Colour meets Flavour

Siegen, October 14, 2011

Magnetic and Electric Dipole Moments

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Happy Birthday, Alex !

Outline

- Experimental data on g and EDM
- CPV scales and EDM calculations
- Anatomy of muon g-2, electroweak corrections
- Triangle amplitudes, nonrenormalization theorem
- Hadronic light-by-light scattering
- Do we see New Physics in the muon g-2?

Lepton magnetic moments

$$H = -\boldsymbol{\mu}\,\boldsymbol{B} \qquad \boldsymbol{\mu} = g\,\frac{e\hbar}{2mc}\,\boldsymbol{s}$$

The present experimental values

Electron: Hanneke, Fogwell, and Gabrielse '08

g/2=1.001 159 652 180 73 (28)

0.28×10^{-12} [0.28 ppt]

New value of α follows

$1/\alpha =$ 137.035 999 084 (51) [0.37 ppb]

Muon: BNL E821 '06

g/2=1.001 165 920 80 (63) [630 ppt]

Tau: Delphi at LEP2 '04

g/2=0.982(17)

Electric dipole moments

$$H = -\boldsymbol{d}\,\boldsymbol{E}$$

Neutron: Baker et al '06

$$|d| < 2.9 \times 10^{-26}\ e \cdot cm$$

Electron: Regan et al '06

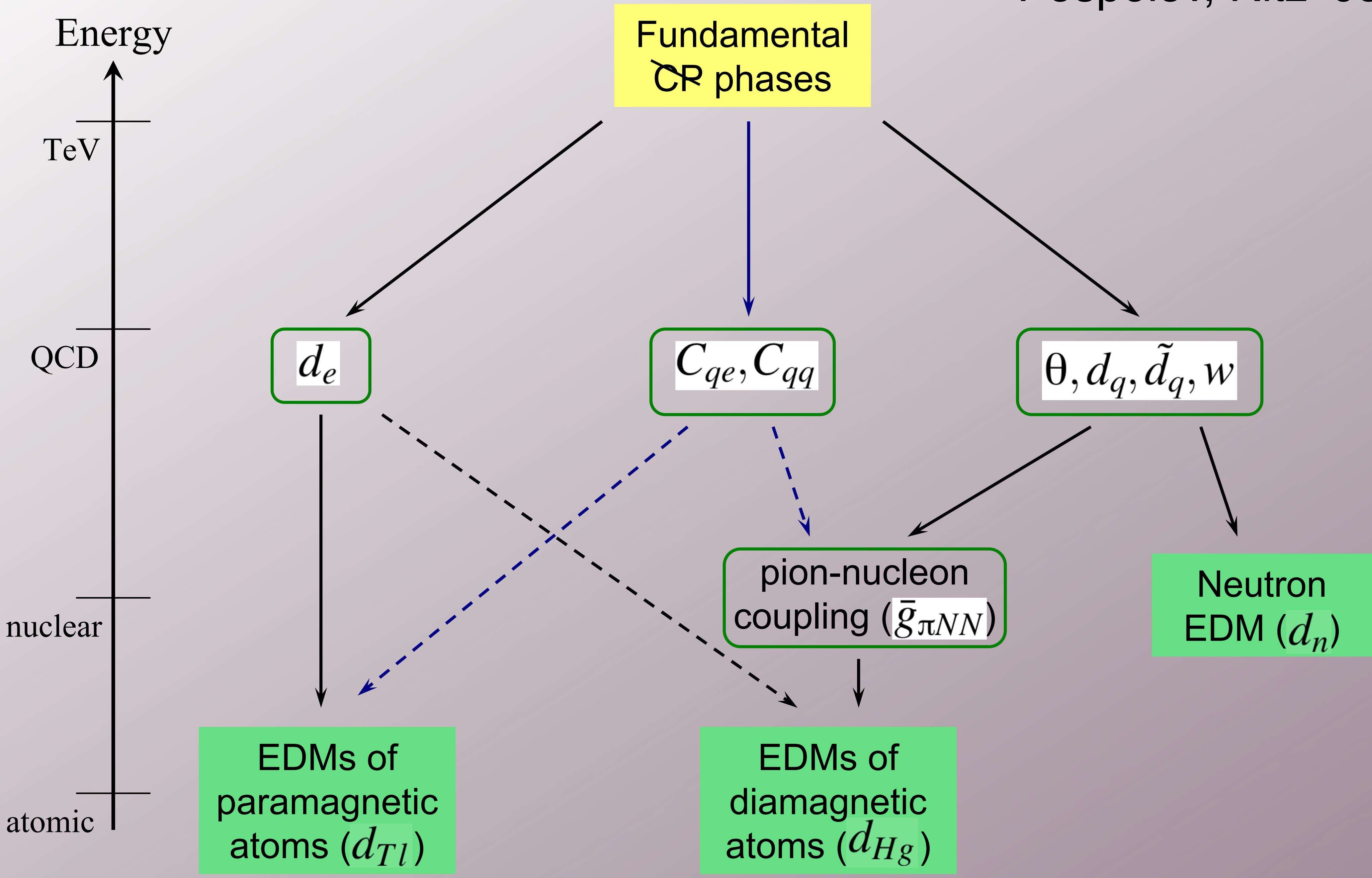
$$|d| < 1.6 \times 10^{-27}\ e \cdot cm$$

Muon: Muon g-2 Collaboration '04

$$|d| < 2.8 \times 10^{-19}\ e \cdot cm$$

Scales hierarchy

Pospelov, Ritz ’05



Observables

$$\mathcal{L}_{eff}^{\rm nuclear} = \mathcal{L}_{\rm edm} + \mathcal{L}_{\pi NN} + \mathcal{L}_{eN},$$

$$\mathcal{L}_{\rm edm} = -\frac{i}{2} \sum_{i=e,p,n} d_i \; \overline{\psi}_i (F\sigma)\gamma_5 \psi,$$

$$\mathcal{L}_{\pi NN} = \bar{g}^{(0)}_{\pi NN} \bar{N} \tau^a N \pi^a + \bar{g}^{(1)}_{\pi NN} \bar{N} N \pi^0 + \bar{g}^{(2)}_{\pi NN} (\bar{N} \tau^a N \pi^a - 3 \bar{N} \tau^3 N \pi^0),$$

$$\mathcal{L}_{eN} = C_S^{(0)} \bar{e} i \gamma_5 e \bar{N} N + C_P^{(0)} \bar{e} e \bar{N} i \gamma_5 N + C_T^{(0)} \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma^{\mu\nu} e \bar{N} \sigma^{\alpha\beta} N + C_S^{(1)} \bar{e} i \gamma_5 e \bar{N} \tau^3 N + C_P^{(1)} \bar{e} e \bar{N} i \gamma_5 \tau^3 N + C_T^{(1)} \epsilon_{\mu\nu\alpha\beta} \bar{e} \sigma^{\mu\nu} e \bar{N} \sigma^{\alpha\beta} \tau^3 N.$$

