THEORY OF $B \to K^{(*)} l^+ l^-$ DECAYS AT HIGH q^2

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Khodjamirianfest

Colour meets Flavour

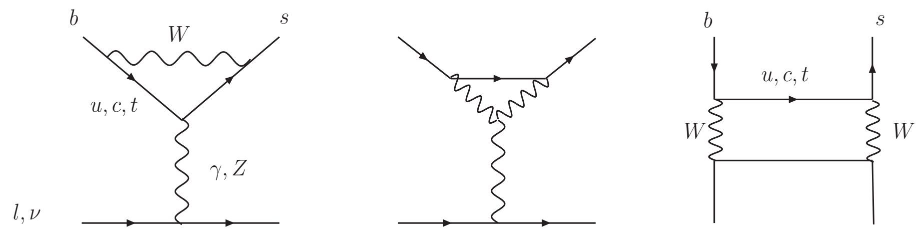
Siegen, 13 – 14 October 2011

- OPE for hadronic contribution to $B \to K^{(*)} l^+ l^-$
- Quark-hadron duality
- Precision flavour physics

Beylich, G.B., Feldmann

G.B., Isidori; Grinstein, Pirjol

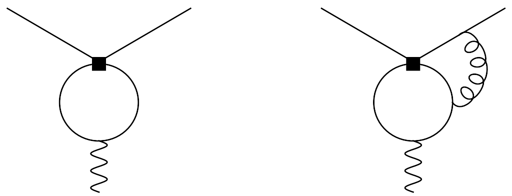
$B \to K^{(*)} ll$ — Basic Structure



$$A(\bar{B} \to \bar{M} l^+ l^-) = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi} \lambda_t \left[A_9^\mu \, \bar{l} \gamma_\mu l + A_{10}^\mu \, \bar{l} \gamma_\mu \gamma_5 l \right]$$

$$A_9^\mu = C_9 \, \langle \bar{M} | \bar{s} \gamma^\mu (1 - \gamma_5) b | \bar{B} \rangle + \langle \bar{M} | \mathcal{K}_H^\mu(q) | \bar{B} \rangle + \ldots$$

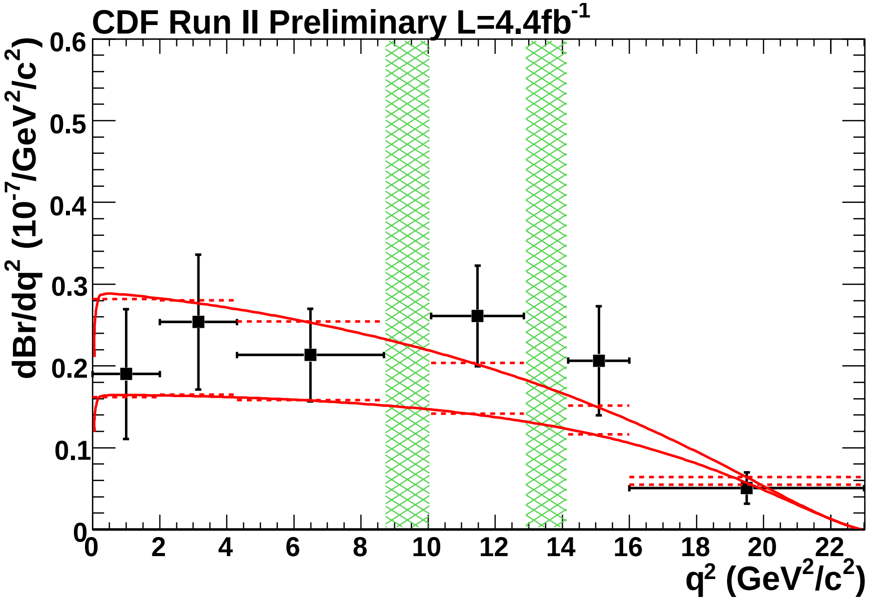
$$\mathcal{K}_H^\mu(q) = -\frac{8\pi^2}{q^2} i \int d^4x \, e^{iq\cdot x} \, T \, j^\mu(x) H^c(0)$$

