

QCD corrections to Kaon Mixing

Khodjamirian Fest

Colour meets Flavour: QCD and Quark Flavour Physics
Siegen 13.10.2011

Based on work done in collaboration with Joachim Brod

Martin Gorbahn
TU München

Excellence Cluster 'Universe'



QCD meets Flavour



QCD meets Flavour



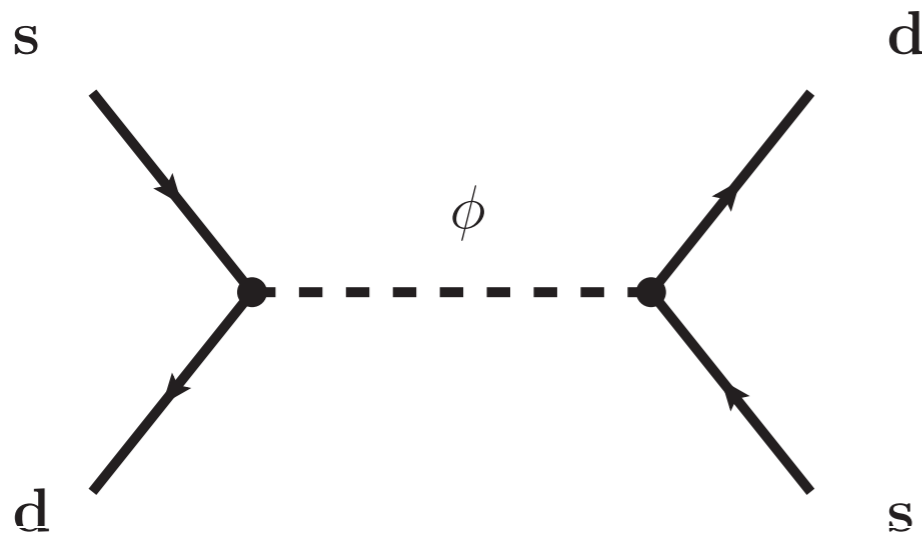
In Flavour Physics:

Find (or build) a path,
which connects **low**
and **high** energies:

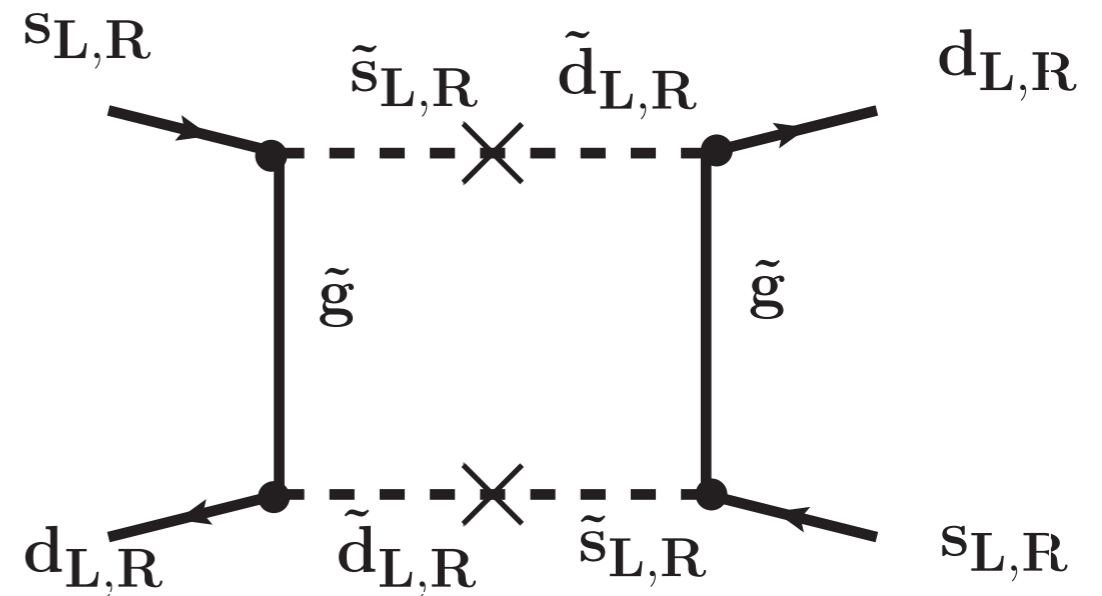
QCD meets Flavour

Kaon Mixing & New Physics

Many more sources of CP & flavour violation:



2HDM Type III



SUSY models

+Technicolour, extra dimensions,

Strong constraints from ϵ_K & ΔM_K !

But we have to understand the standard model background

Kaon Mixing

$$i \frac{d}{dt} \begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix} \right] \begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix}$$

M_{12} is related to important observables

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M_{12} is related to important observables

CP violation in mixing $\text{Re}(\epsilon_K)$ and interference $\text{Im}(\epsilon_K)$

$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left(\frac{\text{Im}(M_{12})}{\Delta m_K} + \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right)$$
$$A_I = \langle (\pi\pi)_I | K^0 \rangle$$

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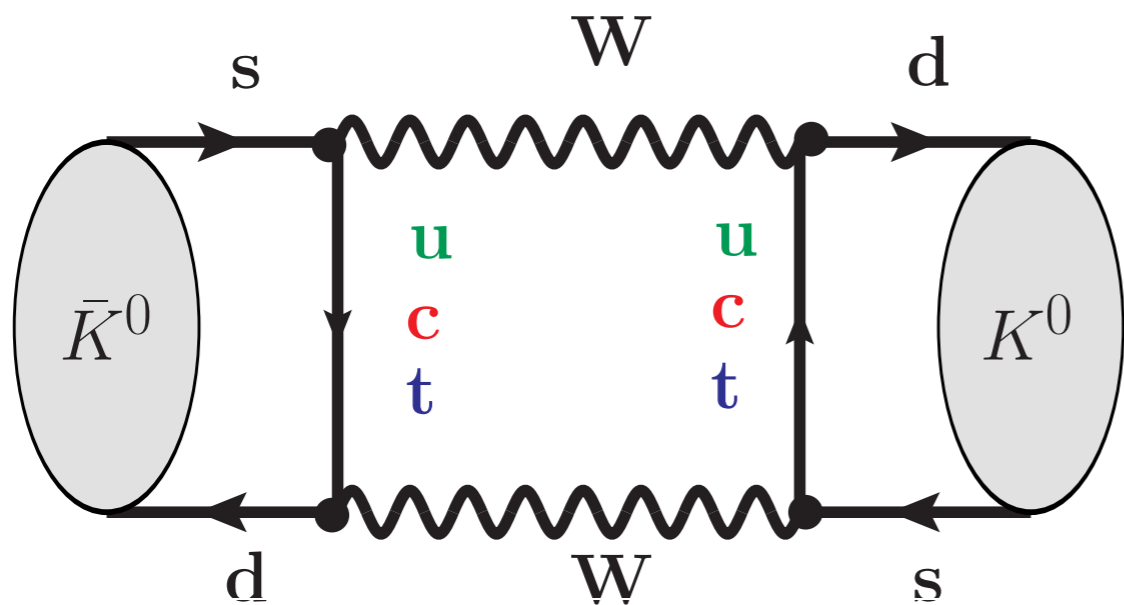
$$A_I = \langle (\pi\pi)_I | K^0 \rangle$$

and K_L - K_S mass difference: $\Delta M_K = 2 \text{Re } M_{12}$

$$2M_K M_{12} = \langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle - \frac{i}{2} \int d^4x \langle K^0 | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | \bar{K}^0 \rangle$$

