

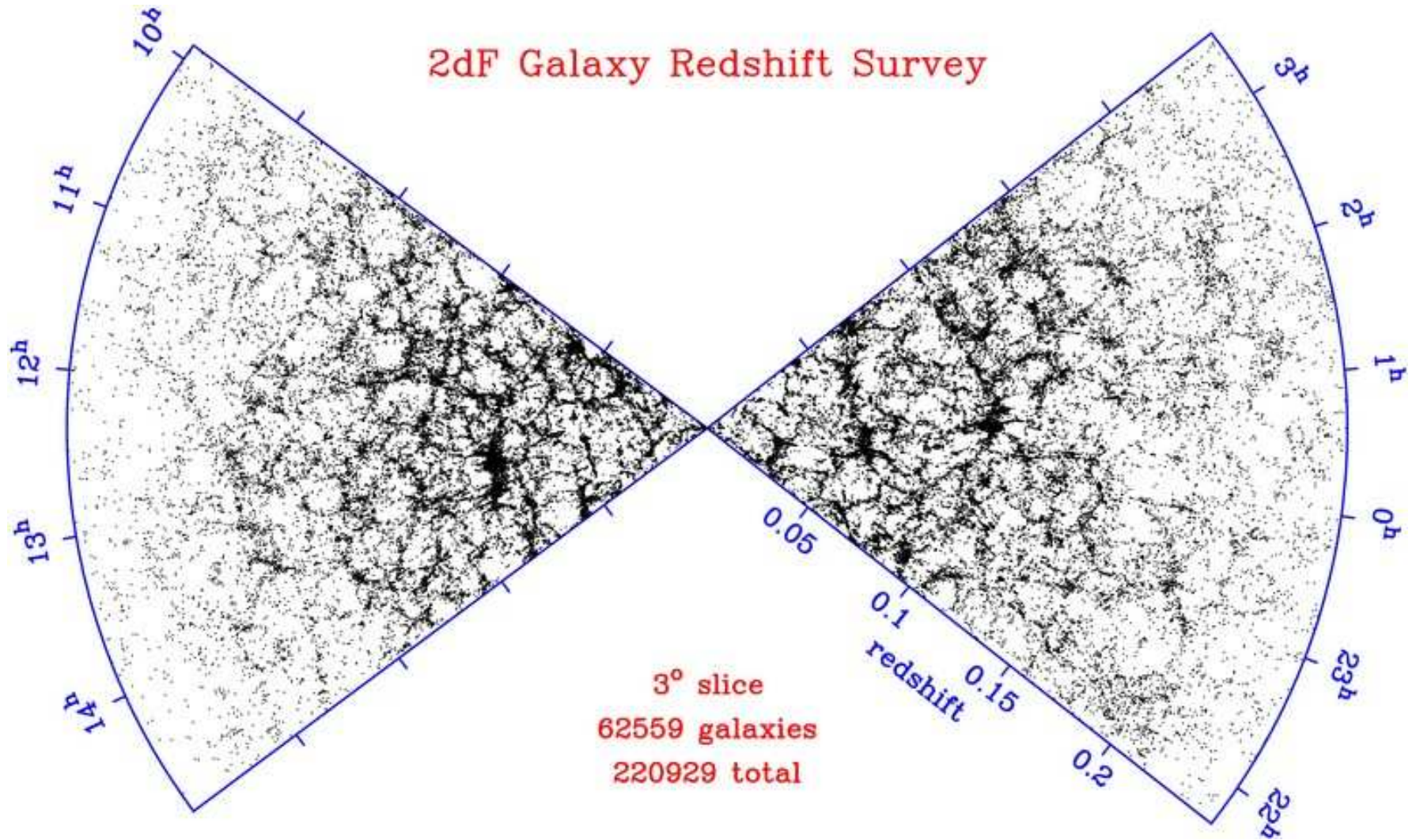
# Pre-inflationary homogenization of the Universe

Krzysztof Bolejko

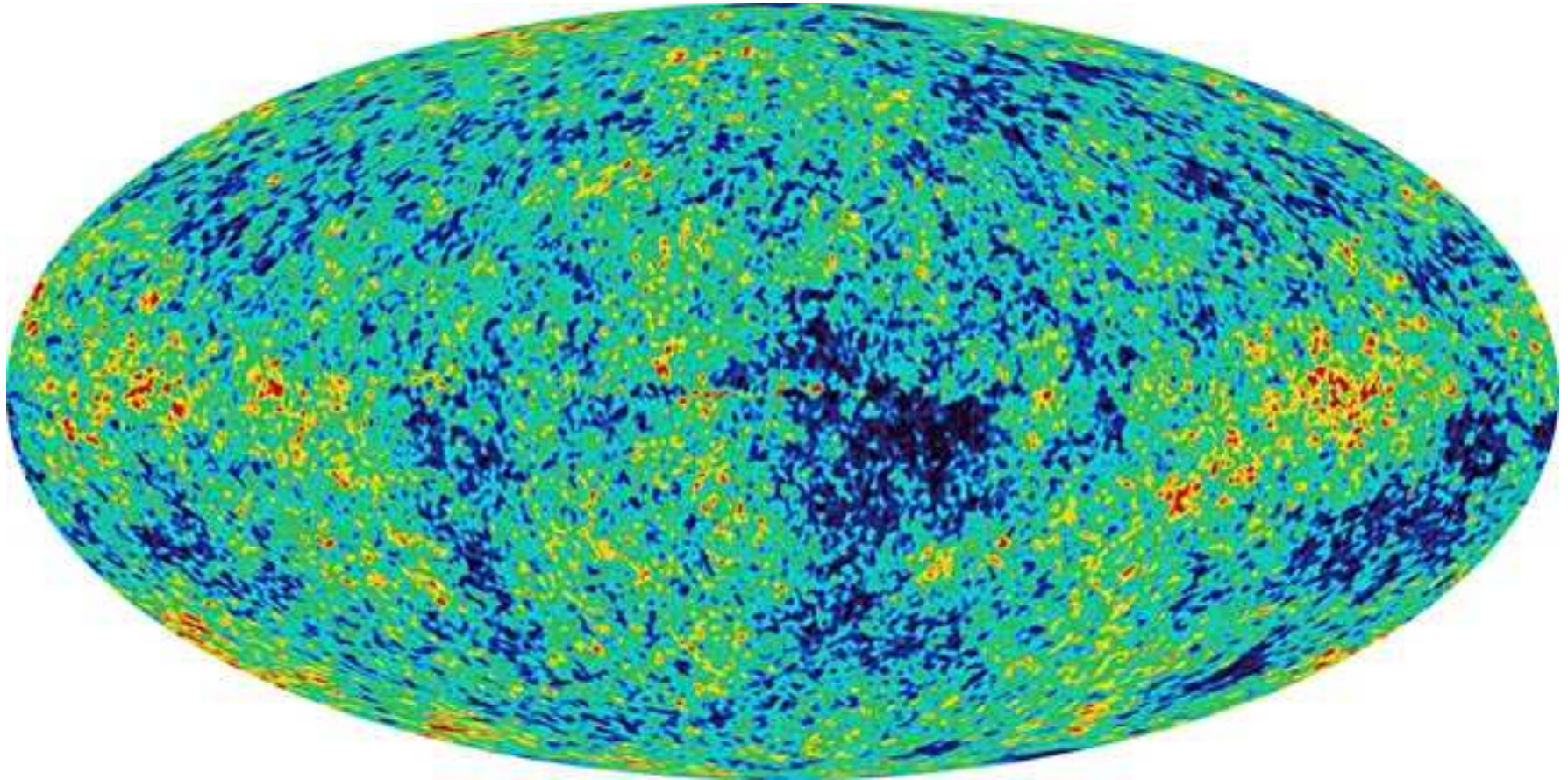
Astrophysics Department  
University of Oxford

