Signatures of conformal window on the Lattice

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14.10.11, Alex Fescht Siegen

Happy Birthday Alex!

$B \to \pi\pi$ Decay in QCD

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A new method is suggested to calculate the $B \to \pi\pi$ hadronic matrix elements from QCD light-cone sum rules. To leading order in α_s and $1/m_b$, the sum rule

A paper I enjoyed reading ....

- trick spurious momentum justify ligth-cone OPE
- spurious momentum removed by suitable analytic continuation

Overview

- ☆ Motivation & introduction conformal window studies

- ☆ Identification on lattice
  - observables in CGT and mass-deformed CGT
  - hyperscaling laws of hadronic observables

Del Debbio & RZ
PRD'10 & PLB'11

- ☆ Brief overview lattice results

Scaling dimension $\Delta_{\mathcal{O}} = d_{\mathcal{O}} + \gamma_{\mathcal{O}}$; naive plus anomalous dimension

$\gamma_m = -\gamma_{\bar{q}q}$, denoted by γ_* at fixed-point $\Rightarrow \Delta_{\bar{q}q} = 3 - \gamma_*$

Motivations for studying gauge theory theories

gauge theories are interesting per se (applications in mathematical physics)

gauge theories are associated with forces in nature (LHC?)

prototype: **Technicolor** *(Susskind/Weinberg '79)*

- Higgs sector ⇨ strongly coupled gauge theory

- Electroweak symmetry breaking due to chiral symmetry breaking - $M_W = g\, f_\pi^{(TC)}$

- Phenomenologically (constraints) advantageous are:
  Walking technicolour *(Holdom '81)*
  Conformal technicolour *(Luty, Okui '04)*

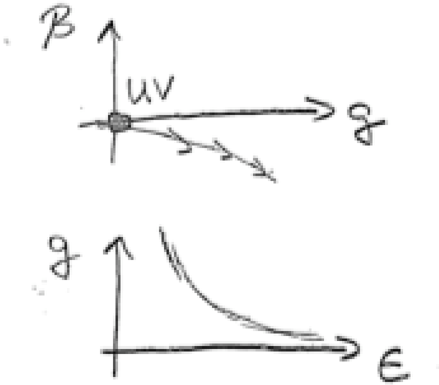
Types of gauge theories

- Adjustable: (1) gauge group e.g. SU(**N_c**)
  (2) **N_f** (massless) fermions
  (3) fermion **representation**

- Focus on asymptotically free theories (not many representations)

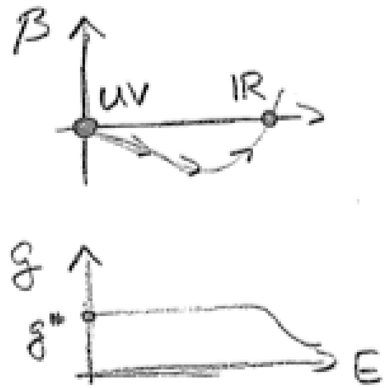
  - well defined on lattice
  - chance for unification in TC

QCD-like (confinement)



Cartoon

IR-conformal



Finding lattice signatures to distinguish the two major importance

Major goals (not exhaustive)

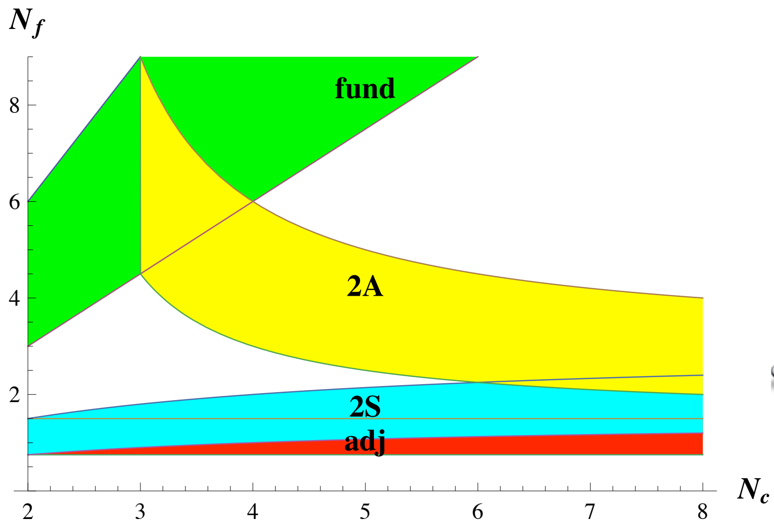
1. How many CGT? size of conformal window
2. Size anomalous dimension γ_m^*: $\Delta_{qq} = 3-\gamma_m^* \geq 1$ -- (γ_m^* large favorable for WTC)
3. The electroweak S-parameter -- (has discredited QCD-like TC)