dispersive part

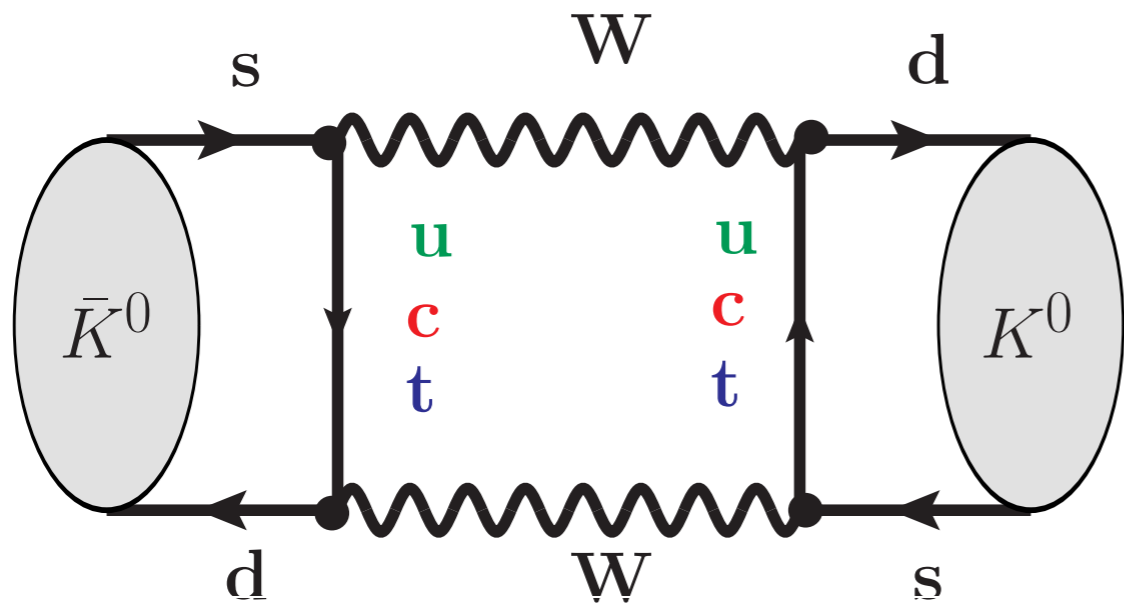
Standard Model Box Diagram



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\chi_i = \frac{m_i^2}{M_W^2} \quad \lambda_i = V_{id}^* V_{is}$$

Standard Model Box Diagram

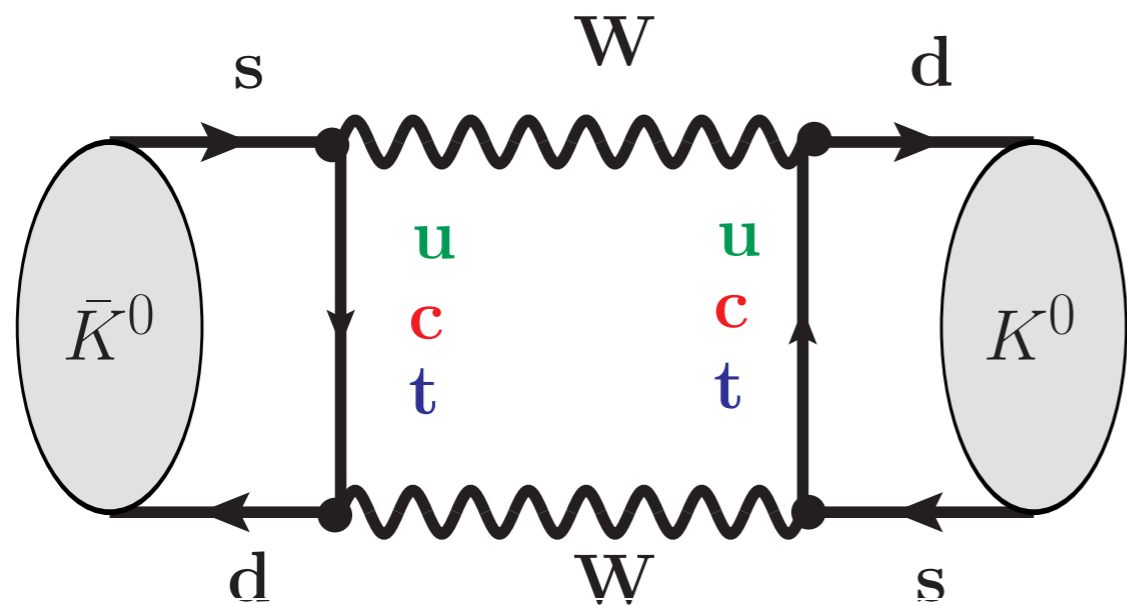


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Three CKM factors: $\lambda_t = O(\lambda^5 e^{i\delta})$, $\lambda_c = O(\lambda + i\lambda^5)$ and $\lambda_u = O(\lambda)$

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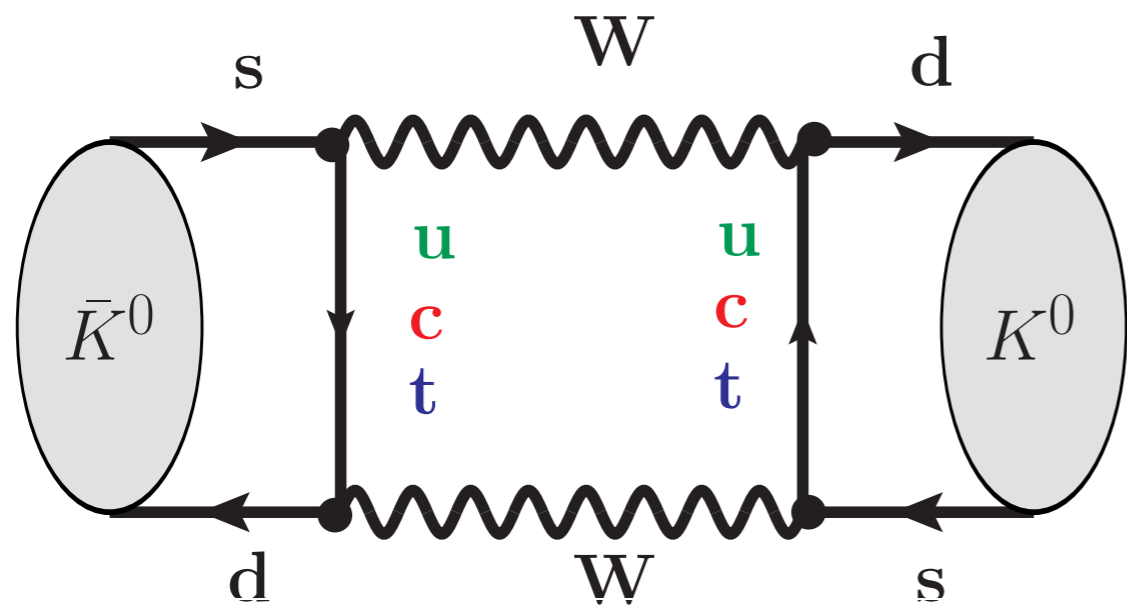
$$x_i = \frac{m_i^2}{M_W^2} \quad \lambda_i = V_{id}^* V_{is}$$

Three CKM factors: $\lambda_t = O(\lambda^5 e^{i\delta})$, $\lambda_c = O(\lambda + i\lambda^5)$ and $\lambda_u = O(\lambda)$

Eliminate $-\lambda_u = \lambda_t + \lambda_c$: For CP violation

$$x_t \lambda_t \lambda_t + x_c \log(x_c) \lambda_c \lambda_t + x_c \lambda_c \lambda_c + \frac{\Lambda_{\text{QCD}}^2}{M_W^2} \lambda_c \lambda_{t/c}$$

Standard Model Box Diagram



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For CP conserving

$$x_c \lambda_c \lambda_c + \frac{\Lambda_{\text{QCD}}^2}{5M_W^2} \lambda_c \lambda_c$$

Formula for ϵ_K

$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left(\frac{\text{Im}(M_{12})}{\Delta m_K} + \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right)$$

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$\Delta m_K, \phi_\epsilon$: Directly from experiment:

$\text{Im}(A_0)/\text{Re}(A_0)$: from ϵ'/ϵ

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$$\text{Im}(M_{12}) = \text{Im}(M_{12})_{\text{SD}} + \text{Im}(M_{12})_{\text{D=8}} + \text{Im}(M_{12})_{\text{Non Local}}$$

Factorize short and long distance: $H^{|\Delta S|=2} = C(\mu)\tilde{Q}$

From lattice: $\hat{B}_K = \frac{3b(\mu)}{2f_K^2 M_K^2} \langle K^0 | \tilde{Q} | \bar{K}^0 \rangle \quad (\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L))$

Experiment & Theory

Experiment:

ϵ_K is measured precisely:

$$|\epsilon_K| = 2.228(11) \times 10^{-3}.$$

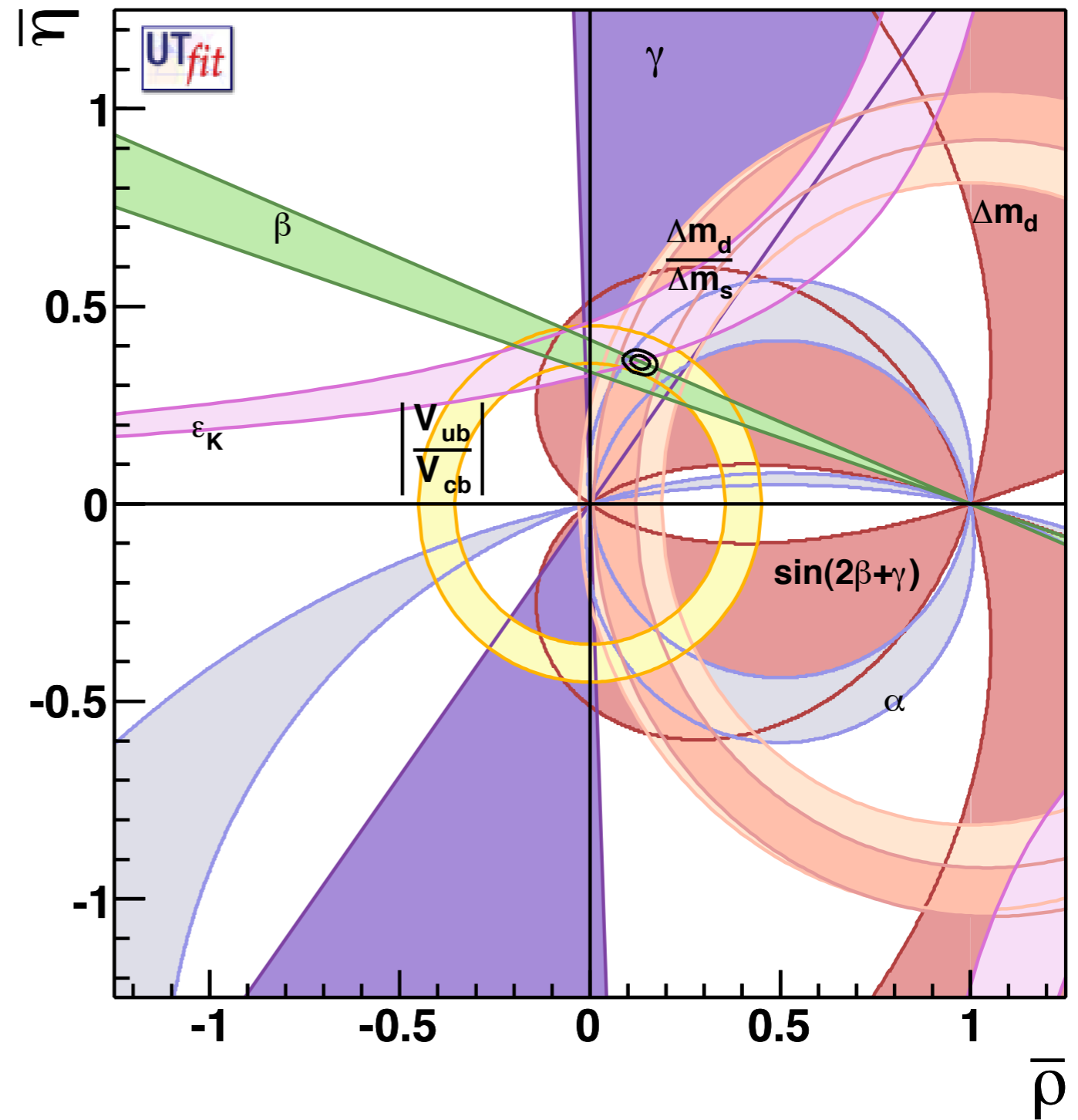
[PDG2010]

Theory:

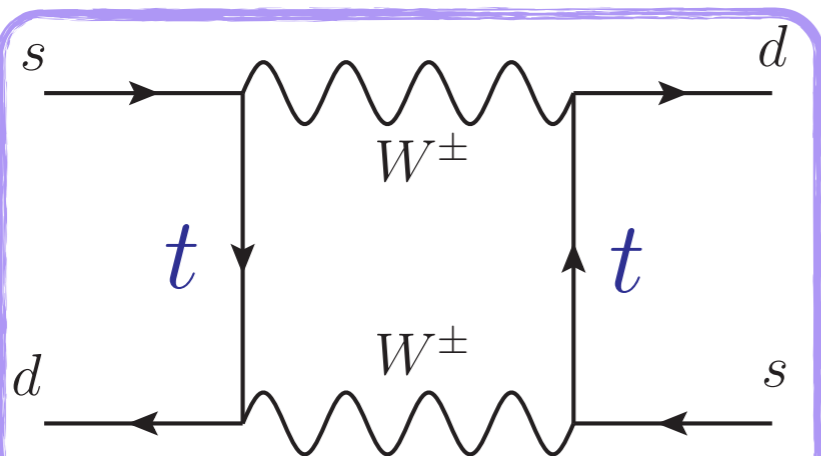
how well can we
calculate ϵ_K ?

$$|\epsilon_K| = 1.83(27) \times 10^{-3}.$$

[NLO SM prediction]



Perturbative Calculation



top: η_{tt}

$\log \chi_t$

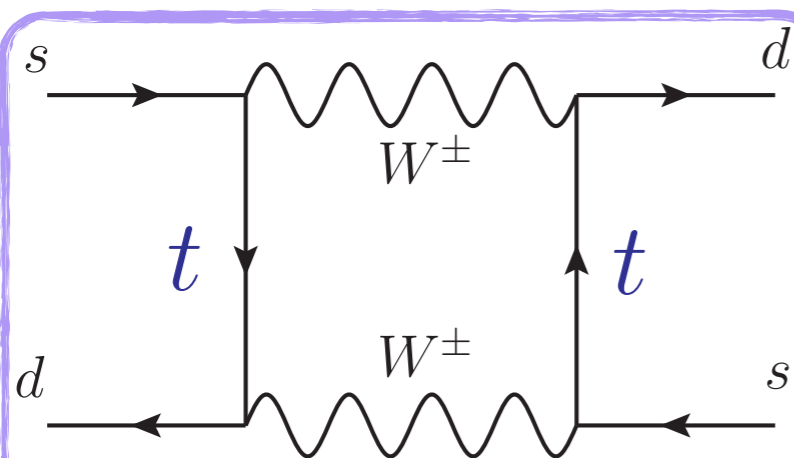
LO $(\alpha_s \log \chi_c)^n$

NLO $\alpha_s (\alpha_s \log \chi_c)^n$

ϵ_K 75%

scale 1.8%

Perturbative Calculation



top: η_{tt}

$\log \chi_t$

LO

$$(\alpha_s \log \chi_c)^n$$

NLO

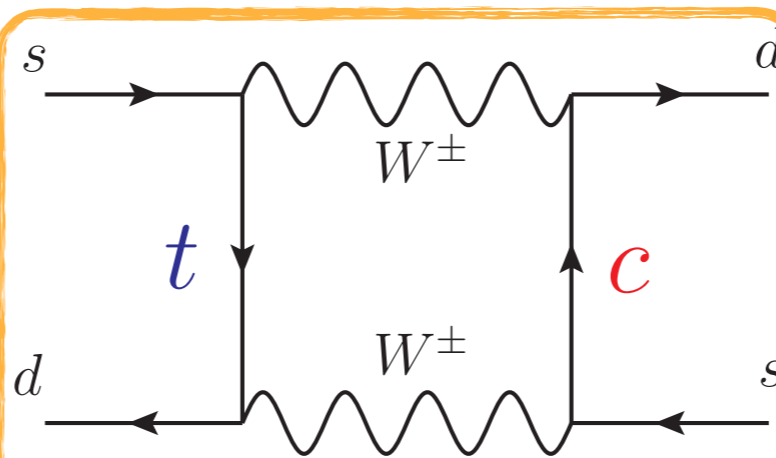
$$\alpha_s (\alpha_s \log \chi_c)^n$$

ϵ_K

75%

scale

1.8%



charm top: η_{tt}

$\log \chi_c$

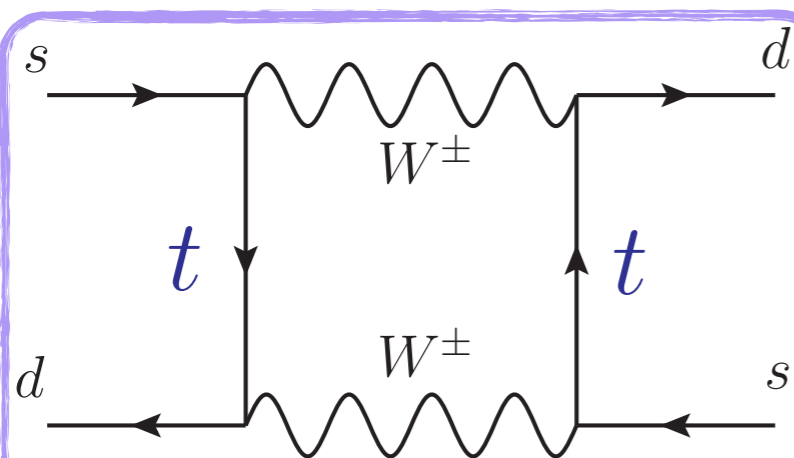
$$(\alpha_s \log \chi_c)^n \log \chi_c$$

