Kinematic power corrections in off-forward hard processes

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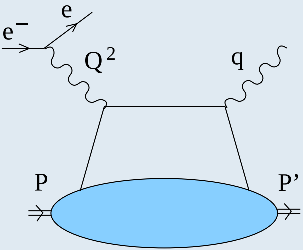
based on

V.M. Braun, A.N. Manashov, arXiv:1108.2394 [hep-ph]

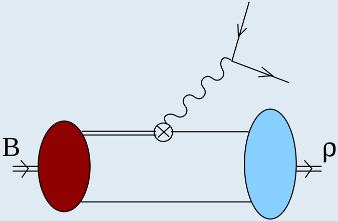
Siegen, 14.10.2011

Hard exclusive processes involve off-forward matrix elements

DVCS: $\gamma^* P \to \gamma P'$



Form factors: $\gamma^* \pi \to \gamma$, $B \to \rho \ell \bar{\nu}_\ell$, ...



Operator Product Expansion

$$J(x)J(0) \sim \sum_N C_N(x^2, \mu^2)\, \mathcal{O}_N(\mu^2)$$

involves

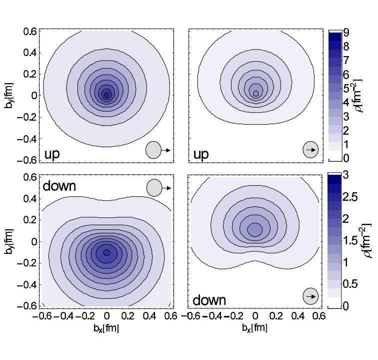
$$\langle P' | \mathcal{O}_N(\mu^2) | P \rangle \qquad \langle \rho(p) | \mathcal{O}_N(\mu^2) | 0 \rangle$$

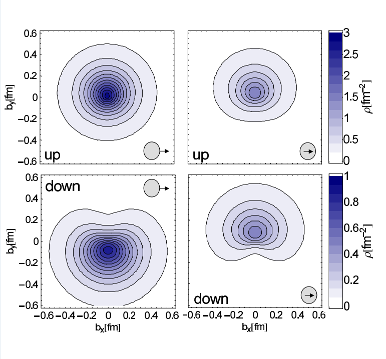
Kinematic variables: hadron mass m^2 momentum transfer $t = (P - P')^2$

How to calculate effects $\sim m^2/Q^2$ and t/Q^2?

Nucleon Tomography

access to three-dimensional picture of the nucleon (M. Burkardt)





↪ **first two moments of transverse spin parton density**

computer simulations: M. Göckeler *et al.*, Phys.Rev.Lett. 98 (2007) 222001

- **paradigm shift: finite t a "nuisance" $\longrightarrow$ important tool**

Where is a problem?

- **Inclusive reactions $\leftrightarrow$ forward matrix elements**

O. Nachtmann, Nucl. Phys. **B63** (1973) 237:
all target mass corrections are due to subtraction of traces in leading twist operators $\mathcal{O}_{\mu\mu_1\ldots\mu_N}$

- **Exclusive reactions $\leftrightarrow$ off-forward matrix elements**

in addition, there are contributions of higher-twist operators that reduce to total derivatives of the operators of leading twist:

$$\partial^2\mathcal{O}_{\mu\mu_1\ldots\mu_N} \qquad \text{and} \qquad \partial^\mu\mathcal{O}_{\mu\mu_1\ldots\mu_N}$$

S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto, Phys. Lett. **B38**, 333 (1972):
matrix elements of $\partial^\mu\mathcal{O}_{\mu\mu_1\ldots\mu_N}$ over free quarks vanish

The same problem in a different language

- **Using EOM $\partial^\mu \mathcal{O}_{\mu\mu_1\dots\mu_N}$ can be expressed in terms of quark-gluon operators:**

$N=1$:

$$\partial^\mu O_{\mu\nu} = 2i\bar q g G_{\nu\mu}\gamma^\mu q\,, \qquad O_{\mu\nu} = (1/2)[\bar q\gamma_\mu \overset{\leftrightarrow}{D}_\nu\, q + (\mu\leftrightarrow\nu)]$$