N.B. unitarity bound for scalar: $\gamma_m^* \leq 2$ Mack'77

Conformal window (the pictures)

“SU(N)”

N=1 SUSY



- upper line AF
- just below pert. BZ/BM fixed pt:

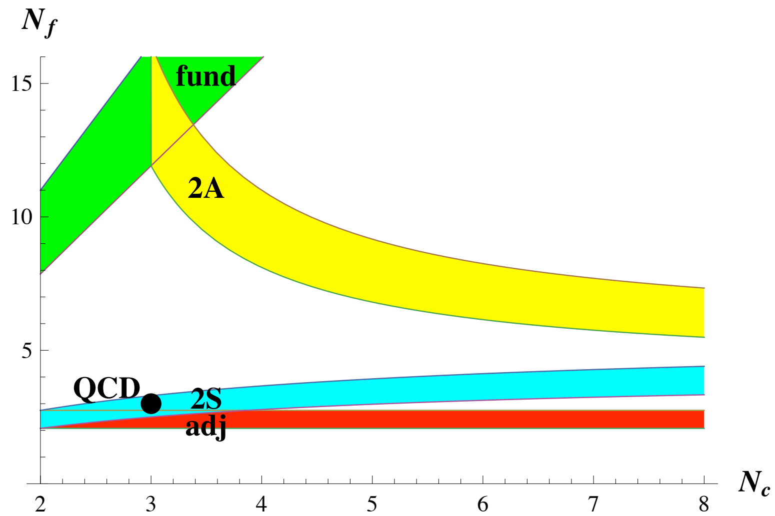
weak coupling

- lower line BZ/BM fixed pt
  “electromagnetic dual”

strong coupling

- assume in between conformal
  use $\beta_{NSVZ}(\gamma^*) = 0$ to get γ^*
- $\gamma^*|_{strong} = 1$ (unitarity bound QQ state)

non-SUSY



$$\beta_0 \text{ tuned small } \frac{\alpha_s^*}{2\pi} = \frac{\beta_0}{-\beta_1} \ll 1$$

- lower line Dyson-Schwinger eqs
  predict chiral symmetry breaking
  (lattice results later ...)
- $\gamma^*\,|_{strong} \approx 1$ DS eqs ladder

Observables in a CFT?

vanishing β-function $\langle O(x)O(0)\rangle \sim (x^2)^{-\Delta}$;$\Delta = d + \gamma_*$

But: Lattice finite m_{quark} (& volume) anyway

⇒ mass-deformed conformal gauge theories (mCGT)* $\mathcal{L} = \mathcal{L}_{\rm CGT} - m\bar{q}q$

Observables in a mCGT

☆ picture due to *Miransky*'98: -- (resembles heavy quark physics ..later)
finite m_q ; quarks decouple ➪ pure YM confines (string tension confirmed lattice)
➪ hadronic spectrum ➪ beloved hadronic observables

signature: leading scaling hadronic observable in m

$$\mathcal{O}_{\rm hadronic} \sim m^{\eta_{\mathcal{O}}}(1 + \dots)\,, \qquad \eta_{\mathcal{O}} > 0\,, \eta = f(\gamma_*)$$

* hardly related to 2D CFT mass deformation a part of algebra and 'therefore' integrability is maintained

Hyperscaling laws

physical states
no anomalous dim.

