On η–η' Mixing: hints for discussion, from colour to flavour

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Workshop "Colour meets Flavour", Oct 14th, 2011,
Khodjamirian's 60th birthday

Outline of the talk

1. Review on theoretical and experimental progresses in $\eta-\eta'$ mixing—probing their gluonic content Do gluons play an independent role also in hadronic spectroscopy?

2. Great effort in literature, at different energy scales
   - electromagnetic and strong decays
   - electroweak D and B decays

3. Pointing out best theoretical and experimental strategies

η–η' Mixing

octet-singlet basis:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_P & -\sin\theta_P \\ \sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

$$\eta_0 = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)$$

$$\eta_8 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$$

quark-flavor basis:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi_P & -\sin\phi_P \\ \sin\phi_P & \cos\phi_P \end{pmatrix} \begin{pmatrix} \eta_{NS} \\ \eta_S \end{pmatrix}$$

$$\eta_{NS} = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

$$\eta_S = s\bar{s}$$

$$\theta_P = \phi_P - \arctan\sqrt{2} = \phi_P - 54.7^\circ$$

Two Mixing Angles Scenario

- From the late 90's *(Leutwyler , Kraiser , Kroll, Stech , Feldmann etc.)* been shown the mixing cannot be adequately described by a single angle; the fact that the decay constants follow the pattern of state mixing is an a-priori assumption

- Due to SU(3) breaking ($f_K \neq f_\pi$), mixing of decay constants does not follow the same pattern of state mixing

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}$$

$$f_\eta^8 \neq f_\eta^0 \neq f_{\eta'}^8 \neq f_{\eta'}^0$$

The estimated difference θ_8-θ_0 can be large [-12º, -19º]

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix}$$

$$f_\eta^q \neq f_\eta^s \neq f_{\eta'}^q \neq f_{\eta'}^s$$

Quark flavor basis

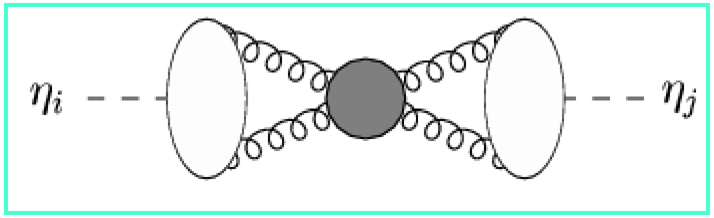
- The smallness of the mixing angles is consistent with the OZI-rule, i.e. amplitudes that involve quark-antiquark annihilation into gluons are suppressed
  – vector meson sector: mixing angle ≈ 3°

    $\omega \approx q\bar{q}$ $\phi \approx s\bar{s}$

$$\begin{cases} \eta_{NS} = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \\ \eta_S = s\bar{s} \end{cases}$$

- In the pseudoscalar sector, $U(1)_A$ anomaly allows larger mixing angle (≈ 40°) between η_{NS} and η_S .

$$\partial^\mu J^u_{\mu 5} = \partial^\mu(\bar{u}\,\gamma_\mu\gamma_5\,u) = 2\,m_u\,(\bar{u}\,i\gamma_5\,u) + \frac{\alpha_s}{4\pi}\,G\,\tilde{G}$$



the difference between the two mixing angles still determined by OZI-rule violating contribution

$$\phi_q \approx \phi_s \equiv \phi_P$$

Mix with Gluonium

- The η′ meson is a good candidate to have a sizeable gluonic content, (while the η meson is well understood as an SU(3)-flavor octet with a small singlet admixture)

$$\begin{cases} \eta' = X_{\eta'} \dfrac{1}{\sqrt{2}} | u\bar{u} + d\bar{d} > + Y_{\eta'} | s\bar{s} > + Z_{\eta'} | glue > \\ \eta = X_{\eta} \dfrac{1}{\sqrt{2}} | u\bar{u} + d\bar{d} > + Y_{\eta} | s\bar{s} > \end{cases}$$

$$\begin{cases} X_{\eta'} = \cos\phi_G \sin\phi_P ; Y_{\eta'} = \cos\phi_G \cos\phi_P ; Z_{\eta'} = \sin\phi_G \\ X_{\eta} = \cos\phi_P ; Y_{\eta} = -\sin\phi_P \end{cases}$$