$$(\alpha_s \log \chi_c)^n$$

40%

25%

Perturbative Calculation



top: η_{tt}

$\log \chi_t$

LO

$$(\alpha_s \log \chi_c)^n$$

NLO

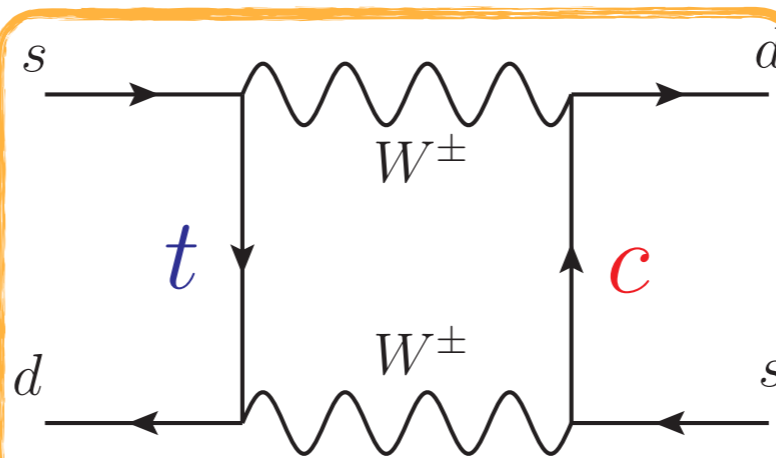
$$\alpha_s (\alpha_s \log \chi_c)^n$$

ϵ_K

75%

scale

1.8%



charm top: η_{tt}

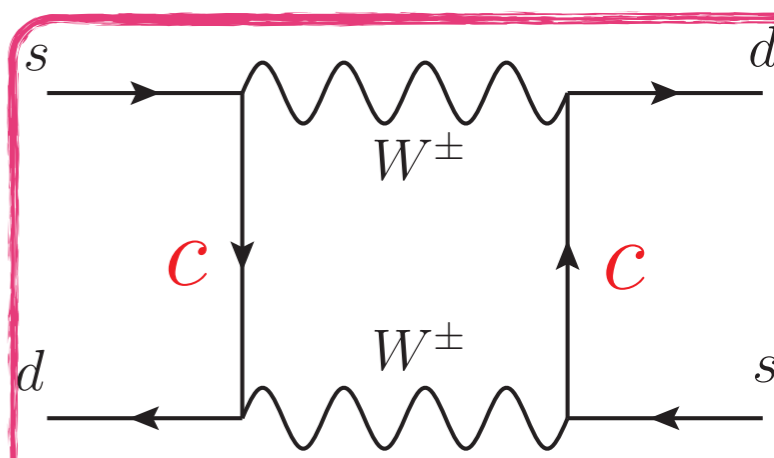
$\log \chi_c$

$$(\alpha_s \log \chi_c)^n \log \chi_c$$

$$(\alpha_s \log \chi_c)^n$$

40%

25%



charm: η_{cc}

$(\log \chi_c)^0$

hard GIM

$$(\alpha_s \log \chi_c)^n$$

$$\alpha_s (\alpha_s \log \chi_c)^n$$

-15%

38%

Charm & Top Quark

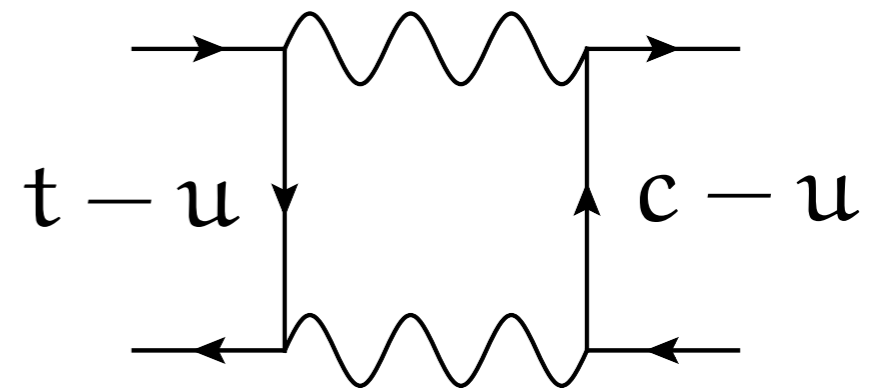
$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S \left(\frac{m_t^2}{M_W^2} \right) + \lambda_c \lambda_t \eta_{ct} S \left(\frac{m_c^2}{M_W^2} \right) + \lambda_c^2 \eta_{cc} S \left(\frac{m_c^2}{M_W^2} \right) \right] \tilde{Q}$$

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1-loop diagram: infrared divergent

$$\approx \lambda_{\text{Cabibbo}}^6 \times \frac{m_c^2}{M_W^2} \log \left(\frac{m_c^2}{M_W^2} \right) \tilde{Q}$$



Charm & Top Quark

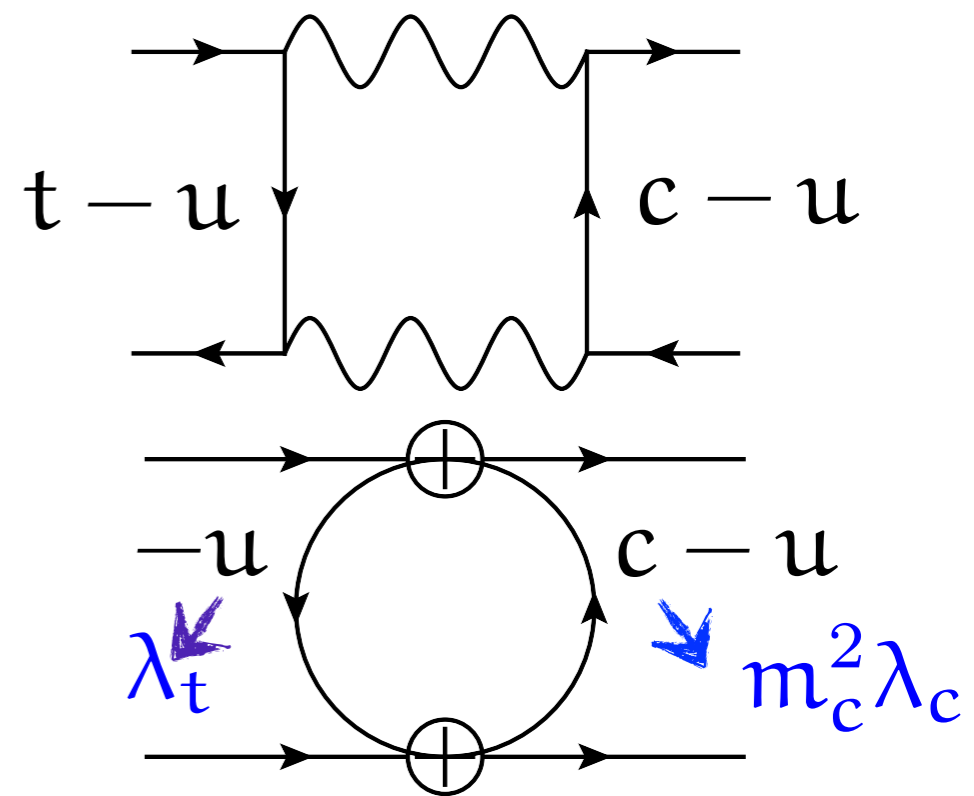
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I-loop diagram: infrared divergent

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Leading Log: I-loop RGE

$$\approx \lambda_{\text{Cabibbo}}^6 \frac{m_t^2}{M_W^2} \tilde{Q} \sum_{i=1}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^{(i-1)} \ln^i \left(\frac{m_c^2}{M_W^2} \right)$$