Consider matrix element: $\mathcal{O}_{12}(g,\hat{m},\mu) \equiv \langle \varphi_2 | \mathcal{O} | \varphi_1 \rangle$

1. $\mathcal{O}_{12}(g,\hat{m},\mu) = b^{-\gamma_{\mathcal{O}}} \mathcal{O}_{12}(g',\hat{m}',\mu')$,

RG -trafo O_{12}
$\mu = b\mu'$

$$g' = b^{y_g} g \quad \hat{m}' = b^{y_m}\hat{m} \ , \quad y_m = 1+\gamma_* \ , \quad y_g < 0 \ \text{(irrelevant)}$$

2. $\mathcal{O}_{12}(\hat{m}',\mu') = b^{-(d_{\mathcal{O}}+d_{\varphi_1}+d_{\varphi_2})} \mathcal{O}_{12}(\hat{m}',\mu)$

change
physical units

3. Choose b s.t. $\hat{m}' = 1 \Rightarrow$ trade b for m

“master equation”

Hyperscaling
relations

$$\Rightarrow \quad \mathcal{O}_{12}(\hat{m},\mu) \sim (\hat{m})^{(\Delta_{\mathcal{O}}+d_{\varphi_1}+d_{\varphi_2})/y_m}$$

* From Weinberg-like RNG eqs on correlation functions (widely used in critical phenomena)

Applications follow:

$$\eta_{\mathcal{O}_{12}} = (\Delta_{\mathcal{O}} + d_{\varphi_1} + d_{\varphi_2})/y_m$$

- vacuum condensates: $\langle \bar{q}q \rangle \sim m^{\frac{3-\gamma_*}{1+\gamma_*}}$, $\langle G^2 \rangle \sim m^{\frac{4}{1+\gamma_*}}$

more later on...

- decay constants:
  $|\varphi\rangle = |H(\text{adronic})\rangle$

  N.B. ($\Delta_H = d_H = -1$ choice)

$\mathcal{O}$	def	$\langle 0 \vert \mathcal{O} \vert J^{P(C)}(p)\rangle$	$J^{P(C)}$	$\Delta_{\mathcal{O}}$	$\eta_{G[F]}$
S	$\bar{q}q$	G_S	0^{++}	$3-\gamma_*$	$(2-\gamma_*)/y_m$
S^a	$\bar{q}\lambda^a q$	G_{S^a}	0^+	$3-\gamma_*$	$(2-\gamma_*)/y_m$
P^a	$\bar{q}i\gamma_5 q$	G_{P^a}	0^-	$3-\gamma_*$	$(2-\gamma_*)/y_m$
V	$\bar{q}\gamma_\mu q$	$\epsilon_\mu(p) M_V F_V$	1^{--}	3	$1/y_m$
V^a	$\bar{q}\gamma_\mu \lambda^a q$	$\epsilon_\mu(p) M_V F_{V^a}$	1^-	3	$1/y_m$
A^a	$\bar{q}\gamma_\mu\gamma_5\lambda^a q$	$\epsilon_\mu(p) M_A F_{A^a}$	1^+	3	$1/y_m$
		$ip_\mu F_{P^a}$	0^-	3	$1/y_m$

Hadronic mass I -- from trace anomaly

- trace anomaly: $\theta_\alpha{}^\alpha|_{\rm on-shell} = \frac{1}{2}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$

Adler et al, Collins et al
N.Nielsen '77 Fujikawa '81

- in our context set: β=0

.

Taking matrix element with hadron H
and the convention : $\langle H(p)|H(k)\rangle = 2E_p\delta^{(3)}(p-k) \Rightarrow$

"master equation"

$$2M_H^2 = N_f(1+\gamma_*)m\langle H|\bar{q}q|H\rangle \sim \mathbf{m}^{\frac{2}{(1+\gamma_*)}}$$

relation reminiscent GMOR-relation

Scaling also Miransky'98 using "pole mass"

Hadronic mass II -- without RG

Del Debbio, RZ Sep'10

Hellmann-Feynman-Thm

$$\frac{\partial E_\lambda}{\partial \lambda} = \langle \psi(\lambda) | \frac{\partial \hat{H}(\lambda)}{\partial \lambda} | \psi(\lambda) \rangle$$

idea: $\frac{\partial \langle \psi(\lambda) | \psi(\lambda) \rangle}{\partial \lambda} = 0$