QCD Low Energy Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{dim}=4} + \mathcal{L}_{\text{dim}=5} + \mathcal{L}_{\text{dim}=6} + \cdots .$$

$$\mathcal{L}_{\text{dim}=4} = \frac{g_s^2}{32\pi^2}\,\bar{\theta} G^a_{\mu\nu}\widetilde{G}^{\mu\nu,a},$$

$$\mathcal{L}_{\text{dim}=5} = -\frac{i}{2}\sum_{i=u,d,s,e,\mu} d_i\,\overline{\psi}_i(F\sigma)\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\tilde{d}_i\,\overline{\psi}_i g_s(G\sigma)\gamma_5\psi_i,$$

Effective dim=6. Thus, we need to add

$$\mathcal{L}_{\text{dim}=6} = \frac{1}{3}w\; f^{abc}G^a_{\mu\nu}\widetilde{G}^{\nu\beta,b}G_\beta{}^{\mu,c} + \sum_{i,j} C_{ij}\;(\bar{\psi}_i\psi_i)(\bar{\psi}_j i\gamma_5\psi_j) + \cdots$$

Examples of calculation

$$d_n(\bar{\theta}) = (1 \pm 0.5)\frac{|\langle \overline{q}q \rangle|}{(225\text{MeV})^3}\bar{\theta} \times 2.5\ 10^{-16} e\,\text{cm},$$

$$d_n^{\text{PQ}}(d_q, \tilde{d}_q) = (1 \pm 0.5)\frac{|\langle \overline{q}q \rangle|}{(225\text{MeV})^3}\left[1.1e(\tilde{d}_d + 0.5\tilde{d}_u) + 1.4(d_d - 0.25 d_u)\right]$$

In the Standard Model

Khriplovich '86

$$d_d = e\frac{m_d m_c^2 \alpha_s G_F^2 J_{CP}}{108\pi^5}\ln^2(m_b^2/m_c^2)\ln(M_W^2/m_b^2).$$

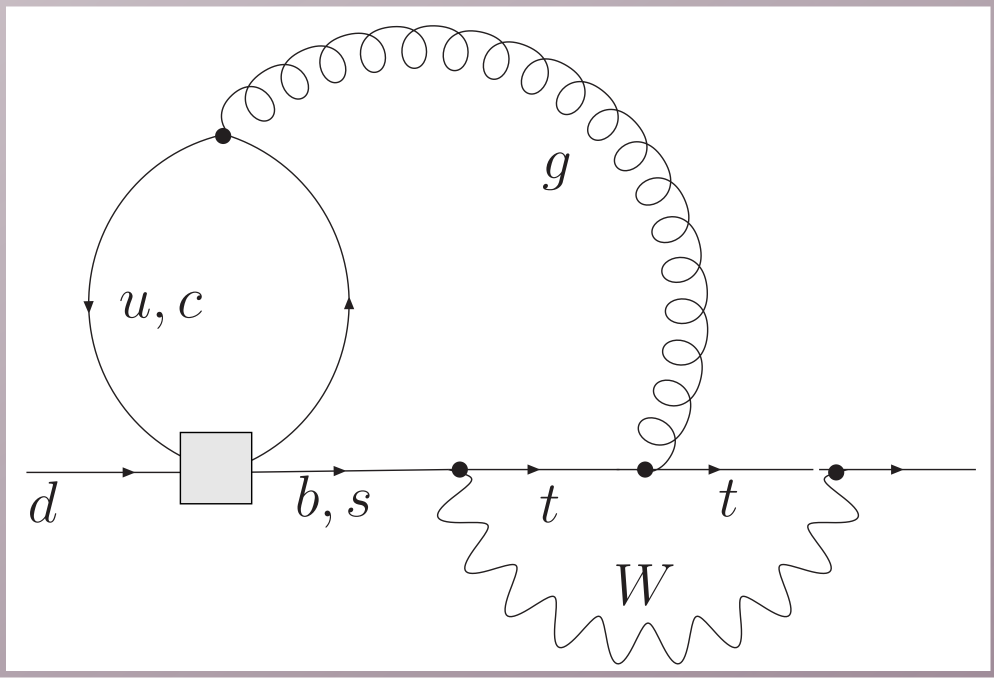
$$d_d^{\text{KM}} \simeq 10^{-34} e\ \text{cm}.$$

$$d_n^{\text{KM}} \simeq 10^{-32} e\ \text{cm}.$$

Gavela; Khriplovich, Zhitnitsky '82

$$d_e^{\text{KM}} \leq 10^{-38} e\ \text{cm}.$$

Khriplovich, Pospelov '91



Potential for NP to show up!

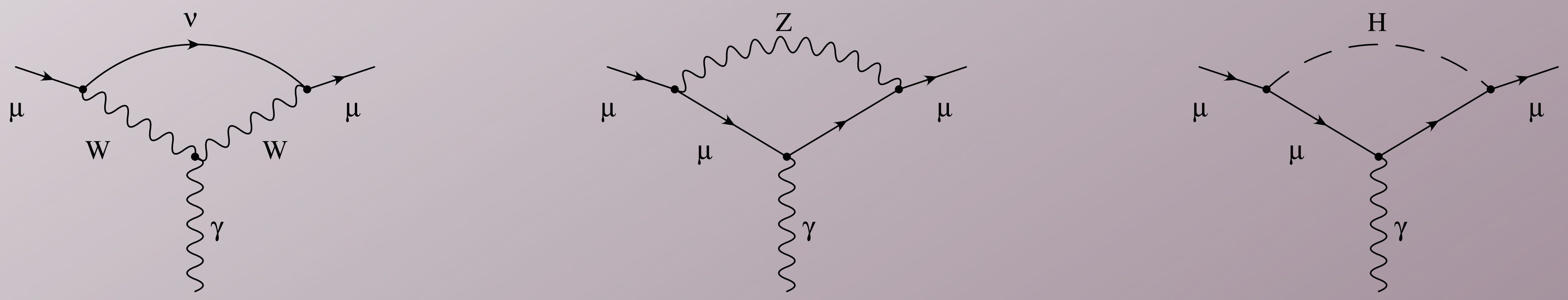
Anatomy of muon g-2

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm EW}$$

$$a_\mu^{\rm QED} = 116\ 584\ 719.58(1.5) \times 10^{-11}$$ Kinoshita et al

$$a_\mu^{\rm EW} = 154(1)(2) \times 10^{-11}$$ Czarnecki, Marciano, AV '02

$$a_\mu^{\rm EW}(1\text{-loop}) = \frac{5\,G_\mu\, m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\,(1-4\sin^2\theta_W)^2 + \mathcal{O}\left(\frac{m_\mu^2}{m_{W,H}^2}\right)\right] = 194.8 \times 10^{-11}$$

