreases as a function of time (loss of particles) : ATLAS CMS

except if one moves the beams (LHCb)

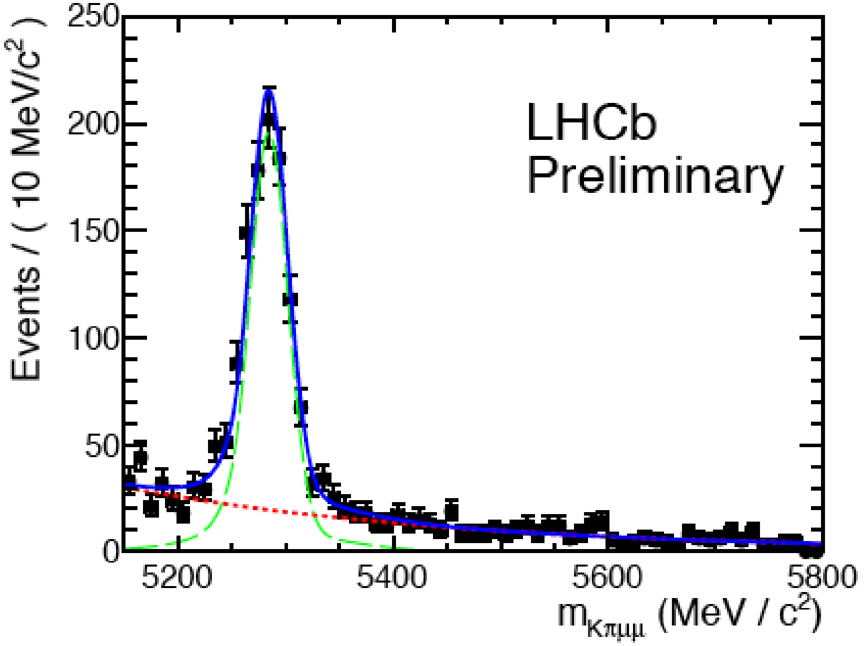
bb production cross section is huge : 290 μb ….
but the inelastic cross section is about 300 times larger
Should trigger on interesting events





Physics consequences : signal selection

At the LHC : ‘standard procedure’ : use the B invariant mass



At B factories : use the additional Υ(4S) constraint. The Υ(4S) decays into 2 B mesons at rest.

2 variables ΔE and m_{ES}
From the lab frame boost all tracks back in the Υ(4S) rest frame where :

$$\sqrt{s} = 2E^{\star}_{\mathrm{beam}}$$

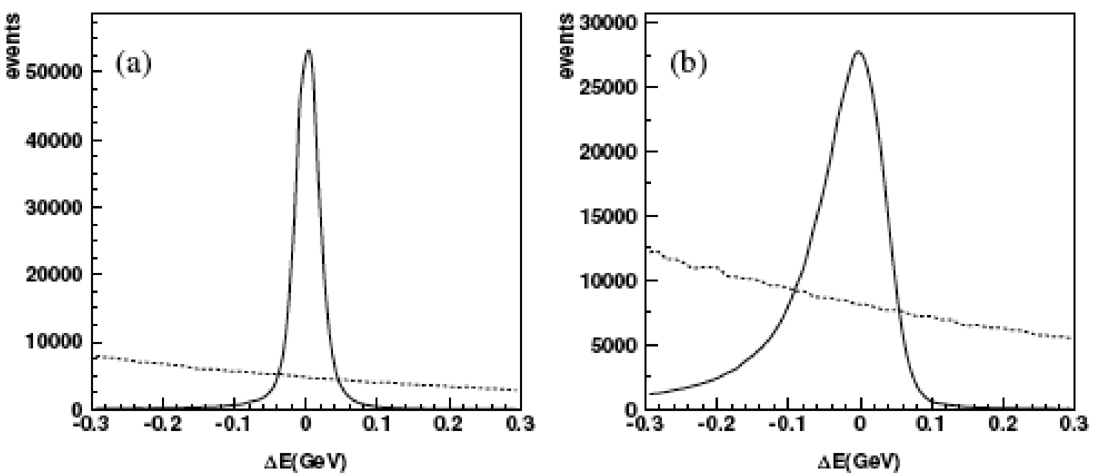
$$\Delta E = E^{\star}_{B} - E^{\star}_{\mathrm{beam}}, \qquad \sigma^2_{\Delta E} = \sigma^2_{E^{\star}_{B}} + \sigma^2_{E^{\star}_{\mathrm{beam}}}$$

reconstruction (dominant) — beam energy spread

charged tracks only | charged tracks + neutral

ΔE

σ= 15 MeV



This is similar to what can be obtained from a standard invariant mass plot

However one can also use :

$$m_{\text{es}} = \sqrt{E_{\text{beam}}^{\star 2} - \boldsymbol{p}_B^{\star 2}},$$

from detector measurement

independent of the mass hypothesis of the particles

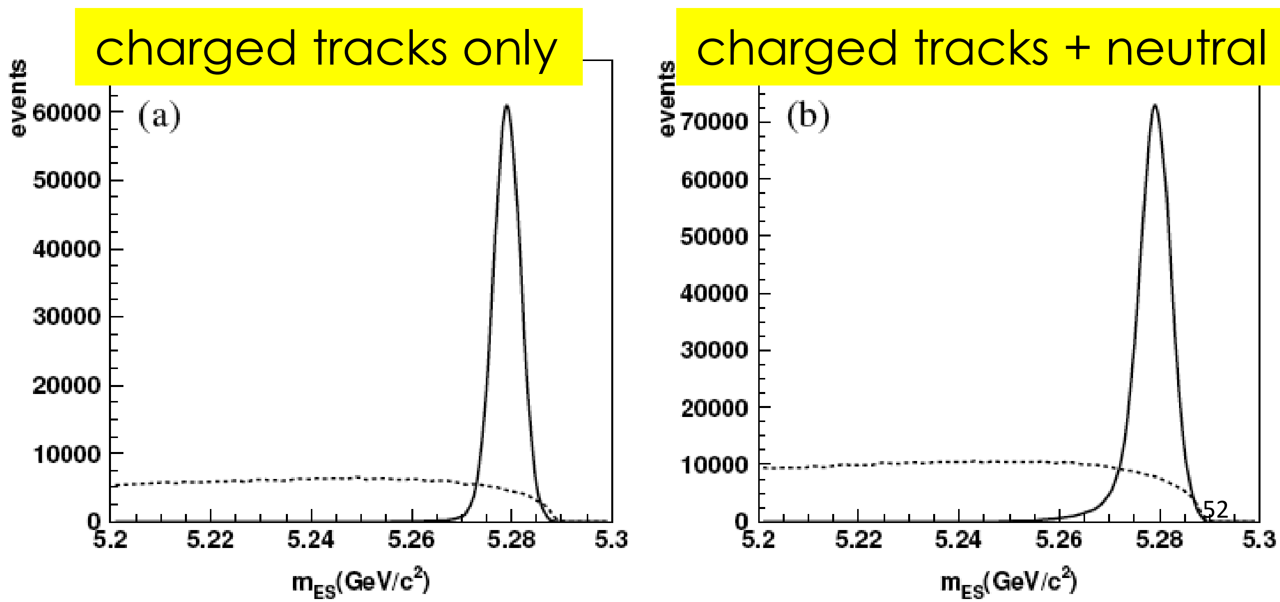
$$\sigma^2_{m_{\text{ES}}} \approx \sigma^2_{E^\star_B} + \left(\frac{p^\star_B}{m_B}\right)^2 \sigma^2_{p^\star_B}$$