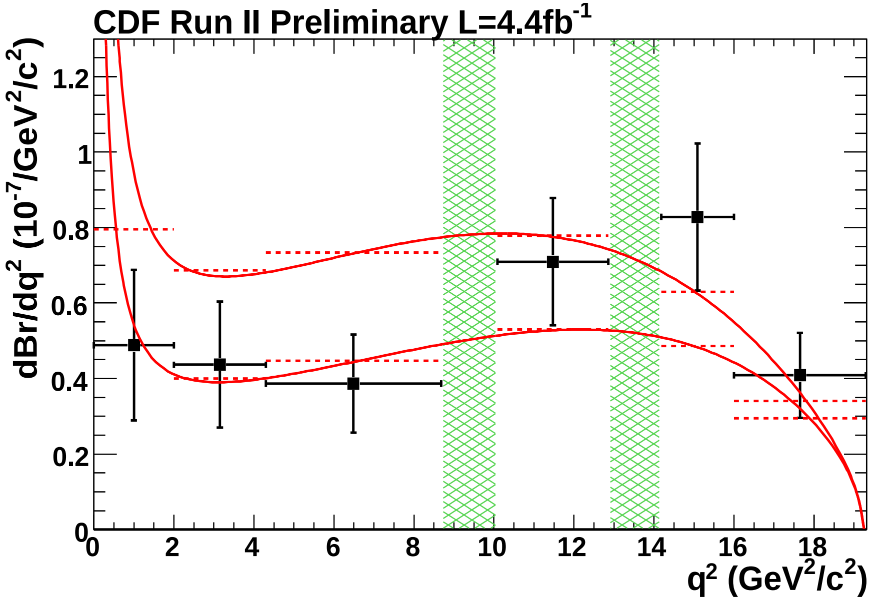


semileptonic contribution $\leftrightarrow$ hadronic contribution ("charm loops")

$B^- \to K^- \mu^+ \mu^-$

$\bar{B}^0 \to \bar{K}^{*0} \mu^+ \mu^-$

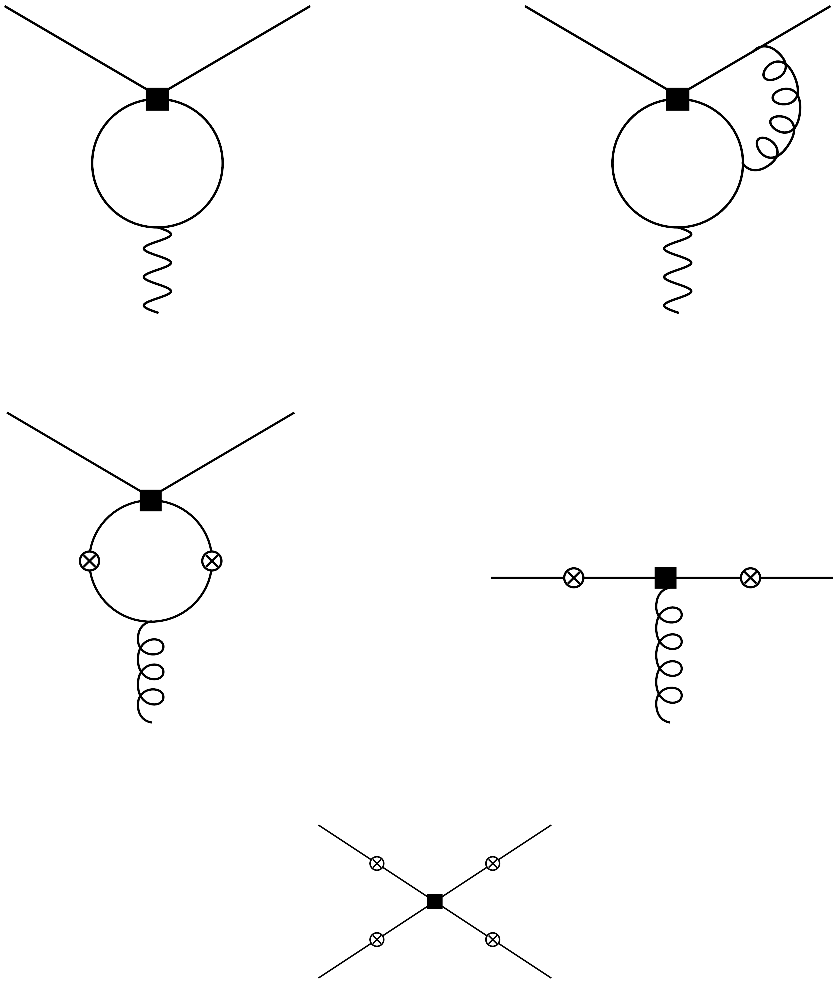




$BR/10^{-6} =$	$0.38 \pm 0.05 \pm 0.03$	$1.06 \pm 0.14 \pm 0.09$	*CDF*
	$0.39 \pm 0.07 \pm 0.02$	$1.11 \pm 0.19 \pm 0.07$	*BaBar*
	$0.48 \pm 0.05 \pm 0.03$	$1.07 \pm 0.11 \pm 0.09$	*Belle*