$N=2$:

$$\frac{4}{5}\partial^\mu \mathcal{O}_{\mu\alpha\beta} = -12i\bar q\gamma^\rho \left\{ gG_{\rho\beta}\vec{D}_\alpha - \overset{\leftarrow}{D}_\alpha\, gG_{\rho\beta} + (\alpha\leftrightarrow\beta)\right\} q - 4\partial^\rho \bar q(\gamma_\beta g\widetilde{G}_{\alpha\rho} + \gamma_\alpha g\widetilde{G}_{\beta\rho})\gamma_5 q$$
$$- \frac{8}{3}\,\partial_\beta\bar q\gamma^\sigma\widetilde{G}_{\sigma\alpha}\gamma_5 q - \frac{8}{3}\,\partial_\alpha\bar q\gamma^\sigma\widetilde{G}_{\sigma\beta}\gamma_5 q + \frac{28}{3}\,g_{\alpha\beta}\partial_\rho\bar q\gamma^\sigma\widetilde{G}_{\sigma\rho}q\,,$$

where

$$\mathcal{O}_{\mu\alpha\beta} = \mathrm{Sym}_{\mu\alpha\beta}\left[\frac{15}{2}\bar q\gamma_\mu \overset{\leftrightarrow}{D}_\alpha \overset{\leftrightarrow}{D}_\beta\, q - \frac{3}{2}\partial_\alpha\partial_\beta\bar q\gamma_\mu q\right] - \text{traces}$$

etc.

- **How to distinguish between "kinematic" and genuine "dynamic" degrees of freedom?**
- **Are matrix elements of twist-four $\bar q G q$ operators $\sim \Lambda^2_{QCD}$ or are they $\sim t \gg \Lambda^2_{QCD}$?**

Let G_{Nk} be the complete basis of twist-four operators

general structure of such relations:

$$(\partial\mathcal{O})_N = \sum_k a_k^{(N)}\, G_{Nk}$$

and simultaneously

$$T\{j(x)j(0)\}^{t=4} = \sum_{N,k} c_{N,k}(x)\, G_{Nk}$$

A separation of "kinematic" and "dynamic" contributions implies rewriting expansion of T-product in such a way that the particular combination appearing in $(\partial\mathcal{O})_N$ is separated from the "remainder".

The "kinematic" approximation would correspond to taking into account this term only, and neglecting contributions of "genuine" quark-gluon operators.

Guidung principle:

- **"Kinematic" and "Dynamic" contributions must have autonomous scale-dependence**

Let $\mathcal{G}_{N,k}$ be the set of *multiplicatively renormalizable* twist-four operators

$$\mathcal{G}_{N,k} = \sum_{k'} \psi^{(N)}_{k,k'}\, G_{N,k'}$$

- **One solution of the RG equations in known without calculation !**

$$(\partial\mathcal{O})_N = \sum_k a^{(N)}_k\, G_{Nk}$$

assume $(\partial O)_N$ corresponds to $k=0$, so $\mathcal{G}_{N,k=0} \equiv (\partial\mathcal{O})_N$ and $\psi^{(N)}_{k=0,k'} = a_{k'}$

Inverting the matrix of coefficients $\psi^{(N)}_{k,k'}$

$$G_{N,k} = \phi^{(N)}_{k,0}(\partial\mathcal{O})_N + \sum_{k'\neq 0} \phi^{(N)}_{k,k'}\, \mathcal{G}_{N,k'}$$

leading to

$$T\{j(x)j(0)\}^{\rm tw-4} = \sum_{N,k} c_{N,k}(x)\phi^{(N)}_{k,0}\, (\partial\mathcal{O})_N + \dots$$

the ellipses stand for the contributions of “genuine” twist-four operators

The problem is that finding $\phi^{(N)}_{k,0}$ requires the knowledge of the full matrix $\psi^{(N)}_{k,k'}$, alias explicit solution of the twist-four RG equations.

Solution:

Bukhvostov, Frolov, Lipatov, Kuraev, Nucl. Phys. **B258** (1985) 601

- **Four-particle twist-4 operators have autonomous scale-dependence $\longrightarrow$ irrelevant**

Braun, Manashov, Rohrwild, Nucl. Phys. **B807** (2009) 89; Nucl. Phys. **B826** (2010) 235.