BritGrav2011, Glasgow, 6/4/11

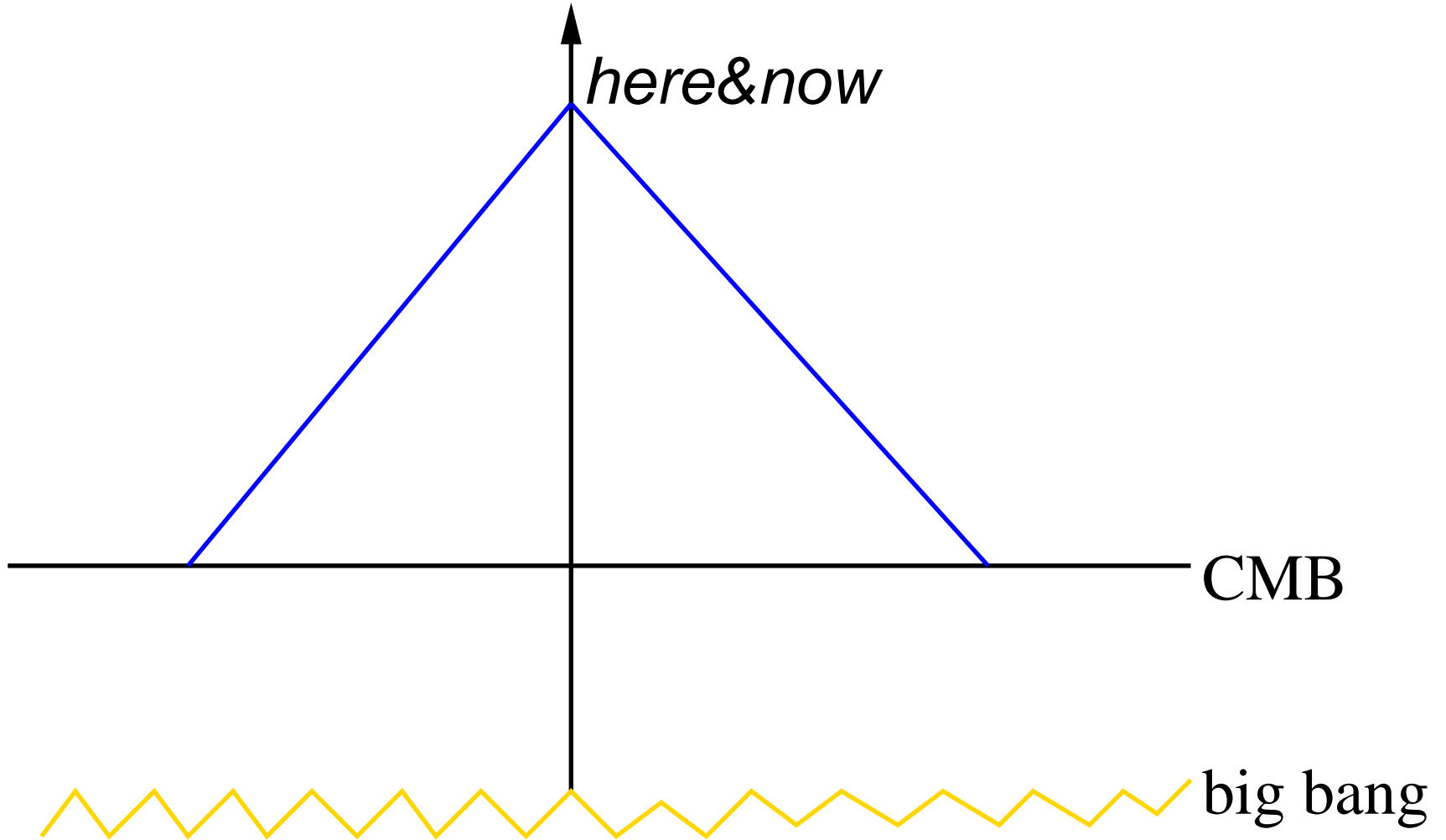
# Our Universe



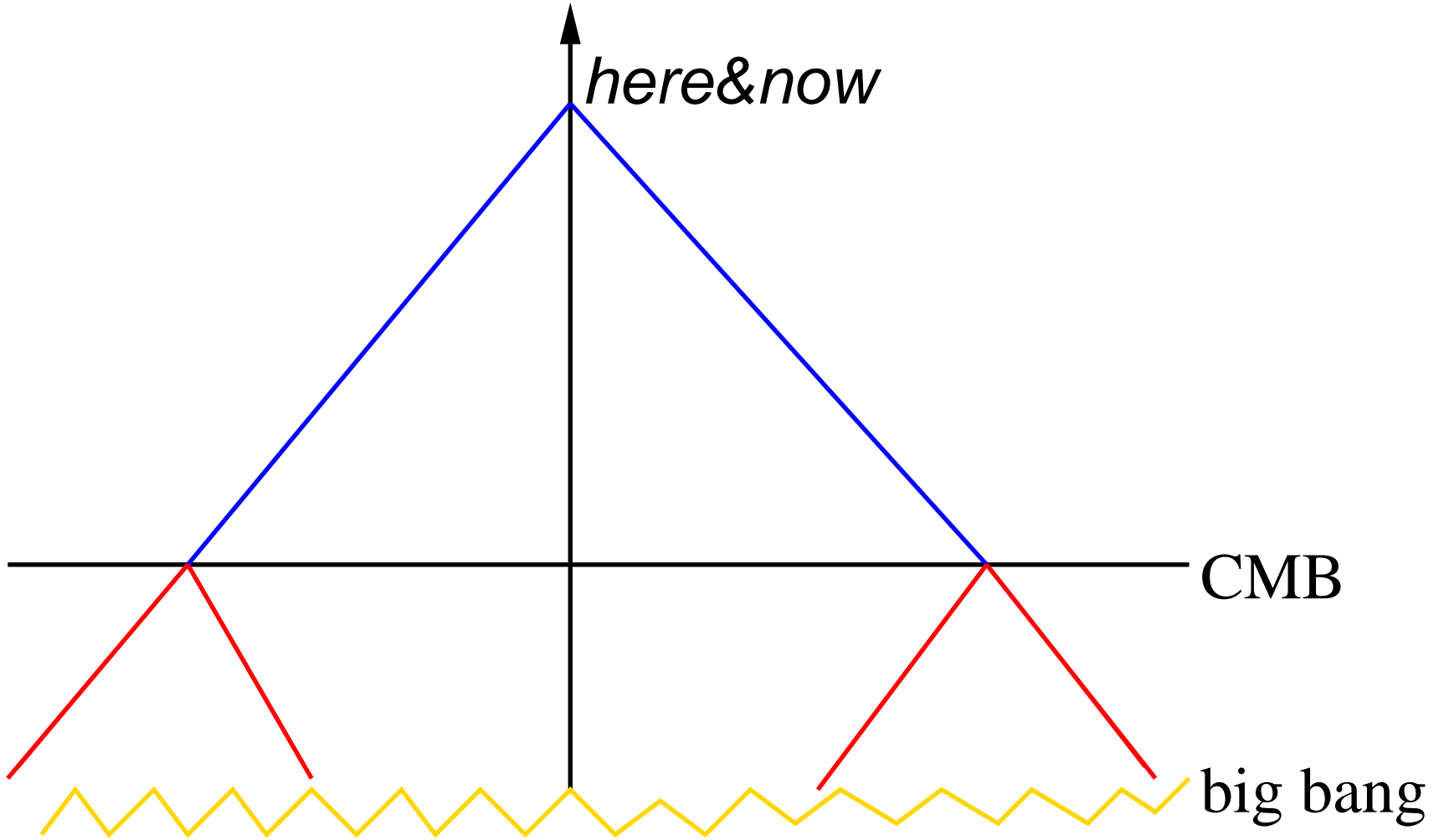
# CMB



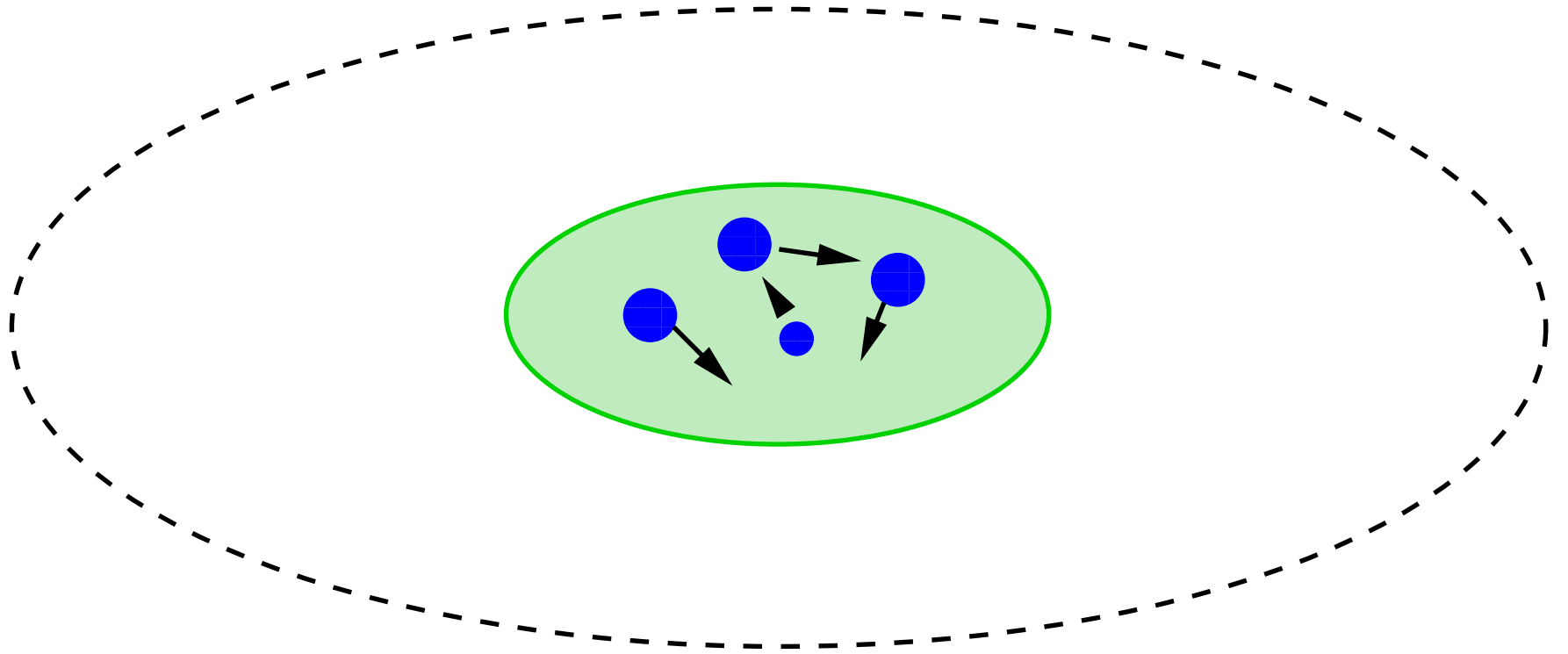
# CMB



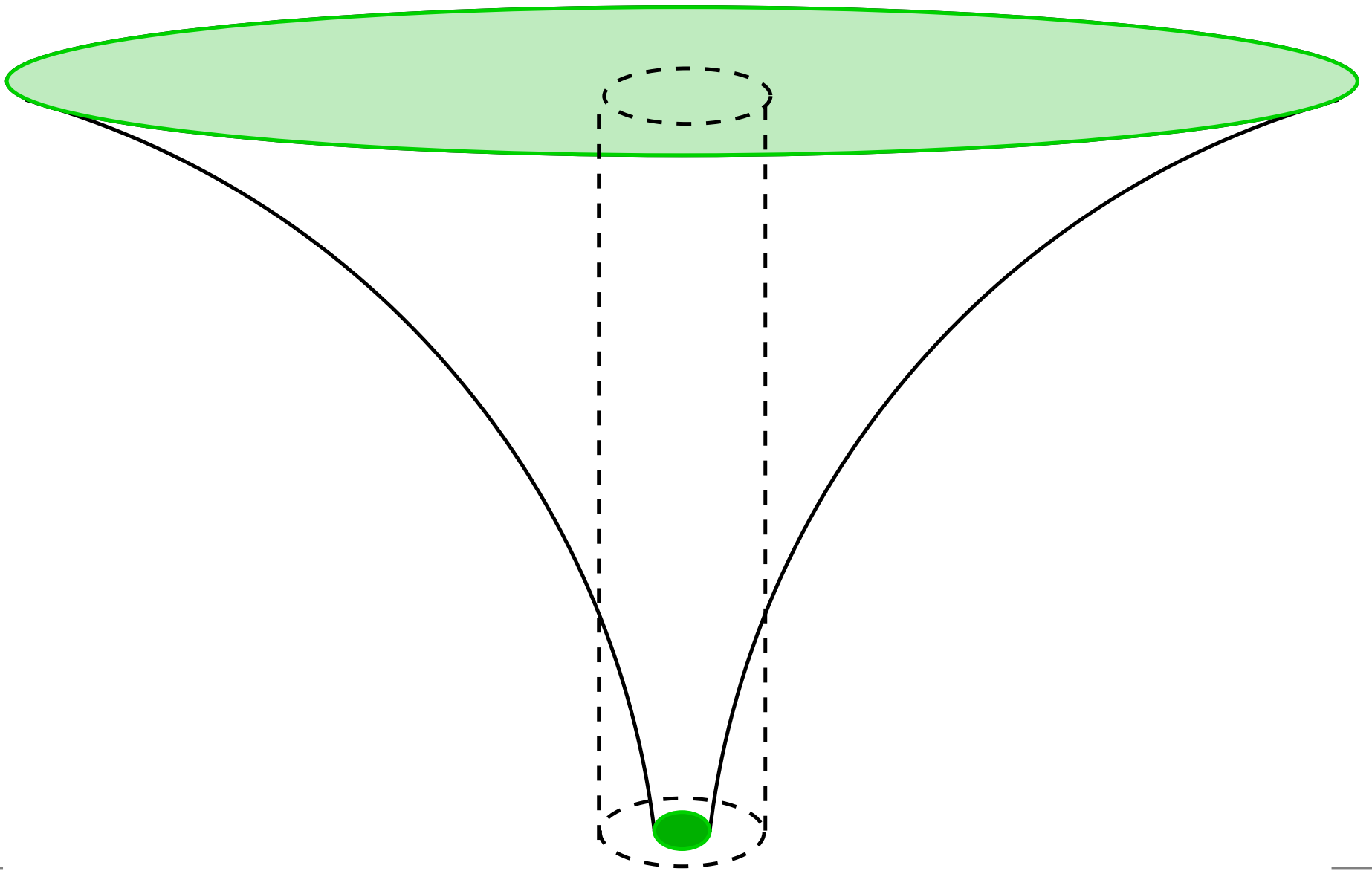
# CMB



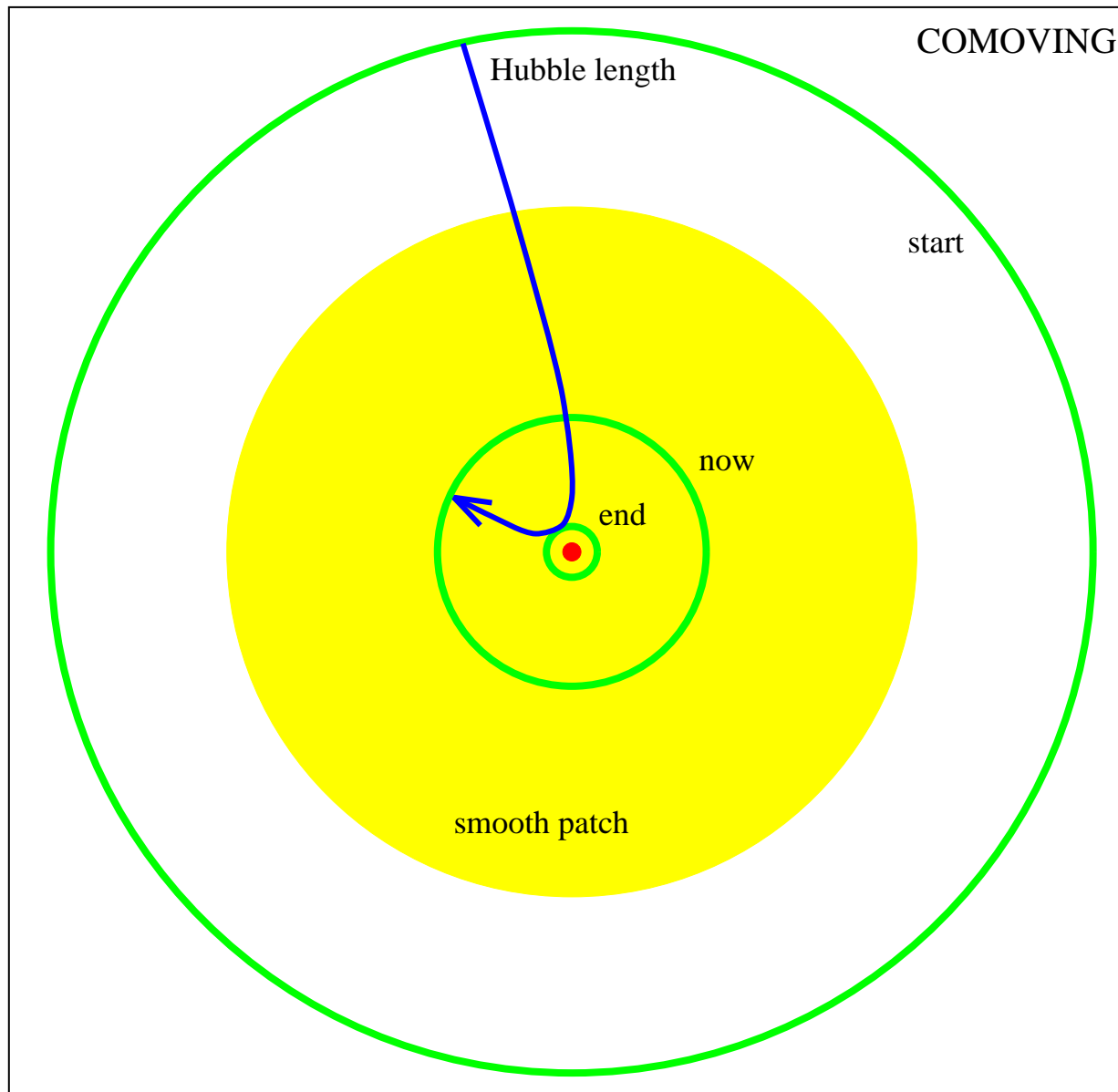
# Inflation



# Inflation

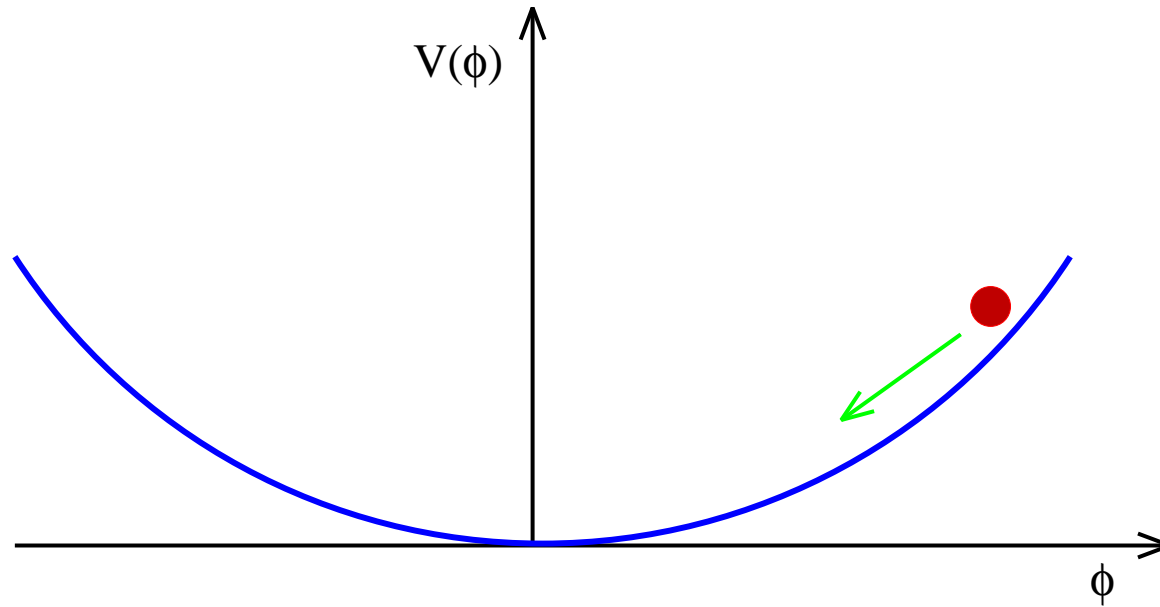


# Inflation



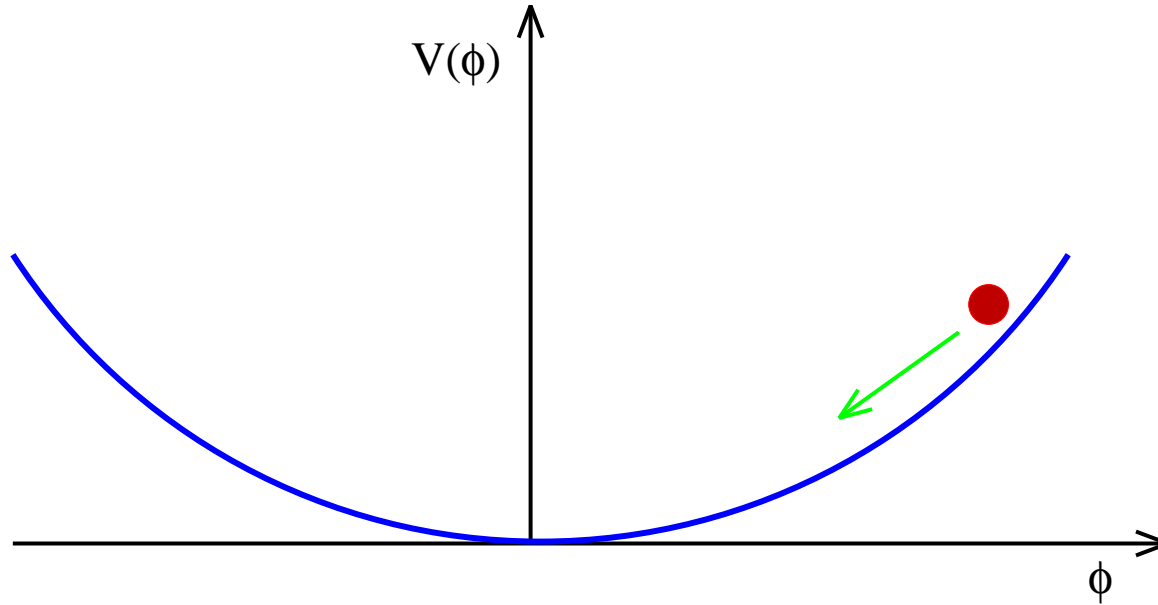


# Scalar field dynamics



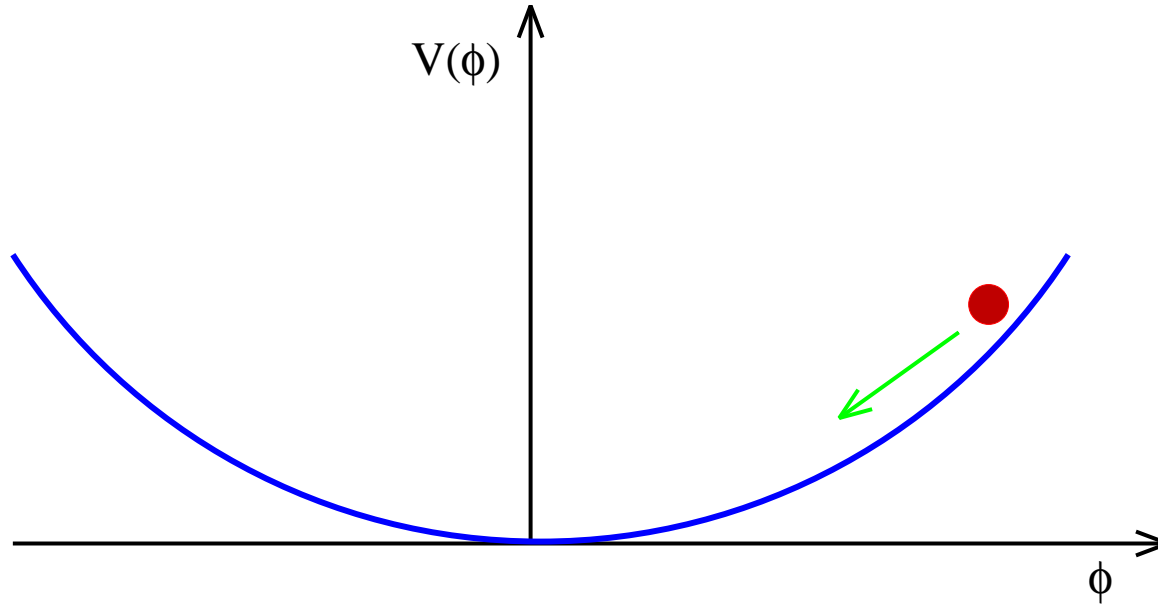
$$\text{EFE} \quad \& \quad \phi^{;\alpha}{}_{;\alpha} + \frac{\partial V}{\partial \phi} = 0$$