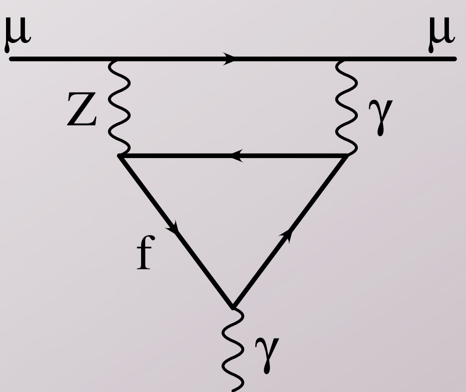


Two-loop corrections are more involved

$$a_\mu^{\mathrm{EW}}(2\text{-loop})_{LL} = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\cdot\frac{\alpha}{\pi}\left\{-\frac{43}{3}\ln\frac{m_Z}{m_\mu}+\frac{36}{5}\sum_{f\in F}N_f\,Q_f^2\,I_f^3\ln\frac{m_Z}{m_f}\right\}$$

$$\approx -37\times 10^{-11} \qquad F=\tau,\,u\,,d,\,s,\,c,\,b$$

Kukhto, Kuraev, Schiller, Silagadze '92
Peris, Perrottet, Rafael '95
Czarnecki, Krause, Marciano '95

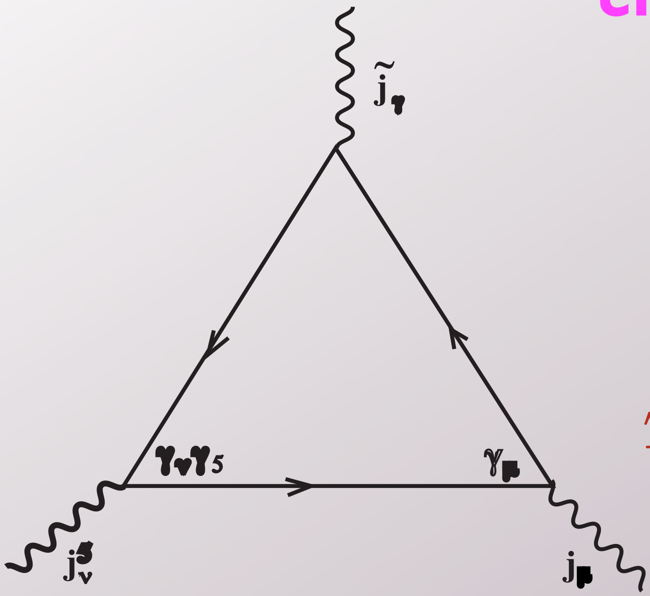


$$m_{u,d}=0.3\ \mathrm{GeV},\ m_s=0.5\ \mathrm{GeV},$$
$$m_c=1.5\ \mathrm{GeV},\ m_b=4.5\ \mathrm{GeV}$$

Fermion triangles ($Z^*\gamma\gamma^*$ vertex)

Total: $a_\mu^{\mathrm{EW}} = 152(4)\times 10^{-11}$ Czarnecki, Marciano '01

Perturbative and nonperturbative triangle amplitudes



$$j_\mu = \bar{q}\, V \gamma_\mu q, \qquad j^5_\nu = \bar{q}\, A \gamma_\nu \gamma_5 q$$

$$T_{\mu\gamma\nu} = -\int \mathrm{d}^4x \mathrm{d}^4y\, \mathrm{e}^{iqx-iky} \langle 0|\, T\{j_\mu(x)\, \tilde{j}_\gamma(y)\, j^5_\nu(0)\}|0\rangle$$

$$T_{\mu\nu} = T_{\mu\gamma\nu} e^\gamma(k) = i \int \mathrm{d}^4x\, \mathrm{e}^{iqx} \langle 0|\, T\{j_\mu(x)\, j^5_\nu(0)\}|\, \gamma(k)\rangle$$

$$T_{\mu\nu} = -\frac{i}{4\pi^2}\left[w_T(q^2)\left(-q^2 \tilde{f}_{\mu\nu} + q_\mu q^\sigma \tilde{f}_{\sigma\nu} - q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right) + w_L(q^2)\, q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right]$$

$$\tilde{f}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\gamma\delta} f^{\gamma\delta}\,, \qquad f_{\mu\nu} = k_\mu e_\nu - k_\nu e_\mu\,.$$

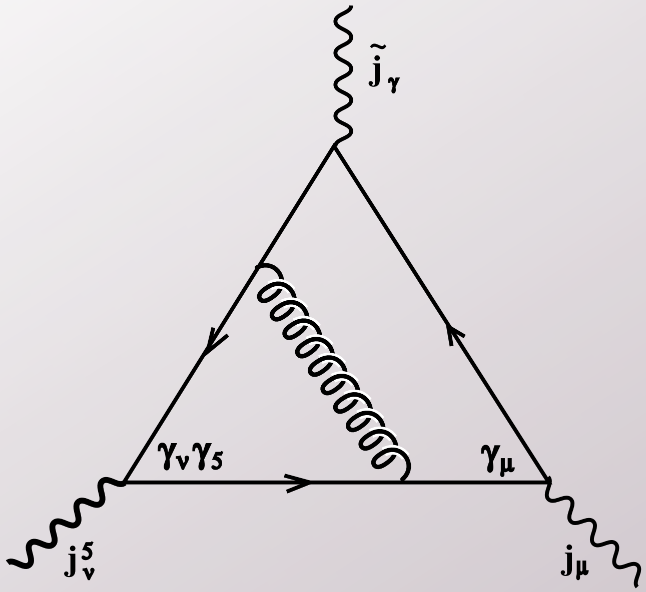
$$w_L^{1-\mathrm{loop}}[m=0] = 2\, w_T^{1-\mathrm{loop}}[m=0] = \frac{2N_c\, \mathrm{Tr}\,(A\, V\, \widetilde{V})}{Q^2}$$