Charm & Top Quark

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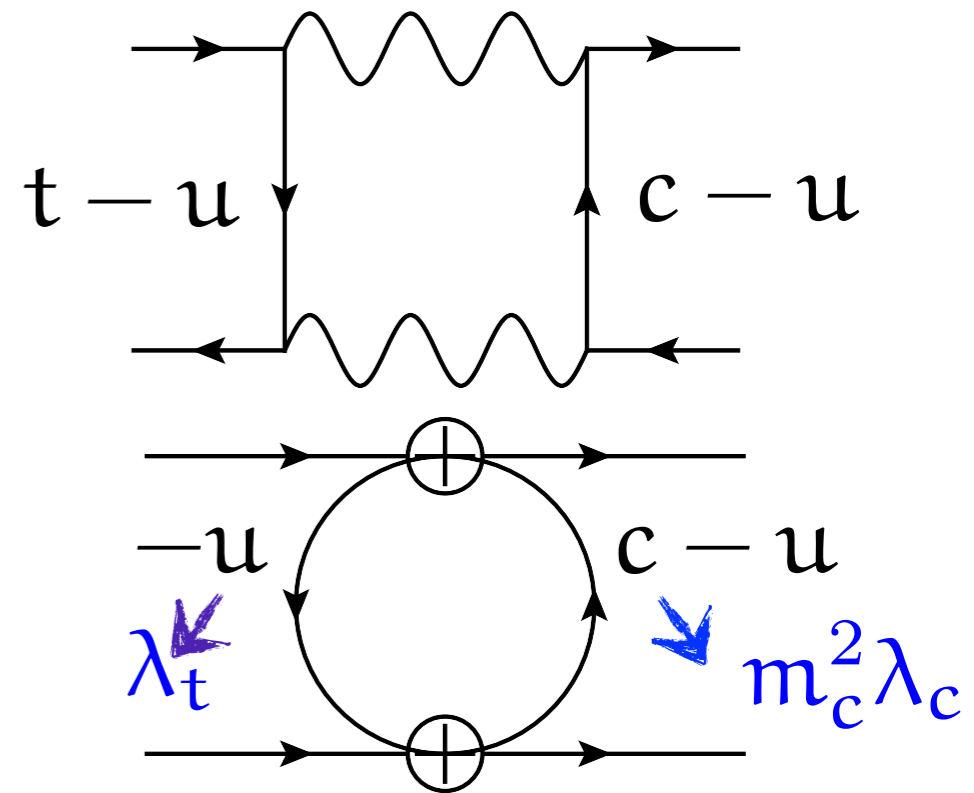
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Leading order:

Tree-level matching &
I-loop RGE and I-loop RGI

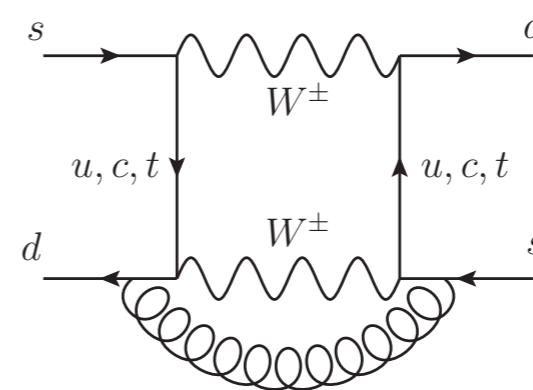


NLO: $\eta_{ct} = 0.457(73)$
+40% Contribution to ϵ_K
(Uncertainty 6%)

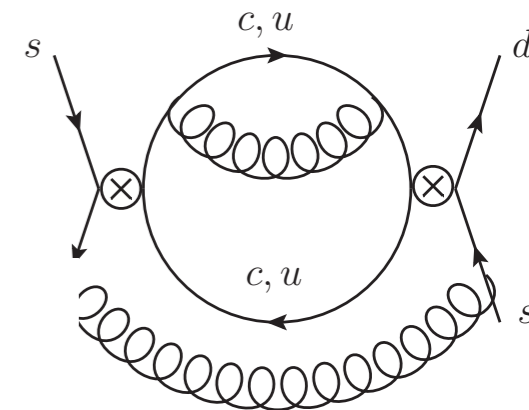
η_{ct} : NNLO Order

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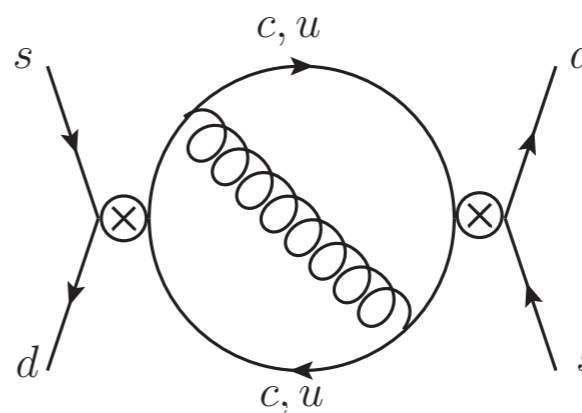
2-loop Matching at the high scale
($\mu = \mu_{t/W}$):



3-loop renormalisation group
equations

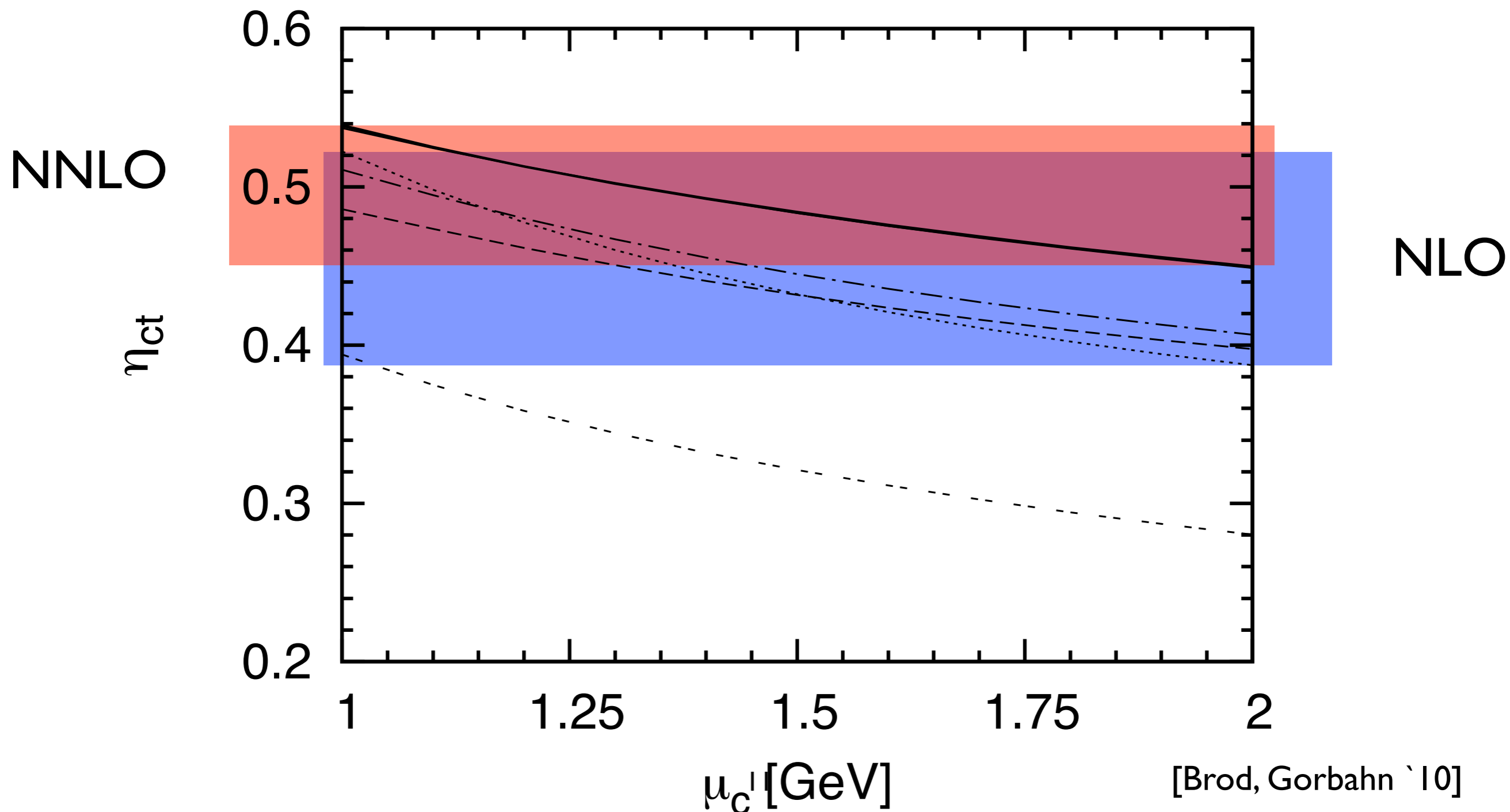


2-loop Matching at ($\mu = \mu_b$)
and ($\mu = \mu_c$)



