Re-evaluation of the muon $g-2$ and $\alpha(M_Z^2)$

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In collaboration with:
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Basics

- Magnetic moment $\boldsymbol{\mu}$ generated by particle spin $\boldsymbol{S} = \hbar \boldsymbol{s}$

$$\boldsymbol{\mu} = g \frac{e\hbar}{2mc} \boldsymbol{s}$$

- g, gyromagnetic factor; m, particle mass
- Anomalous $\longrightarrow$ deviation of $g = 2$ predicted from Dirac theory.

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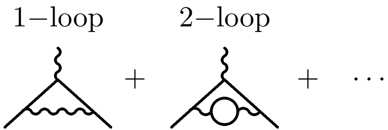

Anomalous magnetic moment (of the muon)

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Motivation

- Strong test of the Standard Model (SM).

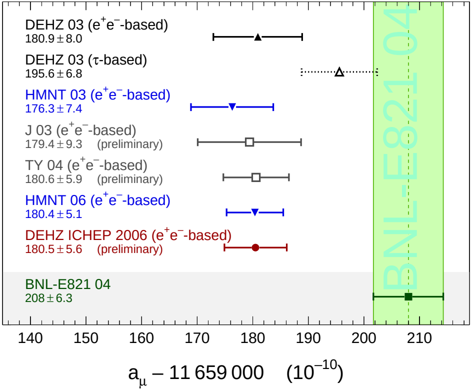

Fig. 1: Discrepancy ($\sim 3\sigma$) between theory and experiment (graph: hep-ph/0701163).

- Beyond SM physics?

$$a_\mu^{\text{BSM?}} = a_\mu^{\text{exp}} - a_\mu^{\text{th}} \sim (29 \pm 9) \times 10^{-10}$$

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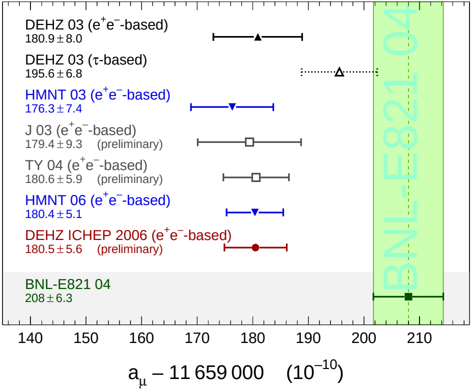

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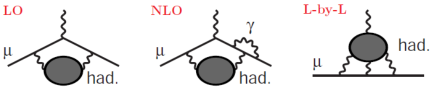
Theoretical contributions

- Contributions to $a_\mu^{\rm th}$

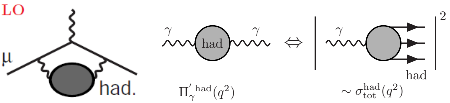
Sectors	Value (10^{-10})	Error	Reference
$a_\mu^{\rm QED}$	11 658 471.81	0.02	Kinoshita et al.
$a_\mu^{\rm had,LO}$	689.4	4.6	Hagiwara et al. (HMNT06)
$a_\mu^{\rm had,NLO}$	-9.79	0.09	Hagiwara et al. (HMNT06)
$a_\mu^{\rm had,LbL}$	10.5	2.6	Prades et al.
$a_\mu^{\rm EW}$	15.4	0.2	Czarnecki et al., Knecht et al.

- QED: incredible precision, numerical 5-loop ongoing
- EW: reliable 2-loop calculation, precision sufficient
- Hadronic: uncertainties dominate



Hadronic contributions

- LO: hadronic vacuum polarisation (hvp) diagram



$$a_\mu^{\mathrm{had,LO}} = \frac{\alpha}{3\pi}\int_{s_0}^{\infty}\frac{\mathrm{d}s}{s}R(s)K(s); \quad K(s) \to 1 \text{ as } s \to \infty$$

- With

$$R(s) = \frac{\sigma^0_{\mathrm{had}}(s)}{\sigma_{\mathrm{pt}}(s)},\ \sigma_{\mathrm{pt}} = \frac{4\pi\alpha^2}{3s}$$

- Utilise the wealth of data on $e^+e^- \to \mathrm{hadrons}$ cross-section.
- NLO: similar method but with different kernel functions.
- LbL: mixed γ virtualities $\to$ low energy effective models.

