Precise light quark masses from lattice QCD in the RI/SMOM scheme

Sebastian Jäger

University of Sussex

Workshop "Colour meets flavour"
in celebration of Alexander Khodjamirian's 60th birthday
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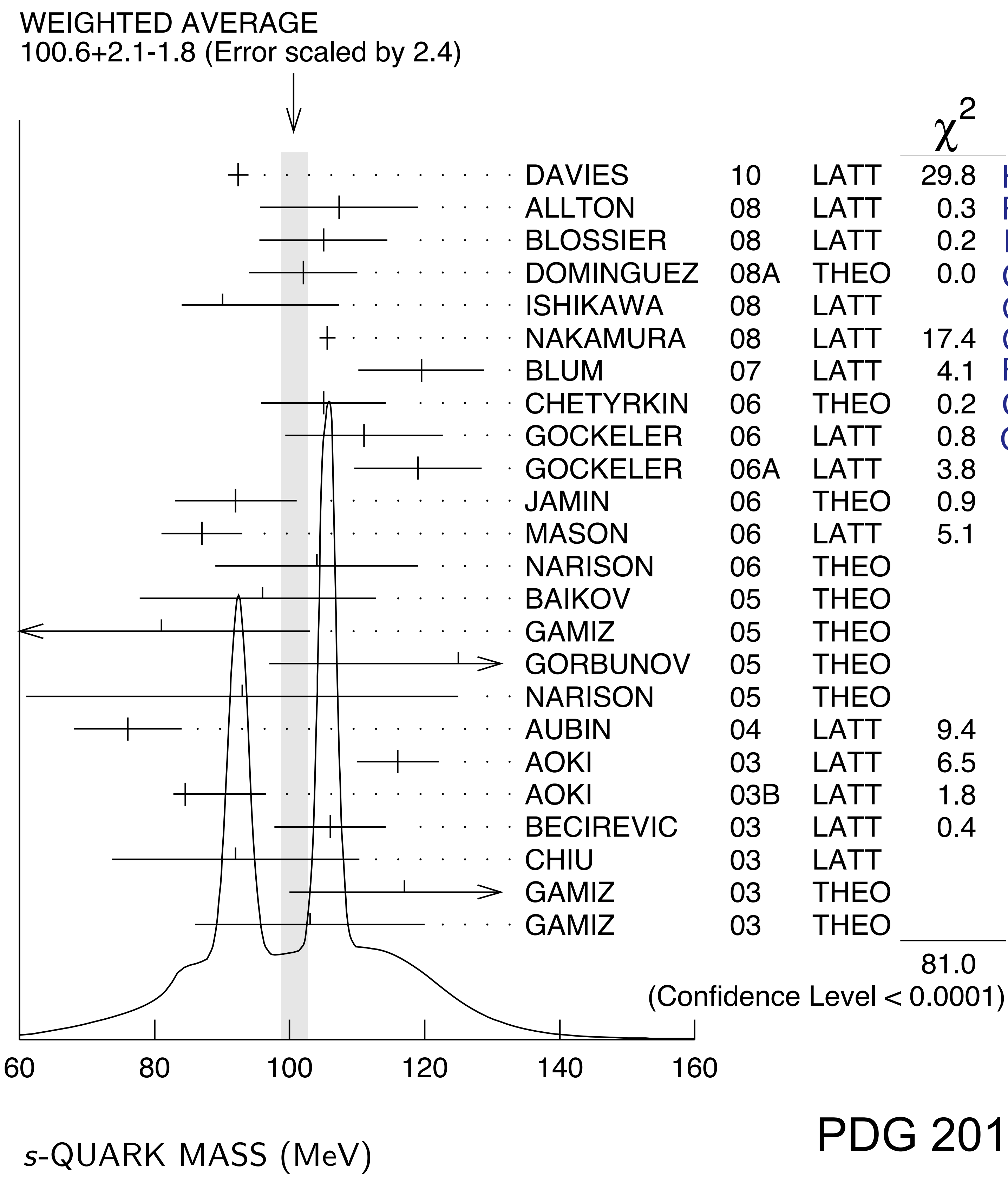
Mass of the strange quark



HPQCD 2+1+1 dyn. flavours, rooted staggered
RBC&UKQCD2+1 dyn. flavours, domain wall ferm.
ETM, 2 dyn. flavours, twisted mass fermions
QCD sum rule
CP-PACS&JLQCD 2+1 dyn. flavours,Wilson quarks
CP-PACS quenched lattice
RBC 2 dyn. flavours, domain wall fermions
QCD sum rule
QCD sum rule

fundamental parameter
(->Yukawa coupling) in SM

- enters predictions for nonleptonic decay matrix elements
- probes of Yukawa unification

...