# Scalar field dynamics



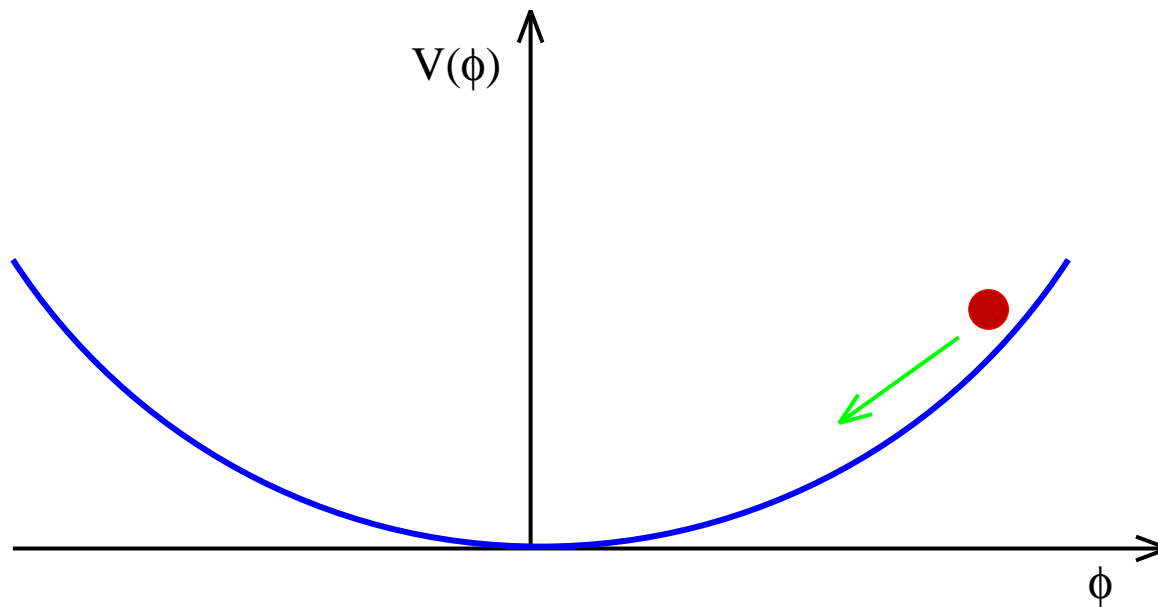
$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi}{3m_{PL}^2} \left[ V(\phi) + \frac{1}{2}\dot{\phi}^2 \right]$$
$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$$

# Scalar field dynamics



$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0$$

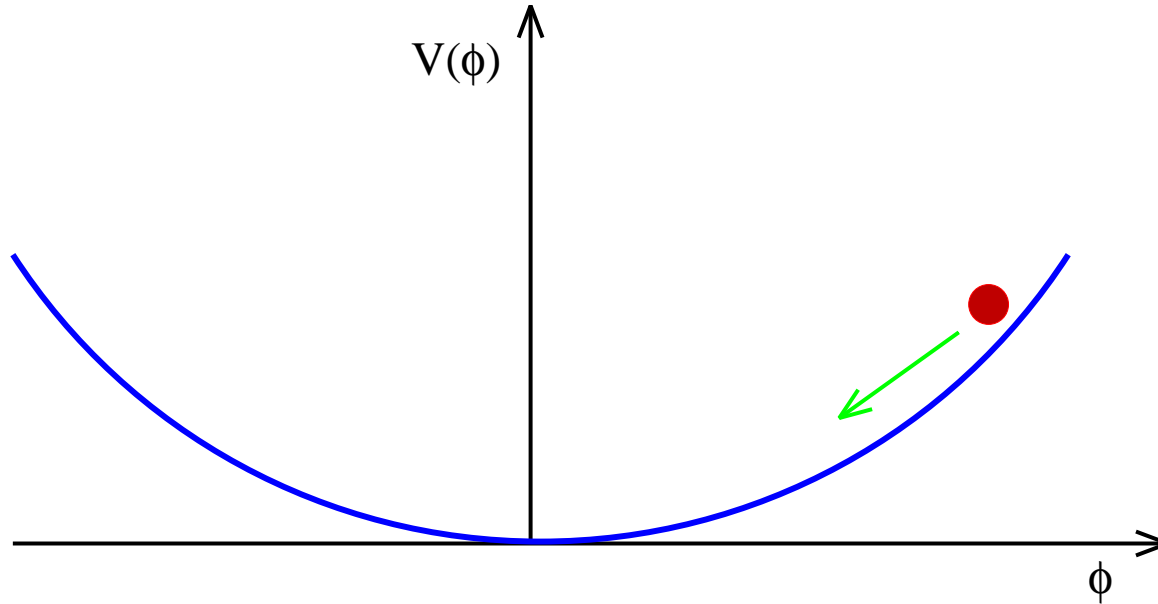
# Scalar field dynamics



EFE

$$\frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu} \phi) + \frac{\partial V}{\partial \phi} = 0$$

# Scalar field dynamics



Stein-Schabes, *Phys. Rev.* **D35**, 2345 (1987)