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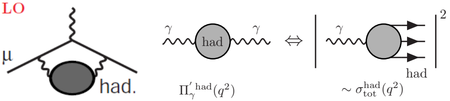

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Data combination

- $0.3 - 2.0$ GeV: 23 final states or channels (2π, 3π, KK etc.)
- $1.4 - 11.09$ GeV: $e^+e^- \rightarrow$ all hadrons
- > 2.0 GeV: can also use pQCD in the continuum
- For each final state, combine data from many experiments:
  - Re-bin $\rightarrow$ non-linear $\chi^2_{\rm min}$ $\rightarrow$ integrate (full cov. mat. for error)
  - Incompatible data $\rightarrow$ *local* (per energy bin) $\sqrt{\chi^2_{\rm min}/{\rm d.o.f.}}$ err. infl. [HMNT06: global $\sqrt{\chi^2_{\rm min}/{\rm d.o.f.}}$ inflation of final error]
- Note: $\sigma^0_{\rm had}$ must
  - *exclude photon vacuum polarisation* to avoid double counting with NLO
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Most important new data since 2006

- KLOE, [DAΦNE], Frascati: {$\pi^+\pi^-(\gamma)$, $\omega\pi^0$}
- BaBar, [PEP-II], SLAC, Stanford: {$\pi^+\pi^-(\gamma)$, $K^+K^-\pi^0$, $K_S^0K\pi$, $2(\pi^+\pi^-)\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $2(\pi^+\pi^-)\eta$, $2(\pi^+\pi^-\pi^0)$, $KK\pi\pi$}
- CMD-2, [VEPP-2M], Novosibirsk: {K^+K^-, $2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-\pi^0)$}
- SND, [VEPP-2M], Novosibirsk: {K^+K^-, $K_S^0K_L^0$}
- BES, [BEPC], Beijing: {inclusive R i.e. $\sigma(e^+e^- \to \text{all hadrons})/\sigma_{pt}$}
- CLEO, [CESR], Cornell: {inclusive R}

Digression: radiative return

- Radative return methods at KLOE and BaBar.
- Meson factory $\longrightarrow$ fixed energy $\longrightarrow$ complements direct scan experiments

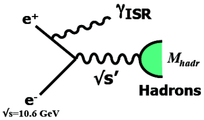


Fig. 2: ISR enables scanning of $\sqrt{s'}$ from threshold upwards.

- High luminosity $\longrightarrow$ much improved statistical error.

The $\pi^+\pi^-$ channel

- $\pi^+\pi^-$ is the most important final state ($\sim 70\%$ of $a_\mu^{\rm had,LO}$)
- New ‘Radiative Return’ data from KLOE08/10 and BaBar 09

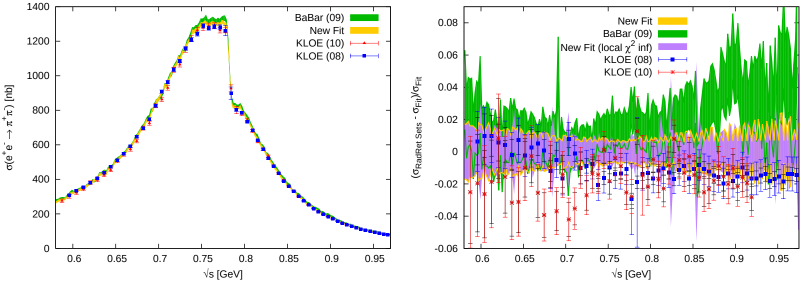
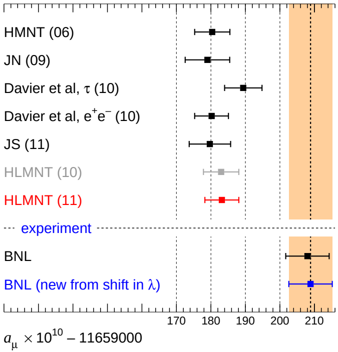


Fig. 3: Cross-section (left) and difference plot (right) for $e^+e^- \to \pi^+\pi^-$, showing the latest data.

- Tensions between new data limit gains in precision.
- HLMNT (11): $a_\mu^{2\pi}(0.32 - 2\text{ GeV}) = (504.23 \pm 2.97)\cdot 10^{-10}$
  [increase of $\sim 5.5\cdot 10^{-10}$ since HMNT (06)]

HLMNT (11) results [in units of 10^{-10}]

- Changes since HMNT (06),
  - $a_\mu^{\rm had,LO}$: 695.4 ± 4.3
    689.4 ± 4.6
  - $a_\mu^{\rm had,NLO}$: -9.85 ± 0.07
    -9.79 ± 0.09
  - $a_\mu^{\rm had,LbL}$: 10.5 ± 2.6 [Prades et al.]
    13.6 ± 2.5 [Melnikov et al.]
- Minor change in BNL result due to shift of muon to proton magnetic ratio from CODATA.
- HLMNT (11) prelim:
  $a_\mu^{\rm exp} - a_\mu^{\rm th} = 25.7 \pm 8.0 \sim 3.2\sigma$