PDG 2010

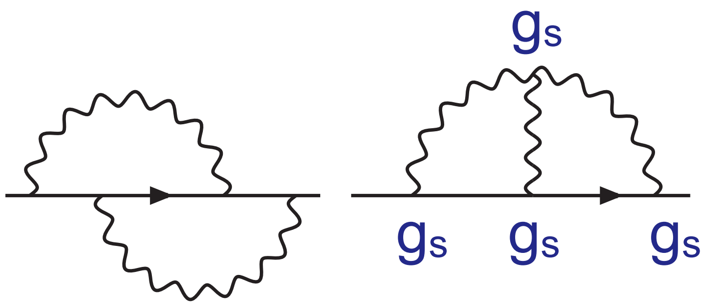
Quantum field theory

- correlation functions given by path integrals

$$\langle 0|O_1(x_1)\cdots O_n(x_n)|0\rangle =$$
$$\int \left(\prod_x dA(x)\right)\left(\prod_x d\psi(x)\, d\bar{\psi}(x)\right) O(x_1)\ldots O(x_n)\, e^{(i/\hbar)\int d^4x \mathcal{L}_{\mathrm{QCD}}}$$

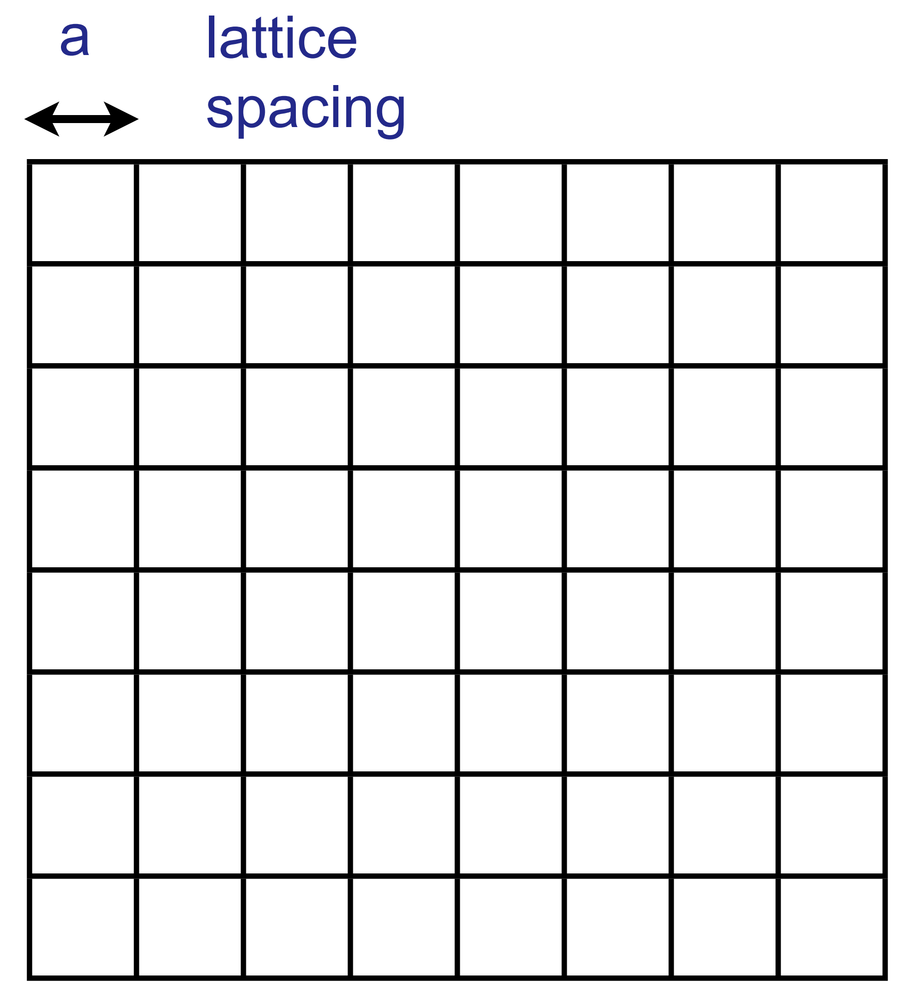
- O_i local operators constructed from quark and gluon fields either gauge invariant; or one has to fix a gauge and the correlation functions depend on the gauge fixing

- perturbation theory (small g_s expansion): Feynman diagrams



- this does not produce confinement, chiral SB, etc, which are non-perturbative phenomena

Lattice QCD



- spacetime replaced by discrete lattice of points

  gives well-defined path integral

- numerical evaluation including non-perturbative physics
- continuum (a->0) and infinite volume limits (extrapolation)
- due to relatively recent progress, chiral symmetry can be preserved by the lattice regularisation
  all symmetries of QCD are then preserved

Quark mass on the lattice

- general idea: mass spectrum depends on quark masses

$$m_{\pi,K,\ldots} = f(m_u, m_d, m_s; g_s)$$

- r.h.s numerically calculated on the lattice
  (by studying suitable 2-point functions)
  use measured meson mass spectrum to determine
  m_u, m_d, m_s

- These parameters are ‘bare’ and need to be renormalized to be of any use outside this particular lattice calculation

Renormalization

- bare parameters depend on details of regularization (lattice), diverge in continuum limit if physical quantities (meson masses) held fixed
- renormalize: $m = Z_m\, m_{bare}$
- properly defining $Z_m(g_s;a)$ gives a finite continuum limit for m
- many ways to specify a renormalization scheme, e.g.

