B and bottomonium spectroscopy from lattice NRQCD

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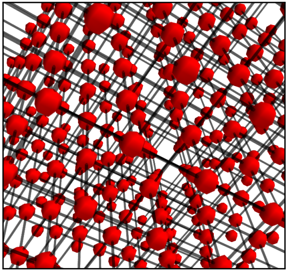
Why do lattice heavy quark physics?

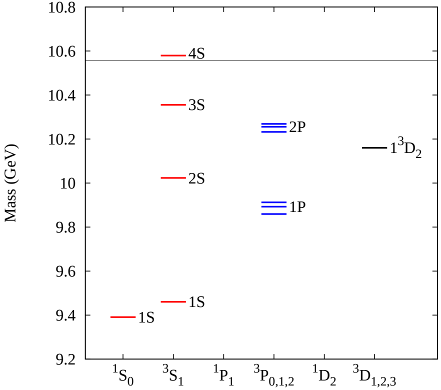
Only way to calculate some quantities is non-perturbatively
How? Discretise QCD on a spacetime lattice of size $= a$
- Monte Carlo simulate

Research aims:

- Spectroscopy of mesons containing b quarks
- B meson mixing
- Semileptonic form factors
- *b* quark mass
- Accurate tests of QCD



Bottomonium spectrum



- Many states accurately known
  - several gaps that the lattice can predict
- First D-wave states (date) and η_b found at CLEO and BABAR
- Check we can reproduce the spectrum before we trust more complicated calculations (mixing, decays)

Status of heavy quark physics on the lattice

Lattice QCD calculations improved significantly in last 5-10 yrs
- Now in the precision era (see C.Davies plenary talk)

- Include effect of u,d,s,c sea quarks
  - very expensive computationally
  - previous calculations were quenched - unknown syst. error
- Stat and syst errors improved - aiming for $\simeq$1% errors
- Spacing still not small enough to handle relativistic b quarks
  - Use effective theories: NRQCD, HQET

Quarks: Non-relativistic QCD

- Effective field theory valid for small v, $v^2 \sim 0.1$ for Upsilon
- Hamiltonian (don't worry about the details!):

$$
\begin{aligned}
aH_0 &= -\frac{\Delta^{(2)}}{2aM_b} \\
a\delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(aM_b)^3} + c_2 \frac{ig}{8(aM_b)^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right) \\
&\quad -c_3 \frac{g}{8(aM_b)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right) \\
&\quad -c_4 \frac{g}{2aM_b}\, \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{a^2 \Delta^{(4)}}{24aM_b} - c_6 \frac{a(\Delta^{(2)})^2}{16n(aM_b)^2}
\end{aligned}
$$

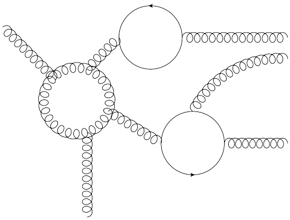
- Expansion up to $\mathcal{O}(v^4)$
- Wilson coeff. $c_i = 1$ at tree level
- c_1, c_5, c_6 are improved to one loop - $\mathcal{O}(\alpha_s v^4)$
- Computationally cheap

Gluons

Gluons included by Monte-Carlo simulation

- We use 5 MILC collaboration ensembles
- Symanzik improved gluon action with one-loop coefficients
- u,d,s,c sea quarks included with HISQ action
- $\sim$ 1000 configurations in each ensemble

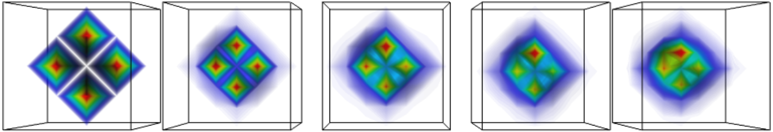
β	a(fm)	m_l/m_s	$L^3 \times T$
5.80	$\sim$ 0.15	0.2	$16^3{\times}48$
5.80	$\sim$ 0.15	0.1	$24^3{\times}48$
6.00	$\sim$ 0.12	0.2	$24^3{\times}64$
6.00	$\sim$ 0.12	0.1	$32^3{\times}64$
6.30	$\sim$ 0.09	0.2	$32^3{\times}96$



Calculation

Spectrum is extracted from meson 2-point functions

$$C(t) = \sum_{\vec{x}} \langle \bar{\psi}(t,\vec{x}) \Gamma \psi(t,\vec{x}) (\bar{\psi}(0)\Gamma\psi(0))^{\dagger} \rangle$$



Energies extracted from simultaneous Bayesian fit to

$$C(t) = \sum_{n=1}^{n_{\rm exp}} A_n \exp(-E_n t)$$

Calculation

Performed on Darwin cluster at Cambridge University

S-waves:

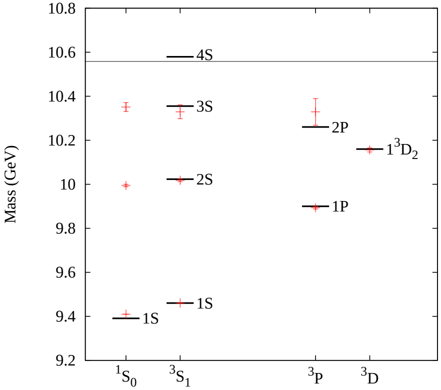
- 16 correlators per configuration
- 5 different smearings per meson

P-waves and D-waves:

- 32 correlators per configuration
- 2 different smearings per meson

Radial and spin independent splittings

$\Upsilon(2S - 1S)$ is used to fix lattice spacings (not a prediction)



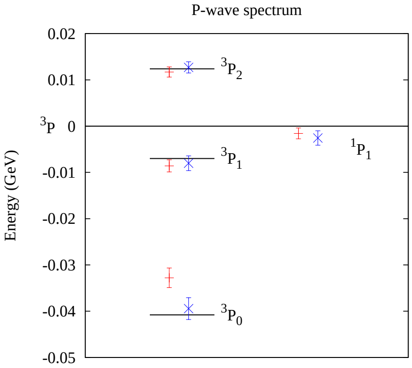
- Results for a single ensemble shown

Dominant syst error from missing $\mathcal{O}(v^6), \mathcal{O}(\alpha_s^2 v^4)$ terms

P-wave splittings

P-wave spectrum is used to non-perturbatively tune c_3, c_4

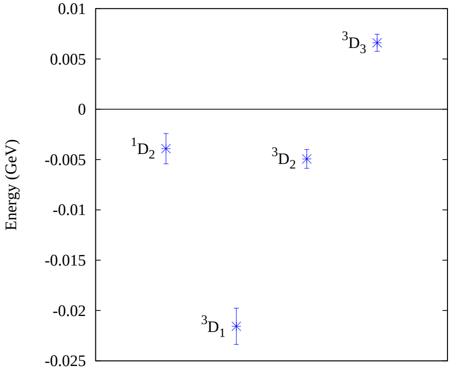
- Plot relative to spin average $\overline{^3P}$ state
- Tree level coefficients give slightly incorrect splittings
- Untuned in red, tuned in blue
- Errors are statistical and lattice spacing only



D-wave splittings prediction from full QCD

Use tuned Wilson coefficients to predict splittings

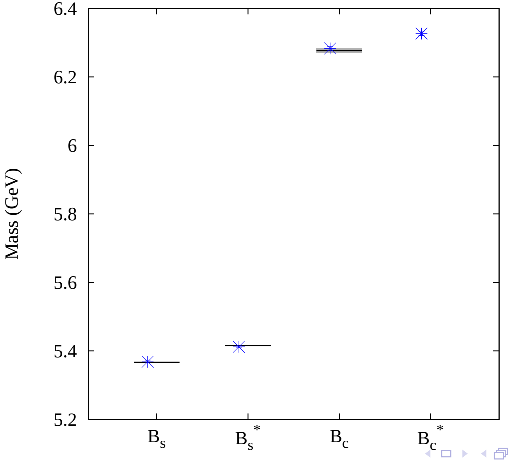
- Statistical errors dominate
- Leading systematic from $\mathcal{O}(v^6)$
- Plot relative to spin average $\overline{^3D}$ state
- All splittings resolved for the first time



B meson spectrum (preliminary)

Also have improved values for B, B_s, B_c meson masses

- Only a few ensembles so far
- Significant improvement on previous HPQCD values
- No free parameters



Summary

- Lattice NRQCD accurately reproduces bottomonium spectrum
- First full prediction of D-wave states
- Systematic errors under much better control
- More than 10$\times$ the statistics of previous bottomonium calculations

Appendix: Gauge action

Symanzik improved - 2 additional terms: Plaquette, Rectangle, Twisted rectangle

$$S_G = \beta\Big[c_P \sum_P \Big(1 - \frac{1}{3}\mathrm{ReTr(P)}\Big) + c_R \sum_R \Big(1 - \frac{1}{3}\mathrm{ReTr(R)}\Big) + c_T \sum_T \Big(1 - \frac{1}{3}\mathrm{ReTr(T)}\Big)\Big]$$

Coefficients calculated to one loop in gluons and quarks

$$C_P = 1.0$$
$$C_R = \frac{-1}{20u_{0P}^2}(1 - (0.6264 - 1.1746N_f)\log(u_{0P}^2))$$
$$C_T = \frac{1}{u_{0P}^2}(0.0433 - 0.0156N_f)\log(u_{0P}^2)$$

u_{0P} removes tadpole diagrams