- applied to our case: $$m \frac{\partial M_H^2}{\partial m} = N_f m \langle H | \bar{q} q | H \rangle$$

- combined with GMOR-like .. $$m \frac{\partial M_H}{\partial m} = \frac{1}{1+\gamma_*} M_H$$

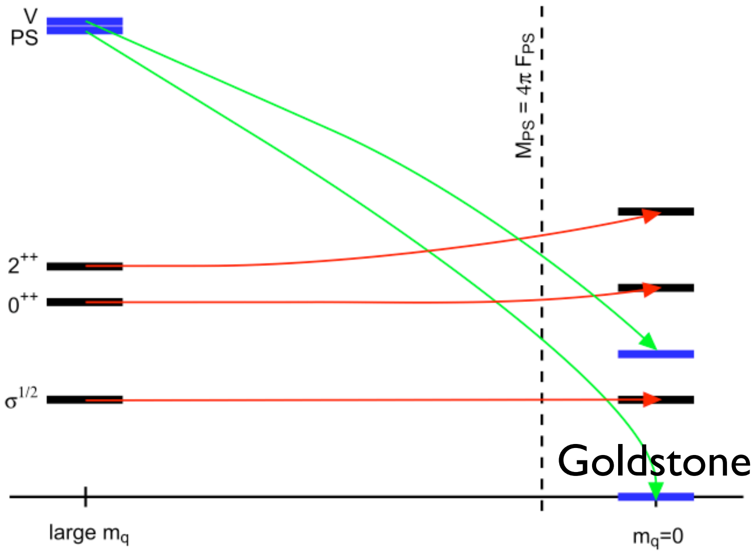
  $2M_H^2 = N_f(1+\gamma_*) m \langle H | \bar{q} q | H \rangle$

$$M_H \sim m^{\frac{1}{1+\gamma_*}}$$

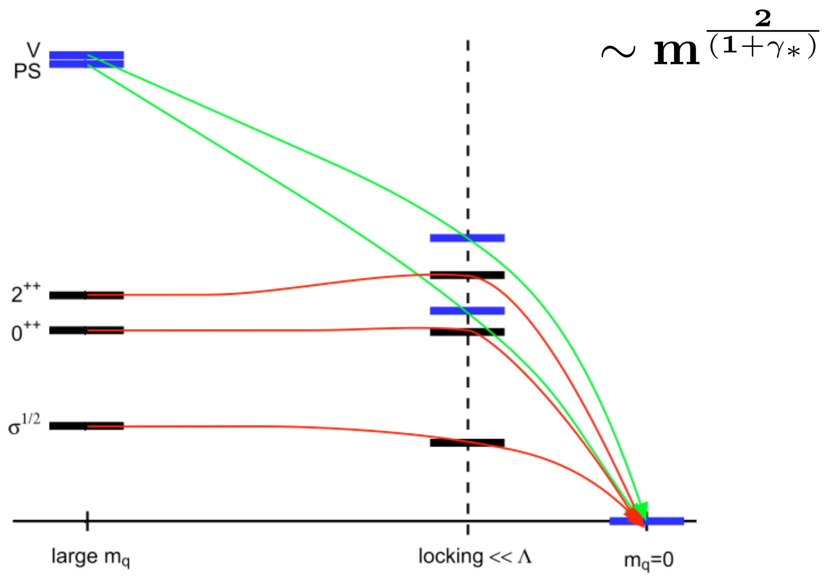
scaling law without using RG!

Let's pause: comparison with QCD-like spectrum

QCD-like (χ-symmetry broken)



"here" mCGT spectrum



⭐ No SSB of χ-symmetry breaking (no goldstone boson)
since condensate triggered by explicit χ-breaking $L = m q_L q_R$ +h.c.