Mixing with heavier pseudoscalar mesons is ignored

Electromagnetic and strong transitions

1. Radiative vector and pseudoscalar meson decays

$$\psi',\, \psi,\, \phi \to \gamma\eta' \;\; vs. \;\; \gamma\eta$$

$$\rho,\, \omega \to \gamma\eta$$

$$\eta' \to \gamma\omega,\, \gamma\rho$$

2. Decays into two photons or production in collisions:

$$\eta' \to 2\gamma \quad vs. \quad \eta \to 2\gamma$$

$$\gamma\gamma \to \eta \quad vs. \quad \gamma\gamma \to \eta'$$

3. Decays of ψ into PV final states with the vector meson acting as a \`flavor filter':

$$\psi \to \rho/\omega/\phi + \eta \;\; vs. \;\; \eta'$$

Radiative ρ/ω/ϕ Decays

	no gluonium hyphotesis	allowing for gluonium
KLOE 07:	$\phi_P = (41.3 \pm 0.3_{stat} \pm 0.7_{sys} \pm 0.6_{th})^\circ$	$\begin{cases} \phi_P = (39.7 \pm 0.7)^\circ \\ Z^2_{\eta'} = 0.14 \pm 0.04 \end{cases}$
Escribano, Nadal 07	$\phi_P = (42.7 \pm 0.7)^\circ$	$\begin{cases} \phi_P = (42.6 \pm 1.1)^\circ \\ Z^2_{\eta'} = 0.01 \pm 0.07 \end{cases}$
Thomas 07: no form factors	$\phi_P = (41.7 \pm 0.5)^\circ$	$\begin{cases} \phi_P = (41.7 \pm 0.5)^\circ \\ Z^2_{\eta'} = 0.04 \pm 0.04 \end{cases}$
with form factors $\exp\left(-\lvert p \rvert^2 / (8\beta^2)\right)$	$\phi_P = (42.8 \pm 0.8)^\circ$	$\begin{cases} \phi_P = (41.9 \pm 0.7)^\circ \\ Z^2_{\eta'} = 0.10 \pm 0.08 \end{cases}$

Latest KLOE data (2009)

- Message not truly inconsistent (considering the stated uncertainties), but ambivalent
  - Some studies (KLOE) point to a significant gluonic component, others not

- the th discussion has prompted a new KLOE update (2009)
  - Results confirmed
    - no gluons $\phi = (41.4 \pm 0.5)^\circ$
    - allowing gluons $\begin{cases} \phi = (40.4 \pm 0.6)^\circ \\ Z_{\eta'}^2 = 0.115 \pm 0.036 \end{cases}$

- the actual difference with KLOE values appears <u>not due</u> to a wrong set of variables, but to the inclusion in the analysis of $\eta' \to 2\gamma$

η' decays

- In lowest order all possible η' strong and first order electromagnetic decays are forbidden by discrete symmetries or occur at a suppressed rate
  - key role second-order electromagnetic transition

$$\boxed{\eta' \longrightarrow 2\gamma}$$

  - MD-1@Novosibirsk (1985), ASP@SLAC 85 $\phi_P = 34.9^\circ \pm 2.2^\circ$

NEW Results BaBar (2011) disagree with theoretical prediction: (*Bakulev et al 2001*): admixture of the two-gluon component?

- can be exploited also looking at the inverse processes, namely $\boxed{\gamma\gamma \longrightarrow \eta^{(\prime)}}$
  - Crystal Ball Collab@DESY (1983)