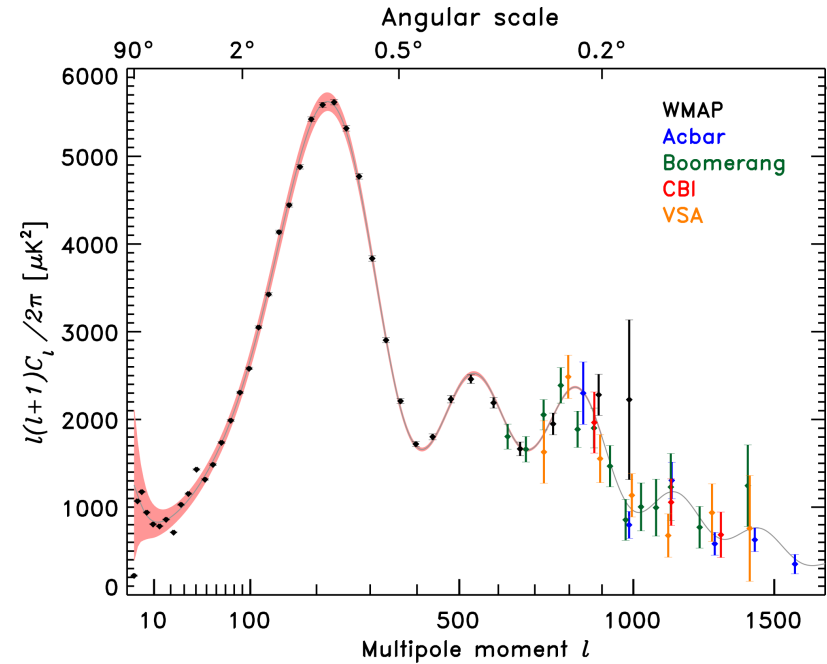
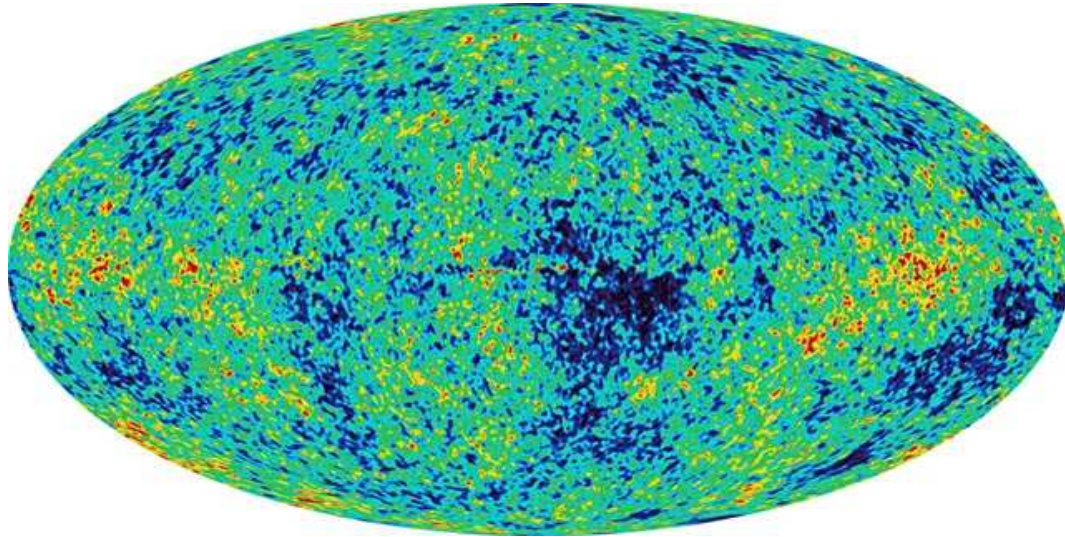
Goldwirth & Piran, *Phys. Rev.* **D40**, 3263 (1989)

Goldwirth & Piran, *Phys. Rev. Lett.* **64**, 2852 (1990)

Goldwirth & Piran, *Phys. Rept* **214**, 223 (1992)

Calzetta & Sakellariadou, *Phys. Rev.* **D45**, 2802 (1992)

# CMB



- does *not* solve the homogenization problem, *but*
- solves the horizon problem
- explains the origin of primordial fluctuations
- solves exotic particle problem
- ...

# Evolution

$$\left\{ \begin{array}{l}
 \dot{\Theta} = -\frac{1}{3} \Theta^2 - \frac{1}{2} (\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda \\
 \dot{\sigma}_{\langle ab \rangle} = -\frac{2}{3} \Theta \sigma_{ab} - \sigma_{c\langle a} \sigma^c{}_{b \rangle} - \omega_{\langle a} \omega_{b \rangle} + D_{\langle a} A_{b \rangle} + A_{\langle a} A_{b \rangle} \\
 \quad - E_{ab} + \frac{1}{2} \pi_{ab} \\
 \dot{\omega}_{\langle a \rangle} = -\frac{2}{3} \Theta \omega_a - \frac{1}{2} \text{curl} A_a + \sigma_{ab} \omega^b
 \end{array} \right.$$
  

$$\left\{ \begin{array}{l}
 \dot{\rho} = -\Theta(\rho + p) - \sigma^{ab} \pi_{ab} \\
 (\rho + p) A_a = -D_a p - D^b \pi_{ab} - \pi_{ab} A^b
 \end{array} \right.$$
  

$$\left\{ \begin{array}{l}
 \dot{E}_{\langle ab \rangle} = -\Theta E_{ab} - \frac{1}{2} (\rho + p) \sigma_{ab} + \text{curl} H_{ab} - \frac{1}{2} \dot{\pi}_{ab} - \frac{1}{6} \Theta \pi_{ab} \\
 \quad + 3\sigma_{\langle a}{}^c (E_{b \rangle c} - \frac{1}{6} \pi_{b \rangle c}) + \varepsilon_{cd\langle a} [2A^c H_{b \rangle}{}^d - \omega^c (E_{b \rangle}{}^d + \frac{1}{2} \pi_{b \rangle}{}^d)] \\
 \dot{H}_{\langle ab \rangle} = -\Theta H_{ab} - \text{curl} E_{ab} + \frac{1}{2} \text{curl} \pi_{ab} + 3\sigma_{\langle a}{}^c H_{b \rangle c} - \\
 \quad - \varepsilon_{cd\langle a} (2A^c E_{b \rangle}{}^d + \omega^c H_{b \rangle}{}^d)
 \end{array} \right.$$

# Evolution

$$2\nabla_{[a}\nabla_{b]}u_c = R_{abcd}u^d$$

$$\left\{ \begin{array}{l} \dot{\rho} = -\Theta(\rho + p) - \sigma^{ab}\pi_{ab} \\ (\rho + p)A_a = -D_a p - D^b\pi_{ab} - \pi_{ab}A^b \end{array} \right.$$

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# Evolution

$$\left\{ \begin{array}{l} \dot{\Theta} = -\frac{1}{3} \Theta^2 - \frac{1}{2} (\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda \\ \dot{\sigma}_{\langle ab \rangle} = -\frac{2}{3} \Theta \sigma_{ab} - \sigma_{c\langle a} \sigma^c_{b \rangle} - \omega_{\langle a} \omega_{b \rangle} + D_{\langle a} A_{b \rangle} + A_{\langle a} A_{b \rangle} \\ \quad - E_{ab} + \frac{1}{2} \pi_{ab} \\ \dot{\omega}_{\langle a \rangle} = -\frac{2}{3} \Theta \omega_a - \frac{1}{2} \text{curl} A_a + \sigma_{ab} \omega^b \end{array} \right.$$

$$\nabla^b T_{ab} = 0$$

$$\left\{ \begin{array}{l} \dot{E}_{\langle ab \rangle} = -\Theta E_{ab} - \frac{1}{2} (\rho + p) \sigma_{ab} + \text{curl} H_{ab} - \frac{1}{2} \dot{\pi}_{ab} - \frac{1}{6} \Theta \pi_{ab} \\ \quad + 3\sigma_{\langle a}{}^c (E_{b \rangle c} - \frac{1}{6} \pi_{b \rangle c}) + \varepsilon_{cd\langle a} [2A^c H_{b \rangle}{}^d - \omega^c (E_{b \rangle}{}^d + \frac{1}{2} \pi_{b \rangle}{}^d)] \\ \dot{H}_{\langle ab \rangle} = -\Theta H_{ab} - \text{curl} E_{ab} + \frac{1}{2} \text{curl} \pi_{ab} + 3\sigma_{\langle a}{}^c H_{b \rangle c} - \\ \quad - \varepsilon_{cd\langle a} (2A^c E_{b \rangle}{}^d + \omega^c H_{b \rangle}{}^d) \end{array} \right.$$

# Evolution

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$$\nabla^d C_{abcd} = \nabla_{[b} R_{a]c} + \frac{1}{6} g_{c[b} \nabla_{a]} R$$

# Evolution

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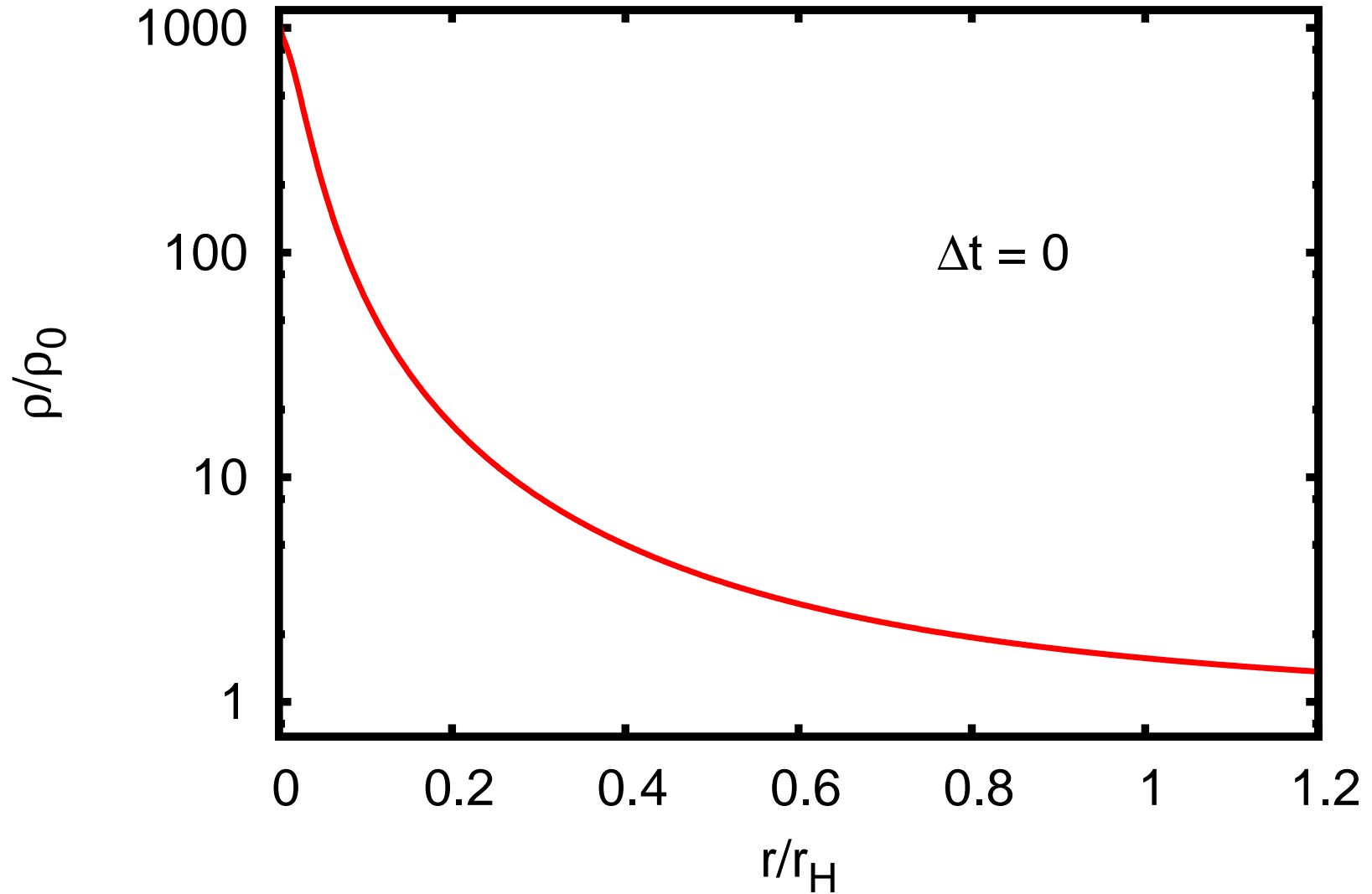
# Evolution

$$\begin{cases} \dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}(\rho + 3p) - 2\sigma^2 + D^a A_a + A_a A^a \\ \dot{\sigma} = \mathcal{F} - \frac{1}{\sqrt{3}}\sigma^2 - \frac{2}{3}\Theta\sigma - \frac{1}{2\sqrt{3}}(\rho - \bar{\rho}) \end{cases}$$

