$B \to \gamma\ell\nu$ and the B meson distribution amplitude

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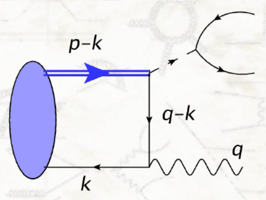
"Colour meets Flavour"
Workshop on Quantumchromodynamics and Quark Flavour Physics
On the occasion of Alex Khodjamirian's 60th birthday
Siegen, 13-14 October 2011

Work in progress with J. Rohrwild, and C. Hellmann and J. Rohrwild

Introduction



$$\Gamma(\ell\nu) \propto f_B^2 \left(\frac{m_\ell}{m_B}\right)^2$$

$$\Gamma(\gamma\ell\nu) \propto f_B^2 \, \frac{\alpha_{\rm em}}{4\pi} \left(\frac{m_B}{\lambda_B}\right)^2$$

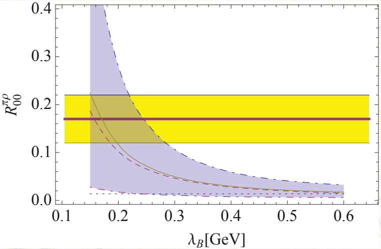
No helicity suppression.
Simplest, non-trivial, hard-exclusive B decay when $2E_\gamma = \mathcal{O}(m_b)$.
Involves B meson light-cone distribution amplitude:

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \, \Phi_{B+}(\omega,\mu)$$

Compare

$$\Gamma(\pi\ell\nu) \propto \left[\xi^{B\pi} + \frac{\alpha_s f_B f_\pi \Phi_\pi}{m_B \lambda_B}\right]^2 \qquad \Gamma(\pi\pi) \propto f_\pi \Phi_\pi \left[C_2 F^{B\pi} + C_1 \, \frac{\alpha_s f_B f_\pi \Phi_\pi}{m_B \lambda_B}\right]^2$$

λ_B and hadronic B decays



(MB, Huber, Li, 2009)

$$R_{00}^{\pi\rho} = \frac{2\,\Gamma(B^0 \to \pi^0\rho^0)}{\Gamma(B^0 \to \pi^+\rho^-) + \Gamma(B^0 \to \pi^-\rho^+)}$$

Whether QCD factorization for colour-suppressed, non-leptonic, charmless B decays works, depends on the value of λ_B.

B meson light-cone distribution amplitude

(Grozin, Neubert, 1997; BBNS 1999/2000; MB, Feldmann, 2000)

$$iF_{\rm stat}(\mu)\Phi_{B+}(\omega,\mu) = \frac{1}{2\pi}\int dt\, e^{it\omega}\,\langle 0|(\bar q_s Y_s)(tn_-)\not{n}_-\gamma_5(Y_s^\dagger h_v)(0)|\bar B_v\rangle_\mu$$

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\Phi_{B+}(\omega,\mu), \qquad \sigma_n(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\,\ln^n\frac{\mu_0}{\omega}\,\Phi_{B+}(\omega,\mu)$$

- Need only inverse, inverse-log moments, since $\omega \sim \Lambda_{\rm QCD}$.
  Not related to local operators.
- 1-loop renormalization known (Lange, Neubert, 2003).
  Not related to moments, since $\int_0^\infty d\omega\,\omega^n \ldots$ does not commute with renormalization.
- Three-particle amplitudes have been studied (Kawamura et al, 2001–; Huang et al, 2006; Khodjamirian, Mannel, Offen, 2006; Geyer, Witzel, 2007; Descotes-Genon, Offen, 2009)
- λ_B from
  - QCD sum rules of various forms (Braun, Korchemsky, Ivanov, 2003; Khodjamirian, Mannel, Offen, 2005; Ball, Kou, 2003): $\lambda_B(1\,\text{GeV}) = (460 \pm 110)\,\text{MeV}$ [BKI]
  - Models of the entire distribution amplitude (Lee, Neubert, 2005; Kawamura, Tanaka, 2009/2010)
  - Upper limits on $B \to \gamma\ell\nu$: $\lambda_B > 669\,\text{MeV}$ ($591\,\text{MeV}$) (BABAR, 2007);
    $\lambda_B > 300\,\text{MeV}$ (BABAR, 2009);