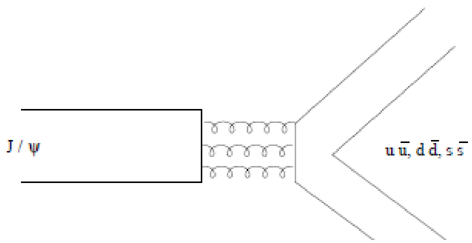
$$\phi_P = 32.3^\circ \pm 1.2^\circ$$

Significantly below previous results

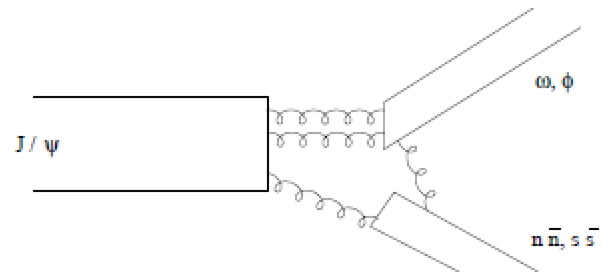
$\psi \to PV$: also ambivalent results

- Mark III (1985) : (35±18)% of the η' wave function can be attribute to gluonium or radial excitation
  - – analysis is based on the assumption that decays proceed via singly disconnected diagram (SOZI), omitting the doubly disconnected (DOZI) diagram
- Mark III (1988) : any gluonium contribution to the η' wave function is ruled out
  - –including DOZI diagrams

SOZI



DOZI



- More recent re-analyses of the hadronic J/ψ and ψ′ decays (including DOZI) furnish a consistent description in terms of one mixing angle with a suggestion of some gluonic component of the η'. F.i.
  - – no form factors $\phi_P = (45 \pm 4)^\circ$ $Z^2_{\eta'} = (0.30 \pm 0.21)$
  - – form factors $\phi_P = (46^{+4}_{-5})^\circ$ $Z^2_{\eta'} = (0.48 \pm 0.16)$ *(Thomas 2007)*

Possible future experimental scenarios

Processes	$(\delta\Gamma/\Gamma)_I$	$(\delta\Gamma/\Gamma)_{II}$	$(\delta\Gamma/\Gamma)_{III}$	$(\delta\Gamma/\Gamma)_{IV}$	$(\delta\Gamma/\Gamma)_V$
$\phi \to \eta'\gamma$	3.5%	3.5%	1.7%	1.7%	1%
$\phi \to \eta\gamma$	2%	2%	1%	1%	1%
$\eta' \to \omega\gamma$	9%	4.5%	9%	4.5%	1.7%
$\eta' \to \rho\gamma$	5%	5%	5%	2.5%	1.7%
$\rho \to \eta\gamma$	7%	7%	7%	3.4%	7%
$\omega \to \eta\gamma$	9%	9%	9%	4.5%	9%
ϕ_P	$(40.6 \pm 0.9)^\circ$	$(40.1^{+1.0}_{-0.8})^\circ$	$(40.6^{+0.6}_{-0.5})^\circ$	$(40.4 \pm 0.5)^\circ$	$(40.1^{+0.3}_{-0.4})^\circ$
$Z^2_{\eta'}$	(0.09 ± 0.05)	(0.13 ± 0.06)	(0.09 ± 0.03)	(0.10 ± 0.03)	(0.13 ± 0.02)

- "*I*" *:* actual uncertainties in the exp input values (PDG 2010)
- "*II*" *:* improvement by studying $\eta' \to \omega\gamma$ using 1) 20 fb^{-1} (KLOE2) 2) selection efficiency 20% 3) neglecting background subtraction
  - limiting factor : uncertainty in the total η' width
- "*III*" *:* improvement in determination of the partial widths for $\phi \to \eta^{(\prime)}\gamma$
- "*IV*" *:* improvement in determination of the partial widths for $\eta^{(\prime)} \to \rho\gamma$
- "*V*" *:* un uncertainty of 1% on the measure of branching ratios for η' decays and of 1.4% for the η' full width.

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$\eta' \to \rho\gamma$	5%	5%	5%	2.5%	1.7%
$\rho \to \eta\gamma$	7%	7%	7%	3.4%	7%
$\omega \to \eta\gamma$	9%	9%	9%	4.5%	9%
ϕ_P	$(40.6 \pm 0.9)^\circ$	$(40.1^{+1.0}_{-0.8})^\circ$	$(40.6^{+0.6}_{-0.5})^\circ$	$(40.4 \pm 0.5)^\circ$	$(40.1^{+0.3}_{-0.4})^\circ$
$Z^2_{\eta'}$	(0.09 ± 0.05)	(0.13 ± 0.06)	(0.09 ± 0.03)	(0.10 ± 0.03)	(0.13 ± 0.02)

- $\phi \to \eta^{(')}\gamma$ partial width mainly due to KLOE07; error dominated by systematics due to the secondary η' branching ratio
- $\eta' \to \omega\gamma$ partial width with relative error 9% (PDG 2010) Relevant experiment (ANL-E-397, 1977) bases on 68 events.
- $\eta' \to \rho\gamma$ partial width with relative error 5% (PDG 2010) Relevant experiment (1969) bases on 298 events
- $\rho \to \eta\gamma$, $\omega \to \eta\gamma$ from SND07