Nonrenormalization theorem

AV ‘02

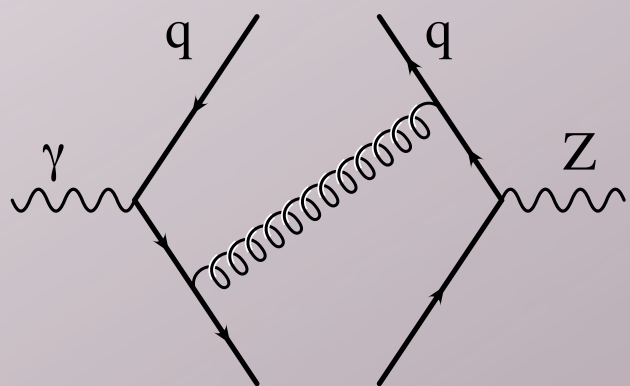
Czarnecki, Marciano, AV ‘02

Knecht, Peris, Perrottet, de Rafael ’03



No pertubative corrections both in longitudinal and transversal parts in the chiral limit. Pole in the longitudinal part corresponds to massless pion.

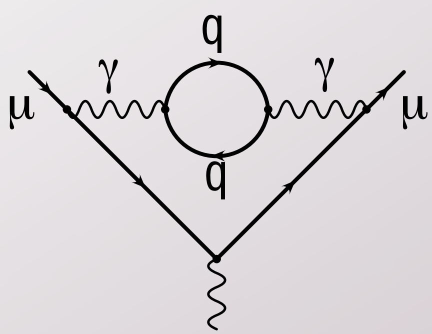
But it should be no massless pole in the transversal part. A shift from zero is provided by nonperturbative effects. Four-fermion operators in OPE.



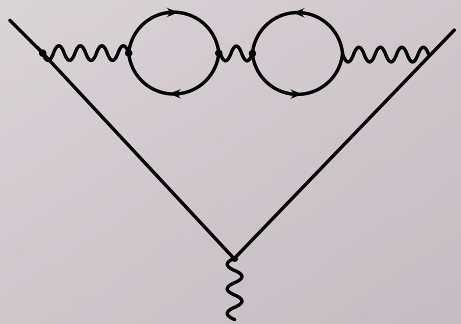
$$w_T[u,d] = \frac{1}{m_{a_1}^2 - m_\rho^2}\left[\frac{m_{a_1}^2 - m_\pi^2}{Q^2 + m_\rho^2} - \frac{m_\rho^2 - m_\pi^2}{Q^2 + m_{a_1}^2}\right]$$

Hadronic contributions

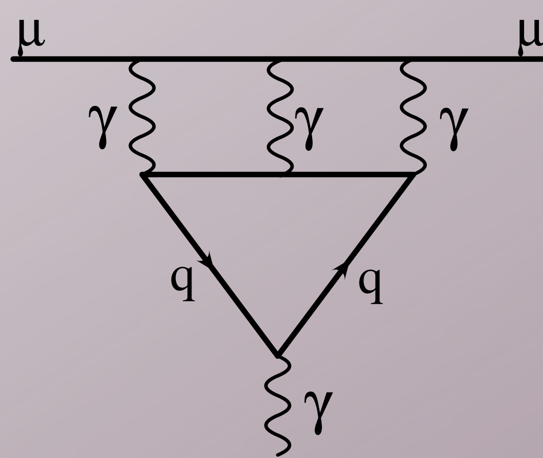
$$a_\mu^{\mathrm{had}} = a_\mu^{\mathrm{had,LO}} + a_\mu^{\mathrm{had,HO}} + a_\mu^{\mathrm{LBL}}$$



Lowest order hadronic contribution represented by a quark loop



An example of higher order hadronic contribution



Light-by-light scattering contribution

In theory

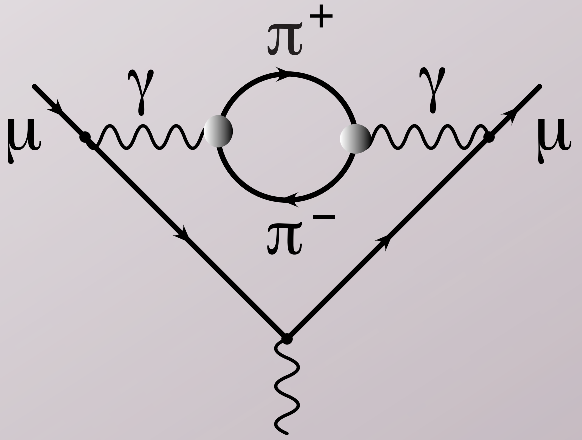
$$a_\mu^{\mathrm{had,LO}} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{\mathrm{d}s}{s^2} K(s) R(s)$$

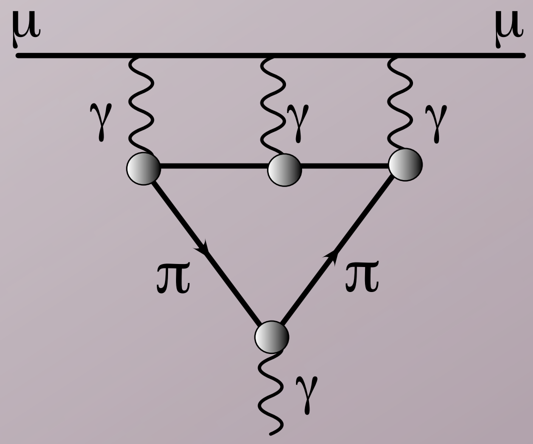
$K(s)$ is the known function, $K(s) \to 1, \quad s \gg m_\mu^2$
$R(s)$ is the cross section of e^+e^- annihilation into hadrons in units of $\sigma(e^+e^- \to \mu^+\mu^-)$.

In difference with $a_\mu^{\mathrm{had,LO}}$ there is no experimental input for the light-by-light contribution.What are possible theoretical parameters to exploit?

