Black holes with SU(N) gauge fields and superconducting horizons

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Outline

1. Introduction to superconductivity and the AdS/CFT correspondence
2. Planar black holes with $\mathfrak{su}(N)$ gauge fields in AdS
3. Properties of the black hole solutions

Superconductivity

Properties

Superconductors are materials that have a phase transition at a critical temperature T_C, such that for $T < T_C$:

- An electron condensate can be formed
- The state with the electron condensate has lower free energy than the normal state
- This state has zero DC resistivity

Most metals superconduct at $T_C \sim 5K$ and are described by BCS theory. The layered cuprates have $T_C \sim 100K$, but no theory to describe them, since:

- The pairing mechanism is not well understood
- Pairs are thought to be strongly coupled

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The AdS/CFT correspondence

The AdS/CFT correspondence relates gravitational theories in $d+1$ dimensional AdS space, to d dimensional conformal field theories on the boundary.

- Original correspondence relates type IIB string theory on $AdS_5 \times S^5$ to 4 dimensional CFTs.
- We will use the classical limit on the gravity side
- Since the layered cuprates are effectively $2+1$ dimensional we will use a gravitational theory in AdS_4

We will use the AdS/CFT correspondence to construct gravitational analogues to superconductors

The model

- Planar black hole line element in AdS is

$$ds^2 = -\sigma^2 \mu \, dt^2 + r^2 dx^2 + r^2 dy^2 + \mu^{-1} dr^2,$$

where $\mu = -\frac{2m(r)}{r} - \frac{\Lambda r^2}{3}$.

- The gauge potential

$$A = \frac{1}{g} \sum_{k=1}^{N-1} \left(h_k(r) H_k dt + \frac{i}{2} \omega_k(r) F_k dx + \frac{i}{2} \omega_k(r) G_k dy \right).$$

- The Einstein-Yang-Mills (EYM) equations are

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \qquad D_\mu F^{\mu\nu} = 0$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g \left[A_\mu, A_\nu \right]$$

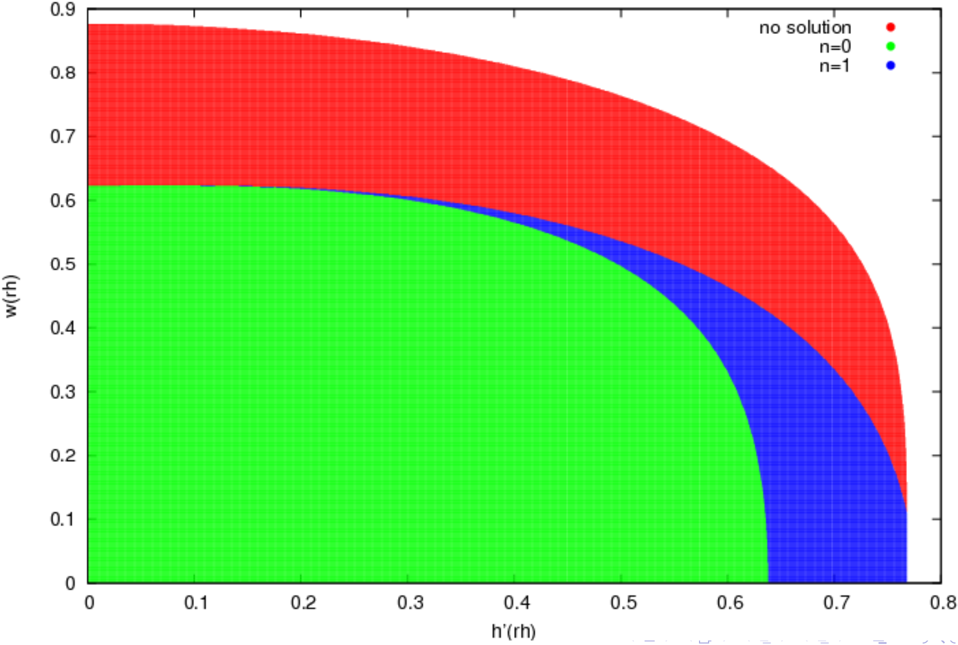
Solutions

Solutions to the field equations

- The EYM equations give us expressions for ω_k'', h_k'', m' and σ', plus some boundary conditions at r_h
- We require $h_k(r_h) = 0$ and $\omega_k \to 0$ as $r \to \infty$ for all k
- We are looking for solutions where ω_k have no nodes
- Solutions are then uniquely determined by Λ and $\omega_k(r_h)$

Known solutions

The $\mathfrak{su}(2)$ case, with a slightly different gauge potential, has been considered by S. Gubser et al.