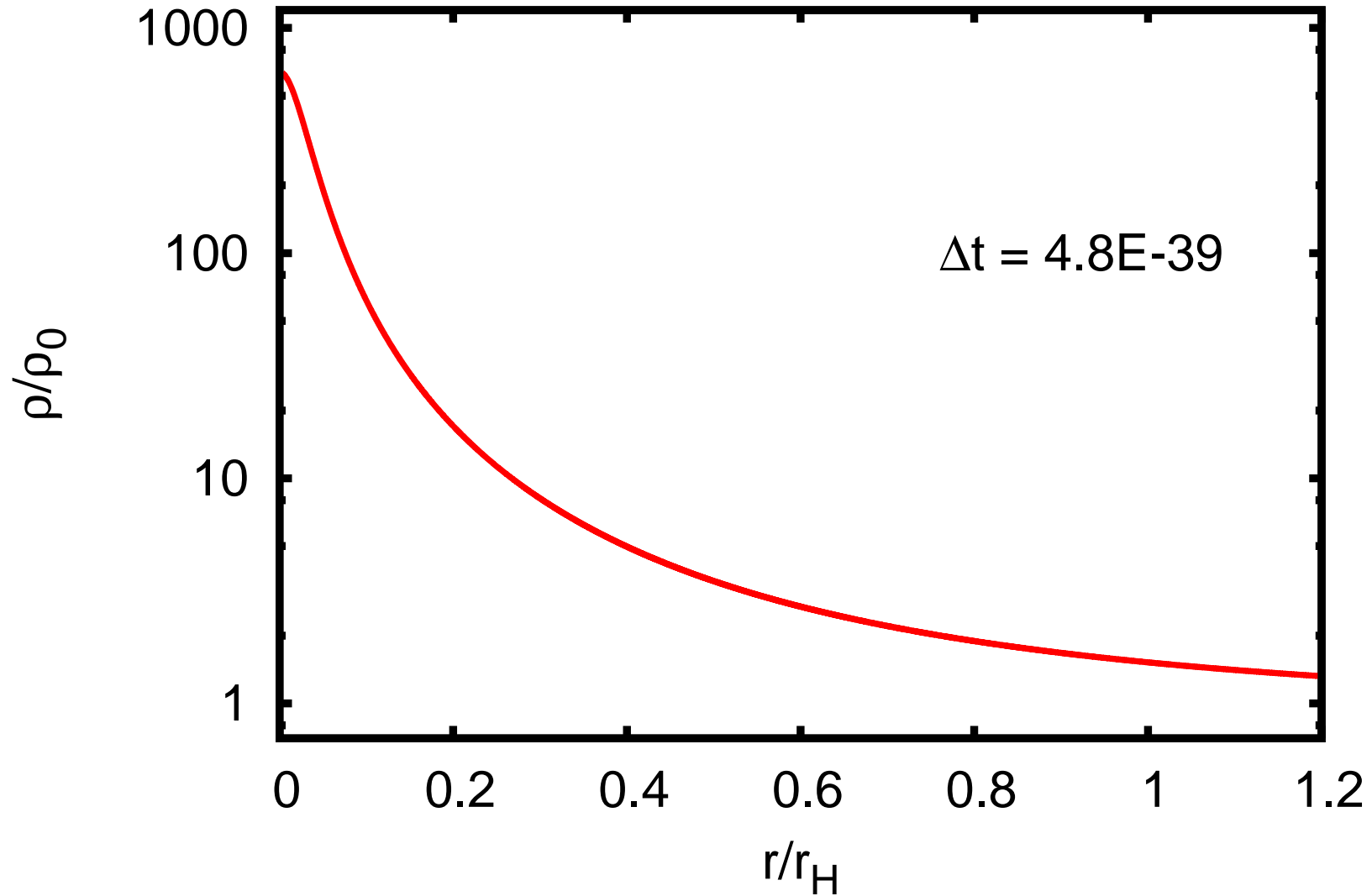
$$\begin{cases} \dot{\rho} = -\Theta(\rho + p) + \lambda\sigma^2 \\ (\rho + p)A = -p' + \frac{2}{\sqrt{3}}(\lambda\sigma)' + 2\sqrt{3}\lambda\sigma R'/R \end{cases}$$

$$\begin{cases} E = \frac{1}{4}(\rho - \bar{\rho}) \\ H = 0 \end{cases}$$

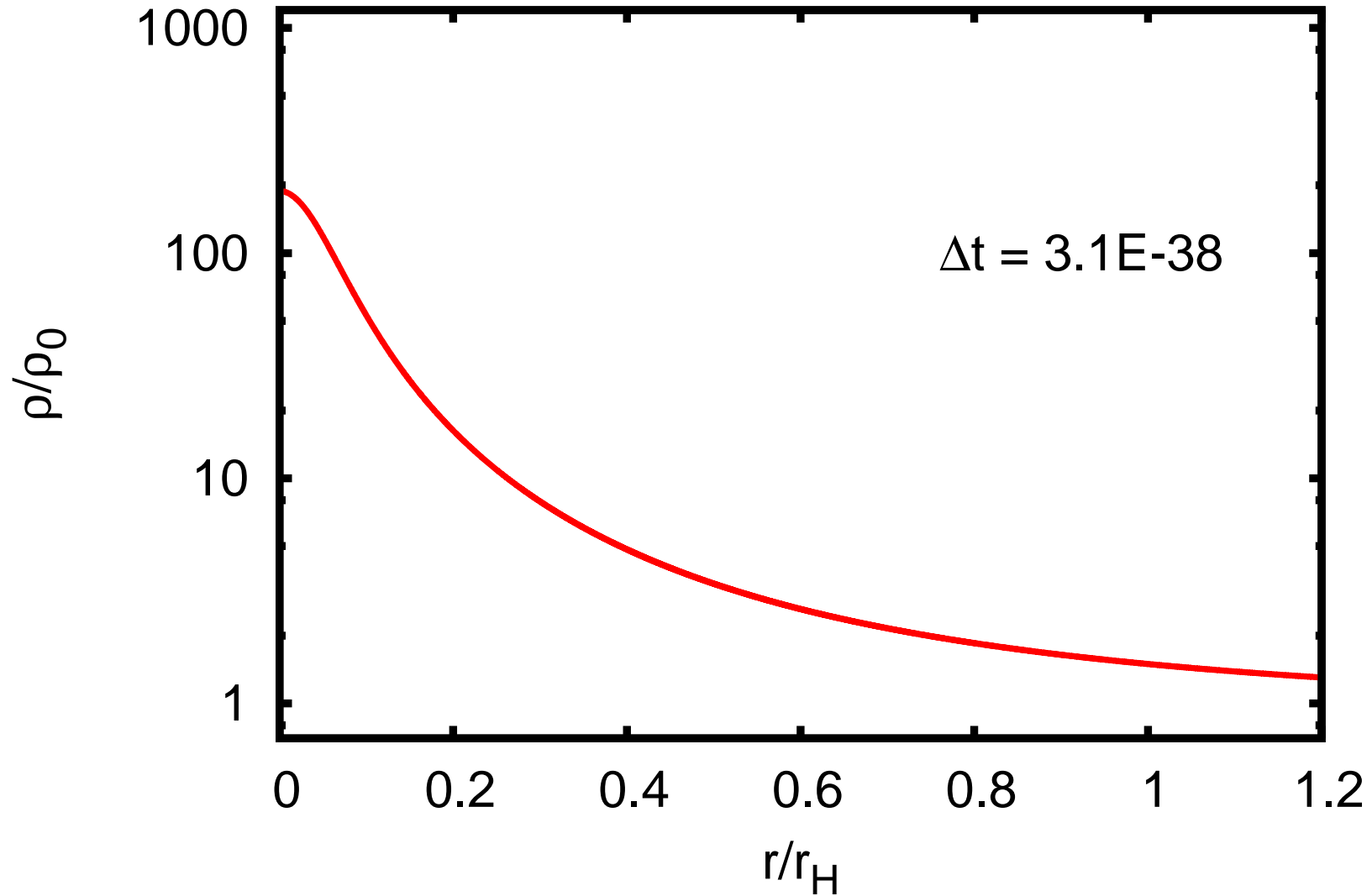
$$t_i = 10^{-34} s \quad T = 10^{14} GeV$$



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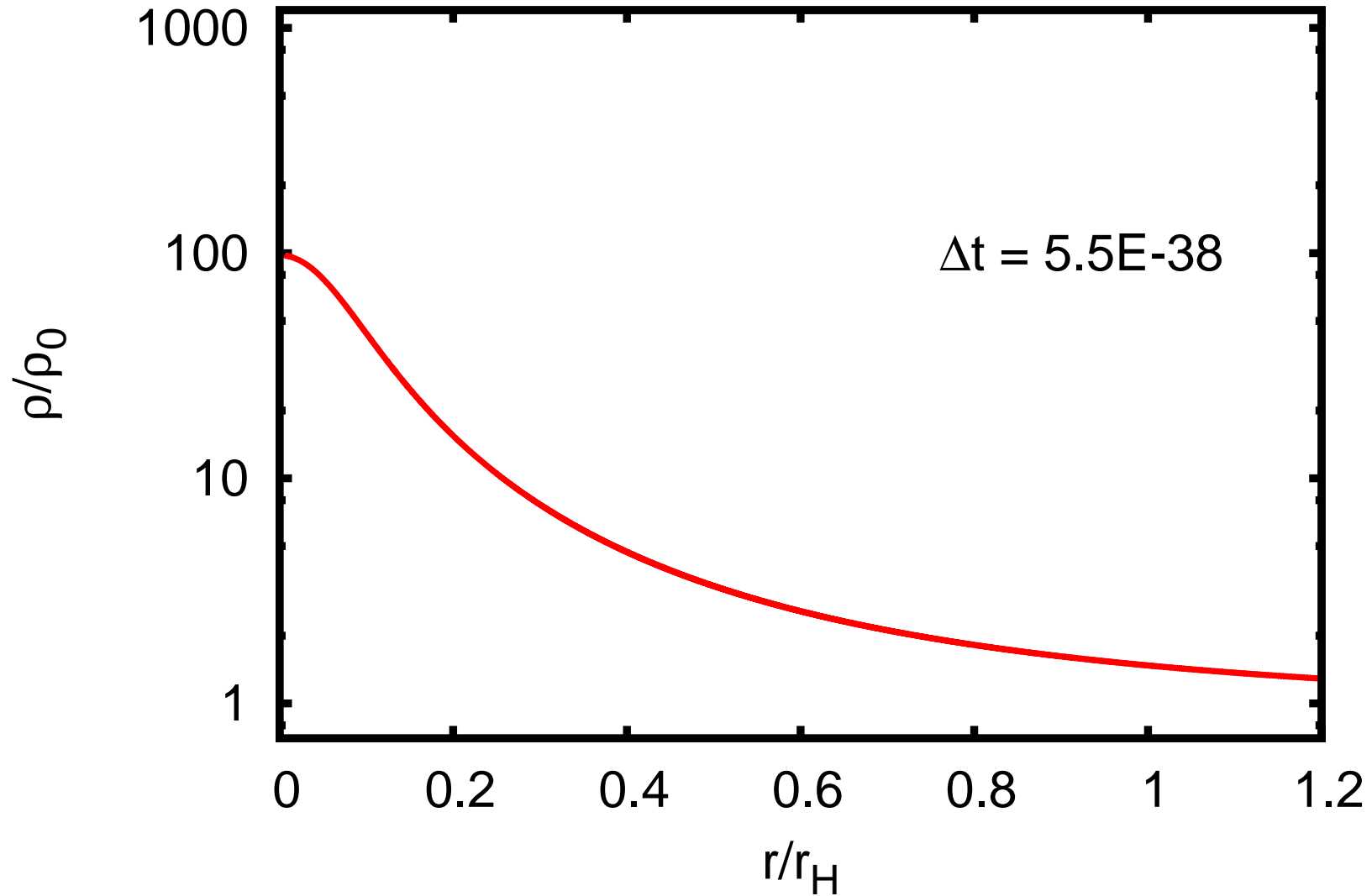