  - physical renormalization scheme (e.g. mass parameter = observed particle mass)
    not possible for confined quarks
  - minimal subtraction $Z_m = 1 - g_s^2/(2\pi^2) \ln(a) + \ldots$
    divergent terms only
    preferred in perturbation theory ($\overline{MS}$), not defined beyond
  - Schroedinger functional method
    difficult/impractical to implement perturbatively
    possible in principle to determine RGI quark mass from step scaling process

Momentum-space subtraction

- Renormalization conditions imposed on Green's functions
- consider two-point function

$$-iS(p) = \int dx e^{ipx} \langle T[\Psi(x)\overline{\Psi}(0)] \rangle = \frac{i}{\not{p} - m + i\epsilon - \Sigma(p)}$$

- The RI-MOM and RI'-MOM schemes renormalize the fields and masses by requiring, in Landau gauge,

$$\lim_{m_R \to 0} \frac{1}{12 m_R} \mathrm{Tr}[S_R^{-1}(p)] \Big|_{p^2=-\mu^2} = 1$$

$$\lim_{m_R \to 0} \frac{1}{48} \mathrm{Tr}\left[\gamma^\mu \frac{\partial S_R^{-1}(p)}{\partial p^\mu}\right] \Big|_{p^2=-\mu^2} = -1$$ RI-MOM

$$\lim_{m_R \to 0} \frac{1}{12 p^2} \mathrm{Tr}[S_R^{-1}(p) \not{p}] \Big|_{p^2\to-\mu^2} = -1$$ RI'-MOM

[Martinelli et al 1995]

Conversion to $\overline{\text{MS}}$ scheme

- The MOM renormalization prescription can be implemented in continuum perturbation theory, most conveniently in dimensional regularization. Then the quark mass can be converted from a MOM scheme to e.g. MS-bar

$$C_m^{\text{scheme}} = \frac{m^{\overline{\text{MS}}}}{m^{\text{scheme}}} = \frac{Z^{\overline{\text{MS}}}}{Z_m^{\text{scheme}}}$$

- In practice, the conversion has been done for RI'-MOM up to three loops, and the perturbation expansion does not behave well:

$C_m^{(\text{RI}')} = 1 - 0.127_{\,[\text{NLO}]} - 0.069_{\,[\text{NNLO}]} - 0.046_{\,[\text{NNNLO}]}$

[Chetyrkin & Retey 1999, Gracey 2003]

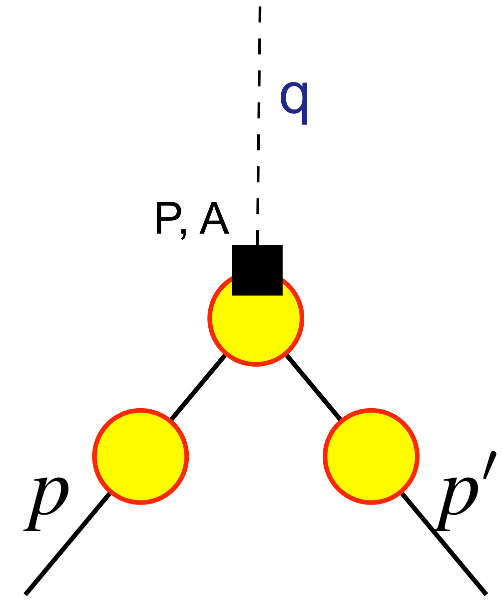
ok sizable large

for a perturbative correction

Ward identity

- By the (non-singlet) axial-vector ward identity,

$$-iq_\mu \Lambda^\mu_{A,B}(p_1,p_2) = 2m_B \Lambda_{P,B}(p_1,p_2) - i\gamma_5 S_B^{-1}(p_1) - S_B^{-1}(p_2) i\gamma_5$$



[fig. due to C Sachrajda]

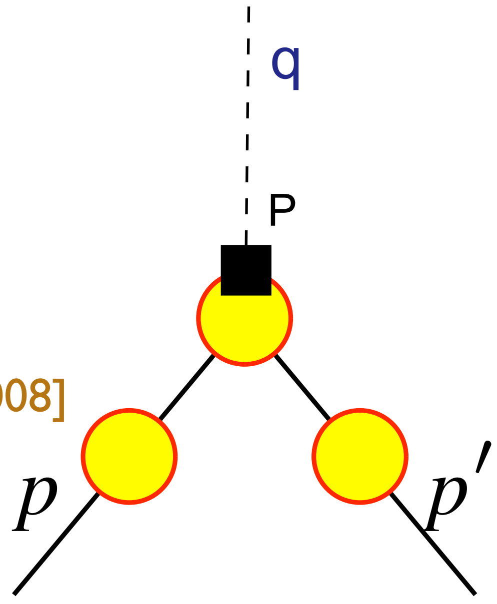
where $\Lambda_{A,B}$ and $\Lambda_{P,B}$ are the bare three-point Green's functions involving the axial current and pseudoscalar density, respectively, the RI/MOM mass renormalization can alternatively be computed as

$$Z_m^{\mathrm{RI}'/\mathrm{MOM}} = \frac{-p^2\, \mathrm{tr}[\Lambda_{P,B}(p,p')\gamma_5]|_{p^2=p'^2=-\mu^2}}{\mathrm{tr}[S_B^{-1} \not{p}]}$$

where $q^2 = (p-p')^2 = 0$

RI-SMOM

- Origin of the bad perturbative behaviour unclear. However, nonperturbatively, at q^2=0 there are $1/p^2$ power corrections, and also the chiral limit does not exist because of a "pseudo-goldstone pole" term
  [eg Aoki et al 2008]



$$\Lambda(p, p'; q^2 = 0) = \frac{\text{const}}{m_K^2} \langle K^+ | s(p) \bar{u}(p') | 0 \rangle + \ldots$$