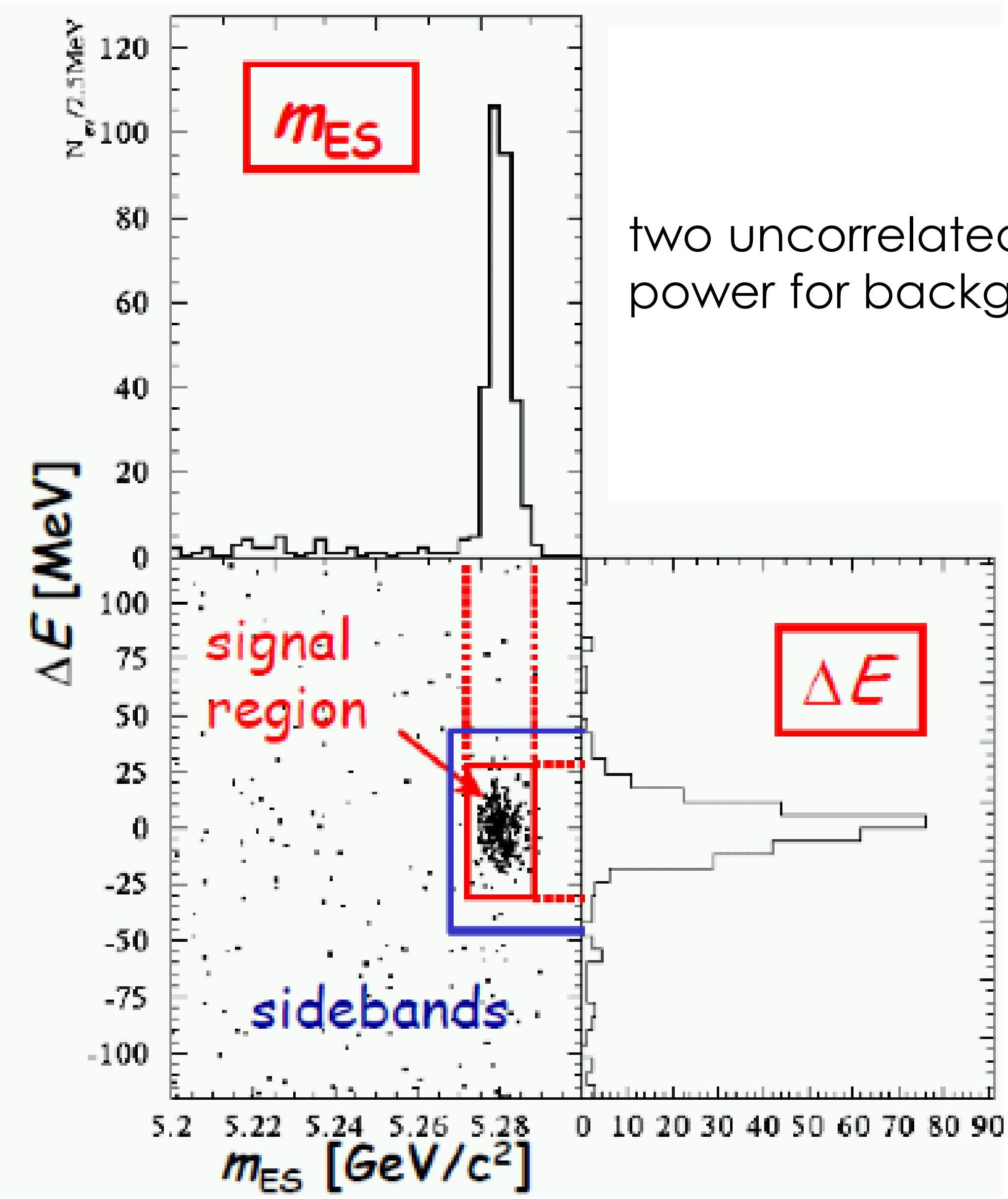
dominated by the beam energy knowledge

0.06^2

m_{ES}

σ= 3 MeV





two uncorrelated variables : additional power for background rejection

Physics consequences : full Breco

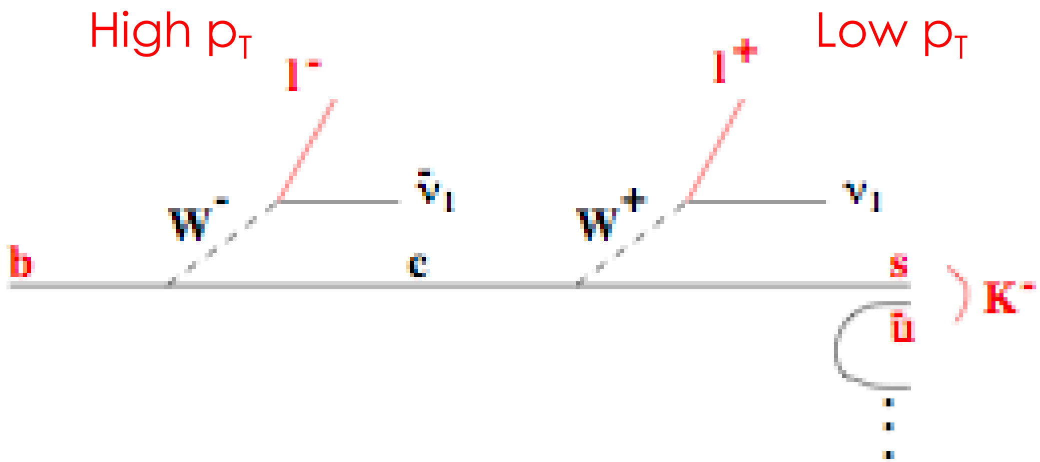
At B-Factories all the tracks are from the two B (no hadronization) :

Can reconstruct B then all the rest is from the other one

=> allow to perform very delicate analyses with neutrinos.

Physics consequences : tagging

Tagging : determination of the flavour of the B (B or $\overline{B}$) at the production time

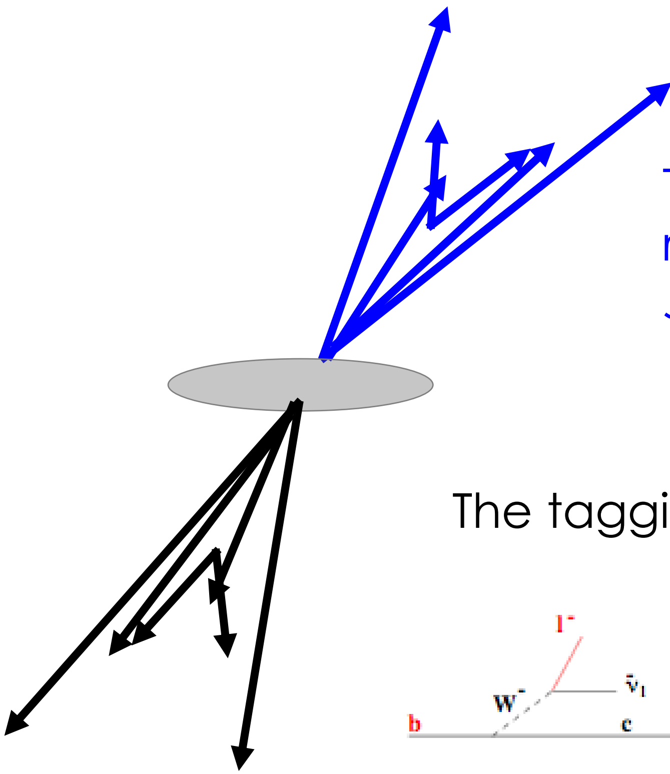


The charge of the lepton or of the kaon gives information on the b :

a high p_T l^- or a K^- probably come from a b quark (and thus a $\overline{B}$ meson)

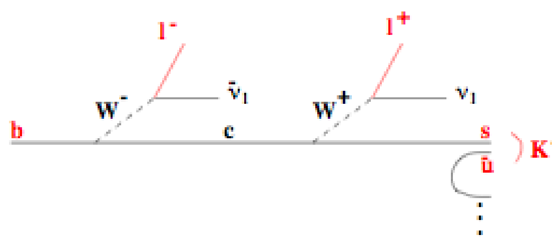
a high p_T l^+ or a K+ probably come from a $\overline{b}$ quark (and thus a B meson)

Two main techniques : Opposite Side Tagging or Same Side Tagging.