- $\Gamma_{\eta'}$ = 0.194 ±0.009 (PDG 2010); $\Gamma_{\eta'}$ = 0.30 ±0.09 direct meas.(1996,1979)
  - Crystal Ball@MAMI (started 2009)
  - New insight could come from production in γγ fusion (KLOE)

NOW COLOUR MEETS FLAVOUR

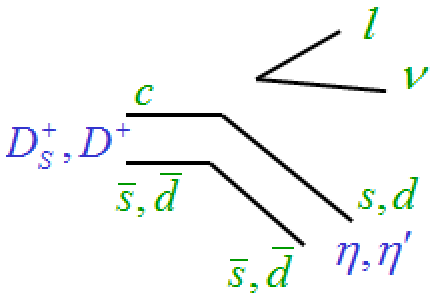
Weak Decays of Charm and Beauty Hadrons

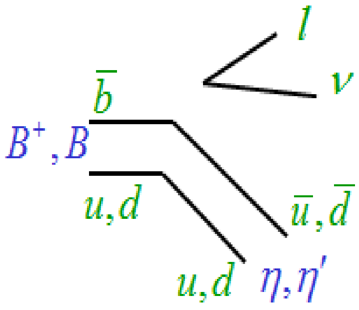
- η/η' wave functions important input for several weak D and B decays
  - CP asymmetries involving η/η' in the final states
  - Control NP vs SM hadronic uncertainties
- Lattice
  - RBC-UKQCD of η/η' masses and mixing using Nf = 2+1 (2010)
    $\theta_P = -14.1(2.8)^\circ$ $(\phi_P \approx 40.7^\circ)$
- Hadron Spectrum Collab. , unphysically heavy light (up, down) quarks and a single lattice spacing (2011) $\phi_P \approx 42(1)^\circ$
- Phenomenological approach

Light Flavour Spectroscopy in Semileptonic Decays

$D_s^+ \to \eta^{(\prime)} l^+ \nu$ $D^+ \to \eta^{(\prime)} l^+ \nu$ $B^+ \to \eta^{(\prime)} l^+ \nu$

- **Spectator diagram dominance**
- Cabibbo allowed c →s, suppressed c →d and CKM suppressed b →u





- No gluons and pole ansatz for form factors

$$\Gamma\left(D_S \to \eta' e^+ \nu\right) / \Gamma\left(D_S \to \eta e^+ \nu\right) \propto \cot^2 \phi$$

$\phi = (41.3 \pm 5.3)°$ *Feldmann, Kroll, Stech 98*

- Allowing gluonic content

$$|\eta'\rangle \simeq \cos\phi_G \sin\phi_P |\eta_q\rangle + \cos\phi_G \cos\phi_P |\eta_s\rangle + \sin\phi_G |gg\rangle$$
$$|\eta\rangle \simeq \cos\phi_P |\eta_q\rangle - \sin\phi_P |\eta_s\rangle$$

$$\Gamma\left(D_S \to \eta' e^+ \nu\right) / \Gamma\left(D_S \to \eta e^+ \nu\right) \propto \cot^2\phi_P \cos^2\phi_G$$

Given ϕ_P=(37.74±2.6)° ⟶ $\phi_G \approx$ 20.3 *Anisovic al. 97*

- new (compatible) CLEO-c 09 data

$$\frac{\Gamma(D_s^+ \to \eta' l^+ \nu)/\Gamma(D_s^+ \to \eta l^+ \nu)}{\Gamma(D^+ \to \eta' l^+ \nu)/\Gamma(D^+ \to \eta l^+ \nu)} \simeq \cot^4\phi_P$$