Scale Dependence of η_{ct}

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S \left(\frac{m_t^2}{M_W^2} \right) + \lambda_c \lambda_t \eta_{ct} S \left(\frac{m_c^2}{M_W^2} \right) + \lambda_c^2 \eta_{cc} S \left(\frac{m_c^2}{M_W^2} \right) \right] \tilde{Q}$$



Charm Quark Contribution

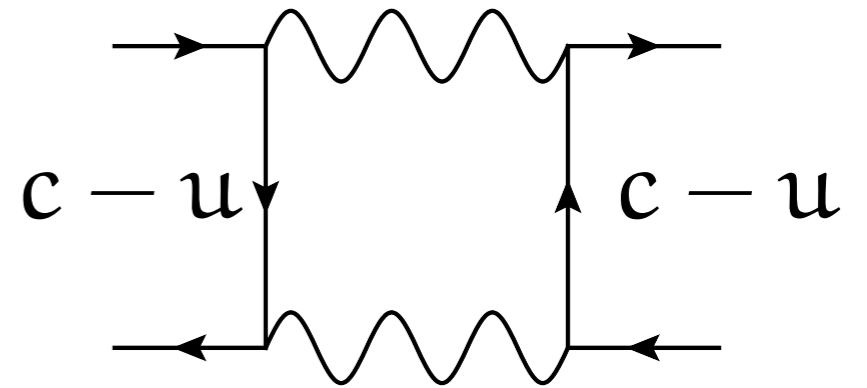
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Matching box diagrams at $(\mu = \mu_t)$
and expand in m_c^2 :

Diagram with 2 charm quarks cancels
with 1 charm 1 up quark diagrams



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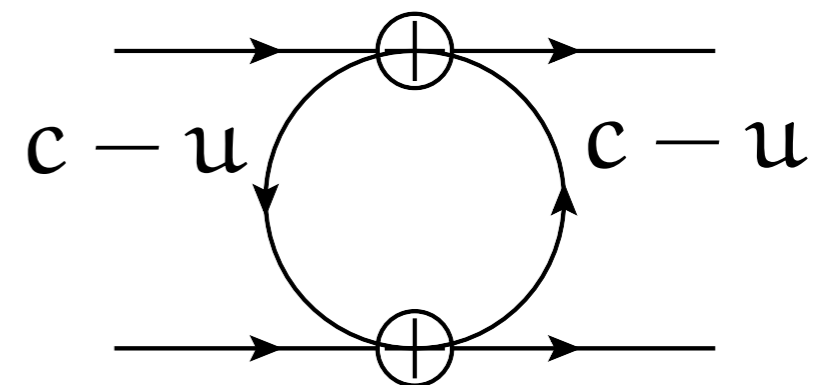
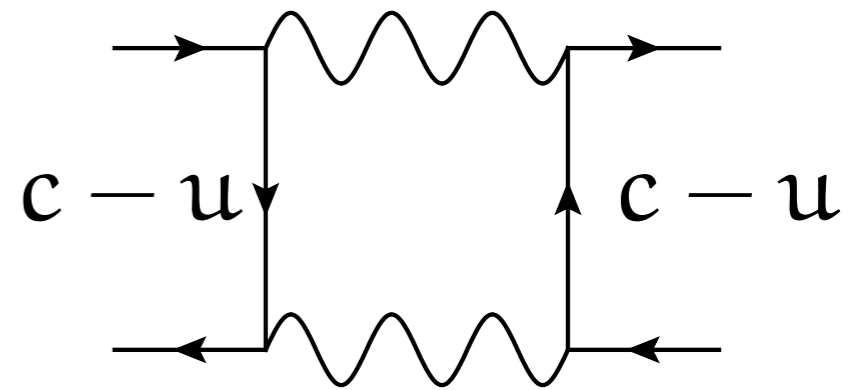
Matching box diagrams at ($\mu = \mu_t$)
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Diagram with 2 charm quarks cancels
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→ RGE & matching ($\mu = \mu_{t/W}$) only for:
current-current operators

Leading order ($\mu = \mu_c$): 1-loop matching

$$\approx \lambda_{\text{Cabibbo}}^6 \eta_{cc} \frac{m_c^2}{M_W^2} \tilde{Q}$$



η_{cc} at NNLO

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S \left(\frac{m_t^2}{M_W^2} \right) + \lambda_c \lambda_t \eta_{ct} S \left(\frac{m_c^2}{M_W^2} \right) + \lambda_c^2 \eta_{cc} S \left(\frac{m_c^2}{M_W^2} \right) \right] \tilde{Q}$$

$\eta_{cc} = 1.40(37)$ at NLO QCD [Herrlich, Nierste '94]
– Using there new estimation of the uncertainty

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A few weeks of 3-loop matching:

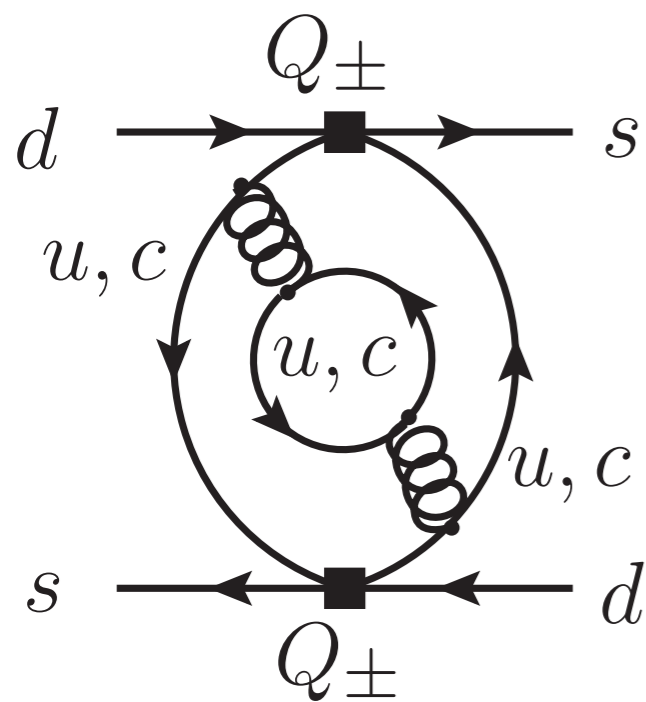
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 – Using there new estimation of the uncertainty

A few weeks of 3-loop matching:

We find a sizable NNLO correction
 [Brod, Gorbahn '11]



η_{cc} at NNLO

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– Using there new estimation of the uncertainty

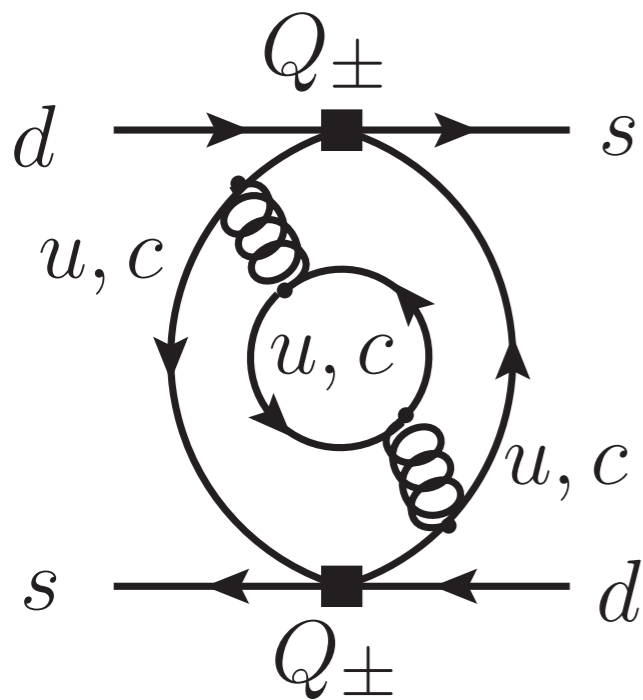
A few weeks of 3-loop matching:

We find a sizable NNLO correction
[Brod, Gorbahn '11]