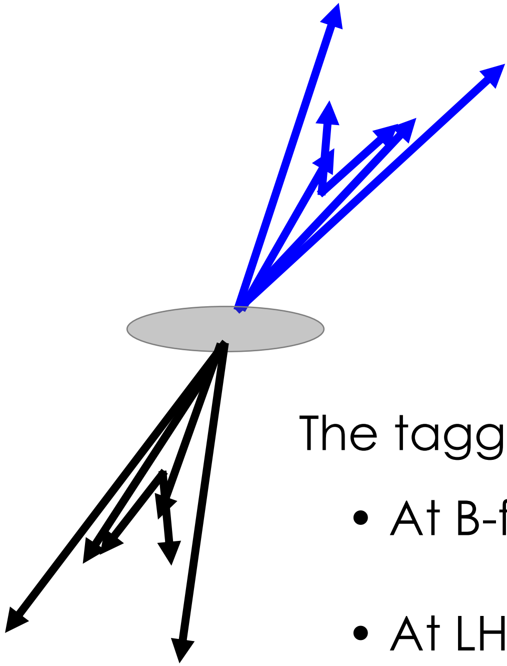


The B meson fully reconstructed (eg $D^{*+}\pi$, J/Ψ Ks ….)

The tagging B



This is opposite side tagging.
It can be performed both at B-factories and LHC, but fundamental differences due to the production mechanism

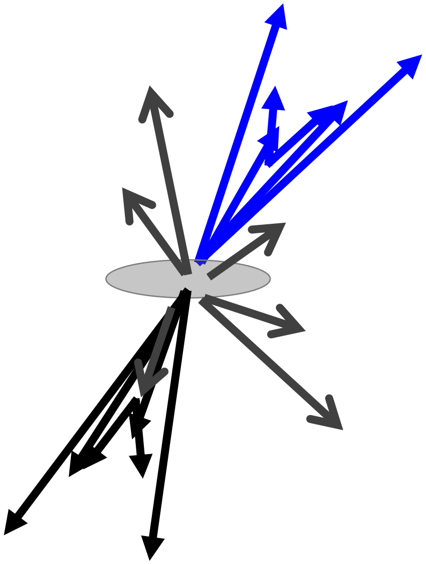


The B meson fully reconstructed (eg $D^{*+}\pi$, J/Ψ Ks ….)

The tagging B

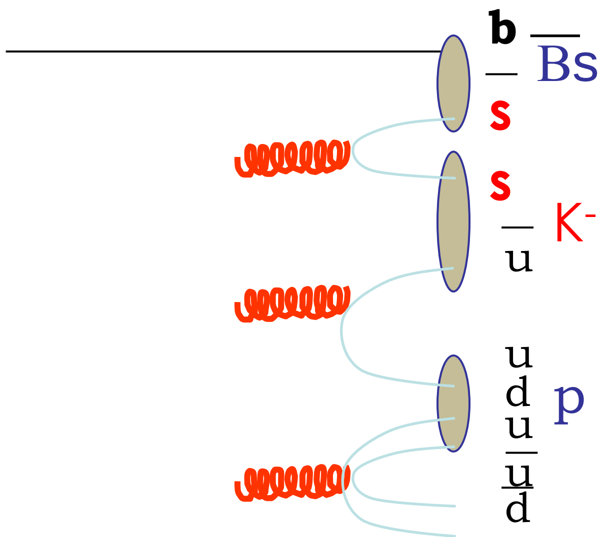
- At B-factories : coherent $B^0 \bar{B}^0$ production

- At LHC if a $\bar{B}^0$ is produced, at the same time one can have at the same time a Bs, a B+ , a Λ_b
The Bs oscillates many time before decaying and does not keep track of its flavour at the production time : information is lost



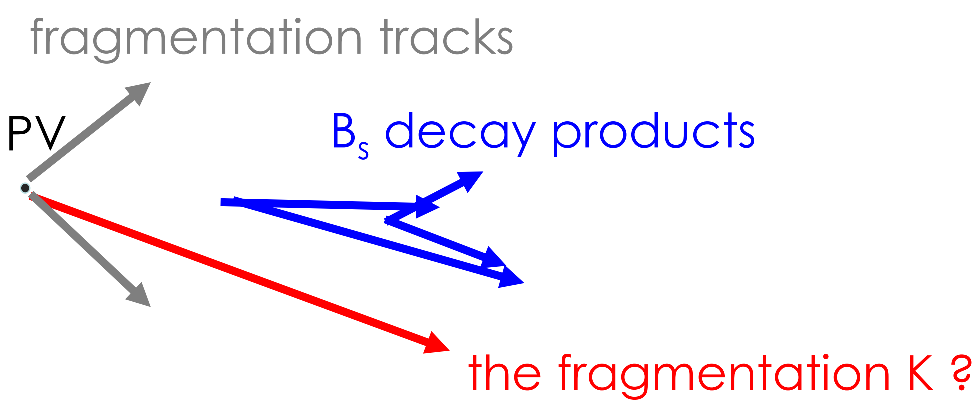
In addition at LHC they are all the fragmentation tracks and the tracks from the other interaction

The fragmentation tracks can however helps the tagging : Same Side Tagging



Search for a track attached to the primary vertex (not to the B decay vertex), close to the B and not too slow

cannot be done at B-factories !



Tagging performances :

$$Q = \varepsilon(1-2\omega)^2 = \varepsilon D^2$$

tagging efficiency ε

mistag probability w ('wrong')

QxN : equivalent number of events perfectly tagged

B-Factories typical result (here BaBar)

Category	ε(%)	ω(%)	Q(%)
Lepton	8.6±0.1	3.2±0.4	7.5±0.2
Kaon I	10.9±0.1	4.6±0.5	9.0±0.2
Kaon II	17.1±0.1	15.6±0.5	8.1±0.2
K-π	13.7±0.1	23.7±0.6	3.8±0.2
Pion	14.5±0.1	33.9±0.6	1.7±0.1
Other	10.0±0.1	41.1±0.8	0.3±0.1
Total	**74.9±0.2**		**30.5±0.4**

LHCb (Tevatron similar)

Taggers	ε_{tag} (%)	ω (%)	$\varepsilon_{tag}\cdot(1-2\omega)^2$ (%)
μ	4.8±0.1	29.9±0.7	0.77±0.07
e	2.2±0.1	33.2±1.1	0.25±0.04
K	11.6±0.1	38.3±0.5	0.63±0.06
Q_{vtx}	15.1±0.1	40.0±0.4	0.60±0.06

Total : 2.3 %

SSK tagging adds about 1.3 %

1000 events reconstructed are equivalent to

- 300 perfectly tagged at B-Factories
- 30 perfectly tagged at LHCb/Tevatron colliders

Putting all together : comparison

	$\sigma(b\bar{b})$	$\sigma(\text{inel})/\sigma(bb)$	$\int L dt$	Number of B produced in the detector acceptance
LHCb	~290 μb	~300	1fb^{-1} (2011) + 2fb^{-1} (2012) +	150 10^9 b bbar pairs (2011)
BaBar BELLE	~1 nb	~4	425 fb^{-1} (BaBar) 700 fb^{-1} (BELLE)	1.1 10^9 B Bbar pairs

Super B factories :
~ 80 10^9 B Bbar pairs

But for LHCb

- trigger efficiency : from 90-95 % efficiency to 30 % efficient depending on the mode
- acceptance : depends on the decay mode (40% - 20%)

- for mode requiring tagging : a factor 1/10 wrt B-factories for LHCb

1. What is the value of the B lifetime ?
   What is the average path in a detector of a B meson with boost of 10?

2. Do you understand why the lifetime of a D meson is smaller than the lifetime of a B meson ?

3. Why the B-factory have two asymmetric beams ?

4. What is the observable of the meson oscillation ?

5. x defined as $\cos \Delta m t = \cos\left(\frac{\Delta m}{\Gamma}\right)\left(\frac{t}{\tau}\right) \quad ; \quad x \equiv \left(\frac{\Delta m}{\Gamma}\right)$