- **RG equations for three-particle (non-quasipartonic) operators are hermitian w.r.t. a certain scalar product**

Hence different solutions are mutually orthogonal w.r.t. a certain weight function:

$$\sum_k \mu_k^{(N)} \psi_{l,k}^{(N)} \psi_{m,k}^{(N)} \sim \delta_{l,m}$$

so that

$$\phi_{k,0}^{(N)} = a_k^{(N)} ||a^{(N)}||^{-2}, \qquad ||a^{(N)}||^2 = \sum_k \mu_k^{(N)} (a_k^{(N)})^2$$

and finally

$$T\{j(x)j(0)\}^{\text{tw}-4} = \sum_N \left(\sum_k \frac{c_{N,k}(x)\, a_k^{(N)}}{||a^{(N)}||^2} \right) (\partial\mathcal{O})_N + \text{dynamic higher twist}$$

Results: Time-ordered product of electromagnetic currents

$$S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\nu\alpha}g_{\mu\beta} - g_{\mu\nu}g_{\alpha\beta}$$

$$\begin{aligned} i\,T\Big\{j^{em}_\mu(x)j^{em}_\nu(0)\Big\} \quad = \quad & -\frac{1}{\pi^2 x^4}\Big\{x^\alpha\Big[S_{\mu\alpha\nu\beta}\mathbb{V}^\beta + i\epsilon_{\mu\nu\alpha\beta}\mathbb{A}^\beta\Big] \\ & + x^2\Big[(x_\mu\partial_\nu + x_\nu\partial_\mu)\mathbb{X} + (x_\mu\partial_\nu - x_\nu\partial_\mu)\mathbb{Y}\Big]\Big\} \end{aligned}$$

twist expansion:

$$\begin{aligned} \mathbb{V}_\beta &= \mathbb{V}_\beta^{t=2} + \mathbb{V}_\beta^{t=3} + \mathbb{V}_\beta^{t=4} + \ldots \\ \mathbb{A}_\beta &= \mathbb{A}_\beta^{t=2} + \mathbb{A}_\beta^{t=3} + \mathbb{A}_\beta^{t=4} + \ldots \\ \mathbb{X} &= \mathbb{X}^{t=4} + \ldots \\ \mathbb{Y} &= \mathbb{Y}^{t=4} + \ldots \end{aligned}$$

Symmetries

- **Conservation of the electromagnetic current**

$$\partial^\mu T_{\mu\nu}(x) = 0\,, \qquad\qquad \partial^\nu T_{\mu\nu}(x) = i[\mathbf{P}^\nu, T_{\mu\nu}(x)]$$

$\hookrightarrow$ **only valid in the sum of all twists but not for each twist separately**

- **Translation invariance**

$$T\left\{j_\mu^{em}(2x) j_\nu^{em}(0)\right\} = e^{-i\mathbf{P}\cdot x}\, T\left\{j_\mu^{em}(x) j_\nu^{em}(-x)\right\} e^{i\mathbf{P}\cdot x}$$

$\hookrightarrow$ **only valid in the sum of all twists but not for each twist separately**

for example

$$\left[T\left\{j_\mu^{em}(2x) j_\nu^{em}(0)\right\}\right]^{twist-2} = \left[e^{-i\mathbf{P}\cdot x}\, T\left\{j_\mu^{em}(x) j_\nu^{em}(-x)\right\} e^{i\mathbf{P}\cdot x}\right]^{twist-2}$$
$$\neq e^{-i\mathbf{P}\cdot x}\left[T\left\{j_\mu^{em}(x) j_\nu^{em}(-x)\right\}\right]^{twist-2} e^{i\mathbf{P}\cdot x}$$

$\Rightarrow$ **OPE for $T\{j(x)j(-x)\}$ and $T\{j(2x)j(0)\}$ look very differently**

only the total finite-t and -m^2 correction from twist 2+3+4 has physical meaning

final answer:

$$
\begin{aligned}
\mathbb{V}_\mu^{t=4} &= \frac{1}{2}\sum_{N,\text{odd}} \varkappa_N \frac{1}{(N+2)^2}\int_0^1 du\,(u\bar u)^{N+1}\Big\{x_\mu\,[(\partial\mathcal{O})_N^V(ux)]_{l.t.} \\
&\qquad + \frac{1}{2}N(N+3)\int_0^1 dv\, v^{N-1}\, x^2\partial_\mu\,[(\partial\mathcal{O})_N^V(uvx)]_{l.t.}\Big\} \\
\mathbb{A}_\mu^{t=4} &= \frac{1}{4}x^2\partial_\mu \sum_{N,\text{even}} \varkappa_N \frac{N(N+3)}{(N+2)^2}\int_0^1 du\,(u\bar u)^{N+1}\int_0^1 dv\, v^{N-1}\,[(\partial\mathcal{O})_N^A(uvx)]_{l.t.}\,, \\
\mathbb{X}^{t=4} &= \frac{1}{4}\sum_{N,\text{odd}} \varkappa_N \frac{N+1}{(N+2)^2}\int_0^1 du\,(u\bar u)^N(u-\bar u)\int_0^1 dv\, v^{N-1}\,[\partial\mathcal{O}_N^V(uvx)]_{l.t.}\,, \\
\mathbb{Y}^{t=4} &= -\frac{1}{4}\sum_{N,\text{odd}} \varkappa_N \frac{N+1}{(N+2)^2}\int_0^1 du\,(u\bar u)^N(u^2+\bar u^2)\int_0^1 dv\, v^{N-1}\,[(\partial\mathcal{O})_N^V(uvx)]_{l.t.}\,.
\end{aligned}
$$

$\varkappa_N = 2(2N+3)/(N+1)!$

$[\ldots]_{l.t.}$: leading-twist projection

Conformal symmetry and $SU(1,1)$ scalar product

collinear conformal transformations $SL(2,\mathbb{R}) \Leftrightarrow SU(1,1)$

$$x_\mu = z n_\mu \,, \qquad z \in \mathbb{R} \to z' = \frac{az+b}{cz+d}\,, \qquad \Leftrightarrow \qquad z \in \mathbb{C} \to z' = \frac{az+b}{\bar{b}z+\bar{a}}$$

representations are labeled by conformal spin

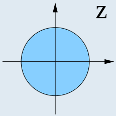
$$\varphi(z) \to T^j \varphi(z) = \frac{1}{(\bar{b}z+\bar{a})^{2j}} \varphi\left(\frac{az+b}{\bar{b}z+\bar{a}}\right)$$

This is a unitary transformation with respect to the following scalar product:

$$\langle \phi, \psi \rangle_j = \frac{2j-1}{\pi} \int_{|z|<1} d^2 z\, (1-|z|^2)^{2j-2} \bar{\phi}(z) \psi(z) \equiv \int_{|z|<1} \mathcal{D}_j z\, \bar{\phi}(z)\psi(z)\,, \qquad ||\phi||^2 = \langle \phi, \phi \rangle$$

similar for several variables

$$\langle \phi, \psi \rangle_{j_1, j_2} = \int_{|z_1|<1} \mathcal{D}_{j_1} z_1 \int_{|z_2|<1} \mathcal{D}_{j_2} z_2\, \bar{\phi}(z_1, z_2) \psi(z_1, z_2)$$



- conformal operator $O_+(z_1,z_2) = \bar\psi_+(z_1 n)\psi_+(z_2 n)$

$$
\begin{aligned}
\mathcal{O}_N &= (-\partial_+)^N \bar\psi_+ C_N^{3/2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)\psi_+ = \rho_N \Big\langle (z_1-z_2)^N, O_+(z_1,z_2)\Big\rangle \\
\partial_+^k \mathcal{O}_N &= \rho_N \Big\langle (S_{12}^+)^k (z_1-z_2)^N, O_+(z_1,z_2)\Big\rangle \equiv \rho_N \Big\langle \Phi_{N,k}(z_1,z_2), O_+(z_1,z_2)\Big\rangle
\end{aligned}
$$

$$S_{12}^+ = z_1^2\partial_{z_1} + z_2^2\partial_{z_2} + 2z_1 + 2z_2$$

- The functions $\Phi_{Nk}(z_1,z_2)$ form an orthogonal basis

$$\Big\langle \Phi_{Nk}, \Phi_{N'k'}\Big\rangle = \delta_{NN'}\delta_{kk'}\,||\Phi_{Nk}||^2$$

so that the light-ray operator can be expanded as

$$\bar\psi_+(z_1 n)\psi_+(z_2 n) = \sum_{N=0}^{\infty}\sum_{k=0}^{\infty} \rho_N^{-1}\frac{1}{||\Phi_{Nk}||^2}\Phi_{Nk}(z_1,z_2)\partial_+^k \mathcal{O}_N$$