  (the pseudoscalar density P has the correct quantum numbers to create pions or kaons, which become massless in the chiral limit)

- a practical issue in lattice simulations involving light quarks
  [Aoki et al 2008]

- the nonperturbative issues can be addressed by going to more general kinematics
  [Sturm et al 2009]
  $p^2 = (p')^2 = -\mu^2$, $q^2 = \omega\, p^2$
  SMOM ("symmetric MOM") : $\omega = 1$

RI-SMOM to $\overline{\text{MS}}$

- one-loop conversion factor at the SMOM point

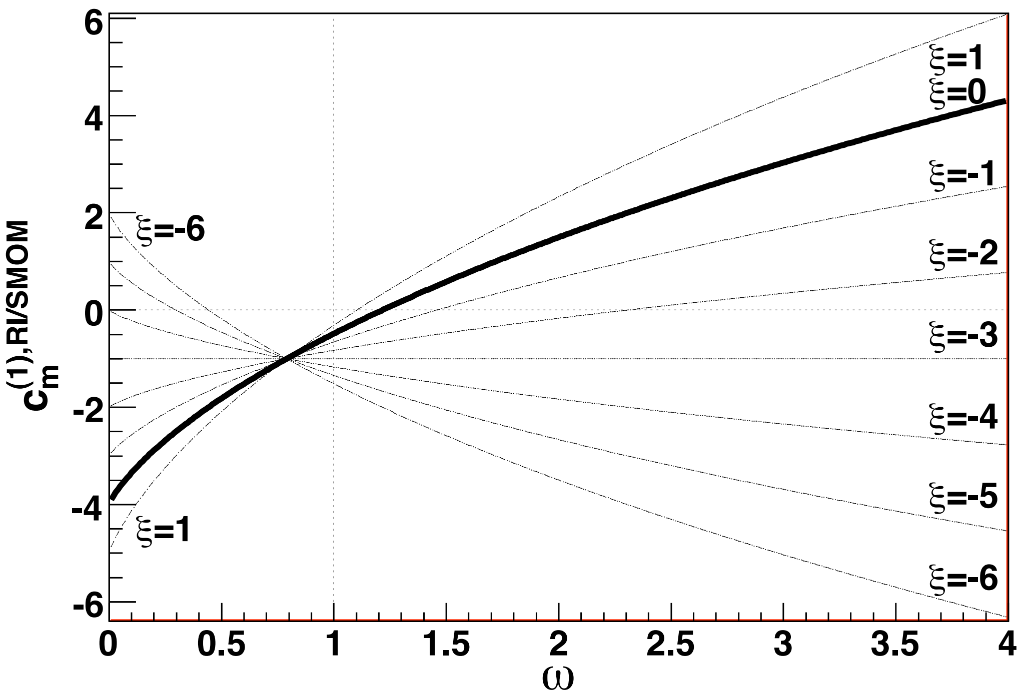
[Sturm et al 2009]

$C_m^{(\text{RI-SMOM})} = 1 - 0.015$ [NLO]

a tiny one-loop correction

(recall $C_m^{(\text{RI'})} = 1 - 0.127$ [NLO] $- 0.069$ [NNLO] $- 0.046$ [NNNLO])

- as a function of $\omega = q^2/p^2$ and the gauge parameter

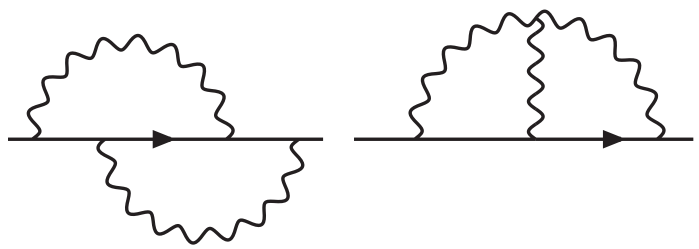


[Sturm et al 2009]