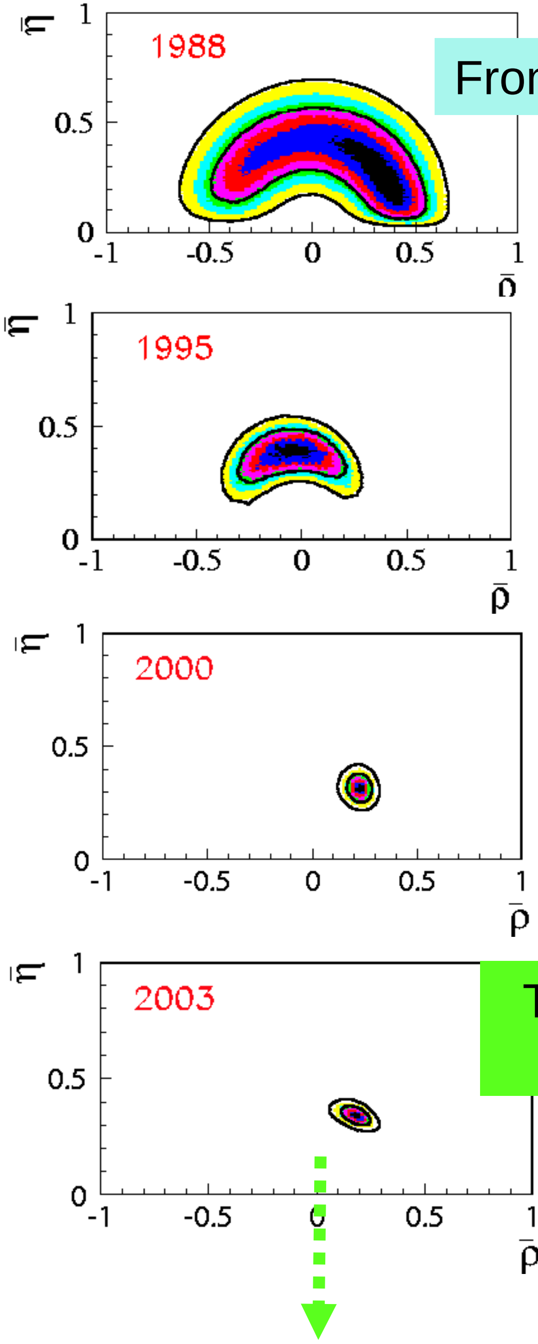
   x is small for K, intermediate for Bd, large for Bs.
   Which is the most difficult to measure ?

6. CP violation is observed in K and B and « suspected » in D sector.
   Does it come from the same CKM matrix element ?

APPENDIX IV

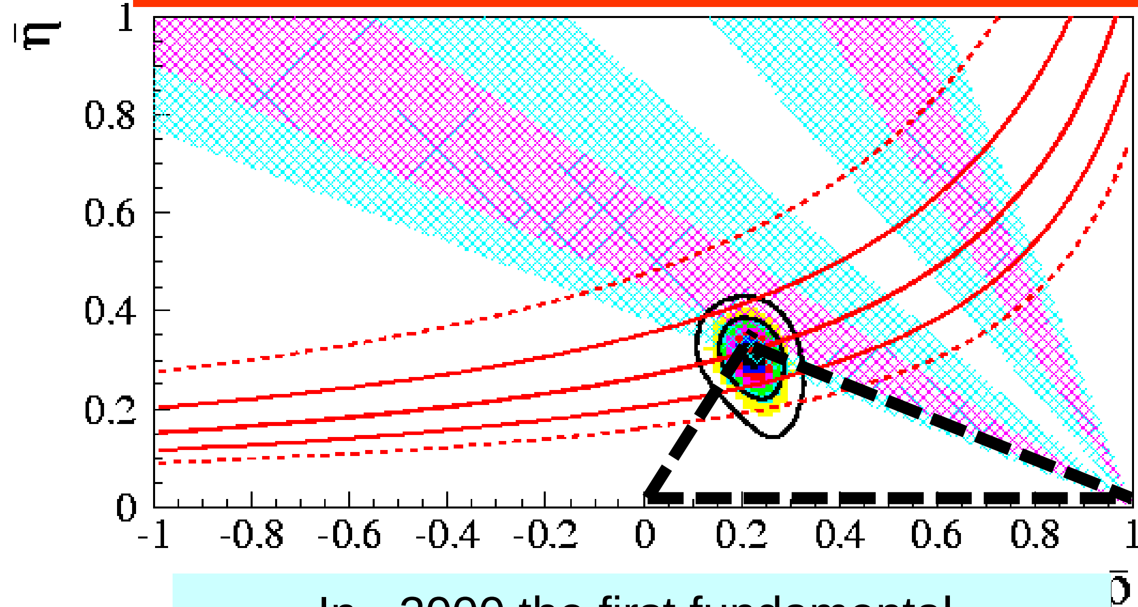
More on CKM

From Childhood



To precision era

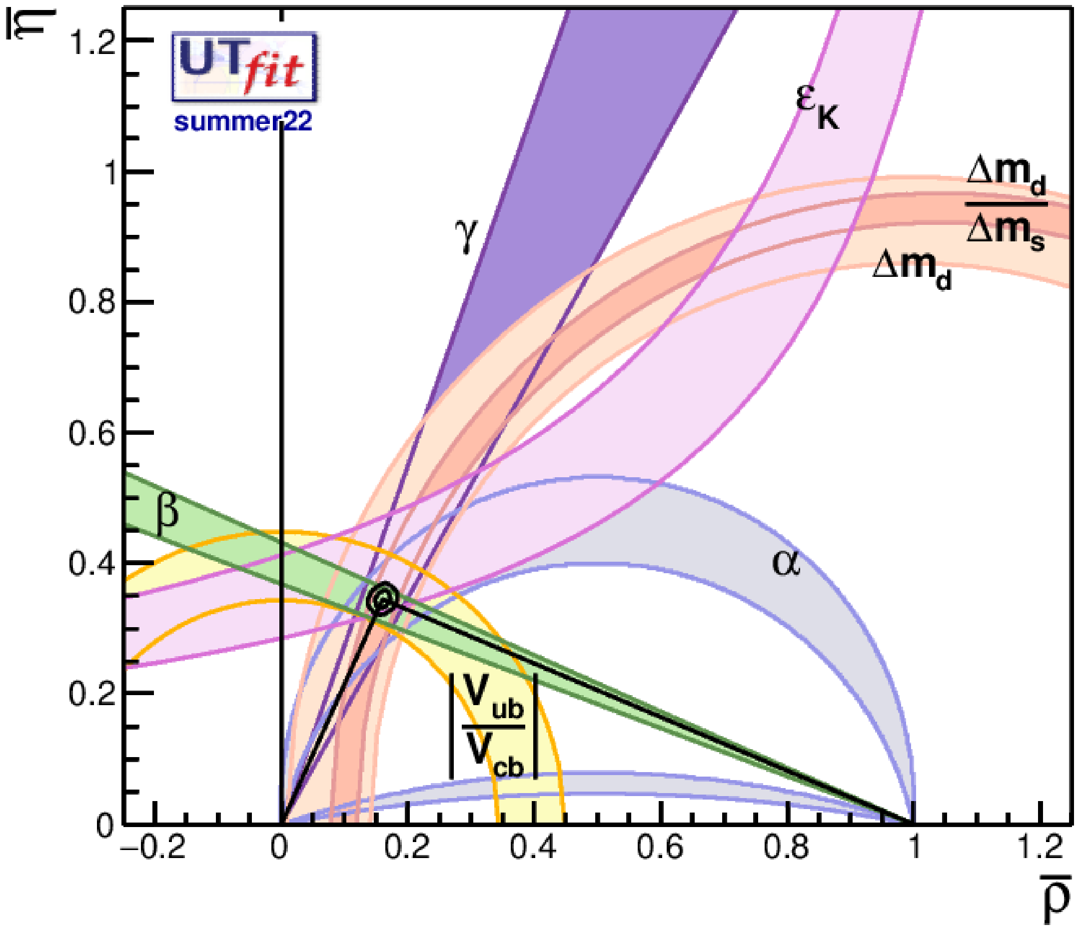
Dominated by
Δm_d, V_{ub}, V_{cb}, εK, limit on Δm_s and Lattice



In ~2000 the first fundamental
test of agreement between
direct and indirect measurements of sin2β

Unitarity Triangle analysis in the SM:

zoomed in..