Expanding in an ordinary perturbation theory at $\mu = m_c$

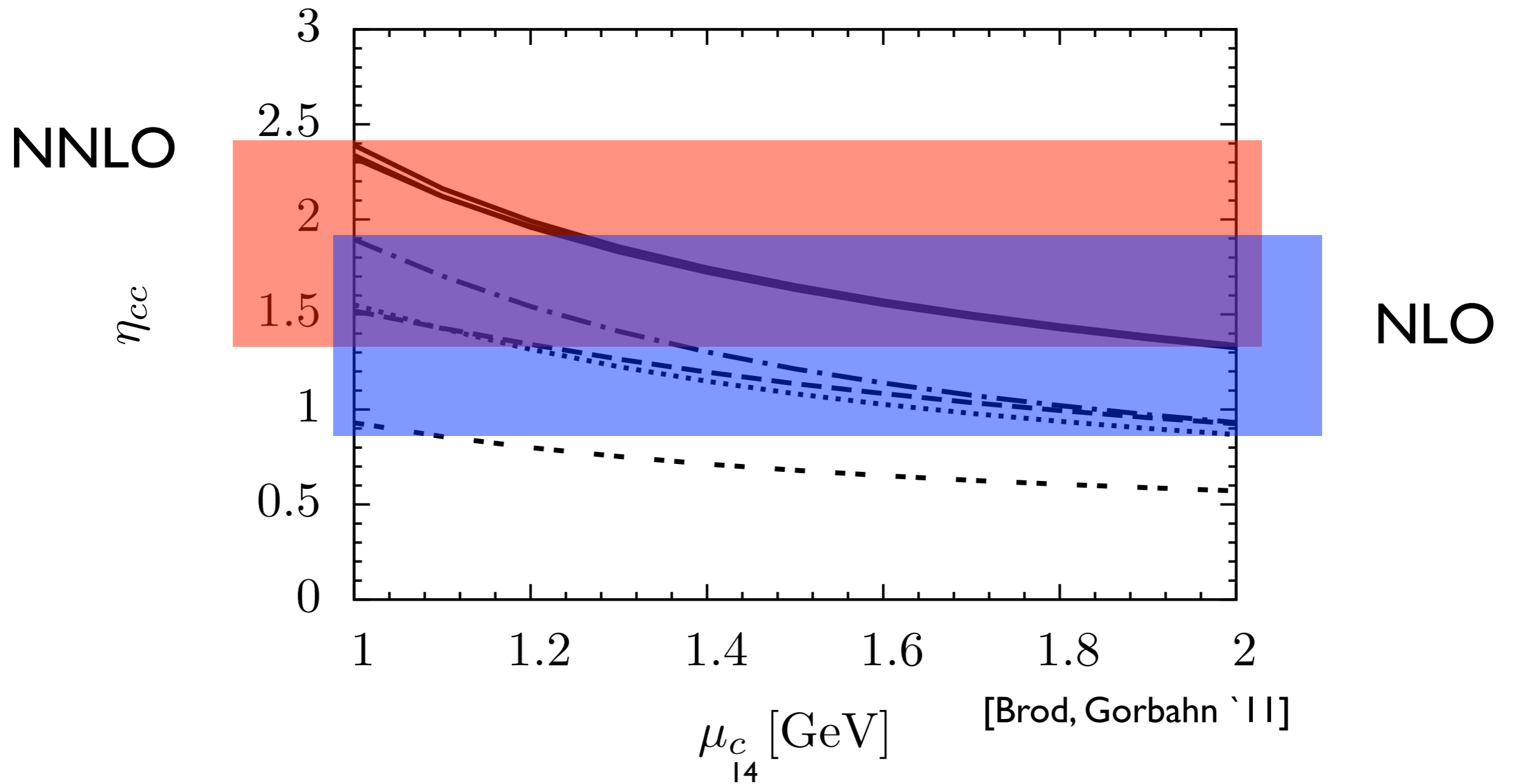
$$\eta_{cc}/\alpha_s^{2/9} = 1 + \alpha_s (0.25 + 0.32L_c) + \alpha_s^2 (1.20 + 0.03L_b + 0.22L_c + 0.27L_c^2)$$

$$\alpha_s = \alpha_s(m_c), \quad L_{3i} = \log(m_i^2/M_W^2)$$



Scale Dependence of η_{cc}

$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S \left(\frac{m_t^2}{M_W^2} \right) + \lambda_c \lambda_t \eta_{ct} S \left(\frac{m_c^2}{M_W^2} \right) + \lambda_c^2 \eta_{cc} S \left(\frac{m_c^2}{M_W^2} \right) \right] \tilde{Q}$$



η_{ct} & η_{cc} : Numbers

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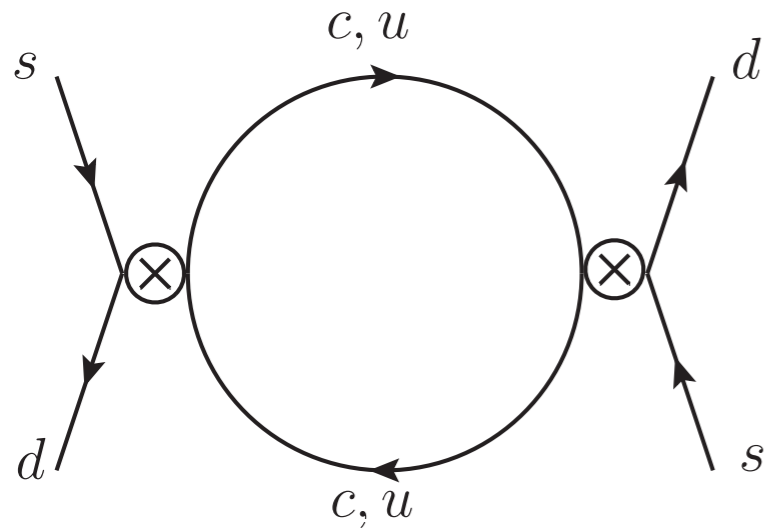
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η_{cc} : Finite shift and scale uncertainty of same size – added in squares

$$\eta_{cc}^{\text{NLO}} = 1.38(53) \xrightarrow{\text{scale}} \eta_{cc}^{\text{NNLO}} = 1.86(54) \xrightarrow{\text{scale+finite}} \eta_{cc}^{\text{NNLO}} = 1.87(76)$$

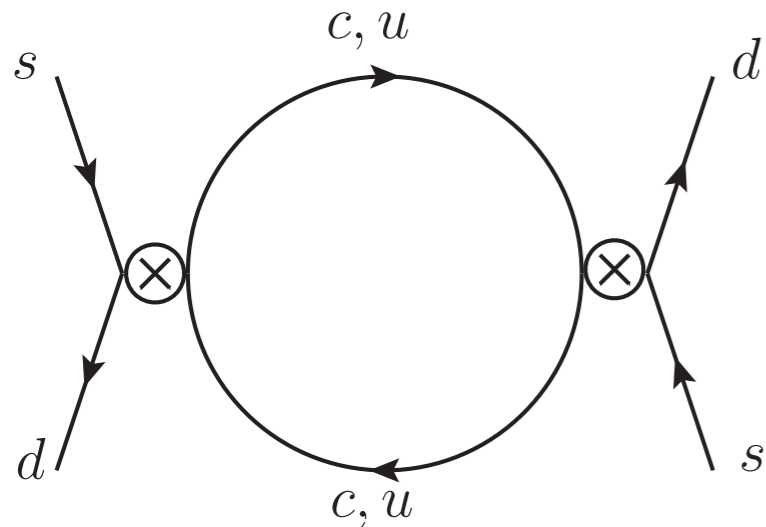
Long Distance Contribution



$$\int d^4x \langle K^0 | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | \bar{K}^0 \rangle$$

Higher dimensional operator [Cata Peris`04]