Smallness of chiral symmetry breaking, $m_\rho^2/m_\pi^2 \gg 1$

$$a_\mu^{(n)} \sim c_1 \left(\frac{\alpha}{\pi}\right)^n \frac{m_\mu^2}{m_\pi^2}\,, \qquad \mathrm{LO}: n=2\,, \quad \mathrm{LbL}: n=3$$

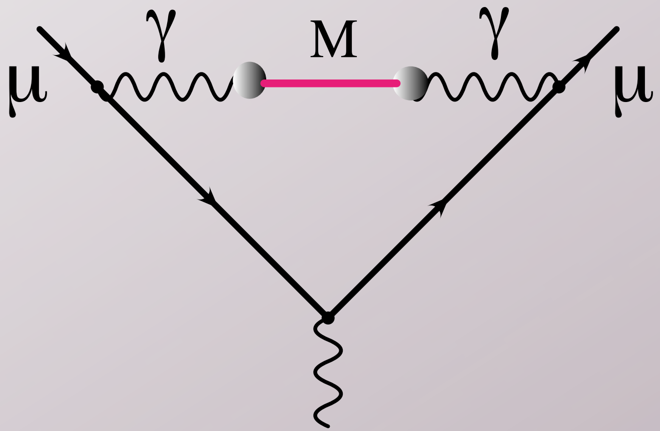


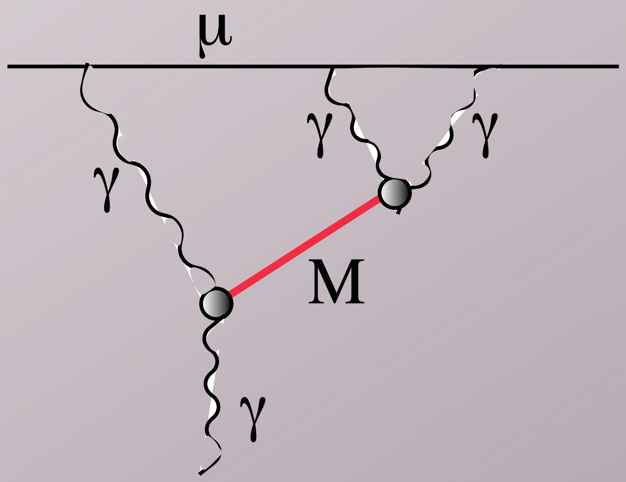


The Goldstone nature of pion implies $m_\pi^2 \propto m_q$ much less than typical $M_{\mathrm{had}}^2 \sim m_\rho^2$. Thus, the threshold range in pion loops produces the $1/m_\pi^2$ enhancement.

Large number of colors, N_c

Quark loops clearly give $a_\mu \propto N_c$. Dual not to pion loops but to meson exchanges.





No continuum in the large N_c limit.
$M = \rho^0, \omega, \phi, \rho', \ldots$ for the polarization operator
$M = \pi^0, \eta, \eta', a_0, a_1, \ldots$ (and any C-even meson) for the light-by-light

$$a_\mu^{(n)} \sim c_2 \left(\frac{\alpha}{\pi}\right)^n N_c \, \frac{m_\mu^2}{m_\rho^2}$$

We can check for $a_\mu^{\rm had,LO}$

Two regions. The threshold region $s \sim 4m_\pi^2$ where

$$R(s) \approx \frac{1}{4}\left(1 - \frac{4m_\pi^2}{s}\right)^{3/2}$$

and the resonance region $s \sim m_\rho^2$ where by quark-hadron duality on average

$$R(s) \approx N_c \sum Q_q^2$$

The chirally enhanced threshold region gives numerically

$$a_\mu^{\rm had,LO}(4m_\pi^2 \leq s \leq m_\rho^2/2) \approx 400 \times 10^{-11}$$

Compare with the N_c enhanced ρ peak,

$$a_\mu^{\rm had,LO}(\rho) = \frac{m_\mu^2\,\Gamma(\rho \to e^+e^-)}{\pi\, m_\rho^3} \approx 5000 \times 10^{-11}$$

This contribution is enhanced by N_c,

$$a_\mu(\rho) \sim c_2 \left(\frac{\alpha}{\pi}\right)^2 N_c \frac{m_\mu^2}{m_\rho^2}$$

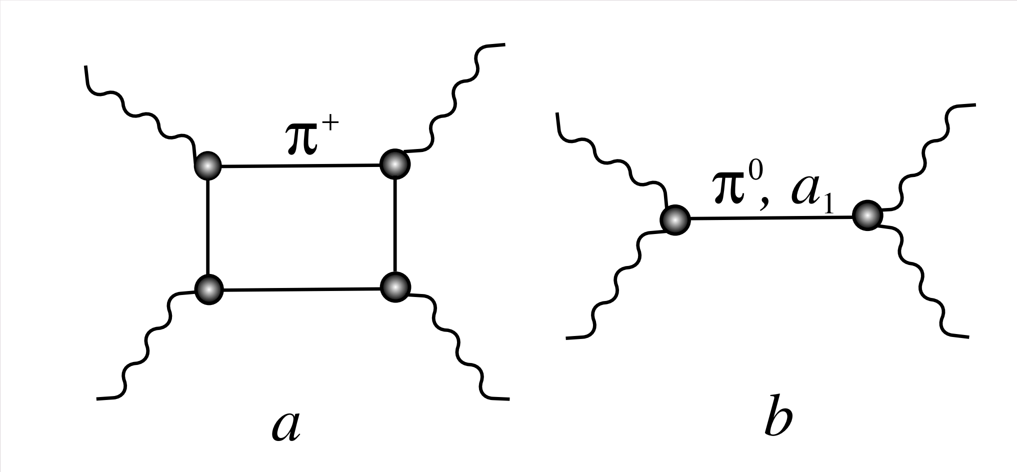
What is a lesson from this exercise? We see that the large N_c enhancement prevails over chiral one.

In the chiral perturbation theory

$$
\begin{aligned}
a_\mu^{2\pi} &= \frac{1}{40}\left(\frac{\alpha}{3\pi}\right)^2 \frac{m_\mu^2}{m_\pi^2}\left[1 + 40\, m_\pi^2 F'_{\pi\pi\gamma*}(0)\ln\frac{m_\rho}{2m_\pi}\right] \\
&= \frac{1}{40}\left(\frac{\alpha}{3\pi}\right)^2 \frac{m_\mu^2}{m_\pi^2}\left[1 + 40\,\frac{m_\pi^2}{m_\rho^2}\ln\frac{m_\rho}{2m_\pi}\right]
\end{aligned}
$$

Chiral perturbation theory does not work. The leading term is suppressed by p-wave nature.