levels @
95% Prob

~6%

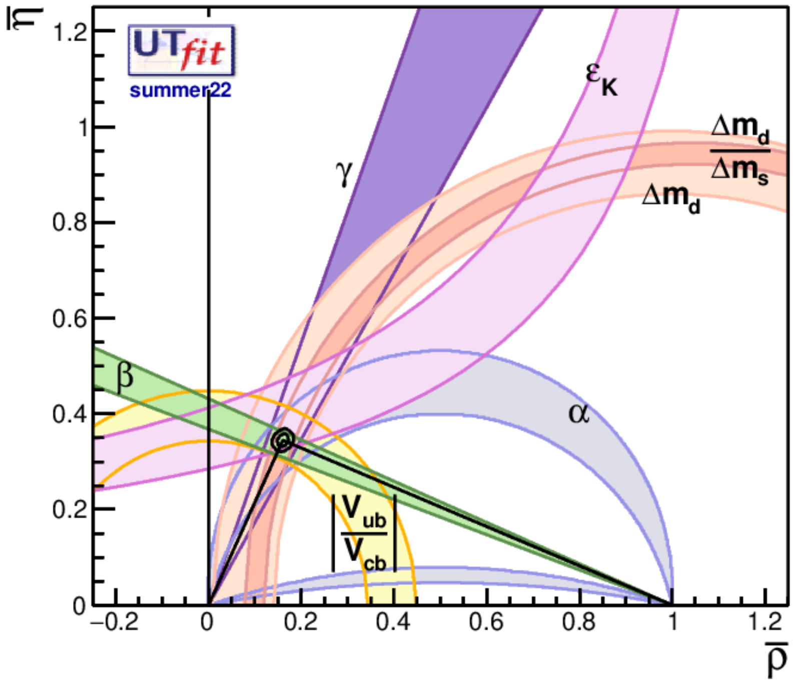
$\rho = 0.160 \pm 0.009$

$\eta = 0.345 \pm 0.009$

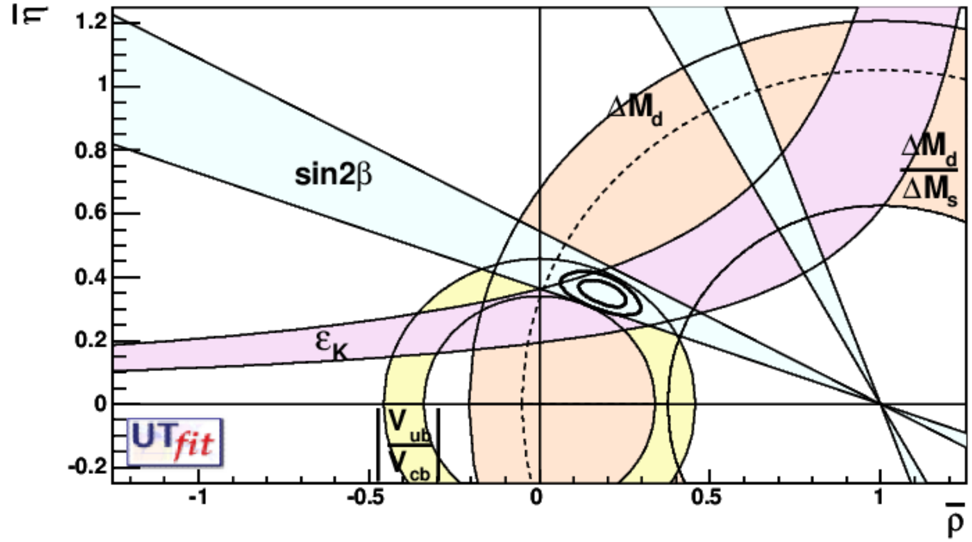
~3%

Unitarity Triangle analysis in the SM:

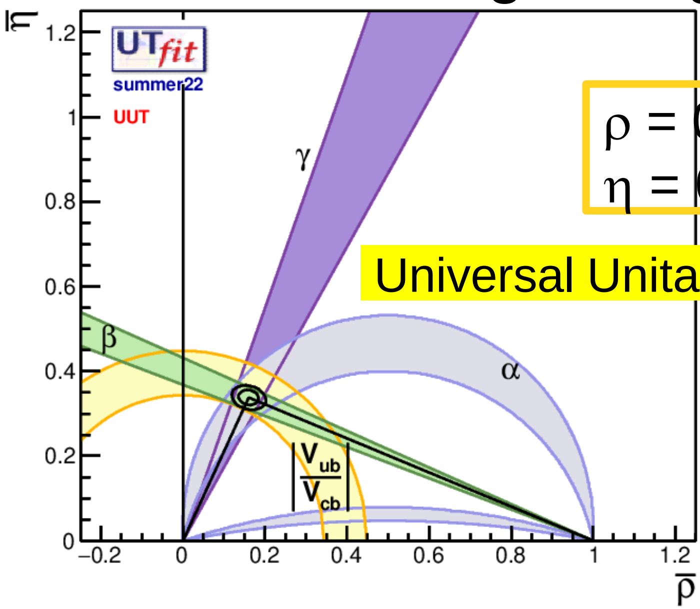
2022



2004



Some interesting configurations

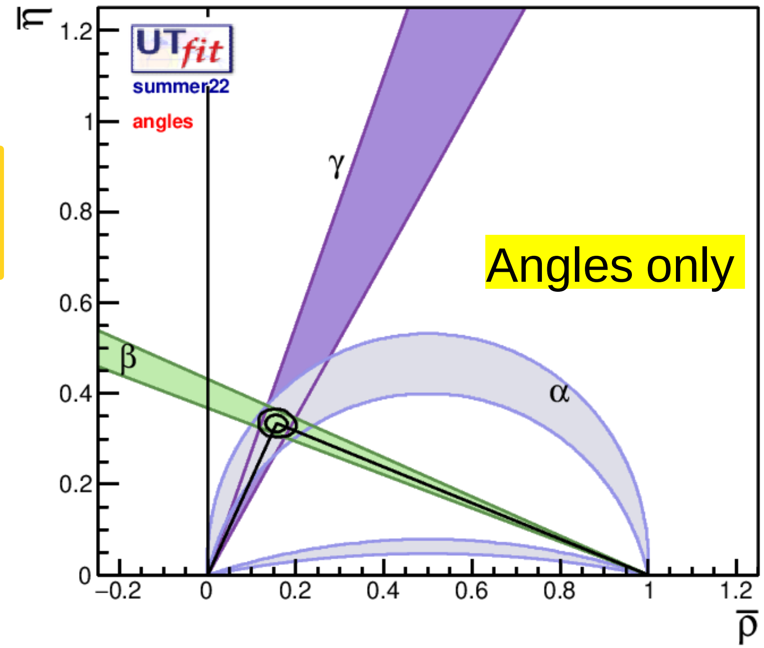


~11%

$\rho = 0.160 \pm 0.017$
$\eta = 0.338 \pm 0.011$

~3%

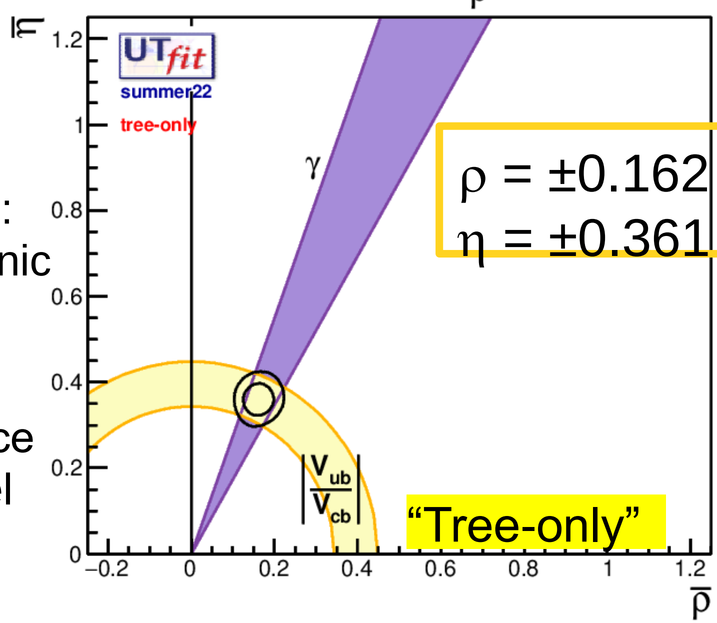
Universal Unitary Triangle



Angles only

Tree-level processes: Semileptonic and DK B decays

→ reference for model building

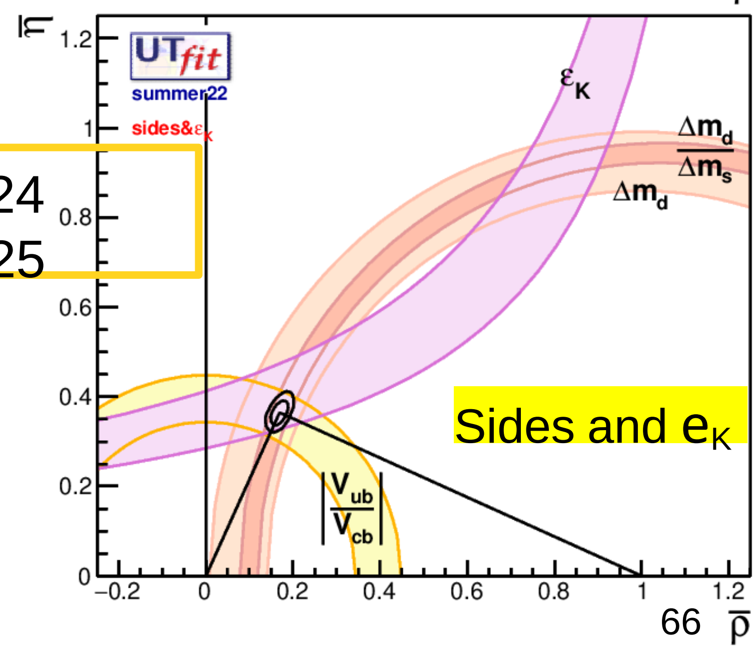


~15%

$\rho = \pm 0.162 \pm 0.024$
$\eta = \pm 0.361 \pm 0.025$

~7%

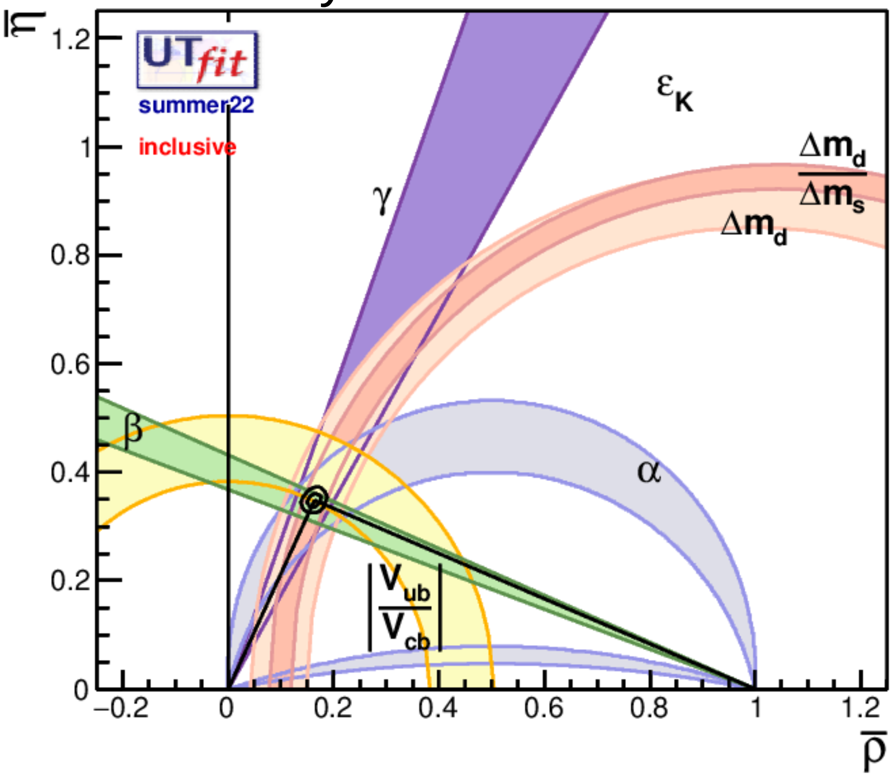
"Tree-only"



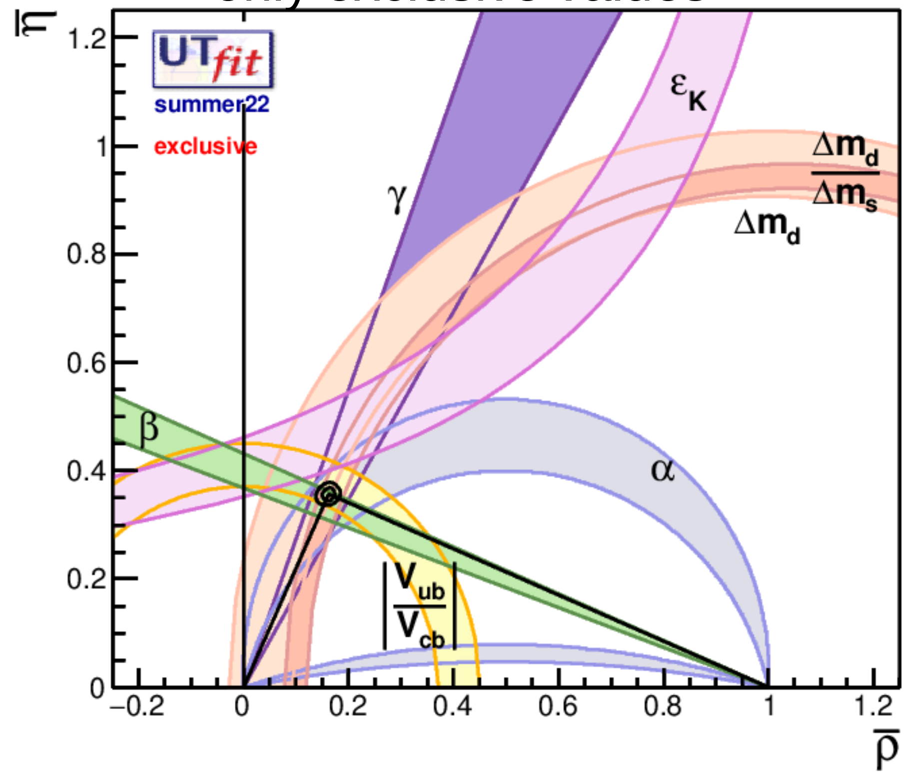
Sides and e_K

Inclusive vs Exclusive

only inclusive values



only exclusive values



$$\rho = 0.164 \pm 0.009$$
$$\eta = 0.348 \pm 0.009$$
$$\sin 2\beta = 0.753 \pm 0.028$$

$$\rho = 0.162 \pm 0.009$$
$$\eta = 0.356 \pm 0.009$$
$$\sin 2\beta = 0.755 \pm 0.020$$

Details if you want ot see how it works

UT analysis including new physics

fit simultaneously for the CKM and
the NP parameters (generalized UT fit)

- add most general loop NP to all sectors
- use all available experimental info
- find out NP contributions to ΔF=2 transitions

B_d and B_s mixing amplitudes
(2+2 real parameters):

$$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}} = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$$

$$\Delta m_{q/K} = C_{B_q/\Delta m_K} (\Delta m_{q/K})^{SM}$$
$$A_{CP}^{B_d \to J/\psi K_S} = sin2(\beta + \phi_{B_d})$$
$$A_{SL}^q = \mathrm{Im}(\Gamma_{12}^q / A_q)$$

$$\varepsilon_K = C_\varepsilon \varepsilon_K^{SM}$$
$$A_{CP}^{B_s \to J/\psi\phi} \sim sin2(-\beta_s + \phi_{B_s})$$
$$\Delta\Gamma^q / \Delta m_q = \mathrm{Re}(\Gamma_{12}^q / A_q)$$