➩ **no chiral perturbation theory** (no parametric suppression of the "pion mass")

⭐ A point that can be clarified: $M_H \sim m^{1/(1+\gamma^*)}$ looks a bit like heavy quark physics
Settled by observing: $f_{P(B\text{-meson})} \sim m^{-1/2}$ whereas $f_{P(mCGT)} \sim m^{(2-\gamma^*)/(1+\gamma^*)}$

Generalized Banks-Casher relation

- Banks & Casher ’80 a la Leutwyler & Smilga 92’:

Green’s function: $\langle q(x)\bar{q}(y)\rangle = \sum_n \frac{u_n(x)u_n^\dagger(y)}{m - i\lambda_n}$, where $\not{D} u_n = \lambda_n u_n$

$$\langle \bar{q}q\rangle_V = \frac{1}{V}\int dx\,\langle \bar{q}(x)q(x)\rangle = -\frac{2m}{V}\sum_{\lambda_n > 0}\frac{1}{m^2+\lambda_n^2} \overset{V\to\infty}{=} -2m\int_0^\infty d\lambda \frac{\rho(\lambda)}{m^2+\lambda^2}$$

- UV-divergences later -- focus IR-physics

$$\langle \bar{q}q\rangle \overset{m\to 0}{\sim} m^{\eta_{\bar{q}q}} \quad \Leftrightarrow \quad \rho(\lambda) \overset{\lambda\to 0}{\sim} \lambda^{\eta_{\bar{q}q}}$$