In light-by-light



The chirally enhanced pion box contribution does not result in large number, it is actually rather small,

$$a_\mu^{\rm LbL}(\text{pion box}) \approx -4\times 10^{-11}$$

Hayakawa, Kinoshita, Sanda; Melnikov

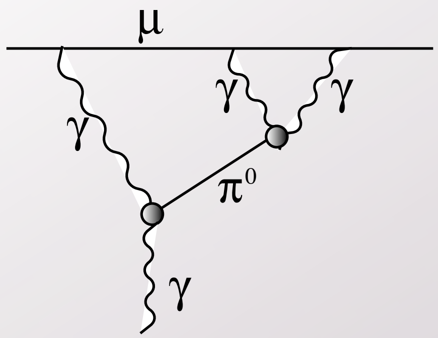
similarly to the hadronic polarization case above.
A larger value (-19) for the pion box was obtained by Bijnens, Pallante, Prades

Instability of the number is due to relatively large pion momenta in the loop, of order of $4m_\pi$ as we estimated. Then details of the model becomes important and theoretical control is lost. In HSL model few first terms of m_π^2/m_ρ^2 expansion are

$$a_\mu(\text{charged pion loop})\times 10^{11} = -46.37 + 35.46 + 10.98 - 4.7 + \ldots = -4.9$$

If momenta were small compared with m_ρ the result would be close to the leading term – free pion loop.

In case of polarization operator the suppression of the leading term in the chiral expansion (larger momenta) can be related to the p-wave p^3 suppression. There is a suppression for s-wave in two-pion intermediate state near threshold in the case of LbL.



Hayakawa, Kinoshita, Sanda
Bijnens, Pallante, Prades
Barbieri, Remiddi
Pivovarov
Bartos, Dubničkova,Dubnička, Kuraev, Zemlyanaya
Knecht, Nyffeler
Knecht, Nyffeler, Perrotttet, de Rafael
Ramsey-Musolf, Wise
Blokland, Czarnecki, Melnikov
Melnikov, A.V.

Different models: constituent quark loop, extended Nambu–Jano-Lasinio model (ENJL), hidden local symmetry (HLS) model . . .

The π^0 pole part of LbL contains besides N_c the chiral enhancement in the logarithmic form, leading to the model-independent analytical expression

$$a_\mu^{\rm LbL}(\pi^0) = \left(\frac{\alpha}{\pi}\right)^3 N_c \frac{m_\mu^2 N_c}{48\pi^2 F_\pi^2} \ln^2 \frac{m_\rho}{m_\pi} + \ldots$$

However next, model dependent, terms are comparable with the the leading or

Numerically

$$a_\mu^{\rm LbL}(\pi^0) = 58(10)\times 10^{-11}$$

Knecht, Nyffeler

Massive quark loop (Laporta, Remiddi '91)

$$a^{\mathrm{HLbL}}(\text{quark loop}) = \left(\frac{\alpha}{\pi}\right)^3 N_c Q_q^4 \left\{ \underbrace{\left[\frac{3}{2}\zeta(3) - \frac{19}{16}\right]}_{0.62} \frac{m_\mu^2}{m_q^2} + \mathcal{O}\left[\frac{m_\mu^4}{m_q^4}\log^2\frac{m_\mu^2}{m_q^2}\right] \right\}$$

For c-quark with $m_c \approx 1.5\ \mathrm{GeV}$,

$$a^{\mathrm{HLbL}}(c) = 2.3 \times 10^{-11}$$

Light quark estimate for the constituent mass 300 MeV

$$a^{\mathrm{HLbL}}(u,d,s) = 64 \times 10^{-11}$$

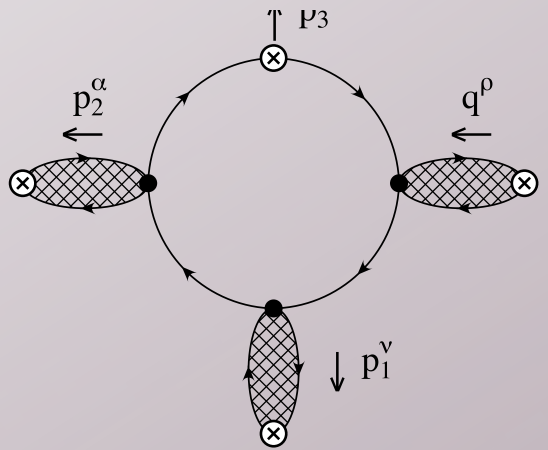
Together with the neutral pion exchange it gives

$$a^{\mathrm{HLbL}} \approx 120 \times 10^{-11}$$

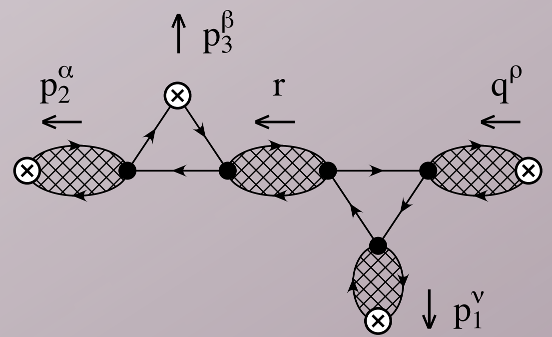
Models

HLS model is a modification the Vector Meson Dominance model.

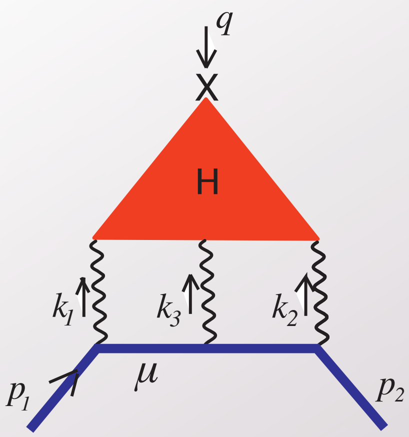
ENJL model is represented by the following graphs



(a)



(b)



OPE constraints and hadronic model

$\epsilon_i^{\mu}(q_i)\,, \quad i=1,2,3,4, \quad \sum q_i = 0$

ϵ_4 represents the external magnetic field $f^{\gamma\delta} = q_4^{\gamma}\epsilon_4^{\delta} - q_4^{\delta}\epsilon_4^{\gamma}, \quad q_4 \to 0.$