OPE $\quad \Lambda,\, E_K \ll \sqrt{q^2}$

$$\mathcal{K}_H^\mu(q) = -\frac{8\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x}\, T\, j^\mu(x) H^c(0) = \sum_{d,n} C_{d,n}(q)\, \mathcal{O}^\mu_{d,n}$$

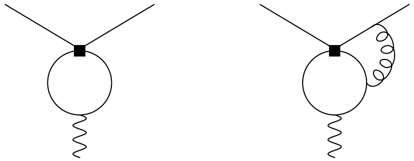


$$\mathcal{O}^\mu_{3,1} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \bar{s}\gamma_\nu(1-\gamma_5)b$$

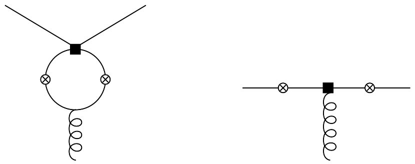
$$\mathcal{O}^\mu_{3,2} = \frac{i m_b}{q^2} q_\lambda\, \bar{s}\sigma^{\lambda\mu}(1+\gamma_5)b$$

$$\mathcal{O}^\mu_{5,n} = \bar{s}(gG\Gamma_n)^\mu b$$

$$\mathcal{O}^\mu_{6ann,n} = (\bar{r}\Gamma_1 b\, \bar{s}\Gamma_2 r)^\mu_n$$



$$\mathcal{K}^{\mu}_{H3} = \left(g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q^2}\right) \bar{s}\gamma_{\nu}(1-\gamma_5)b \cdot h(x)(C_1 + 3C_2) + \ldots, \qquad x = \frac{4m_c^2}{q^2}$$



$$\mathcal{K}^{\mu}_{H5} = \left[\varepsilon^{\alpha\beta\lambda\rho}\frac{q_{\beta}q^{\mu}}{q^2} + \varepsilon^{\beta\mu\lambda\rho}\frac{q_{\beta}q^{\alpha}}{q^2} - \varepsilon^{\alpha\mu\lambda\rho}\right] \bar{s}\gamma_{\lambda}(1-\gamma_5)gG_{\alpha\rho}b\,\frac{C_1 Q_c}{q^2} f(x)$$
$$- \frac{q_{\lambda}}{m_B}\bar{s}gG_{\alpha\beta}(g^{\alpha\lambda}\sigma^{\beta\mu} - g^{\alpha\mu}\sigma^{\beta\lambda})(1+\gamma_5)b\,\frac{4C_8 Q_b}{q^2}$$

$$f(x) = \frac{x}{\sqrt{1-x}}\left(\ln\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}} + i\pi\right) - 2$$

G.B., Isidori, Rey

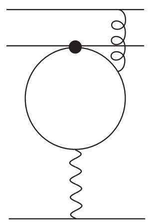
Voloshin; Khodjamirian et al.

$a_9 \approx 4$

$$\Delta a_{9,H5}(K) = -\frac{\pi\alpha_s(E_K)C_F}{2N} C_1 Q_c f(x) \frac{m_B f_B f_K}{\lambda_B\, f_+(q^2)\, q^2} \approx 0.019 - 0.012i$$

$$\Delta a_{9,H5}(K^*_\perp) = -\frac{\pi\alpha_s(E_K)C_F}{4N} \frac{m_B f_B f_\perp}{\lambda_B\, V(q^2)\, q^2} \left(C_1 Q_c f(x) + 8 C_8 Q_b\right) \approx 0.008 - 0.006i$$

(for $q^2 = 15\,\mathrm{GeV}^2$)



- explicit calculation of 2nd order power corrections for $\Lambda \ll E_K \ll \sqrt{q^2}$ ($q^2 \gtrsim 15\,\mathrm{GeV}^2$)
- impact below 1%