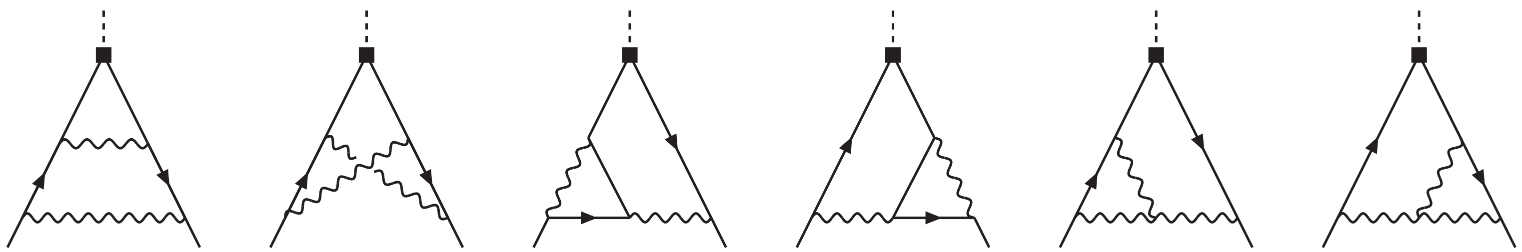
Two-loop (NNLO) calculation

[Gorbahn, SJ 1004.3997]

[Almeida, Sturm 1004.4613]



\+ recursively one-loop



\+ recursively one-loop

- straightforward evaluation of traces over numerators
- express numerators as polynomials of the denominators
- Feynman integrals with general propagator powers

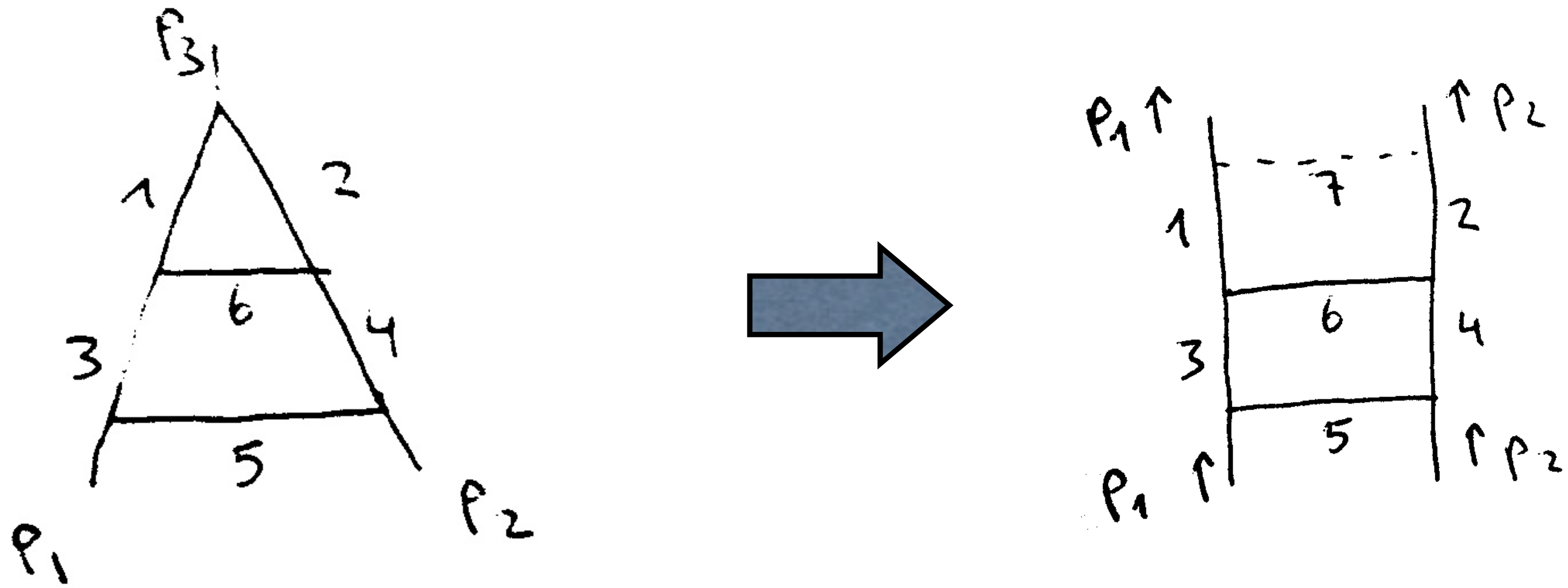
$$\int \frac{d^dk}{(2\pi)^d}\,\frac{d^dl}{(2\pi)^d}\,\prod_i (P_i^n)^{-a_i} \equiv \left(\frac{i}{16\pi^2}\right)^2 \left(\frac{\mu^2}{4\,\pi}e^{\gamma}\right)^{2\epsilon} k_n(a_1,\ldots,a_m;\{s_k\})$$

topology

propagator powers

kinematic invariants

Integral reduction



- integration by part identities via Laporta's algorithm [S Laporta 2001] use the public Mathematica implementation FIRE [A V Smirnov 2008]
- two two-loop master integrals - known in terms of higher polylogarithms [Davydychev and Usyukina 1994]
- recursively one-loop diagrams with spurios poles -> sensitivity to higher orders in ϵ

Masterly inactivity

- only unknown "master" ingredient is one-loop integral !

$$j(d;\nu_1,\nu_2,\nu_3;p_1^2,p_2^2,p_3^2) \equiv$$
$$\left(\frac{i}{16\pi^2}\right)^{-1}\left(\frac{\mu^2}{4\,\pi}e^{\gamma}\right)^{\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{1}{[-k^2]^{\nu_3}[-(k+p_1)^2]^{\nu_2}[-(k-p_2)^2]^{\nu_1}}$$