- Other groups, Davier et al.: 3.6σ; JN (09): 3.2σ; JS (11): 3.3σ

Overview and results

- Running of $\alpha(s)$ is due to VP and can be written as,

$$\alpha(s) = \alpha/(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha_{\rm had}(s))$$

- So similar to $a_\mu^{\rm had,LO}$ use dispersion relation,

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi} {\rm P} \int_{s_0}^{\infty} {\rm d}s' \frac{R(s')}{s'(s'-s)}$$

- Least precise of the EW parameters $\{G_\mu, M_Z, \alpha(M_Z^2)\}$, is $\alpha(M_Z^2)$ due to uncertainties of $\Delta\alpha^{(5)}_{\rm had}(s)$.
- HLMNT (11) prelim.: $\Delta\alpha^{(5)}_{\rm had}(s) = 0.02764 \pm 0.00010$
  [HLMNT (10): 0.02759 ± 0.00015, HMNT (06): 0.02768 ± 0.00022]
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Summary and outlook

- $(g-2)_\mu$ strongly tests all sectors of the SM.
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  [Small decrease in error, but result consolidated by many groups using indept. methods]
- Theory side:
  - New data from CMD-3 and SND [VEPP-2000] in Novosibirsk.
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- Experiment side:
  - New $(g-2)_\mu$ experiments at Fermilab [E-989] (stage-1 approval) and J-PARC (proposed).
  - Aiming for ~ 4 fold increase in precision compared to BNL.
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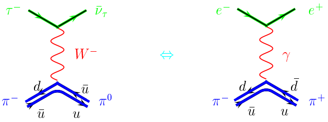
Spares

τ data

- Data from hadronic τ-decays can relate to e^+e^- cross section e.g.



- Charged current $\longrightarrow$ isospin breaking corrections needed
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- Davier et al. [EPJC66 (2010) 1] $\longrightarrow$ better agreement
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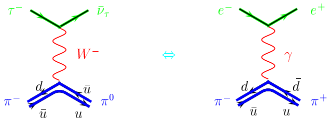

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$\pi^+\pi^-$: new data and updated treatment

- 2 new data sets with full stat+syst covariance matrices.
  - BaBar (09): $0.3 - 3.0$ GeV, 334 points
  - KLOE (10): indept. from KLOE (08), $0.32 - 0.92$ GeV
- Updated χ^2 function

$$\chi^2(R_m, f_k) = \sum_{k=1}^{N_{\rm exp}} \left(\frac{1-f_k}{df_k}\right)^2 + \left\{\sum_{m=1}^{N_{\rm clu}} \sum_{i=1}^{N(k,m)} \left(\frac{R_i^{(k,m)} - f_k R_m}{dR_i^{(k,m)}}\right)^2\right\}_{\text{w/o cov. mat.}} + \left\{\sum_{m=1}^{N_{\rm clu}} \sum_{i=1}^{N(k,m)} \sum_{j=1}^{N(k,n)} \left(R_i^{(k,m)} - f_k R_m\right) C^{-1}(m_i, n_j) \left(R_j^{(k,n)} - f_k R_n\right)\right\}$$

- Updated re-bining (clustering) algorithm. Adaptive, i.e. more freedom to
  - break up clusters with 'large' number of data
  - or grouping more data together.

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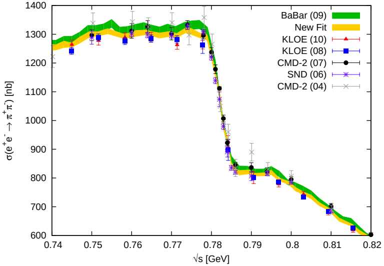
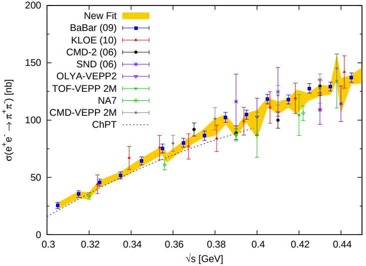
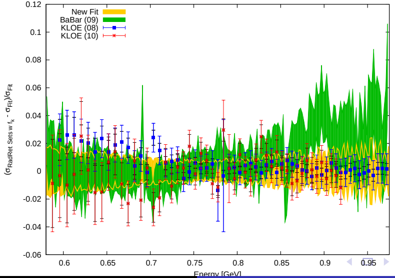
$\pi^+\pi^-$: more graphs







Inclusive vs exclusive

Between 1.43 and 2.0 GeV, we can use either:

- sum of all final states (exclusive): $a_\mu^{\text{excl,new}} = 34.97 \pm 0.85$
  $a_\mu^{\text{excl,old}} = 35.68 \pm 1.71$
- $e^+e^- \to$ all hadrons (inclusive): $a_\mu^{\text{incl}} = 31.99 \pm 2.43$

- Comparision graph

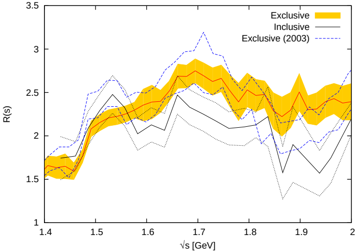


Fig 3. $R(s)$ *behaviour of inclusive data vs sum of exclusive final states.*

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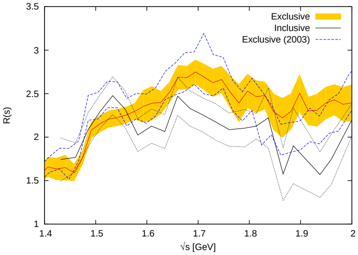

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Sum rule analysis

- Consistency check of data below some s_0 against pQCD. Given $R(s)$, what will the corresponding $\alpha_s(M_z^2)$ be?
- Analyticity of $\Pi(s)$:

$$\int_{s_{th}}^{s_0} \mathrm{d}s R(s) f(s) = \int_C \mathrm{d}s D(s) g(s), \quad D(s) \equiv -12\pi^2 s \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\Pi(s)}{s}\right)$$

- Explicitly $D(s) = D_0(s) + D_m(s) + D_{np}(s)$ where $D_0(s)$ is a series in $\alpha_s(s)$, which is our free parameter to be fitted.
- To maximise the region of interest we take, $f^{(m,n)}(s) = (1 - s/s_0)^m (s/s_0)^n$ where $m + n = 0, 1, 2$.
- This also fixes $g(s)$, e.g. $m = n = 1$ gives,

$$\int_{s_{th}}^{s_0} \mathrm{d}s R(s) \left(1 - \frac{s}{s_0}\right)\left(\frac{s}{s_0}\right) = \frac{i}{2\pi} \int_C \mathrm{d}s D(s) \left[-\frac{1}{3}\left(\frac{s}{s_0}\right)^2 + \frac{1}{2}\frac{s}{s_0} - \frac{1}{6}\frac{s_0}{s}\right]$$

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- Left: using $R(s)$ only. Right: using pQCD above 2 GeV.

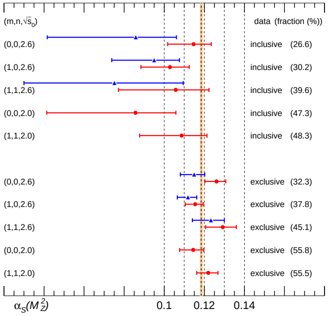

Fig 4. Sum rule results that uses data only (left) and pQCD above 2 GeV (right), red indicates new results.

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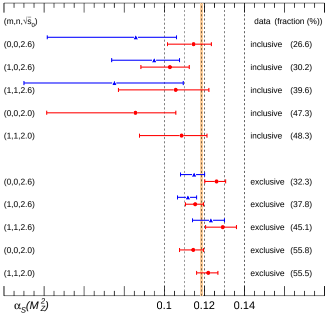

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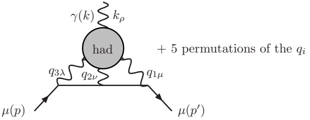

Fig 8. Hadronic light-by-light diagram, note the 3 virtual photons $(q_{1,2,3})$.

- Efforts from Lattice community: Dr. Paul Rakow and Warren Lockhart.
- Low energy effective models $\longrightarrow$ match pQCD at high energies.

Fig 9. Leading contributions: neutral pseudoscalar-exchange diagrams.

- π^0-exchange: all except the $\mathcal{F}_{\pi^0\gamma^{(*)}\gamma^*}$ is known.

Hadronic light-by-light

- Conceptually hard: non-perturbative & no direct experimental input.

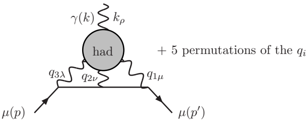

Fig 8. Hadronic light-by-light diagram, note the 3 virtual photons $(q_{1,2,3})$.

- Efforts from Lattice community: Dr. Paul Rakow and Warren Lockhart.
- Low energy effective models $\longrightarrow$ match pQCD at high energies.

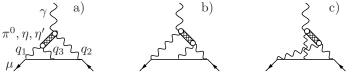

Fig 9. Leading contributions: neutral pseudoscalar-exchange diagrams.

- π^0-exchange: all except the $\mathcal{F}_{\pi^0\gamma^{(*)}\gamma^*}$ is known.