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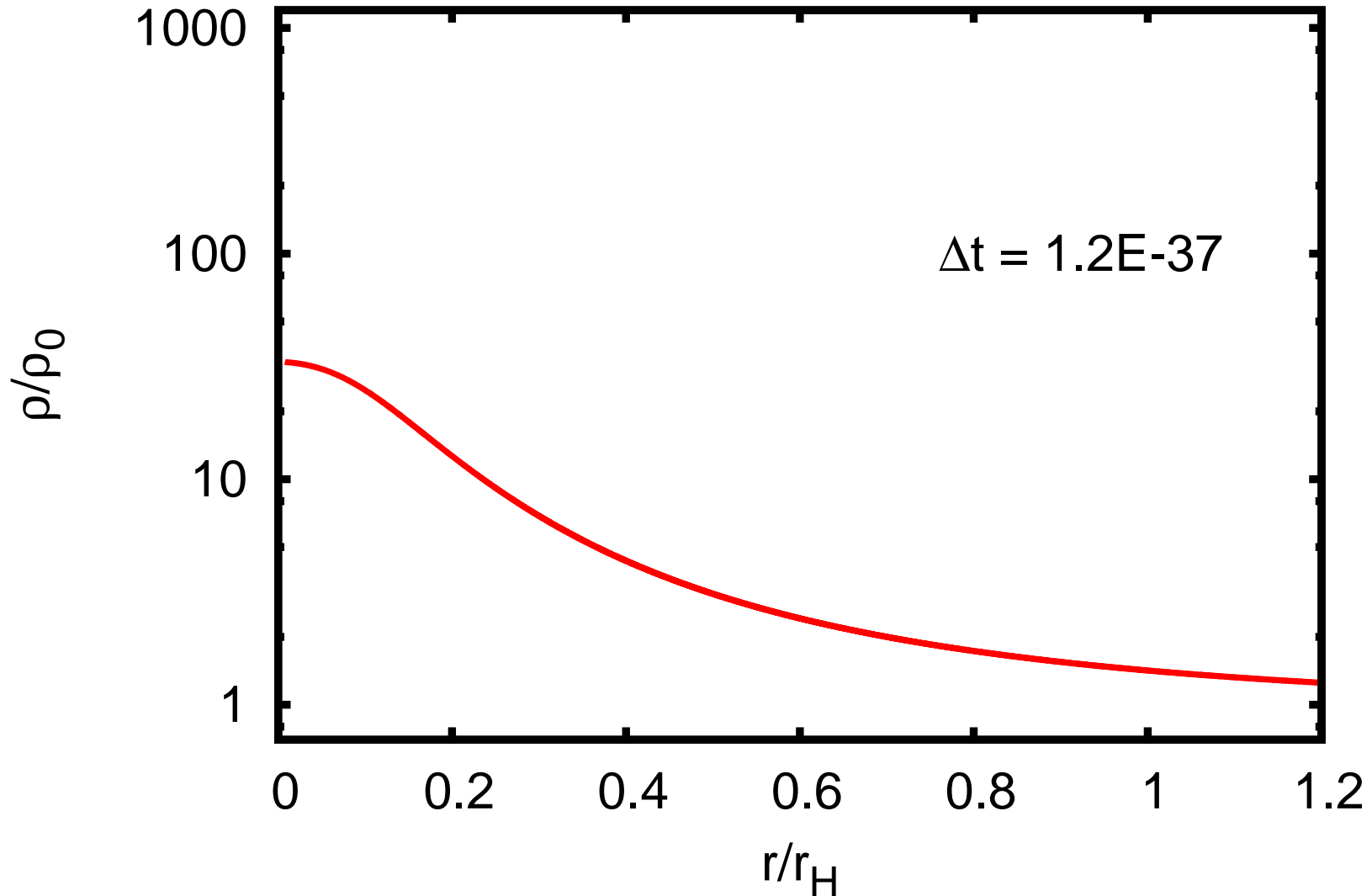




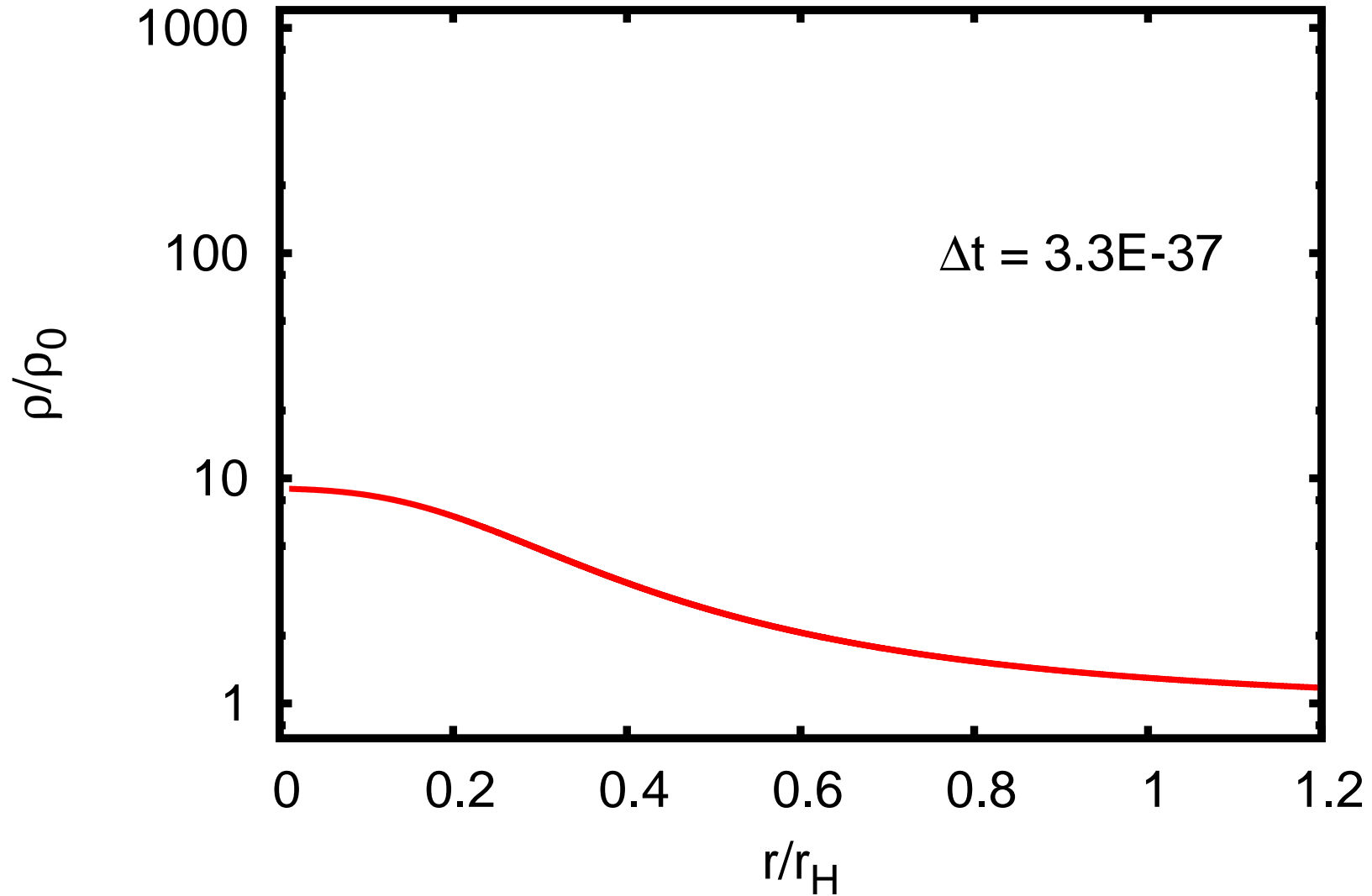
$$t_i = 10^{-34} s \quad T = 10^{14} GeV$$



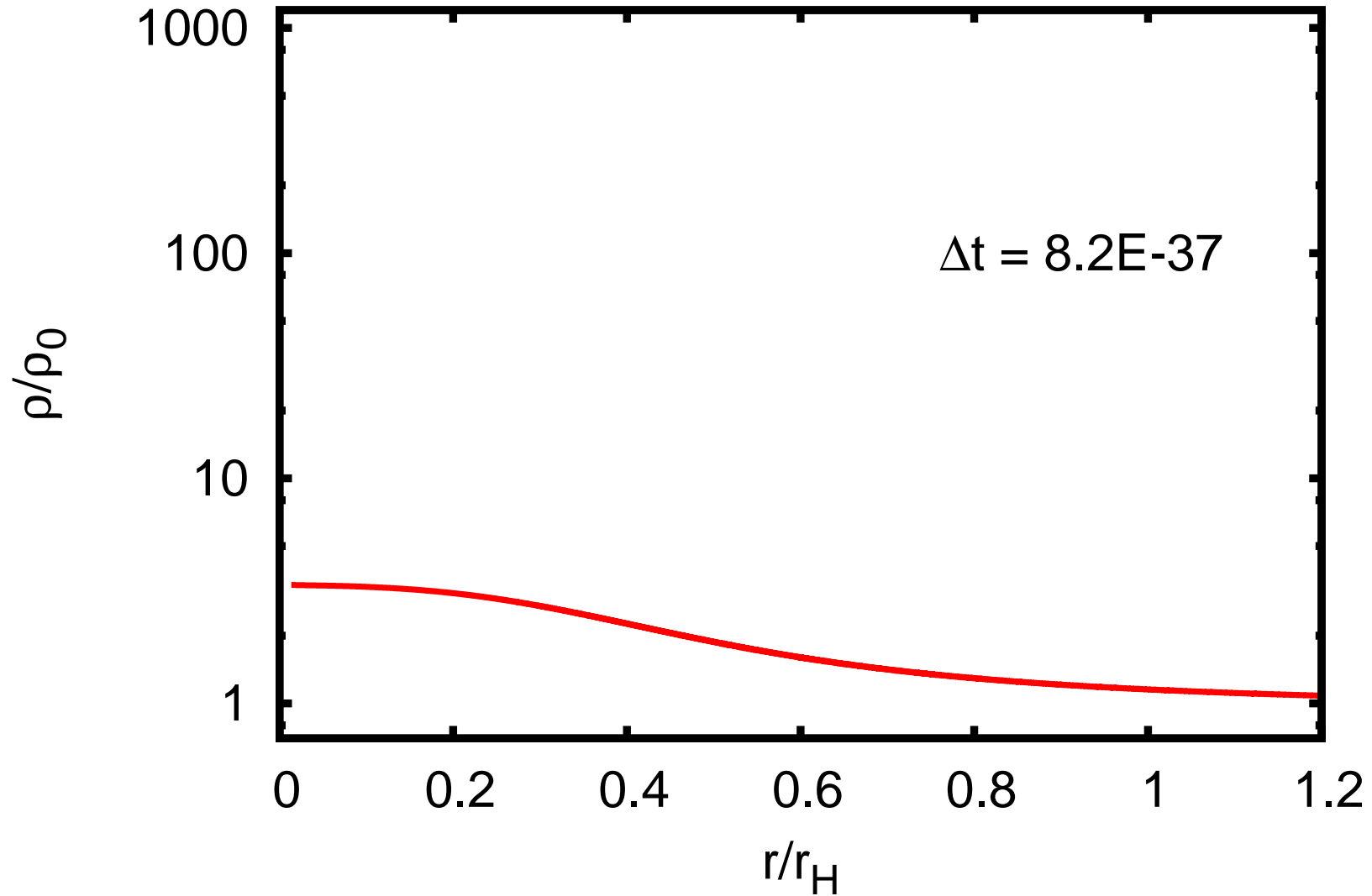
$$t_i = 10^{-34} s \quad T = 10^{14} GeV$$



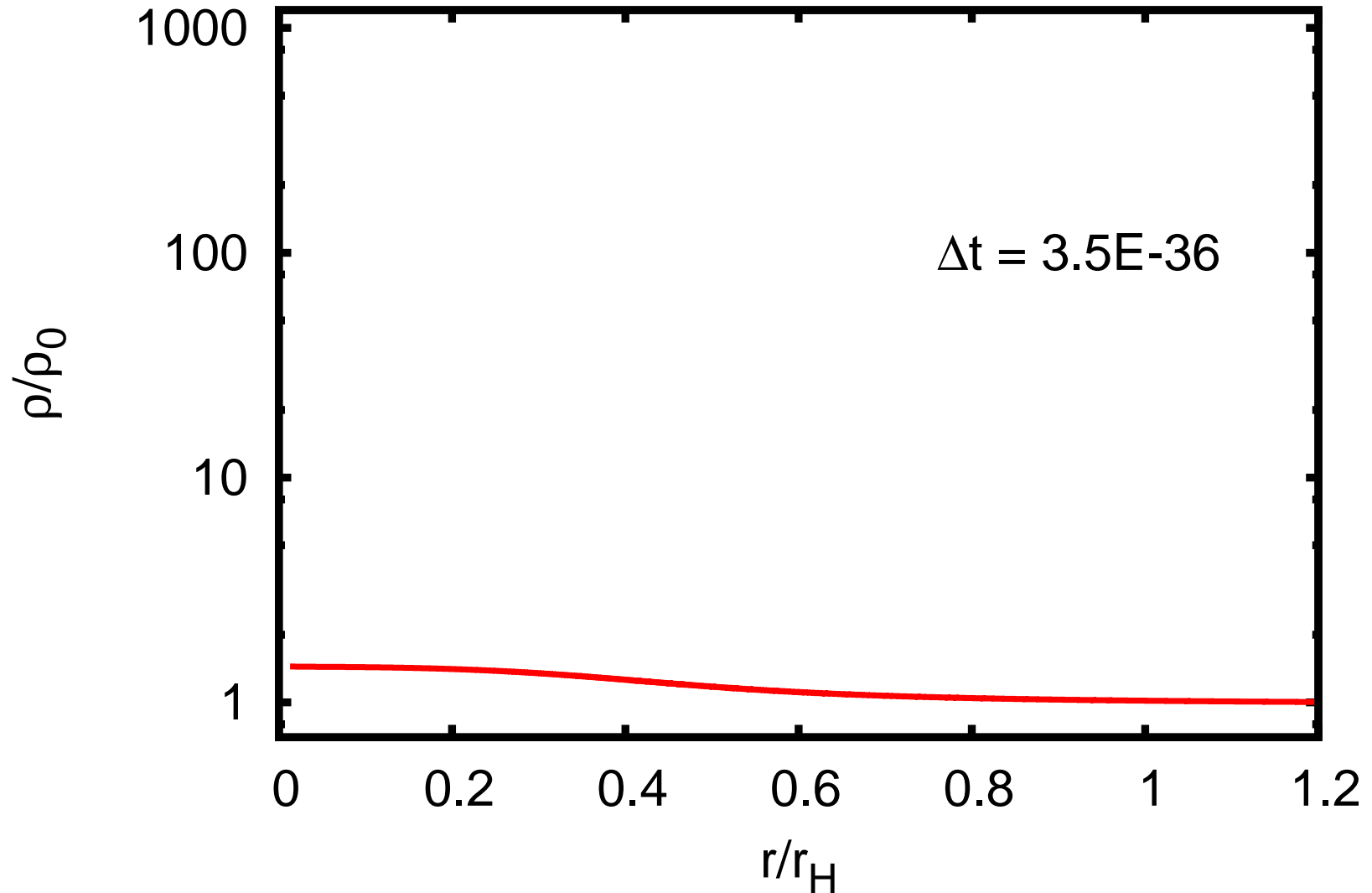
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# Spontaneous homogenization