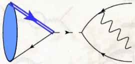
$B \to \gamma \ell \nu$ amplitude

$$\langle \ell\bar{\nu}\gamma|\ell\gamma^\mu(1-\gamma_5)\nu \cdot \bar{u}\gamma_\mu(1-\gamma_5)b|B^-\rangle =$$

$$= -ie\epsilon^\star_\nu \Bigg[\langle \ell\bar{\nu}|\bar{\ell}\gamma^\mu(1-\gamma_5)\nu)|0\rangle \cdot \int d^4x\, e^{iqx}\langle 0|\mathrm{T}\{j^\nu_{\mathrm{em}}(x)(\bar{u}\gamma_\mu(1-\gamma_5)b)(0)\}|B^-\rangle$$

$$+ \int d^4x\, e^{iqx}\langle \ell\bar{\nu}|\mathrm{T}\{j^\nu_{\mathrm{em}}(x)(\bar{\ell}\gamma_\mu(1-\gamma_5)\nu)(0)\}|0\rangle \cdot \langle 0|\bar{u}\gamma^\mu(1-\gamma_5)b|B^-\rangle\Bigg]$$

$$= e\epsilon^\star_\nu\, \bar{u}_\ell\gamma_\mu(1-\gamma_5)u_\nu \cdot T^{\nu\mu}(p,q) - ieQ_\ell f_B \cdot \bar{u}_\ell \not{\epsilon}^\star(1-\gamma_5)u_\nu$$



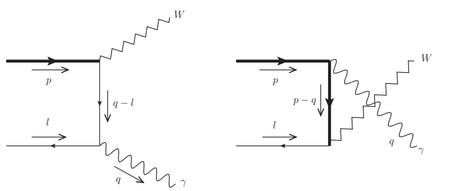
$$iT_{\nu\mu}(p,q) = i\epsilon_{\mu\nu\rho\sigma}v^\rho q^\sigma\, F_V(E_\gamma) + (g_{\mu\nu}v\cdot q - v_\nu q_\mu)\,\hat{F}_A(E_\gamma) + \frac{v_\nu v_\mu}{v\cdot q} f_B m_B + q_\nu\text{-terms}$$

Contact term fixed by the em Ward identity $q_\nu T^{\nu\mu} = -if_B p^\mu$ (Khodjamirian, Wyler, 2001). Replace

$$(g_{\mu\nu}v\cdot q - v_\nu q_\mu)\,\hat{F}_A(E_\gamma) + \frac{v_\nu v_\mu}{v\cdot q} f_B m_B \to (g_{\mu\nu}v\cdot q - v_\nu q_\mu)\,F_A(E_\gamma) + g_{\mu\nu}f_B$$

to cancel the lepton emission term. Then $F_A = \hat{F}_A + \dfrac{Q_\ell f_B}{E_\gamma}$

$B \to \gamma\ell\nu$ in QCD (Korchemsky, Pirjol, Yan, 1999; ...)



Intermediate light-quark propagator has hard-collinear virtuality $m_b\Lambda$

$$\frac{i(\not{q}-\not{l})}{(q-l)^2} = -\frac{i\not{q}}{2q\cdot l} + \ldots \sim \frac{1}{\Lambda}$$

$$F_V = F_A = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B}$$

Photon inherits the helicity of the energetic up quark. Leading power.

Intermediate heavy-quark propagator has hard virtuality m_b^2

$$\frac{i(\not{p}-\not{q})+m_b}{(p-q)^2-m_b^2} = \frac{i\not{q}}{2p\cdot q} + \ldots \sim \frac{1}{m_b}$$

$$F_V = -F_A = \frac{Q_b m_B f_B}{2E_\gamma m_b}$$

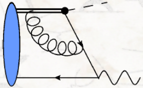
Photon has opposite helicity, because the weak current couples to the spectator quark. Power-suppressed.