Long Distance Contribution



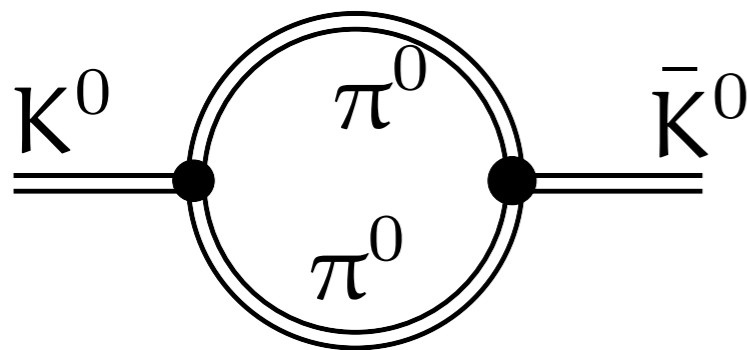
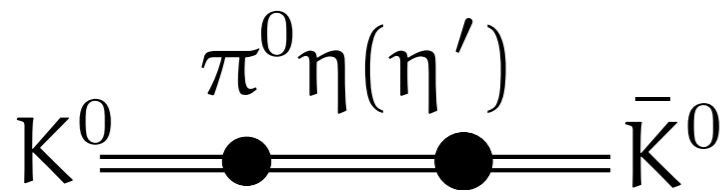
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Light quark loops in CHPT:

π^0, η tree level vanishes (Gell-Mann-Okuba)

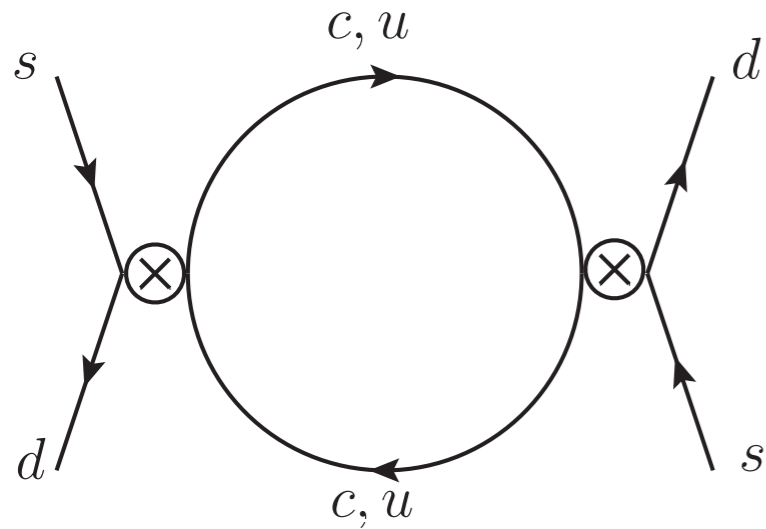
η' comes with zero phase [Gerard et.al. '05]



1-loop diagram divergent:

estimate from $\ln(m_\pi/m_\rho)$ [Buras et.al. '10]

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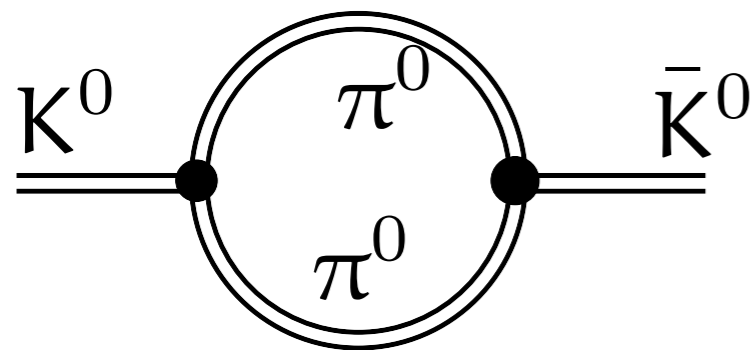
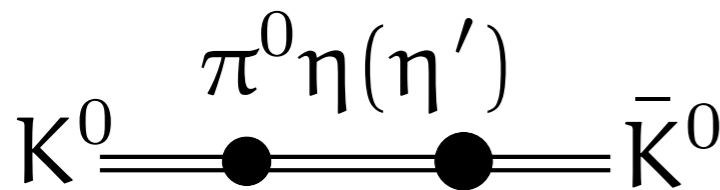
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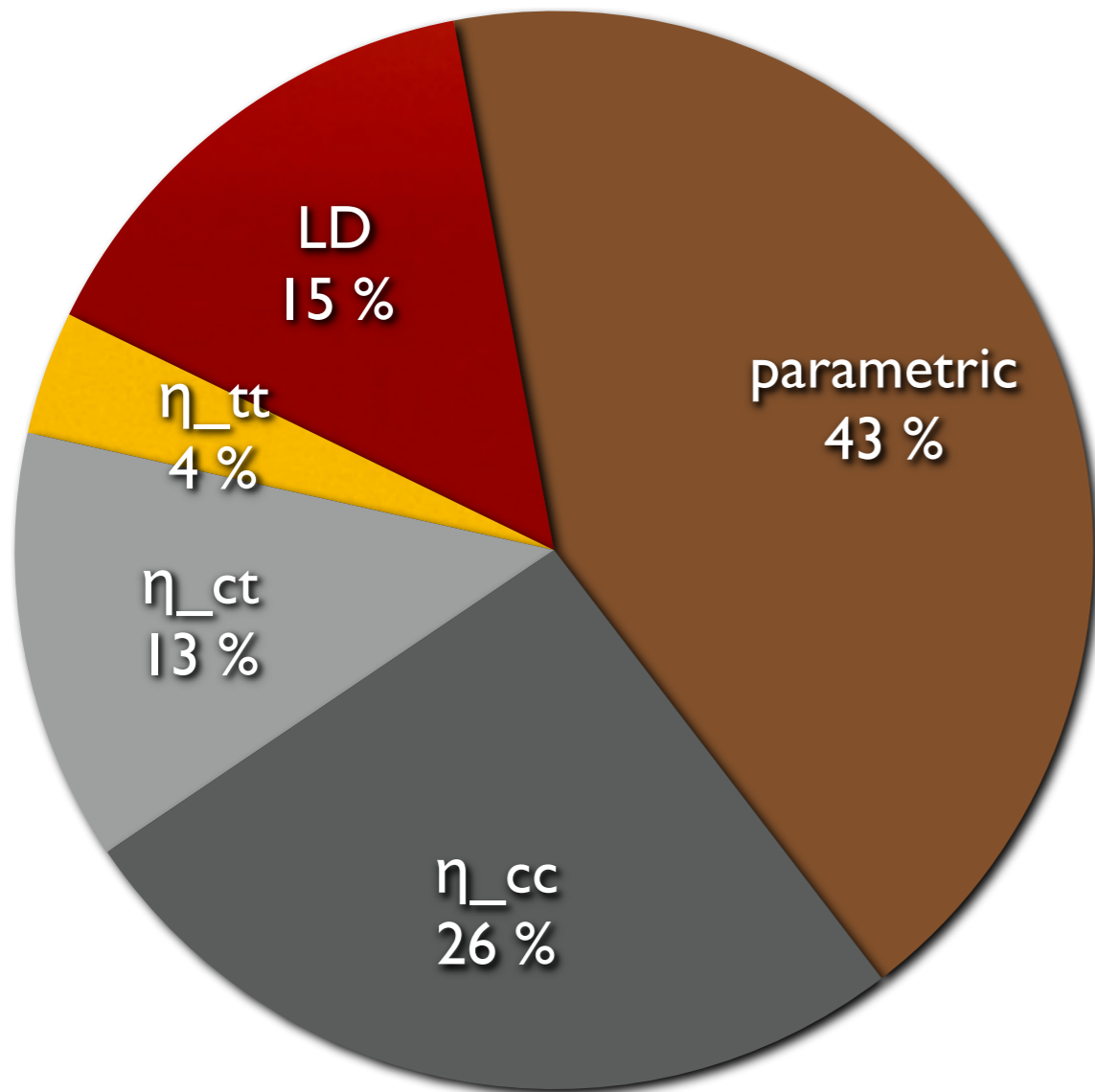
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$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left(\frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right)$$

absorptive part estimated from ϵ'

Results and Error Budget



input [PDG `10]

$$|\epsilon_K| = 1.81(28) \times 10^{-3}$$

$$|V_{cb}| = 406(13) \times 10^{-4}$$

Experiment [PDG `10]:

$$|\epsilon_K|^{\text{exp.}} = 2.228(11) \times 10^{-3}$$

B_K precisely known [Aubin, UKQCD, ETM]
we use 0.725 (26)

The short distance part of ΔM_K saturates 86% of ΔM_K experimental

$$\Delta M_K^{\text{SD}} = (3.0 \pm 1.2) \times 10^{-15} \text{ GeV}$$

Conclusions

Perturbative QCD corrections to Kaon mixing at NNLO

+ Improvements from lattice and experimental input:

Slight tension between the standard model prediction and the experimental value for ϵ_K

Hint for new physics? or rather theory uncertainty still too large