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{-k}{R^2} + \frac{2M}{R^3}$$

$$\frac{|k|}{R^2} \ll \frac{2M}{R^3}$$

$$\frac{|k'|}{R'R} \ll \frac{2M}{R^3}$$

$$\frac{|\dot{k}|}{\dot{R}R} \ll \frac{2M}{R^3}$$

# Spontaneous homogenization

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{-k}{R^2} + \frac{2M}{R^3}$$

$$\frac{|k|}{R^2} \ll \frac{2M}{R^3} \quad \frac{|k'|}{R'R} \ll \frac{2M}{R^3} \quad \frac{|\dot{k}|}{\dot{R}R} \ll \frac{2M}{R^3}$$

$$K^2 \gg {}^3\mathcal{R} \quad \Leftrightarrow \quad {}^3\mathcal{R} \ll {}^4\mathcal{R}$$

# Evolution

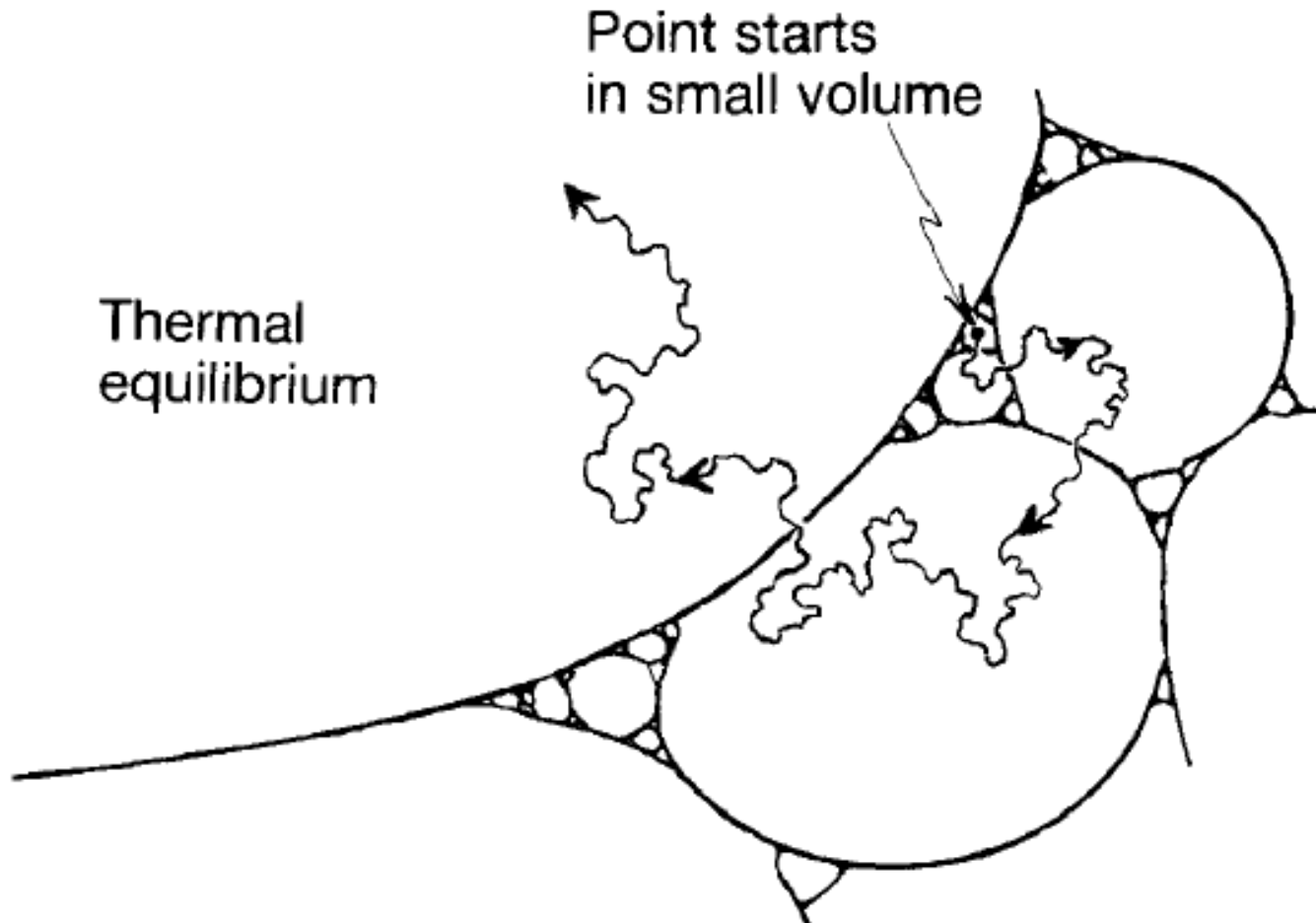
$$\left\{ \begin{array}{l} \dot{\Theta} = -\frac{1}{3} \Theta^2 - \frac{1}{2} (\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda \\ \dot{\sigma}_{\langle ab \rangle} = -\frac{2}{3} \Theta \sigma_{ab} - \sigma_{c\langle a} \sigma^c{}_{b \rangle} - \omega_{\langle a} \omega_{b \rangle} + D_{\langle a} A_{b \rangle} + A_{\langle a} A_{b \rangle} \\ \quad - E_{ab} - \frac{1}{2} \lambda \sigma_{ab} \\ \dot{\omega}_{\langle a \rangle} = -\frac{2}{3} \Theta \omega_a - \frac{1}{2} \text{curl} A_a + \sigma_{ab} \omega^b \end{array} \right.$$

$$\left\{ \begin{array}{l} \dot{\rho} = -\Theta(\rho + p) + \lambda \sigma_{ab} \sigma^{ab} \\ (\rho + p) A_a = -D_a p + D^b \lambda \sigma_{ab} + \lambda \sigma_{ab} A^b \end{array} \right.$$

$$\left\{ \begin{array}{l} \dot{E}_{\langle ab \rangle} = -\Theta E_{ab} - \frac{1}{2} (\rho + p) \sigma_{ab} + \text{curl} H_{ab} + \frac{1}{2} \lambda \dot{\sigma}_{ab} + \frac{1}{6} \Theta \lambda \sigma_{ab} \\ \quad + 3\sigma_{\langle a}{}^c (E_{b \rangle c} + \frac{1}{6} \lambda \sigma_{b \rangle c}) + \varepsilon_{cd\langle a} [2A^c H_{b \rangle}{}^d - \omega^c (E_{b \rangle}{}^d - \frac{1}{2} \lambda \sigma_{b \rangle}{}^c] \\ \dot{H}_{\langle ab \rangle} = -\Theta H_{ab} - \text{curl} E_{ab} - \frac{1}{2} \lambda \text{curl} \sigma_{ab} + 3\sigma_{\langle a}{}^c H_{b \rangle c} - \\ \quad - \varepsilon_{cd\langle a} (2A^c E_{b \rangle}{}^d + \omega^c H_{b \rangle}{}^d) \end{array} \right.$$

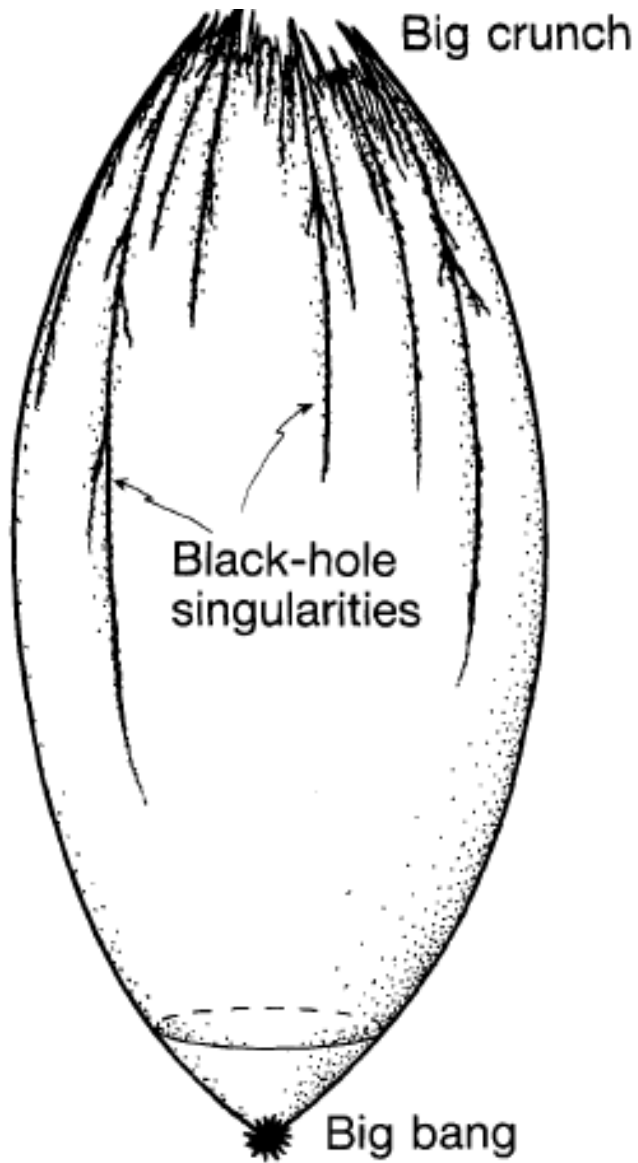


# Weyl curvature hypothesis



Penrose, *The Emperor's New Mind*, (Oxford University Press, 1989)

# Weyl curvature hypothesis



$$S_g = \frac{C_{abcd}C^{abcd}}{R_{ef}R^{ef}}$$

Penrose, *The Emperor's New Mind*, (Oxford University Press, 1989)

# Evolution

$$\left\{ \begin{array}{l} \dot{\Theta} = -\frac{1}{3} \Theta^2 - \frac{1}{2} (\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda \\ \dot{\sigma}_{\langle ab \rangle} = -\frac{2}{3} \Theta \sigma_{ab} - \sigma_{c\langle a} \sigma^c{}_{b \rangle} - \omega_{\langle a} \omega_{b \rangle} + D_{\langle a} A_{b \rangle} + A_{\langle a} A_{b \rangle} \\ \quad - E_{ab} - \frac{1}{2} \lambda \sigma_{ab} \\ \dot{\omega}_{\langle a \rangle} = -\frac{2}{3} \Theta \omega_a - \frac{1}{2} \text{curl} A_a + \sigma_{ab} \omega^b \end{array} \right.$$