Factorization at leading power in the heavy-quark mass expansion

(Lunghi, Pirjol, Wyler, 2002; Bosch, Hill, Lange, Neubert, 2003; Descotes-Genon, Sachrajda, 2002)

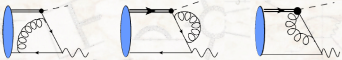
Integrate out hard virtualities:

$$iT_{\nu\mu}(p,q) = C(E_\gamma,\mu)\int d^4x\, e^{iqx}\langle 0|\mathrm{T}\{[j_{\nu,\mathrm{em}}(x)]_{\mathrm{SCET}}, (\bar\xi W_c\gamma_\mu(1-\gamma_5)h_v)(0)_\mu\}|B^-\rangle$$



$$[j_{\nu,\mathrm{em}}(x)]_{\mathrm{SCET}} = (\bar q_s W_c^\dagger \gamma_\nu \xi)(x)$$
$$+\, i\int d^4y\,\mathrm{T}\{\bar\xi\Big[\gamma_\nu \frac{1}{in_+D_c} i\not{D}_\perp + i\not{D}_\perp \frac{1}{in_+D_c}\gamma_\nu\Big]\frac{\not{n}_+}{2}\xi(y), (\bar q_s W_c^\dagger i\not{D}_\perp \xi)(x)\}$$

Integrate out hard-collinear virtualities:



$$F_V(E_\gamma) = F_A(E_\gamma) = C(E_\gamma,\mu)\int dt\, J(E_\gamma,t,\mu)\langle 0|(\bar q_s Y_s)(tn_-)\not{n}_-\gamma_5(Y_s^\dagger h_v)(0)|\bar B_v\rangle_\mu$$
$$= \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)}\, R(E_\gamma,\mu)$$

where R can be calculated in perturbation theory and depends on σ_i, $i\le 2n$ at $\mathcal{O}(\alpha_s^n)$.

Radiative corrections (NLL)

(Descotes-Genon, Sachrajda, 2002; Lunghi, Pirjol, Wyler, 2002; Bosch, Hill, Lange, Neubert, 2003; MB, Rohrwild, 2011)

$$R(E_\gamma,\mu) = C(E_\gamma,\mu_{h1})K^{-1}(\mu_{h2}) \times U(E_\gamma,\mu_{h1},\mu_{h2},\mu) \times J(E_\gamma,\mu)$$

$$C(E_\gamma,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2\ln^2\frac{2E_\gamma}{\mu} + 5\ln\frac{2E_\gamma}{\mu} - \frac{3-2x}{1-x}\ln x - 2\mathrm{Li}_2(1-x) - 6 - \frac{\pi^2}{12}\right)$$

$$J(E_\gamma,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{2E_\gamma\mu_0}{\mu^2} - 2\sigma_1(\mu)\ln\frac{2E_\gamma\mu_0}{\mu^2} - 1 - \frac{\pi^2}{6} + \sigma_2(\mu)\right).$$

$$K(\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right).$$

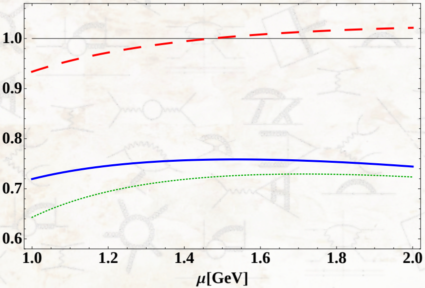
NLL renormalization-group evolution from the hard (m_b) to the hard-collinear scale ($(m_b\Lambda)^{1/2}$)

$$U(E_\gamma,\mu_{h1},\mu_{h2},\mu) = U_1(E_\gamma,\mu_{h1},\mu)U_2(\mu_{h2},\mu)^{-1}$$

$$\mu\frac{d}{d\mu}U_1(E_\gamma,\mu_h,\mu) = \left(\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu}{2E_\gamma} + \gamma(\alpha_s)\right)U_1(E_\gamma,\mu_h,\mu)$$

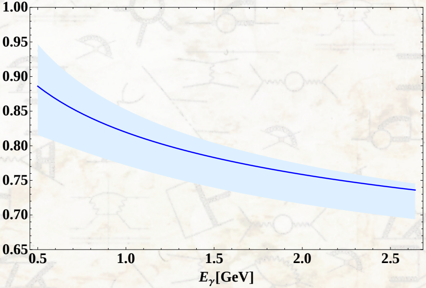
Two-loop anomalous dimension of the heavy-light SCET current from (Bonciani, Ferroglia; Asatrian, Greub, Pecjak; MB, Huber, Li; Bell 2008)