- in particular, need j(d; 1,1,2+ϵ; p^2, x p^2, y p^2), proportional to:

$$\frac{1}{xy}\Big(-\frac{1}{\epsilon}+2\ln x+2\ln y-\epsilon(2\ln^2 x+2\ln^2 y+\ln x\ln y+3(1-x-y)\Phi^{(1)}(x,y)-\frac{\pi^2}{6})+\epsilon^2\beta(x,y)+\mathcal{O}(\epsilon^2)\Big)$$

(actually needed only for y=1 or y=x)

- Need O(ε^2) term β(x,y) - not known and difficult to compute [at least for us]
- can avoid computation - reducing the *known* 2-loop masters & using an identity from rotating the triangle by 120 degrees, one can obtain sufficient (algebraic) constraints on it

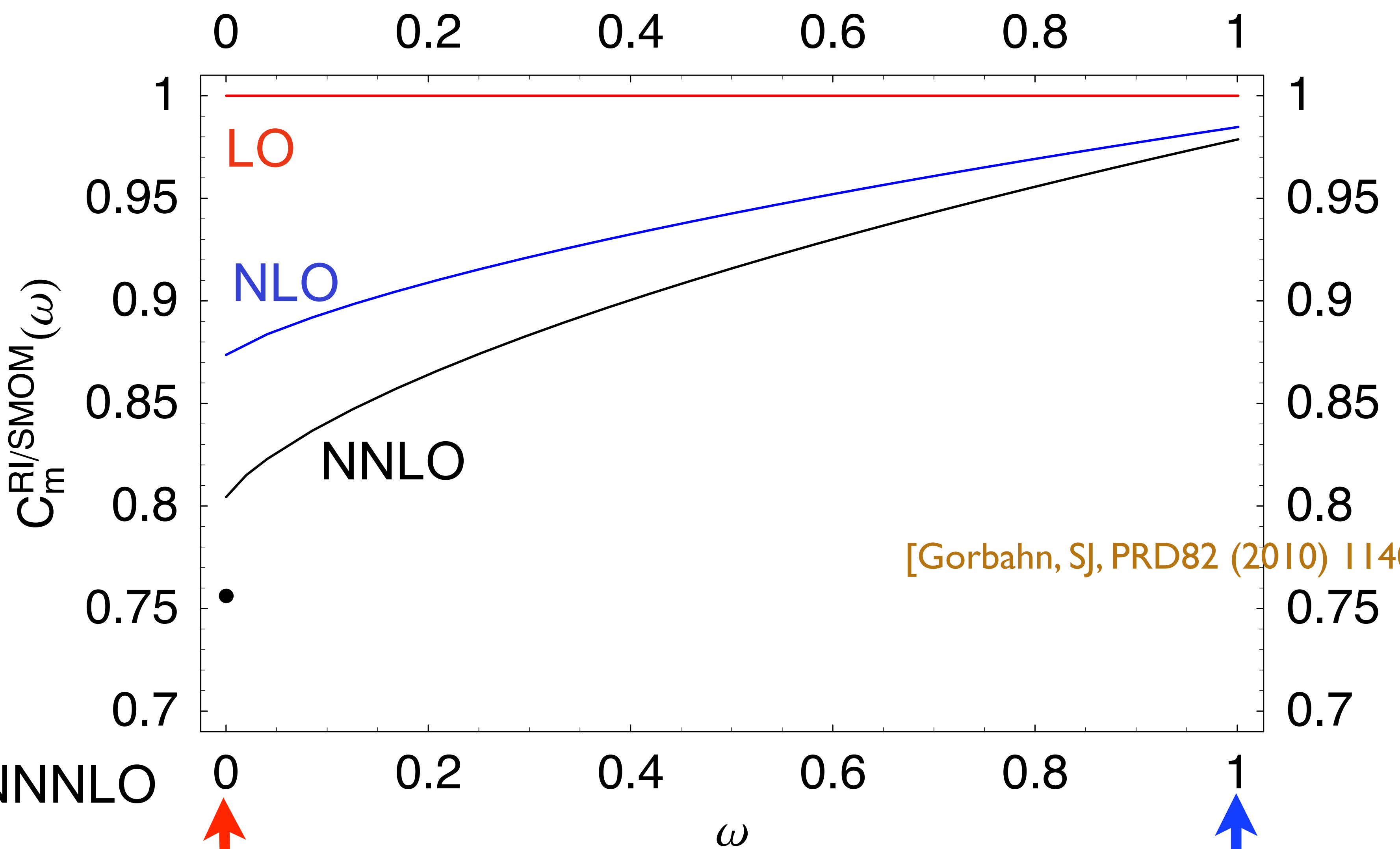
Analytical result

[Gorbahn, SJ, PRD82 (2010) 114001, arXiv:1004.3997]