Spinor Representation

Coordinates:

$$x_{\alpha\dot\alpha} = x_\mu \sigma^\mu_{\alpha\dot\alpha} = \begin{pmatrix} x_0 + x_3 & x_1 - ix_2 \\ x_1 + ix_2 & x_0 - x_3 \end{pmatrix} = \begin{pmatrix} x_+ & w \\ \bar w & x_- \end{pmatrix}, \qquad \sigma^\mu = (\mathbb{1}, \vec\sigma)$$

To maintain Lorentz–covariance, introduce two light-like vectors $n^2 = \tilde n^2 = 0$

$$n_{\alpha\dot\alpha} = \lambda_\alpha \bar\lambda_{\dot\alpha}, \qquad \tilde n_{\alpha\dot\alpha} = \mu_\alpha \bar\mu_{\dot\alpha}$$

with auxiliary spinors λ and μ

$$x_{\alpha\dot\alpha} = z\,\lambda_\alpha\bar\lambda_{\dot\alpha} + \tilde z\,\mu_\alpha\bar\mu_{\dot\alpha} + w\,\lambda_\alpha\bar\mu_{\dot\alpha} + \bar w\,\mu_\alpha\bar\lambda_{\dot\alpha}$$

Fields:

$$q = \begin{pmatrix} \psi_\alpha \\ \bar\chi^{\dot\beta} \end{pmatrix}, \qquad \bar q = (\chi^\beta, \bar\psi_{\dot\alpha}),$$

$$F_{\alpha\beta,\dot\alpha\dot\beta} = \sigma^\mu_{\alpha\dot\alpha}\sigma^\nu_{\beta\dot\beta}F_{\mu\nu} = 2\left(\epsilon_{\dot\alpha\dot\beta}f_{\alpha\beta} - \epsilon_{\alpha\beta}\bar f_{\dot\alpha\dot\beta}\right)$$

$f_{\alpha\beta}$ and $\bar f_{\dot\alpha\dot\beta}$ transform according to $(1,0)$ and $(0,1)$ representations of Lorentz group

Conformal basis for twist-four non-quasipartonic operators

Braun, Manashov, Rohrwild, Nucl. Phys. **B826** (2010) 235.

$$
\begin{aligned}
Q_1(z_1,z_2,z_3) &= \bar\psi_+(z_1) f_{+-}(z_2)\psi_+(z_3)\,, && T^{j=1}\otimes T^{j=1}\otimes T^{j=1}\\
Q_2(z_1,z_2,z_3) &= \bar\psi_+(z_1) f_{++}(z_2)\psi_-(z_3)\,, && T^{j=1}\otimes T^{j=3/2}\otimes T^{j=1/2}\\
Q_3(z_1,z_2,z_3) &= \frac{1}{2}[D_{-+}\bar\psi_+](z_1) f_{++}(z_2)\psi_+(z_3)\,, && T^{j=3/2}\otimes T^{j=3/2}\otimes T^{j=1}
\end{aligned}
$$

and three similar operators with $f\to\bar f$

C.f. in usual notation

$$\bar q_L(z_1)\Big[F_{+\mu}(z_2)+i\widetilde F_{+\mu}(z_2)\Big]\gamma^\mu q_L(z_3) = Q_2(z_1,z_2,z_3)-Q_1(z_1,z_2,z_3)$$

$$\vec{Q}(z_1,z_2,z_3) = \begin{pmatrix} Q_1(z_1,z_2,z_3)\\ Q_2(z_1,z_2,z_3)\\ Q_3(z_1,z_2,z_3)\end{pmatrix} \qquad (1)$$

Kinematic projection operators

$$
\begin{aligned}
2(\partial\mathcal{O})_N &= \frac{ig\rho_N}{(N+1)^2}\Big[\langle\!\langle \vec{\Psi}_N, \vec{Q}\rangle\!\rangle - \langle\!\langle \vec{\bar{\Psi}}_N, \vec{\bar{Q}}\rangle\!\rangle\Big] + \dots \\
\langle\!\langle \vec{\Phi}, \vec{\Psi}\rangle\!\rangle &= 2\langle \Phi_1, \Psi_1\rangle_{111} + \langle \Phi_2, \Psi_2\rangle_{1\frac{3}{2}\frac{1}{2}} + \frac{1}{2}\langle \Phi_3, \Psi_3\rangle_{\frac{3}{2}\frac{3}{2}1}
\end{aligned}
$$

$$
ig\vec{Q}(z_1,z_2,z_3) = \sum_{N=0}^{\infty}\sum_{k=0}^{\infty}\frac{p_{Nk}(N+1)^2}{\rho_N||\Psi_N||^2}\,\vec{\Psi}_{Nk}(z_1,z_2,z_3)\,\partial_+^k(\partial\mathcal{O})_N + \dots
$$