Size of radiative corrections (to the amplitude)



Hard-collinear scale dependence
($E_\gamma = 2\,\mathrm{GeV}$)

Photon-energy dependence (and total scale uncertainty)



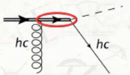
$1/m_b$ power corrections

Integrate out hard virtualities:

$$iT_{\nu\mu}(p,q) = \int d^4x\, e^{iqx} \langle 0|\mathrm{T}\{[j_{\nu,\mathrm{em}}(x)]_{\mathrm{SCET}}, (\bar{u}\gamma_\mu(1-\gamma_5)b)(0)_{\mathrm{SCET}}\}|B^-\rangle + \frac{1}{m_b}\langle 0|iS^{\mathrm{local}}_{\nu\mu}|B^-\rangle$$

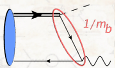
Time-ordered products with sub-leading SCET interactions, subleading corrections to the currents, e.g.

$$[\bar{u}\gamma_\mu(1-\gamma_5)Q](0) \to \int ds \sum_{i=1}^{3} \tilde{C}_i^{(A)}(s) J_\mu^{(A),i}(s) + \int ds\, ds' \sum_{i=1}^{4} \tilde{C}_i^{(B)}(s,s') J_\mu^{(B),i}(s,s')$$



and a local term.
At tree level only emission off the heavy quark and

$$\frac{i(\not{q}-\not{l})}{(q-l)^2} = -\frac{i\not{q}}{2q\cdot l} + \underbrace{\frac{i\not{l}}{2q\cdot l}}_{\text{power suppressed}} = -\frac{i\not{n}_-}{4l_-} + \Big[\underbrace{\frac{il_+\not{n}_-}{4E_\gamma l_-} + \frac{i\not{l}_\perp}{2E_\gamma l_-}}_{\text{SCET time-ordered products}} + \underbrace{\frac{i\not{n}_+}{4E_\gamma}}_{\text{local}}\Big],$$



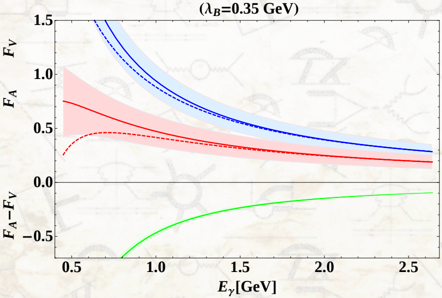
Local operator is $S^{\mathrm{local}}_{\nu\mu} = \bar{q}_s\Gamma_{\mu\nu}\not{n}_\pm h_v \to f_B$.
Beyond tree level: photon structure from collinear quark-photon coupling, three-particle B-meson distribution amplitudes, non-factorizable time-ordered products (work in progress)

$B \to \gamma$ form factors with radiative and power corrections

$$F_V(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \left[\xi(E_\gamma) + \frac{Q_b m_B f_B}{2E_\gamma m_b} + \frac{Q_u m_B f_B}{(2E_\gamma)^2}\right],$$

$$F_A(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \left[\xi(E_\gamma) - \frac{Q_b m_B f_B}{2E_\gamma m_b} - \frac{Q_u m_B f_B}{(2E_\gamma)^2} + \frac{Q_\ell f_B}{E_\gamma}\right].$$

Non-factorizable time-ordered product terms parameterized by $\xi(E_\gamma) = \pm f_B/(2E_\gamma)$. Irreducible uncertainty in determination of λ_B. Drops out in $F_V - F_A$.