- QCD : $\eta_{\bar{q}q} = 0 \Rightarrow \rho(0) = -\pi\langle\bar{q}q\rangle$

  mCGT: another way to measure anomalous dimension

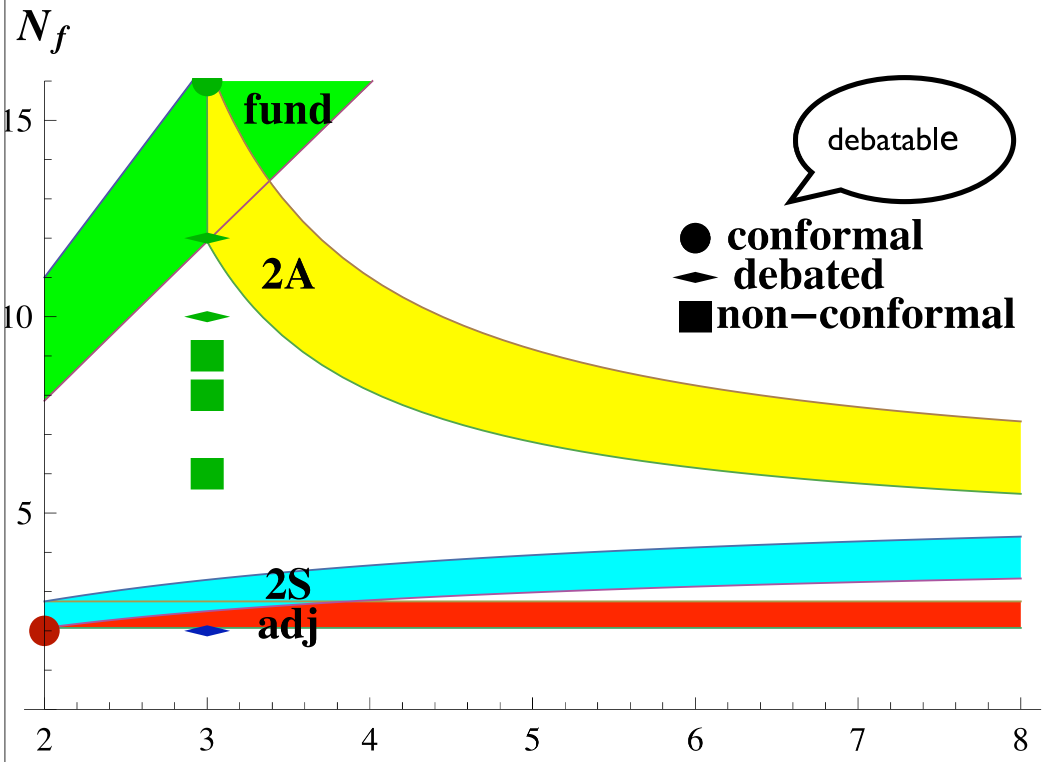
Banks, Casher’80

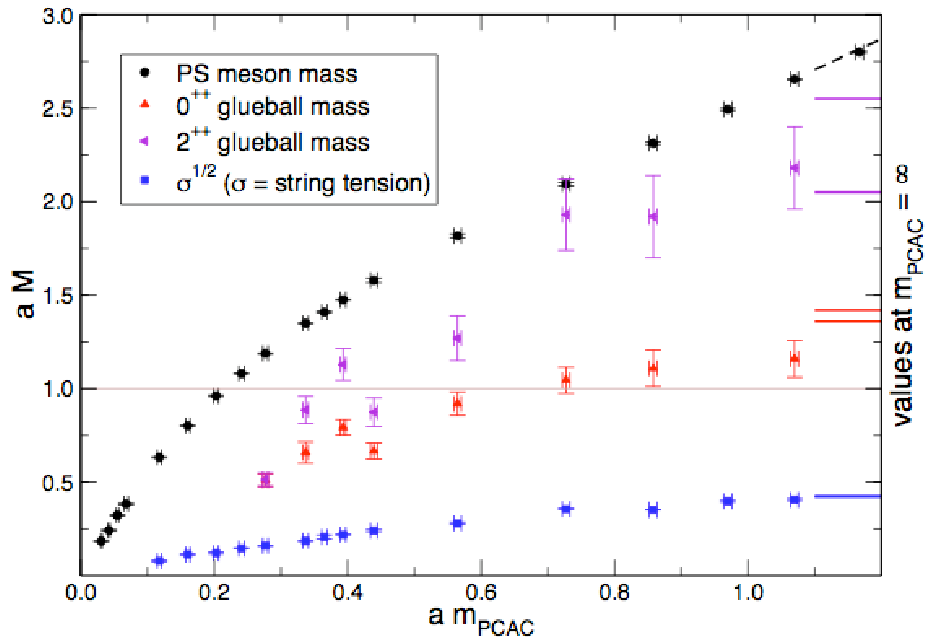
DeGrand’09

DelDebbio RZ’10 May

Lattice results

control systematics $a \to 0$, $V \to \infty$ and scaling m
$\Rightarrow$ challenging





Del Debbio et al SU(2) adj

<u>Methods:</u>

- spectral studies (scaling laws)
- β-fct through stepsize scaling
- enhancement of <qq>/f_π^3 for WTC (no parametric control)

- SU(3) fund. (code exists) (trend towards parity doubling for $N_F = 6$)
- SU(2) adj=2S MWTC consensus $0.05 < \gamma_* < 0.56$ glueballs lighter mesons
- SU(3) 2S (hextet) NMWTC maybe conformal $\gamma_* < 0.6$

Epilogue

- Identified "universal" hyperscaling laws in mass deformation
valid for any conformal theory in the vicinity of the fixed point (small mass)

- Conformal gauge theories have been identified -- non-QCD follows
scaling laws within uncertainties

- People have accepted that identification of conformal window is
more difficult than anticipated (need more CPU-time)

- Anomalous mass dimension significantly below $\gamma^* < 1$
(unitarity bound of $\gamma^* < 2$ not nearly reached so far!)

- Numerical results for S-parameter are eagerly awaited.
First study has been reported by DeGrand'10

Cheerio!

Backup slides ...

Some relevant/useful references

- Miransky hep-ph/9812350 spectrum (with mass) as signal of conformal window works with pole mass -- weak coupling regime $\Lambda_{YM} \cong m \, \mathrm{Exp}[-1/b_{YM} \, \alpha^*]$
  ➪ glueballs lighter than mesons
- Luty Okui JHEP'96 conformal technicolor propose spectrum as signal of cw
- Dietrich/Sannino PRD'07 conformal window SU(N) higher representation using Dyson-Schwinger techniques known from WTC
- Sannino/RZ PRD'08 <qq> done heuristically IR and UV effects understood 0905
- DelDebbio et al ArXiv 0907 Mass lowest state from RGE equation
- DeGrand scaling <qq> stated ArXiv 0910
- DelDebbio RZ ArXiv 0905 scaling of vacuum condensates, all lowest lying states
- DelDebbio RZ ArXiv 0909 scaling extended to entire spectrum and all local matrix elements