The LbL amplitude

$$\begin{aligned}\mathcal{M} &= \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A} = \alpha^2 N_c \,\mathrm{Tr}\,[\hat{Q}^4]\,\mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta}\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3} f^{\gamma\delta} \\ &= -e^3 \int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-iq_1x-iq_2y}\,\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}\epsilon_3^{\mu_3}\langle 0|T\,\{j_{\mu_1}(x)\,j_{\mu_2}(y)\,j_{\mu_3}(0)\}\,|\gamma\end{aligned}$$

The electromagnetic current $j_\mu = \bar{q}\,\hat{Q}\gamma_\mu q, \quad q = \{u,d,s\}$

Three Lorentz invariants: q_1^2, q_2^2, q_3^2

Consider the Euclidian range $q_1^2 \approx q_2^2 \gg q_3^2 \gg \Lambda_{\mathrm{QCD}}^2$

We can use OPE for the currents that carry large momenta q_1, q_2

$$i\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{e}^{-iq_1x-iq_2y}\,T\left\{j_{\mu_1}(x), j_{\mu_2}(y)\right\} =$$

$$\int \mathrm{d}^4z\,\mathrm{e}^{-i(q_1+q_2)z}\,\frac{2i}{\hat{q}^2}\,\epsilon_{\mu_1\mu_2\delta\rho}\,\hat{q}^\delta j_5^\rho(z) + \cdots .$$

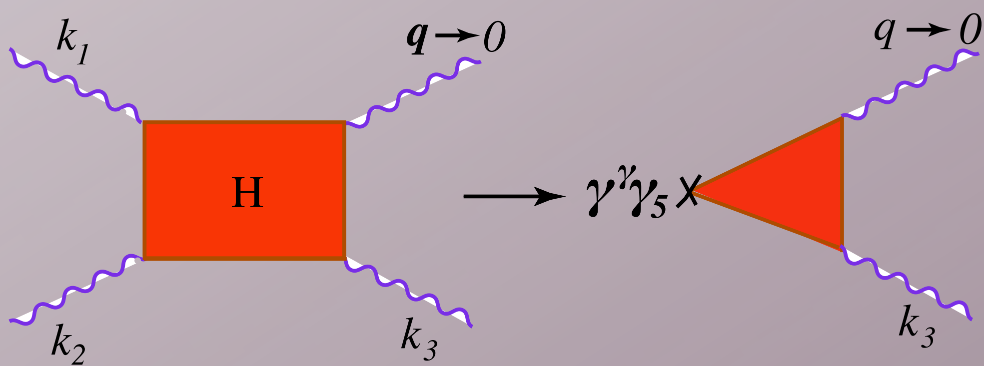
$\hat{q} = (q_1 - q_2)/2$, the axial current $j_5^\rho = \bar{q}\,\hat{Q}^2\gamma^\rho\gamma_5\,q$ is the linear combination of

$j^{(3)}_{5\rho} = \bar{q}\,\lambda_3\gamma^\rho\gamma_5\,q$ isovector

$j^{(3)}_{5\rho} = \bar{q}\,\lambda_8\gamma^\rho\gamma_5\,q$ hypercharge

$j^{(3)}_{5\rho} = \bar{q}\,\gamma^\rho\gamma_5\,q$ singlet

$$j_{5\rho} = \sum_{a=3,8,0} \frac{\mathrm{Tr}\,[\lambda_a\hat{Q}^2]}{\mathrm{Tr}\,[\lambda_a^2]}\,j^{(a)}_{5\rho}$$



The triangle amplitude

$$T^{(a)}_{\mu_3\rho} = i\,\langle 0|\int \mathrm{d}^4 z\, \mathrm{e}^{iq_3 z} T\{j^{(a)}_{5\rho}(z)\, j_{\mu_3}(0)\}|\gamma\rangle$$

kinematically is expressed via two scalar amplitudes

$$T^{(a)}_{\mu_3\rho} = -\frac{ie\,N_c \mathrm{Tr}\,[\lambda_a \hat{Q}^2]}{4\pi^2}\Big\{ w^{(a)}_L(q_3^2)\, q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3} +$$
$$+ w^{(a)}_T(q_3^2)\Big(-q_3^2 \tilde{f}_{\mu_3\rho} + q_{3\mu_3} q_3^\sigma \tilde{f}_{\sigma\rho} - q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu_3}\Big)\Big\}$$

Longitudinal w_L: pseudoscalar mesons exchange

Transversal w_T: pseudovector mesons exchange

In perturbation theory for massless quarks

$$w^{(a)}_L(q^2) = 2w^{(a)}_T(q^2) = -\frac{2}{q^2}$$

Nonvanishing w_L is the signature of the axial Adler–Bell–Jackiw anomaly.

Moreover, for nonsinglet $w_L^{(3,8)}$ it is the *exact* QCD result, no perturbative as well as nonperturbative corrections. So the pole behavior is preserved all way down to small q^2 where the pole is associated with Goldstone mesons $\pi^0,\ \eta$.

Comparing the pole residue we get the famous ABJ result

$$g_{\pi\gamma\gamma} = \frac{N_c \mathrm{Tr}\,[\lambda_3 \hat{Q}^2]}{16\pi^2\, F_\pi}$$

There exists the nonrenormalization theorem for w_T as well but only in respect to perturbative corrections. A.V. '02; Knecht, Peris, Perrottet, de Rafael '03

Higher terms in the OPE does not vanish in this case, they are responsible for shift of the pole $1/q^2 \to 1/(q^2 - m^2_{V,PV})$

Combining we get at $q_1^2 \approx q_2^2 \gg q_3^2$

$$\begin{aligned} \mathcal{A}_{\mu_1\mu_2\mu_3\gamma\delta} f^{\gamma\delta} &= \frac{8}{\hat{q}^2}\epsilon_{\mu_1\mu_2\delta\rho}\hat{q}^\delta \sum_{a=3,8,0} W^{(a)}\Big\{ w_L^{(a)}(q_3^2)\, q_3^\rho q_3^\sigma \tilde{f}_{\sigma\mu_3} \\ &+ \ w_T^{(a)}(q_3^2)\Big(-q_3^2 \tilde{f}^\rho_{\mu_3} + q_{3\mu_3} q_3^\sigma \tilde{f}^\rho_\sigma - q_3^\rho q_3^\sigma \tilde{f}_{\sigma\mu_3}\Big)\ \Big\} + \cdots \end{aligned}$$

where the weights $W^{(3)} = 1/4,\ W^{(8)} = 1/12,\ W^{(0)} = 2/3$.