$\phi_P \approx$ 40°

BESIII expects errors on ϕ going down to 2%

Weak annihilation (WA) diagrams

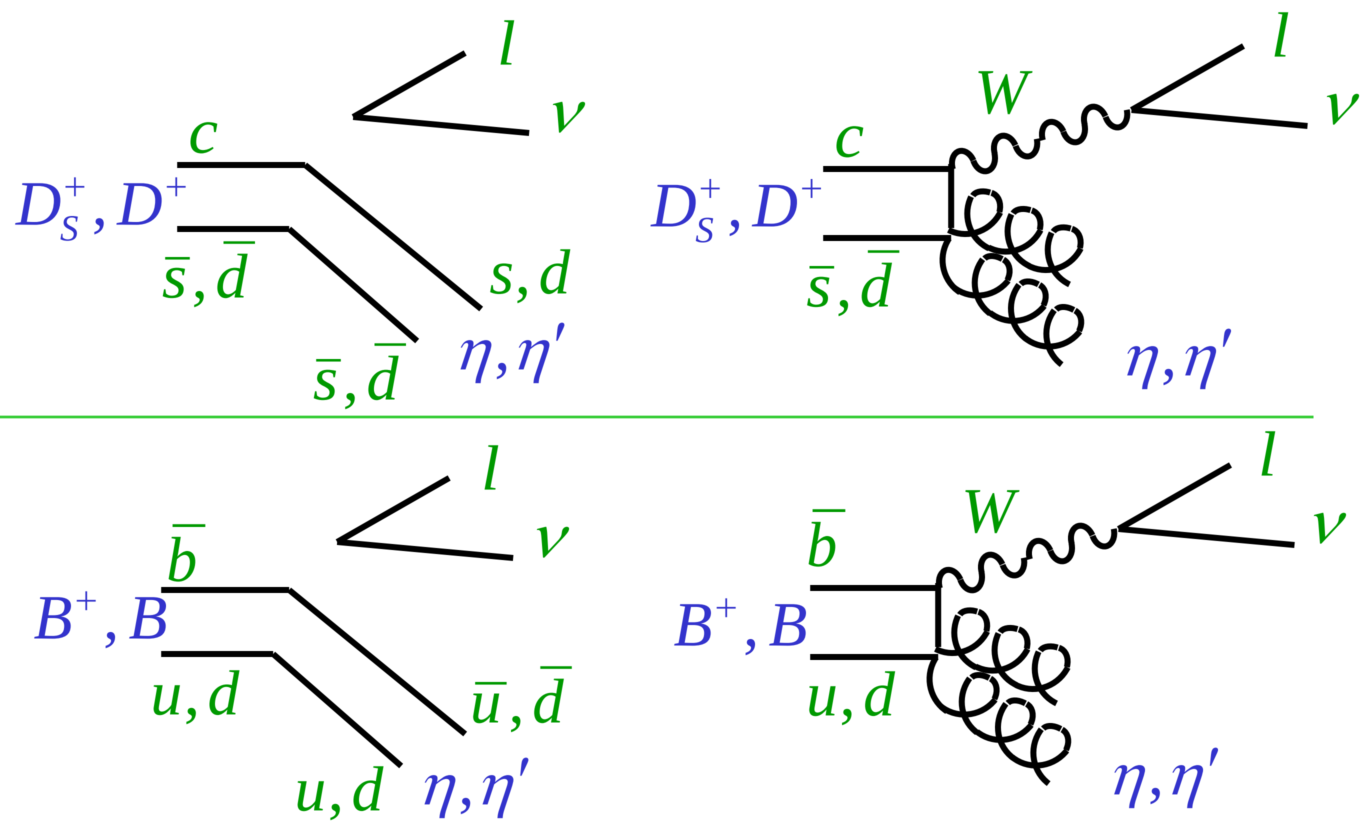
Spectator | WA (gluonic)

$$D_S^+ \to \eta(\eta')l^+\nu$$

$$D^+ \to \eta(\eta')l^+\nu$$

$$B^+ \to \eta(\eta')l^+\nu$$

$$B \to \eta(\eta')l^+\nu$$



WA for precision studies

- WA no more than a nonleading contribution to inclusive rates, BUT could affect exclusive modes considerably

- strength depends on:
  - size of the gg component in the wave functions
  - how much gg radiation one can expect in each semileptonic channel
  - might come from the interference with the spectator amplitude, it can a priori enhance or reduce those

- analysis based on inclusive semileptonic D decays, which considers both the widths and the lepton energy moments shows no clear evidence of WA effects *Gambino, Kamenik 2010*