Del Debbio, RZ Sep'10

remarks S-parameter

Analytical guidance S-parameter: $S = 4\pi\Pi_{V-A}(0) - \text{pion pole}$

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\delta_{ab}\Pi_{V-A}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\left(V_a^\mu(x)V_b^\nu(0) - (V \leftrightarrow A)\right)|0\rangle$$

$$\Pi_{V-A}(q^2) \simeq \frac{f_V^2}{m_V^2 - q^2} - \frac{f_A^2}{m_A^2 - q^2} - \frac{f_P^2}{m_P^2 - q^2} + \dots .$$

(conspiracy) cancellations modulo improve on ...

$$\Pi_{V-A}^{\mathrm{W-TC}}(0) \sim O(m^{-1})$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(0) \sim O(m^0)$$

$$\Pi_{V-A}^{\mathrm{mCGT}}(q^2) \sim \frac{m^{2/y_m}}{q^2}$$

for $-q^2 \gg (\Lambda_U)^2$

Sannino'10 free theory

⇒ lattice determination coming soon (already some market)

Another look at the β-function

- Consider the again the trace (scale) anomaly:

$$\theta_\alpha{}^\alpha|_{\mathrm{on-shell}} = \frac{1}{2g}\beta G^2 + N_f m(1+\gamma_m)\bar{q}q$$

- Evaluate it on any hadronic state |H> and solve for β:

$$\beta = \frac{A_H + \gamma_m B_H}{G_H}$$

$$A_H = 2M_H^2 - mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$B_H = mN_f\langle H|\bar{q}q|H\rangle\,,$$
$$G_H = \langle H|G^2|H\rangle\,.$$

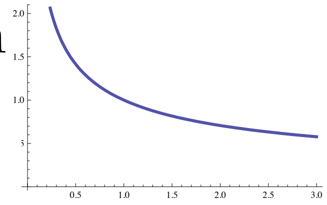
  - Ratios of A_H/G_H & B_H/G_H independent
  - Form β-function close to NSVZ β (for N=1 SUSY gauge theories)

$$\beta_{NSVZ} = -\frac{1}{16\pi^2}\frac{3T_G - T_R(1+\gamma)}{1-T_G/(8\pi^2)}$$

Heuristic look

- Deconstruct the continuous spectrum of a two point function
  Infinite sum of adjusted particles can mimick continuous spectrum

Stephanov'07



$$\bar{q}q(x) \sim \sum_n f_n \varphi_n(x)\,; \qquad \langle \varphi_n | \bar{q}q | 0 \rangle \sim f_n\,, \qquad \begin{cases} f_n^2 = \delta^2\,(M_n^2)^{\Delta_{\bar{q}q}-2} \\ M_n^2 = n\delta^2 \end{cases}$$

- Adding mass term looks like tadpole.
  ⇒ find new minimum -- add M_n to potential

$$\mathcal{L} = -m \textstyle\sum_n f_n \varphi_n - 1/2 \sum_n M_n^2 \varphi_n^2$$

Delgado, Espinoso, Quiros'07

- Solve $\quad m f_n + M_n^2 \varphi_n = 0 \quad \Rightarrow \quad \langle \varphi_n \rangle = -m f_n / M_n^2$ and reinsert:

$$\langle \bar{q}q \rangle \sim \sum_n f_n \langle \varphi_n \rangle = -m \sum_n \frac{f_n^2}{M_n^2} \overset{\delta \to 0}{\to} -m \int_{\Lambda_{\rm IR}^2}^{\Lambda_{\rm UV}^2} s^{\Delta_{\bar{q}q}-3} ds$$

  - Λ_{UV}: $\Delta_{qq} = 3$ find quadratic divergence known from Leutwyler-Smilga rep.
  - Λ_{IR}: 1) $\Lambda_{IR} \sim M_H \sim m^{1/(1+\gamma)}$ or use $(M_{dyn})^{\Delta qq} \sim$ <qq> generalizing Politzer OPE.
    and confirm $\eta_{qq} = \Delta_{qq}/(1+\gamma)$!