Details if you want ot see how it works

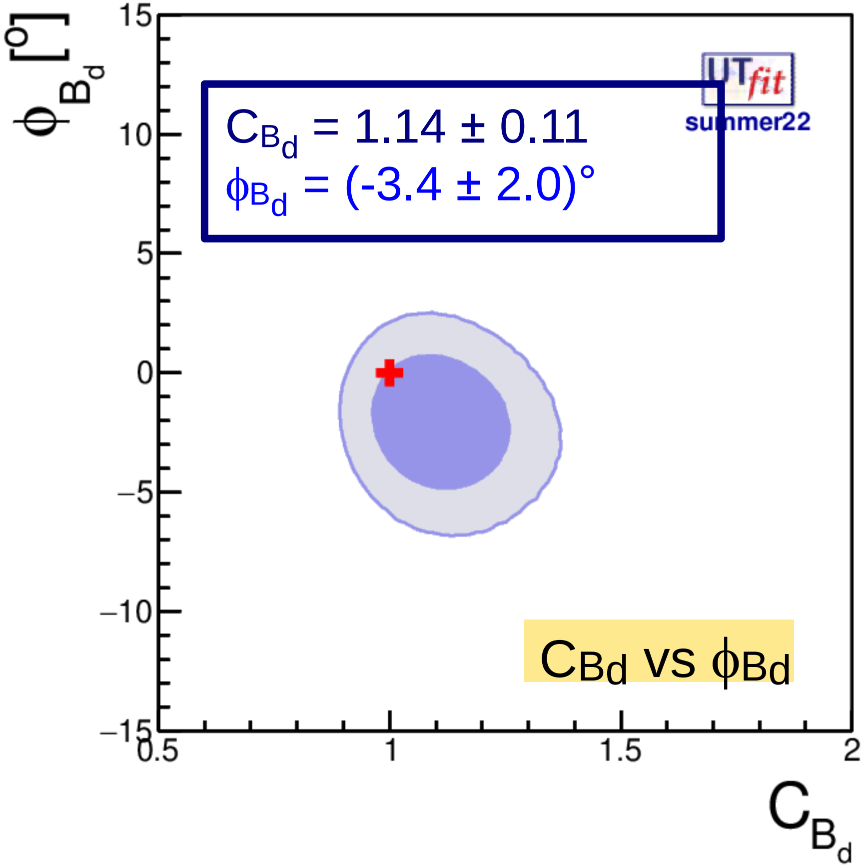
NP parameter results

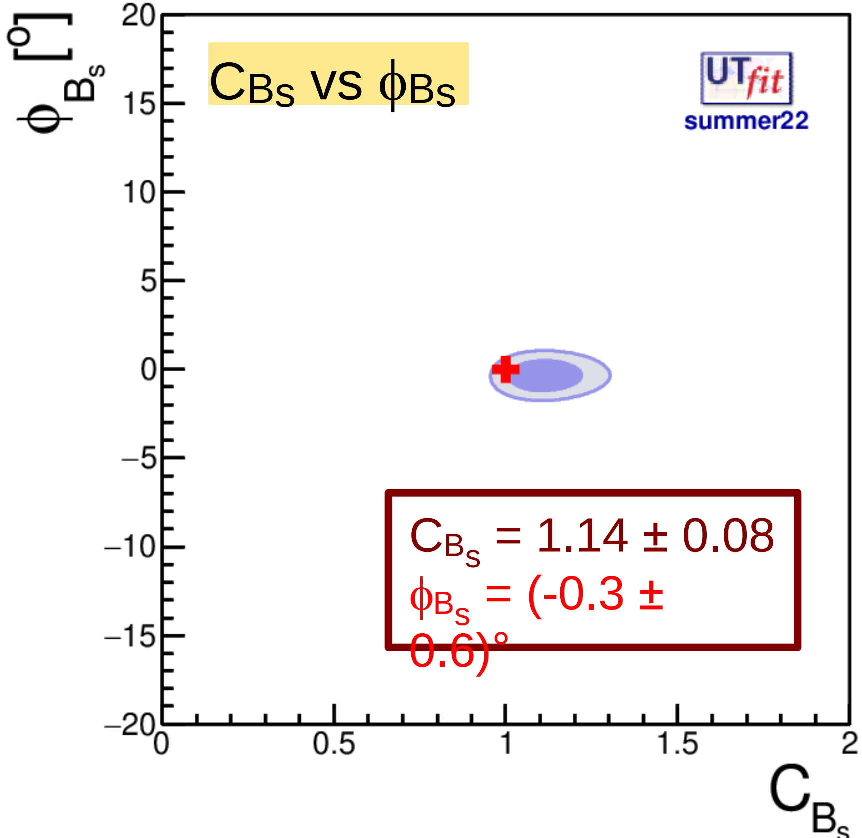
$$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}}$$

dark: 68%
light: 95%
SM: red cross

K system

$C_{\epsilon_K} = 1.12 \pm 0.12$

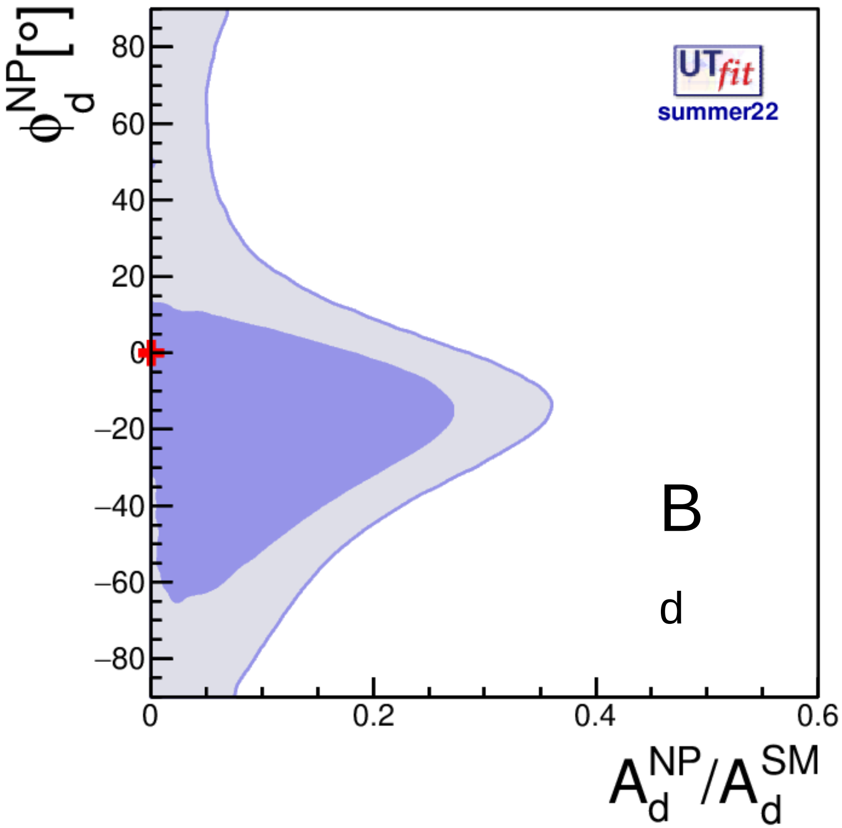


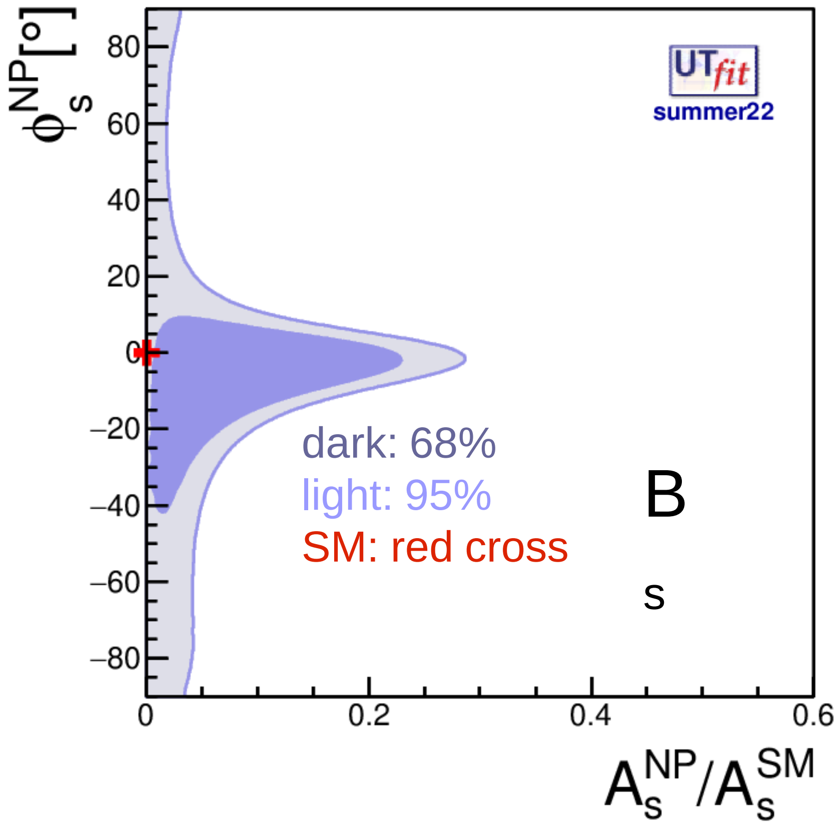


Details if you want ot see how it works

NP parameter results

$$A_q = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$$





The ratio of NP/SM amplitudes is:

< 25% @68% prob. (35% @95%) in B_d mixing

< 25% @68% prob. (30% @95%) in B_s mixing

To evaluate which constraint we can put on contributions from New Physics amplitudes is a delicate problem and often is Model dependent.

Out of these measurement there a general agreement that we have limited the contributions of New Physics amplitudes (A_{NP}) wrt to SM ones (A_{SM}) at the the level of

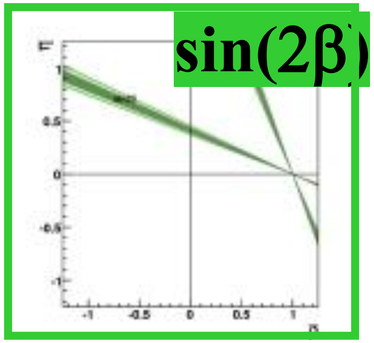
$$R = \frac{A_{NP}}{A_{SM}} < 20\%$$

What does it imply ?

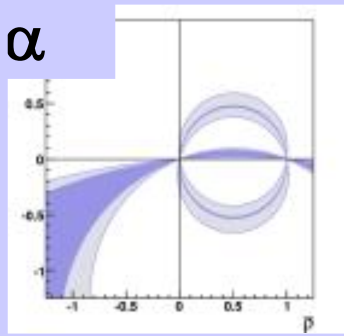
What happened since….

Many new (or more precise) measurements
to constraint UT parameters and test New Physics

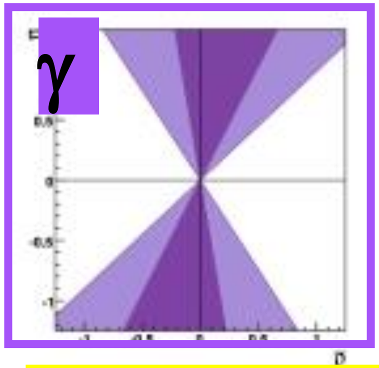
sin(2β)



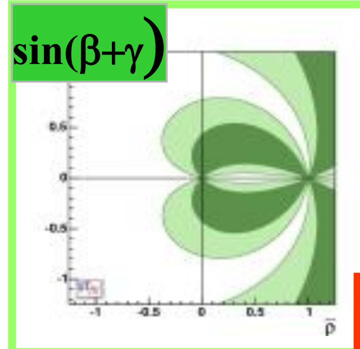
α



γ



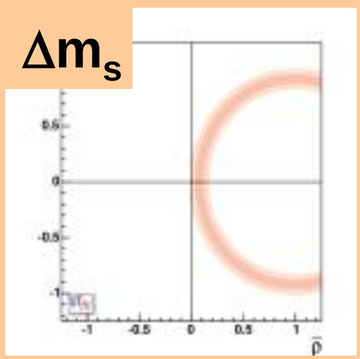
sin(β+γ)



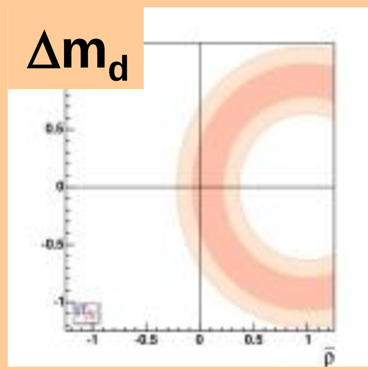
β_s

the angles..

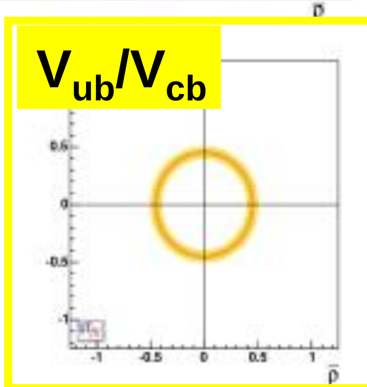
Δm_s



Δm_d

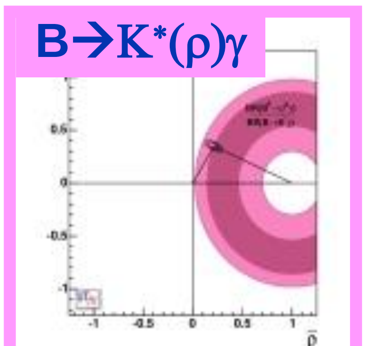


V_{ub}/V_{cb}

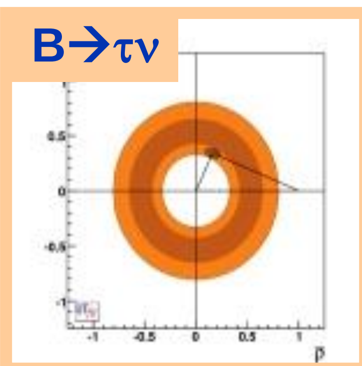


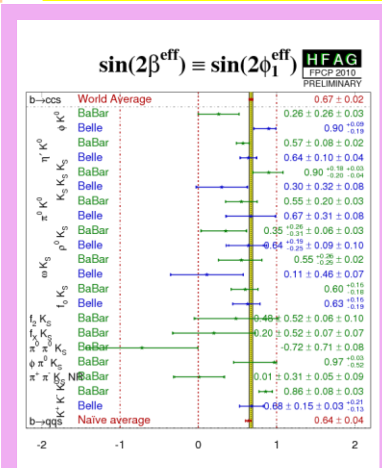
the sides…

B→K*(ρ)γ

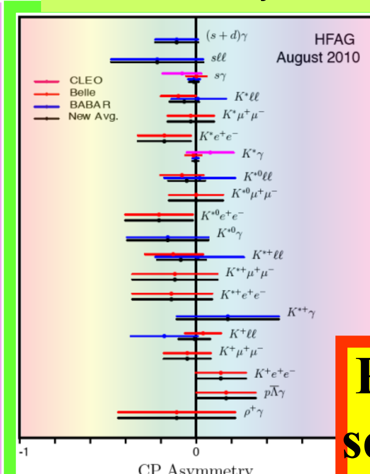


B→τν





CP asymmetries in radiative decays



…

Rare decays… sensitive to NP

Beyond the Standard Model with flavour physics

$$\frac{|\delta_{bq}|}{\Lambda_{eff}}$$

The indirect searches look for "New Physics" through virtual effects from new particles in loop corrections

1. ~1970 charm quark from FCNC and GIM-mechanism $K^0 \rightarrow \mu\mu$
2. ~1973 3rd generation from CP violation in kaon (ε_K) KM-mechanism
3. ~1990 heavy top from B oscillations Δm_B
4. >2010 success of the description of FCNC and CPV in SM

"Discoveries" and construction of the SM Lagrangian

- SM FCNCs and CP-violating (CPV) processes occur at the loop level
- SM quark Flavour Violation (FV) and CPV are governed by weak interactions and are suppressed by mixing angles.
- SM quark CPV comes from a single sources (if we neglect θ_{QCD})

New Physics does not necessarily share the SM behaviour of FV and CPV