BABAR analysis of $B \to \gamma\ell\nu$

$$\Delta\mathcal{B} = \tau_{B_d} \int_{\text{PS-Cuts}} dE_\gamma\, dE_\ell\, \frac{d^2\Gamma}{dE_\gamma\, dE_\ell}$$

- (BABAR, 0704.1478): $E_\gamma \in (0.45, 2.35)$ GeV, $E_\ell \in (1.875, 2.64)$ GeV, $\cos\theta(\ell,\gamma) < -0.36$

$$\Delta\mathcal{B} < 1.7(2.3) \cdot 10^{-6}$$

$\lambda_B > 669(591)$ MeV

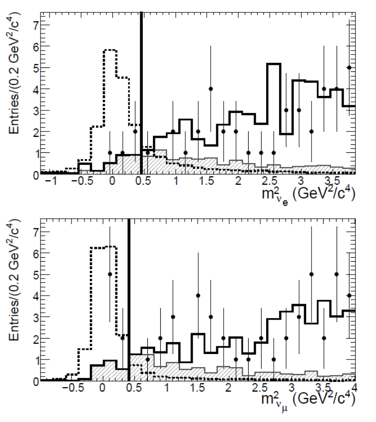
- (BABAR, Phys.Rev. D80 (2009) 111105 [0907.1681]): $E_\gamma > 1.0$ GeV

$$\Delta\mathcal{B} < 14 \cdot 10^{-6}$$

$\lambda_B > 0.3$ GeV

- Nothing from BELLE!?

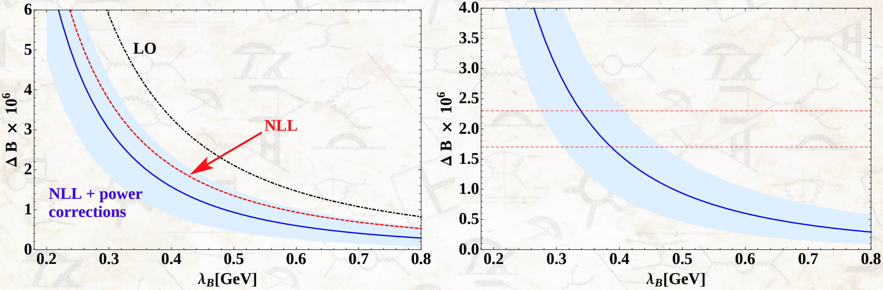
(BABAR, Phys.Rev. D80 (2009) 111105 [0907.1681])



But both analyses do not include radiative corrections and theoretical uncertainties.
Wrong input for power corrections from (Pirjol, Korchemsky, Yan, 1999)

Present limits

Cuts corresponding the BABAR2007



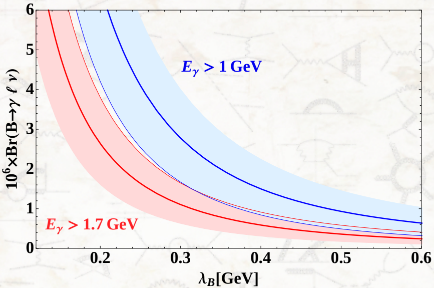
$$\lambda_B > 669\,(591)\,\text{MeV} \longrightarrow 372\,(331)\,\text{MeV}\ \text{[incl. } |V_{ub}|\text{]} \longrightarrow 310\,(274)\,\text{MeV}\ \text{[excl. } |V_{ub}|\text{]}$$

$$\lambda_B > 300\,\text{MeV} \longrightarrow \ldots \longrightarrow 115\,\text{MeV}\ \text{[excl. } |V_{ub}|\text{] with data from BABAR2009}$$

But the first line needs to be taken with caution. Unpublished data analysis, E_γ cut too low.
No stringent constaints yet.

Outlook

SuperB factories should do this. BELLE?



Example:

$$\mathrm{Br}\,(B^- \to \gamma\ell\bar{\nu}, E_\gamma > 1.7\,\mathrm{GeV}) = (2.0 \pm 0.4) \times 10^{-6} \quad \to \quad \lambda_B = 228^{+76}_{-61}\,\mathrm{MeV}$$

Dominant theoretical uncertainty about equally from σ_1, σ_2 and ξ.

Conclusion

- $B \to \gamma \ell \nu$ can determine λ_B at SuperB factories.
  Radiative corrections are under control at NLL.

- Interesting for study of power corrections.
  $F_V - F_A$ is power-suppressed and depends only on f_B (to all orders? – work in progress)