$$
\begin{aligned}
C_m^{\mathrm{RI/SMOM}}(\omega) =\ & 1 + \frac{\alpha_s}{4\pi} C_F \left(\frac{3+\xi}{2} \Phi^{(1)}\left(\frac{1}{\omega},\frac{1}{\omega}\right) - 4 - \xi + 3\ln r \right) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_F \left\{ N_c \left(-\frac{2513}{48} - \frac{3\,\xi}{2} - \frac{\xi^2}{4} + 12\,\zeta(3) \right.\right. \\
& + \frac{307+6\,\xi^2}{12} \ln r - \frac{13}{4} \ln^2 r + \left[\frac{301}{24} + \frac{3\,\xi}{4} - \frac{\xi^2}{8} - \frac{13+\xi^2}{4} \ln r - \frac{7+3\,\xi}{4} \ln\omega \right] \Phi^{(1)}\left(\frac{1}{\omega},\frac{1}{\omega}\right) + \frac{9+6\,\xi+\xi^2}{8} \Phi^{(1)}\left(\frac{1}{\omega},\frac{1}{\omega}\right)^2 \\
& \left. + \omega\, \Phi^{(2)}(1,\omega) - \frac{3+\xi}{2} \Phi^{(2)}\left(\frac{1}{\omega},\frac{1}{\omega}\right) \right) + n_f \left(\frac{83}{12} + \left[\ln r - \frac{5}{3} \right] \Phi^{(1)}\left(\frac{1}{\omega},\frac{1}{\omega}\right) - \frac{13}{3} \ln r + \ln^2 r \right) \\
& + \frac{1}{N_c} \left(-\frac{19}{16} - 2\,\xi - \frac{\xi^2}{2} + \left[\frac{7}{2} + \xi + \frac{\xi^2}{2} - \frac{9+3\,\xi}{4} \ln r + \frac{5+3\,\xi}{4} \ln\omega \right] \Phi^{(1)}\left(\frac{1}{\omega},\frac{1}{\omega}\right) + \frac{21+6\,\xi}{4} \ln r - \frac{9}{4} \ln^2 r \right. \\
& \left.\left. + \frac{1+\xi}{2} \Phi^{(2)}\left(\frac{1}{\omega},\frac{1}{\omega}\right) + \frac{1}{2} \Omega^{(2)}\left(\frac{1}{\omega},\frac{1}{\omega}\right) - \Omega^{(2)}(1,\omega) - \left[\frac{5}{8} + \frac{3\,\xi}{4} + \frac{\xi^2}{8} + \frac{1}{\omega} \right] \Phi^{(1)}\left(\frac{1}{\omega},\frac{1}{\omega}\right)^2 \right) \right\}
\end{aligned}
$$

- The functions $\Phi^{(1)}$, $\Phi^{(2)}$, $\Psi^{(2)}$, $\Omega^{(2)}$ are all given in terms of polylogarithms up to fourth order
- full ω dependence: can interpolate RI/MOM - RI/SMOM

NNLO result: ω dependence



NNNLO

[Gorbahn, SJ, PRD82 (2010) 114001, arXiv:1004.3997]

RI' point: large loop corrections

SMOM point: tiny corrections

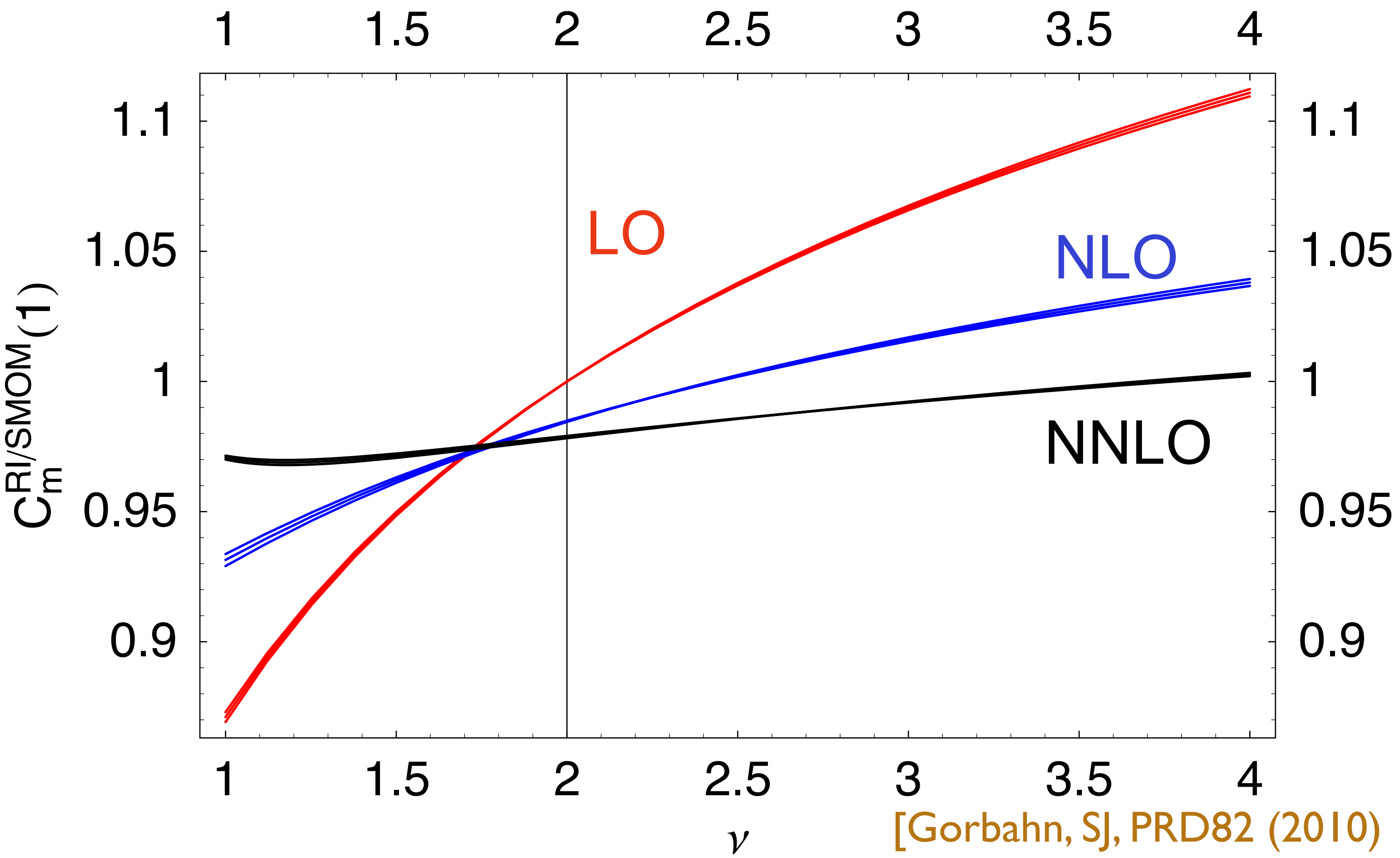
- $C_m^{(SMOM)} = 1 - 0.015$ [NLO] $- 0.006$ [NNLO]