- All entries known explicitly
- The ellipses stand for "dynamic" operators

Outlook

- Done:

A theoretical framework for the calculation of finite t and target mass corrections in hard off-forward processes

- To do:
  - **Factorization of kinematic contributions to DVCS to twist-4 accuracy**
  - **Concrete predictions and applications to data analysis in DVCS, $\gamma^* \to \eta\gamma$**
  - **Meson distribution amplitudes, applications to B-decays**
  - **An alternative derivation**

Supplementary material

Application to GPDs

$$P = (p+p')/2, \quad \Delta = p' - p$$

$$\langle p'|\mathcal{O}_N|p\rangle = \left[\bar{u}(p')\not{n}u(p)\sum_{k=even}^{N} A_{N,k}(t)\Delta_+^k P_+^{N-k} + \frac{\bar{u}(p')u(p)}{m}\sum_{k=even}^{N+1} B_{N,k}(t)\Delta_+^k P_+^{N+1-k}\right]_{l.t.}$$

$A_{N,k}(t)$, $B_{N,k}(t)$: generalized form factors; conformal moments of GPDs

$$\langle P'|(\partial\mathcal{O})_N|P\rangle = i\left[\bar{u}(p')\not{n}u(p)\sum_{k=even}^{N+1} \widehat{A}_{N,k}(t)\Delta_+^{k-1} P_+^{N-k} + \frac{\bar{u}(p')u(p)}{m}\sum_{k=even}^{N+2} \widehat{B}_{N,k}(t)\Delta_+^{k-1} P_+^{N+1-k}\right]_{l.t.}$$

$$\begin{aligned}
\widehat{A}_{N,k}(t) &= t\,A_{N,k}(t)\frac{k(2N+3-k)}{2(N+1)^2} - \left(m^2 - \frac{t}{4}\right) A_{N,k-2}\frac{(N-k+2)(N-k+1)}{2(N+1)^2} \\
\widehat{B}_{N,k}(t) &= t\,B_{N,k}(t)\frac{k(2N+3-k)}{2(N+1)^2} - \left(m^2 - \frac{t}{4}\right) B_{N,k-2}\frac{(N-k+3)(N-k+2)}{2(N+1)^2} \\
&\quad - \frac{m^2}{(N+1)^2}(N-k+2)A_{N,k-2}(t)
\end{aligned}$$

“Plus” and “Minus” components

$$\psi_+ = \lambda^\alpha \psi_\alpha\,, \qquad \chi_+ = \lambda^\alpha \chi_\alpha\,, \qquad f_{++} = \lambda^\alpha \lambda^\beta f_{\alpha\beta}\,,$$
$$\bar\psi_+ = \bar\lambda^{\dot\alpha} \bar\psi_{\dot\alpha}\,, \qquad \bar\chi_+ = \bar\lambda^{\dot\alpha} \chi_{\dot\alpha}\,, \qquad \bar f_{++} = \bar\lambda^{\dot\alpha} \bar\lambda^{\dot\beta} \bar f_{\dot\alpha\dot\beta}\,,$$
$$\psi_- = \mu^\alpha \psi_\alpha\,, \qquad \bar\psi_- = \bar\mu^{\dot\alpha} \bar\psi_{\dot\alpha} \qquad f_{+-} = \lambda^\alpha \mu^\beta f_{\alpha\beta}$$

similar for derivatives $\partial_\mu \to \partial_{\alpha\dot\alpha}$

$$\partial_{++} = 2\partial_z\,, \qquad \partial_{--} = 2\partial_{\bar z}\,, \qquad \partial_{+-} = 2\partial_w\,, \qquad \partial_{-+} = 2\partial_{\bar w}$$

- ψ_+, χ_+, f_{++} and $\bar\psi_+, \bar\chi_+, \bar f_{++}$ are defined as quasipartonic