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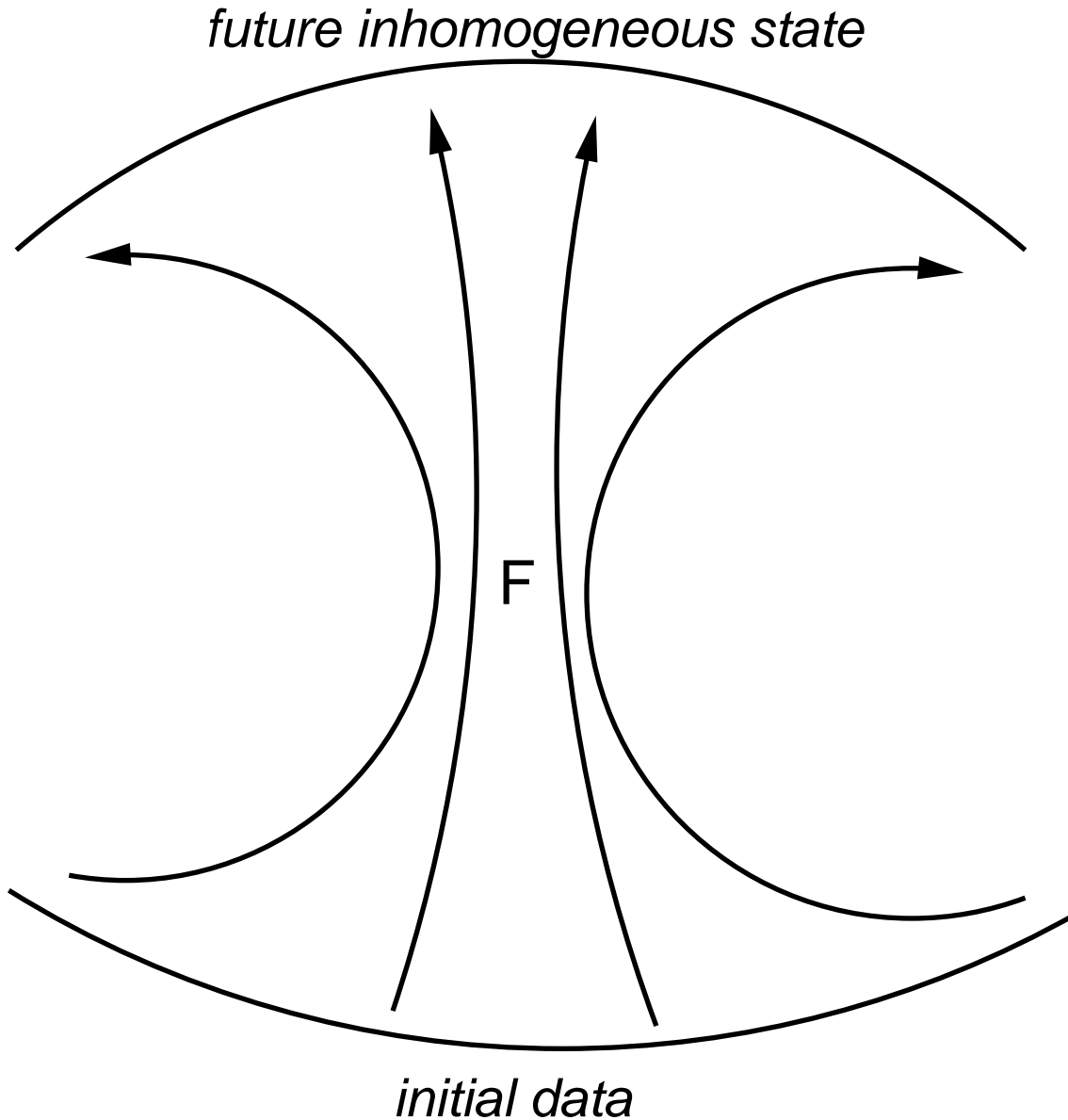
# Evolution

$$\left\{ \begin{array}{l} \dot{\Theta} = -\frac{1}{3} \Theta^2 - \frac{1}{2} (\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda \\ \dot{\sigma}_{\langle ab \rangle} = -\frac{2}{3} \Theta \sigma_{ab} - \sigma_{c\langle a} \sigma^c_{b \rangle} - \omega_{\langle a} \omega_{b \rangle} + D_{\langle a} A_{b \rangle} + A_{\langle a} A_{b \rangle} \\ \quad - E_{ab} - \frac{1}{2} \lambda \sigma_{ab} \\ \dot{\omega}_{\langle a \rangle} = -\frac{2}{3} \Theta \omega_a - \frac{1}{2} \text{curl} A_a + \sigma_{ab} \omega^b \end{array} \right.$$

$$\left\{ \begin{array}{l} \dot{\rho} = -\Theta(\rho + p) + \lambda \sigma_{ab} \sigma^{ab} \\ (\rho + p) A_a = -D_a p + D^b \lambda \sigma_{ab} + \lambda \sigma_{ab} A^b \end{array} \right.$$

$$\left\{ \begin{array}{l} \dot{E}_{\langle ab \rangle} = -\Theta E_{ab} - \frac{1}{2} (\rho + p) \sigma_{ab} + \text{curl} H_{ab} + \frac{1}{2} \lambda \dot{\sigma}_{ab} + \frac{1}{6} \Theta \lambda \sigma_{ab} \\ \quad + 3\sigma_{\langle a}{}^c (E_{b \rangle c} + \frac{1}{6} \lambda \sigma_{b \rangle c}) + \varepsilon_{cd\langle a} [2A^c H_{b \rangle}{}^d - \omega^c (E_{b \rangle}{}^d - \frac{1}{2} \lambda \sigma_{b \rangle}{}^c] \\ \dot{H}_{\langle ab \rangle} = -\Theta H_{ab} - \text{curl} E_{ab} - \frac{1}{2} \lambda \mathbf{curl} \sigma_{ab} + 3\sigma_{\langle a}{}^c H_{b \rangle c} - \\ \quad - \varepsilon_{cd\langle a} (2A^c E_{b \rangle}{}^d + \omega^c H_{b \rangle}{}^d) \end{array} \right.$$

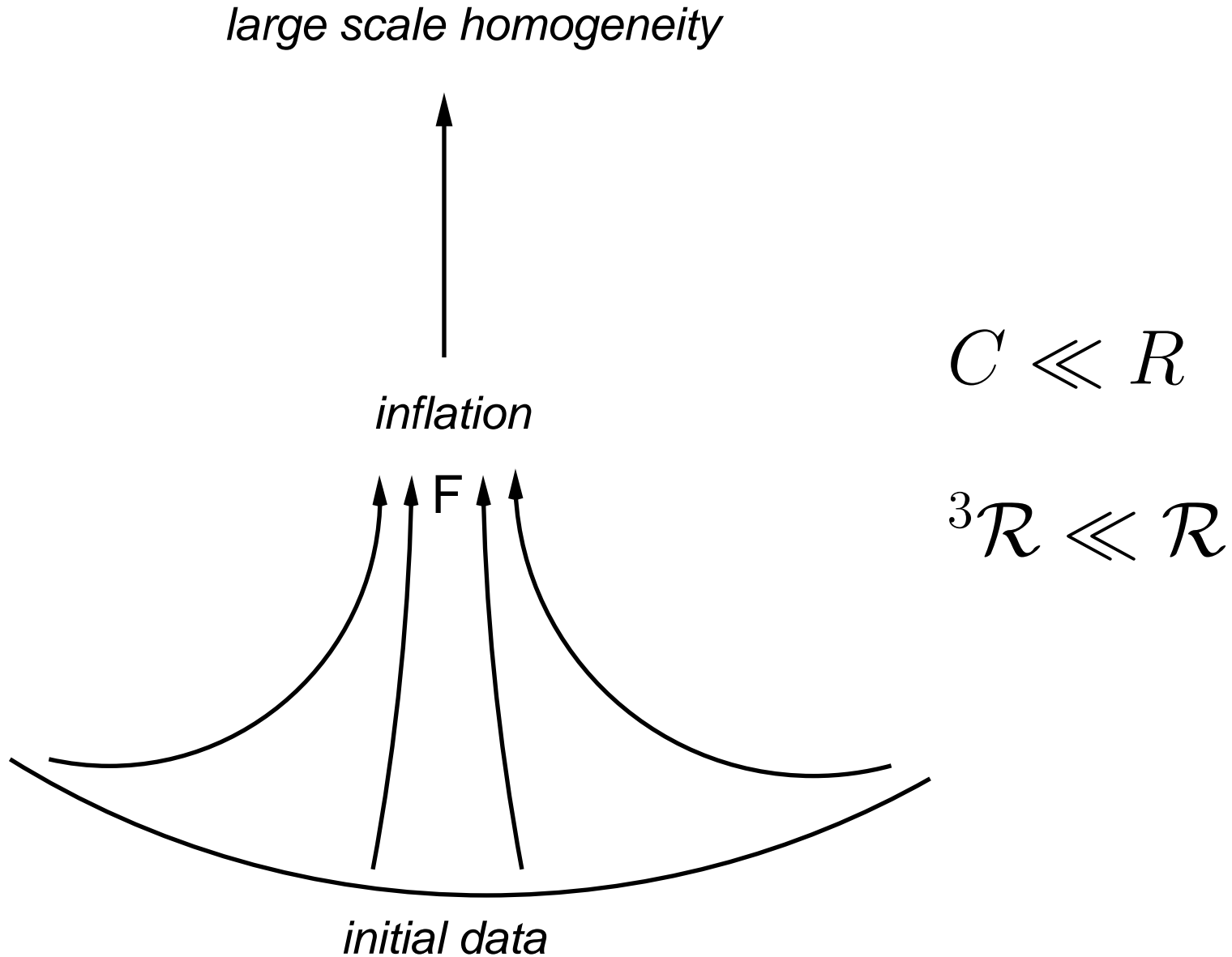
# Intermediate homogenization



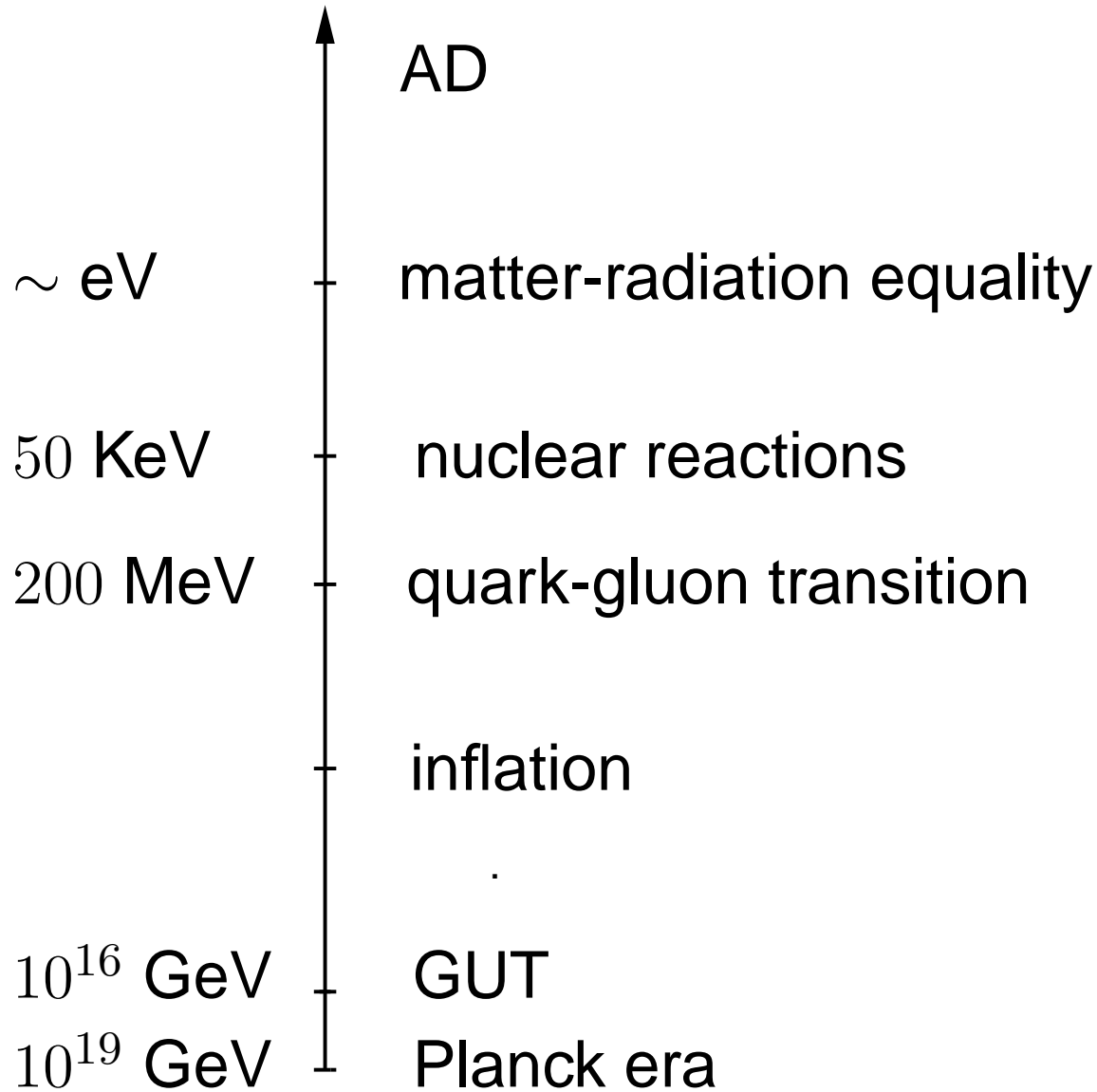
$$C \ll \mathcal{R}$$

$${}^3\mathcal{R} \ll \mathcal{R}$$

# Intermediate homogenization



# Summary



# Summary