confirmed by Almeida, Sturm [at ω=1]

- $C_m^{(RI')} = 1 - 0.127$ [NLO] $- 0.069$ [NNLO] $- 0.046$ [NNNLO]

Residual scale dependence

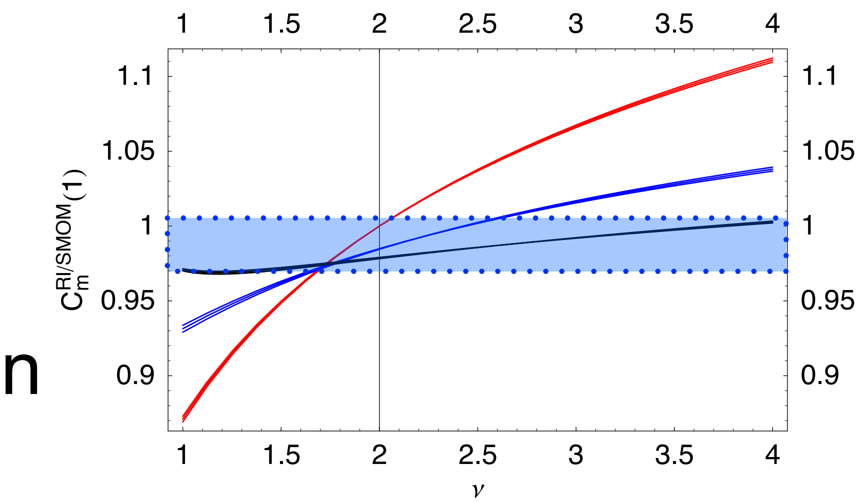
- construct a formally RG-invariant quantity
- e.g. convert to $\overline{\text{MS}}$ mass from fixed MOM scale 2 GeV, varying dim reg scale used in conversion and RG-evolving back to 2 GeV



[Gorbahn, SJ, PRD82 (2010) 114001, arXiv:1004.3997]

- alternatively, consider “RGI” mass, similar picture

NNLO result with error



- take the range of the NNLO band as theoretical range
- symmetrizing around the midpoint, obtain

$$m^{\overline{\rm MS}}(2\,{\rm GeV}) = \left(0.978^{+0.024}_{-0.010}\Big|_{\rm h.o.}{}^{+0.001}_{-0.001}\Big|_{\alpha_s}\right) m^{\rm RI/SMOM}(2\,{\rm GeV})$$

$$m^{\rm RGI} = \left(2.53^{+0.052}_{-0.014}\Big|_{\rm h.o.}{}^{+0.02}_{-0.02}\Big|_{\alpha_s}\right) m^{\rm RI/SMOM}(2\,{\rm GeV})$$

[Gorbahn, SJ, PRD82 (2010) 114001, arXiv:1004.3997]

- 2 percent error !
- error dominated by unknown higher orders, α_s uncertainty subleading
- new RBC/UKQCD result: $m^{\overline{\rm MS}}(2\,{\rm GeV}) = (96.2 \pm 2.7)\,{\rm MeV}$

[Aoki et al, PRD83(2011)074508, arXiv:1011.0892]

- similar stability for other quantities? (B_K/ε_K, ε'/ε, ...)