- No extensive exclusive theoretical analysis yet

$B^{\pm}$ semileptonic decays

Same than $D^{\pm}$ only large q^2 and CKM suppressed by $|V_{ub}|^2$

- first evidence of $B^+ \to \eta' l^+ \nu$ by CLEO in 2008
- newest BaBar results (2011) (with a significance of 3.0 σ).
  - an order of magnitude smaller than the CLEO result

$$\mathcal{B}(B^+ \to \eta' l^+ \nu)/\mathcal{B}(B^+ \to \eta l^+ \nu) = 0.67 \pm 0.24_{stat} \pm 0.11_{syst}$$

seems to allow a large gluonic singlet contribution

- $\mathcal{B}(B_s \to \eta' l^+ l^-)$ potentially informative on the gluonic content, exp challenging. In SM BR 10^{-7}-10^{-8} (Super-flavour factories)

Charmless B decays

Long standing puzzle (from 2004)

$$\mathcal{B}(B^0 \to K^0\eta') = (6.6 \pm 0.4) \times 10^{-5} \quad >> \quad \mathcal{B}(B^0 \to K^0\eta) = (1.1 \pm 0.4) \times 10^{-6}$$

$$\mathcal{B}(B^\pm \to \eta' K^\pm) = (71.5 \pm 1.3 \pm 3.2)10^{-6} \quad >> \quad \mathcal{B}(B^\pm \to \pi^0 K^\pm) = (13.6 \pm 0.6 \pm 0.7)10^{-6}$$

- SU(3)fl singlet penguin (B$_i$ meson triplet, M^l_j meson nonet), including gluonic contributions
- Anomaly effects, large charm content, NP?

More recent data

2008-2010 CLEO/BaBar $\mathcal{B}(B^+ \to \eta' l^+ \nu) < \mathcal{B}(B^+ \to \eta l^+ \nu)$