The model

Melnikov, AV '03

$$\mathcal{A} = \mathcal{A}_{\rm PS} + \mathcal{A}_{\rm PV} \; + \text{permutations},$$

$$\mathcal{A}_{\rm PS} = \sum_{a=3,8,0} W^{(a)} \phi_L^{(a)}(q_1^2, q_2^2)\, w_L^{(a)}(q_3^2)\, \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\},$$

$$\mathcal{A}_{\rm PV} = \sum_{a=3,8,0} W^{(a)} \phi_T^{(a)}(q_1^2, q_2^2)\, w_T^{(a)}(q_3^2) \Big(\{q_2 f_2 \tilde{f}_1 \tilde{f} f_3 q_3\}$$

$$+ \{q_1 f_1 \tilde{f}_2 \tilde{f} f_3 q_3\} + \frac{q_1^2 + q_2^2}{4} \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\} \Big).$$

For π^0

$$w_L^{(3)}(q^2) = \frac{2}{q^2 + m_\pi^2},$$

$$\phi_L^3(q_1^2, q_2^2) = \frac{N_c}{4\pi^2 F_\pi^2} F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)$$

Knecht, Nyffeler

$$= \frac{q_1^2 q_2^2 (q_1^2 + q_2^2) - h_2 q_1^2 q_2^2 + h_5 (q_1^2 + q_2^2) + (N_c M_1^4 M_2^4 / 4\pi^2 F_\pi^2)}{(q_1^2 + M_1^2)(q_1^2 + M_2^2)(q_2^2 + M_1^2)(q_2^2 + M_2^2)}$$

The model results in

$$a_\mu^{\pi^0} = 76.5 \times 10^{-11}, \qquad a_\mu^{\rm PS} = 114(10) \times 10^{-11}$$

A similar analysis for pseudovector exchange gives

$$a_\mu^{\rm PV} = 22(5) \times 10^{-11}$$

and finally

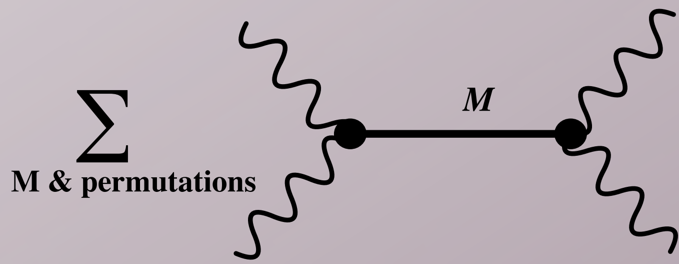
$$a_\mu^{\rm LbL} = 136(25) \times 10^{-11}$$

Summary for LbL

In our '08 mini-review with Prades, de Rafael we combined different calculations with some educated guesses about possible errors to come to:

$$a^{\mathrm{HLbL}} = (105 \pm 26) \times 10^{-11}$$

However the error estimates are quite subjective and further study of different exchanges is certainly needed. Experimental data on radiative decays can be a help.



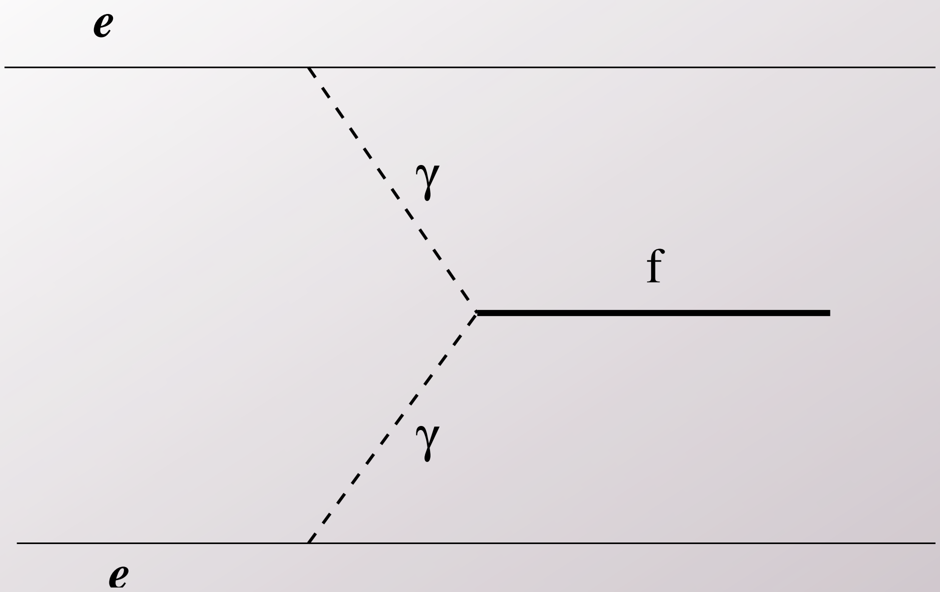
pseudoscalar mesons π^0, η, η'; scalars f_0, a_0; vectors π_1^0; pseudovectors a_1^0, f_1, f_1^*;
spin 2 f_2, a_2, η_2, π_2

Do we see NP in the muon g-2?

QED	$116\,584\,719.58(1.5) \times 10^{-11}$
Electroweak	$154(2)(1) \times 10^{-11}$
Hadronic LO	$6\,901(42)(19)(07) \times 10^{-11}$
Hadronic HO	$-\,97.9(0.9)(0.3) \times 10^{-11}$
Hadronic LbL	$105(26) \times 10^{-11}$

Total SM	$11\,659\,1785\,(52) \times 10^{-11}$	
Experimental a	$11\,659\,2080\,(63) \times 10^{-11}$	
Δa	$300\,(82) \times 10^{-11}$	$3.6\,\sigma$

Both experimental and theoretical uncertainty should be reduced to be sure of NP.



$$\Gamma(f_1(1285) \to \gamma\gamma^*) = (2.8 \pm 0.8)\ \mathrm{keV}$$

This is compatible with our model of pseudovector exchange. However,

$$\frac{\Gamma(f_1(1285) \to \gamma\rho^0)}{\Gamma_{\mathrm{total}}} = (5.5 \pm 1.3) \times 10^{-2}$$

leads to a strong enhancement (of order of 5) for PV exchange. Work in progress.

Conclusions

Having in mind the new g-2 experiment more theoretical efforts are needed to improve accuracy for the hadronic light-by-light contribution.

In my view it should also involve new measurements of hadronic two-photon production which provides a good test of theoretical models for HLbL.

Many happy returns of the day!