Beneke, G.B., Neubert, Sachrajda

$$\mathcal{H}_{\text{eff}} = \frac{G}{\sqrt{2}} \left[(\bar{l}_2 l_1)_{V-A} \, (\bar{c}c)_{V-A} - (\bar{l}_2 l_1)_{V-A} \, (\bar{t}t)_{V-A} \right]$$

$$A(l_1 \to l_2 \, e^+ e^-) = -\frac{G}{\sqrt{2}} \, e_c e^2 \, \Pi(q^2) \, \bar{l}_2 \gamma^\mu (1 - \gamma_5) l_1 \, \bar{e} \gamma_\mu e$$

$$\Pi \equiv \Pi_c - \Pi_t, \qquad \Pi(0) = \frac{N}{12\pi^2} \ln \frac{m_t^2}{m_c^2}$$

$$\frac{d\Gamma(l_1 \to l_2 \, e^+ e^-)}{ds} = \frac{G^2 \alpha^2 m_1^5}{108 \pi^5} (1 - s)^2 \, (1 + 2s) \left| C + \Delta(q^2) \right|^2 , \qquad s = \frac{q^2}{m_1^2}$$

$$C \equiv 2\pi^2 \, \Pi(0) \qquad \Delta(q^2) \equiv 2\pi^2 \, \left(\Pi(q^2) - \Pi(0) \right)$$

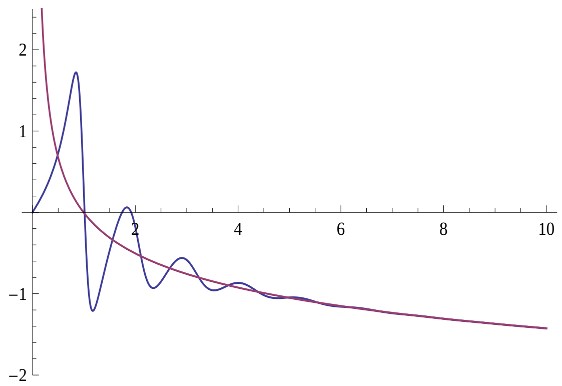
$$|C + \Delta|^2 = C^2 + 2C \, \text{Re}\Delta + |\Delta|^2$$

$$\Pi(q^2) = c \sum_{n=1}^{\infty} \frac{1}{z+n}\,, \qquad z = \left(\frac{-q^2-i\epsilon}{\lambda^2}\right)^{1-b/\pi}\,, \qquad b = \frac{\Gamma_n}{M_n}$$

$$\Rightarrow \Pi(q^2) - \Pi(0) = -\frac{N}{12\pi^2}\frac{1}{1-b/\pi}\left[\psi(z+1)+\gamma\right] \to -\frac{N}{12\pi^2}\ln\frac{-q^2-i\epsilon}{\lambda^2}$$

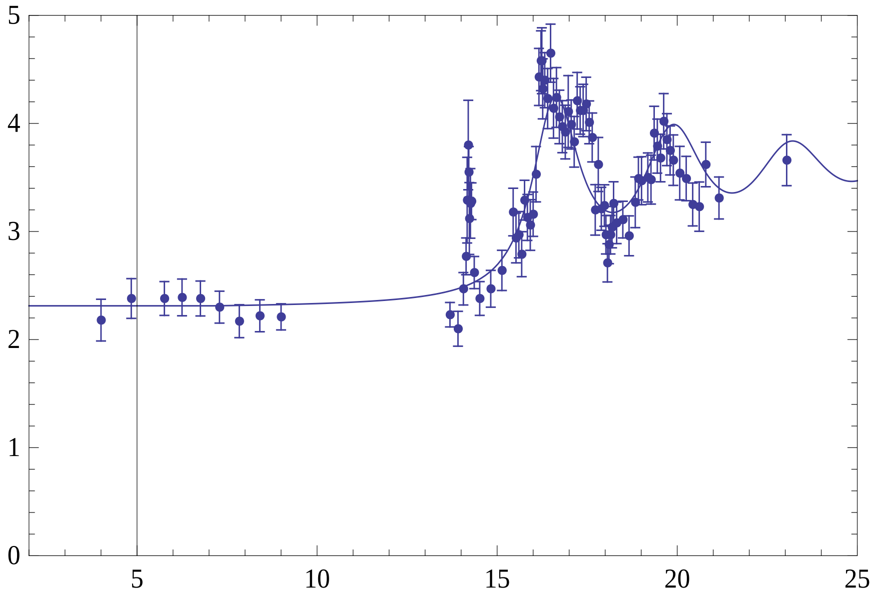
asymptotic series: $\psi(z+1) = \ln z - \sum_{k=1}^{\infty} \frac{B_k}{k}\frac{1}{z^k}$