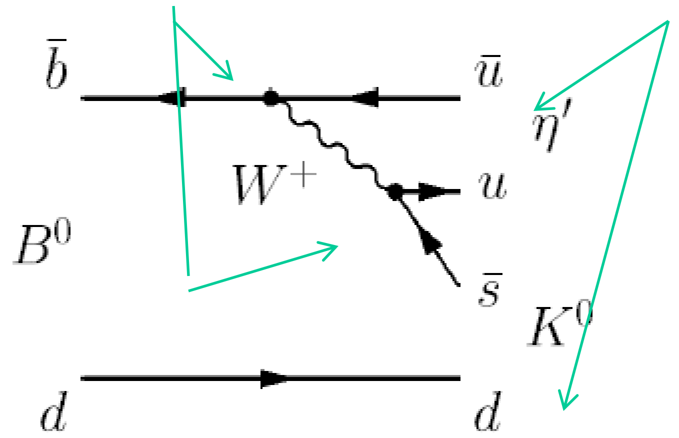
$(3.9 \pm 0.8) \times 10^{-5}$

2010 Babar

$$\Gamma(B \to K^\star \eta') < \Gamma(B \to K^\star \eta)$$

Penguin dominated decays

- Tree level CKM and color suppressed

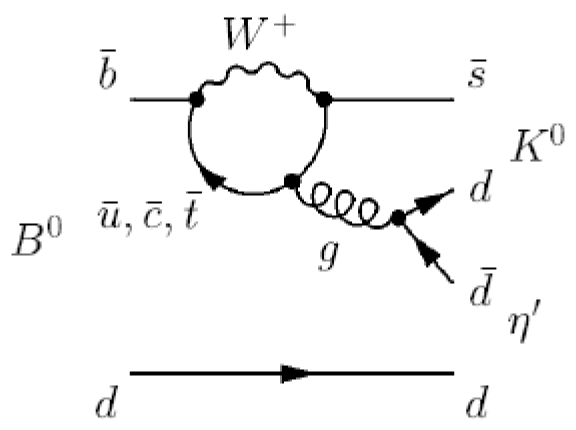


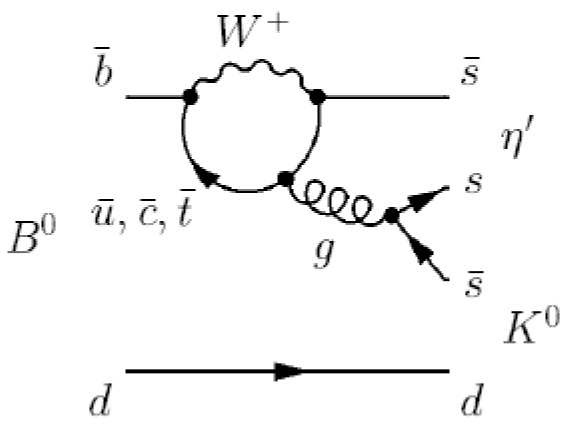
$\lambda \approx 0.22$

$|V_{ub} V_{us}| \approx \lambda^4$

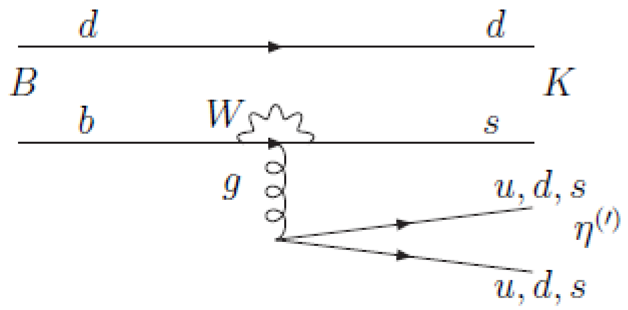
- Penguins CKM not suppressed

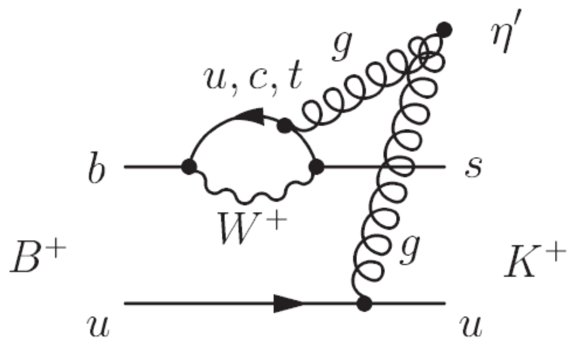
$|V_{tb} V_{ts}| \approx \lambda^2$





Singlet contribution





- Additional contribution to the SU(3) singlet contribution: fusion of gluons, one gluon from $b \rightarrow sg$ process and another one from spectator.

  A sizeable gluonium contribution to the η' meson could play an important role: the contribution of the diagram in which two gluons are directly attached to gluonium in η' in principle important

Theoretical approaches

Within large errors BR compatible with interference among different contributions

- QCDF and SCET : sizable gluonic contributions to the B $\rightarrow \eta'$ form factor

Beneke, Neubert 03;
Williamson, Zupan 06

- pQCD : impact of the gluonic component numerically very small

Y.-Y. Charng, T. Kurimoto, H.-n. Li 06

Major analyses prior to 06, relying on old experimental data

ρωφ→γ P , η'→γ V	ϕ_P	$Z^2_{\eta'}$
KLOE (+η′→γγ) 2009	(39.7±0.7)°	0.14±0.04
Escribano, Nadal 2007	(42.6±1.1)°	0.01±0.07
Thomas 2007	(41.7±0.5)°	0. 04±0.04
Thomas F.F. 2007	(41.9±0.7)°	0.10±0.04
J/ψ→VP	ϕ_P	$Z^2_{\eta'}$
Escribano 2010	(44.6±4.4)°	0.29+0.28/-0.26
Thomas 2007	(45±4)°	0.30±0.21
Thomas F.F. 2007	(46+4/-5)°	0.48±0.16
$D_s^+ \to \eta^{(\prime)} l\ \nu$	ϕ_P	$Z^2_{\eta'}$
Brandeburg 1995	(37.7±2.6)°	0.12
Anisovich 1997		

Summary

- The different determinations of $\eta - \eta'$ mixing angle are generally consistent, but show relevant model and mode dependence
- the message concerning η' gluonium content remains ambivalent
- More dedicated studies are necessary (including theoretical updates with new data) while waiting for lattice (but wait seated…)
- Also D^+, D_S^+ and B decays must be included in traditional investigations to check η' gluonium role

Now about Alex….

As pointed out, Alex is going though many difficult, hard, new paths, but, you know,….

he is smart in choosing how to go!

Happy Birthday, Alex!