Mass & decay constant trajectory

- At large-N_c neglect width $\quad g_{H_n} \equiv \langle 0|\mathcal{O}|H_n\rangle$ (decay constant)

$$\Delta(q^2) \sim \int_x e^{ixq} \langle 0|\mathcal{O}(x)\mathcal{O}(0)|0\rangle = \sum_n \frac{|g_{H_n}|^2}{q^2+M_{H_n}^2}$$

- In limit m → 0 (scale invariant correlator)

$$\Delta(q^2) = \int_0^\infty \frac{ds\, s^{1-\gamma_*}}{q^2+s} + \text{s.t} \propto (q^2)^{1-\gamma_*}$$

- Solution are given by:

$$M_{H_n}^2 \sim \alpha_n m^{\frac{2}{1+\gamma_*}} \;, \quad g_{H_n}^2 \sim \alpha_n'(\alpha_n)^{1-\gamma_*} m^{\frac{2(2-\gamma_*)}{1+\gamma_*}}$$

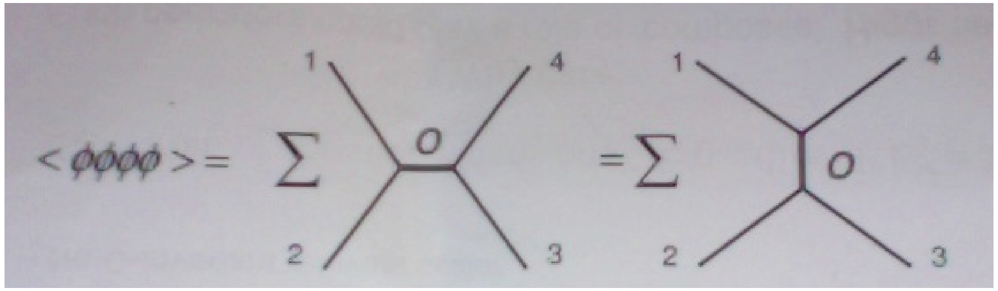
where α_n arbitrary function (corresponds freedom change of variables in ∫)

- QCD expect $\alpha_n \sim n$ (linear radial Regge-trajectory) (few more words)

- For those who know: resembles deconstruction Stephanov'07
  difference physical interpretation of spacing due to scaling spectrum

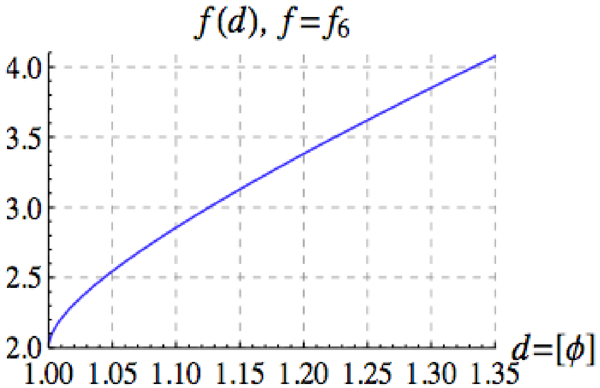
Addendum (bounds scaling dimension)

- assume add L = mqq (N.B. not a scalar under global flavour symmetry!)
- using bootstrap ('associative' OPE on 4pt function) possible to obtain upper-bound on scaling dimension Δ of lowest operator in OPE



non-singlet

Rattazzi, Rychkov Tonni & Vichi'08



1.35 still rather close to unitarity bound

singlet $\Delta \leq 4$
allows for Δ_{qq} to be:

Rattazzi, Rychkov & Vichi '10

G	$U(1) \equiv SO(2)$	$SO(3)$	$SO(4)$	$SU(2)$	$SU(3)$
d_*	1.063 ($k=2$) 1.12 ($k=4$)	1.032 ($k=2$) 1.08 ($k=4$)	1.017 ($k=2$) 1.06 ($k=4$)	1.016 ($k=2$)	1.003 ($k=2$)

very close to unitarity bound!