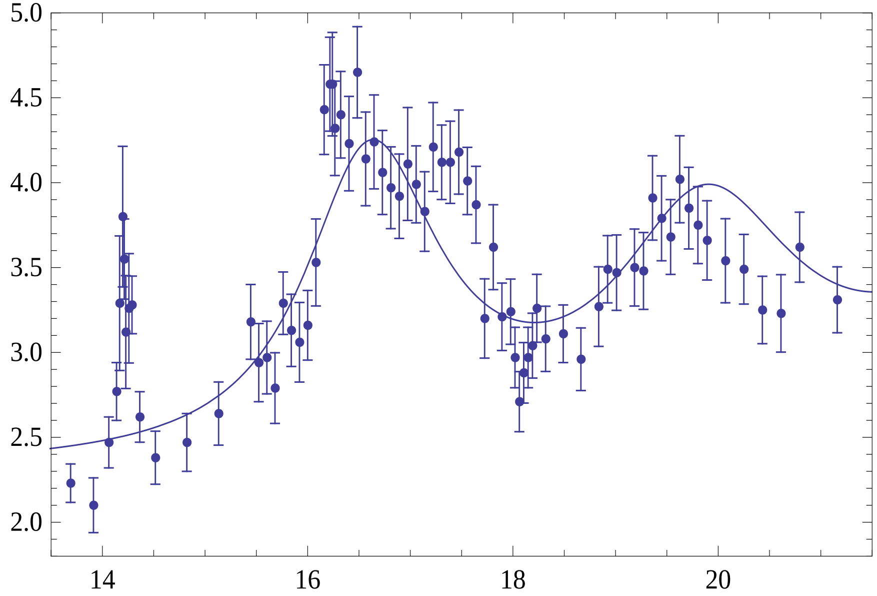
$\mathrm{Re}\Delta(q^2/\lambda^2)$



$$\psi(z+1)+\gamma \equiv \left[\psi(-z)+\gamma-i\pi\right]_{OPE} + \left[-\pi\cot\pi z + i\pi\right]_{DV}$$

$$\left[-\pi\cot\pi z + i\pi\right] \approx 2\pi\exp\left(\frac{-2\pi b q^2}{\lambda^2}\right)\left[\sin\left(\frac{2\pi q^2}{\lambda^2}\right) - i\cos\left(\frac{2\pi q^2}{\lambda^2}\right)\right]$$





$$R = R_{\text{light}} - \frac{4}{3}\,\frac{1}{(1-b/\pi)\,\pi}\,\mathrm{Im}\,\psi(3+z)\,, \qquad z = \left(-\frac{q^2-4m_c^2+i\epsilon}{\lambda^2}\right)^{1-b/\pi}$$

$$\lambda^2 = 3.08\,\mathrm{GeV}^2, \quad m_c = 1.33\,\mathrm{GeV}, \quad b = 0.082$$

$$M_n^2 = n\lambda^2 + M_0^2, \qquad n^3S_1:\ \psi(3097), \psi(3686), \psi(4040), \psi(4415)$$

Gershtein et al.

$$\langle \mathcal{K}_H^\mu \rangle = \tfrac{16\pi^2}{3} a_2 \, \langle (\bar{s}b)_{V-A} \rangle^\mu \, \Pi_c(q^2) \,, \qquad a_2 \approx 0.3$$

$$\Delta a_9 = a_2 d, \qquad d \equiv \tfrac{16\pi^2}{3} \left(\Pi_c(q^2) - \Pi_c(0) \right)$$

$$d = -\tfrac{4}{3} \tfrac{1}{1-b/\pi} \left[\psi(z+3) - \psi(z_0+3) \right]$$

$$\to d_{DV} \approx -\tfrac{8\pi}{3} \exp(-2\pi b r) \left(\sin 2\pi r - i \cos 2\pi r \right)$$

$$r = (q^2 - 4m_c^2)/\lambda^2 \,, \qquad s = q^2/m_B^2$$

$$|R_{DV,1}| = \left| \frac{2a_2}{a_9} \frac{\int_{s_0}^{s_m} ds \, \varphi(s) \, \mathrm{Re} d_2}{\int_{s_0}^{s_m} ds \, \varphi(s)} \right| \lesssim \frac{8}{3} \frac{a_2}{a_9} \frac{\varphi(s_0)}{\int_{s_0}^{s_m} ds \, \varphi(s)} \frac{\lambda^2}{m_B^2} \exp(-2\pi b (q_0^2 - 4m_c^2)/\lambda^2)$$

impact on high-q^2 rate $\sim 1.5\%$

Conclusions

- high q^2: OPE (in Λ/m_b) for hadronic amplitude in $B \to K^{(*)} l^+ l^-$: no 1st order power corrections, 2nd order small

- high q^2: quantitative model estimate of duality violation (small)

- excellent control of $B \to K^{(*)} l^+ l^-$ at high (and low) q^2

Congratulations

Happy Birthday Alexander!!!