no solution
n=0
n=1
w(rh)
h'(rh)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Thermodynamics

- The Hawking temperature is defined by

$$T_H = \frac{\mu'(r_h)\sigma(r_h)}{4\pi}$$

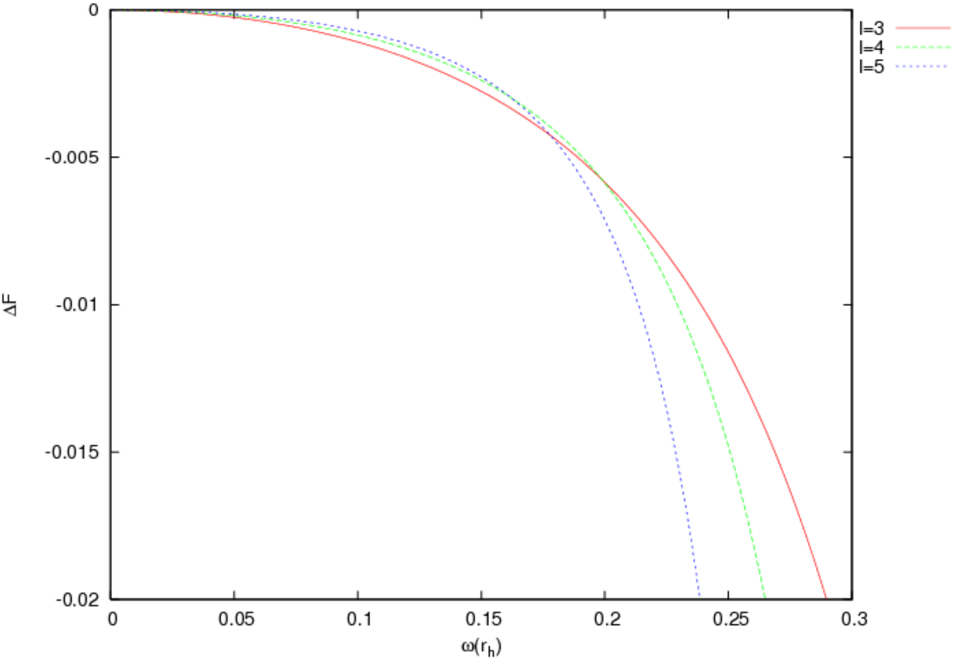
- The free energy is

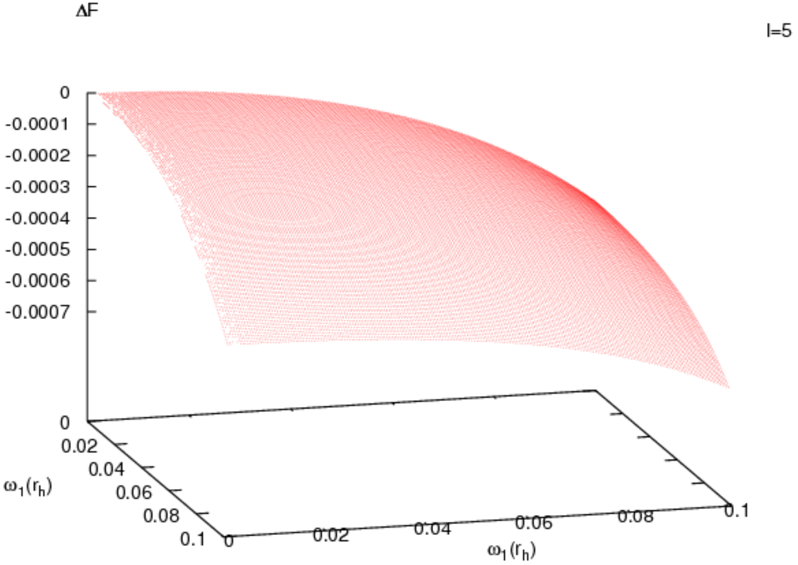
$$F = M - T_H S$$

- We also define

$$\Delta F = F - F_{RN}$$

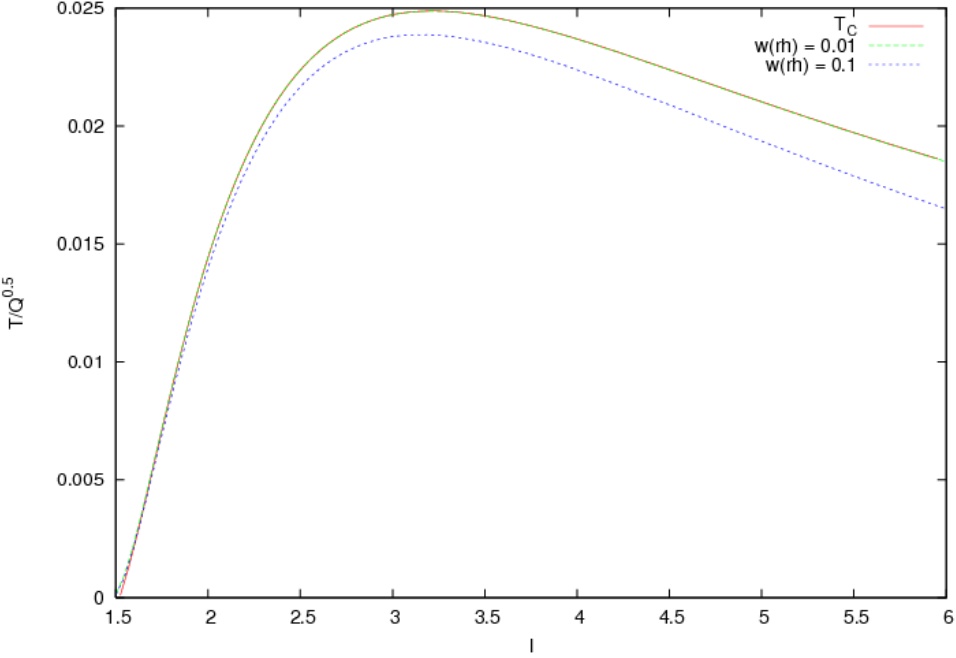
- The superconducting state will become thermodynamically favoured when $\Delta F < 0$


0
-0.005
-0.01
-0.015
-0.02
ΔF
0
0.05
0.1
0.15
0.2
0.25
0.3
ω(r_h)
l=3
l=4
l=5

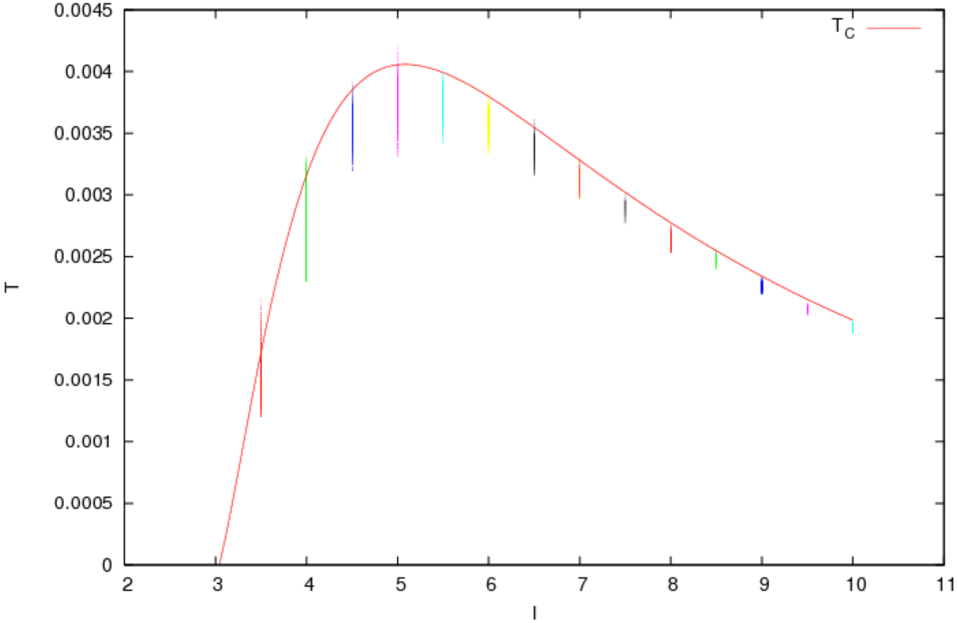

ΔF
l=5
0
-0.0001
-0.0002
-0.0003
-0.0004
-0.0005
-0.0006
-0.0007
0
0.02
0.04
0.06
0.08
0.1
ω1(rh)
0
0.02
0.04
0.06
0.08
0.1
ω1(rh)

Perturbations of the Reissner Nordström solutions

- T_C is the temperature at which the condensate begins to form
- At T_C there are only a few pairs of superconducting electrons
- We therefore consider a small condensate perturbation to the Reissner-Nordström $\delta\omega_k$ and corresponding δh_k
- The solution is then uniquely determined by Λ.



0.025
0.02
0.015
0.01
0.005
0
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
T/Q^0.5
l
T_C
w(rh) = 0.01
w(rh) = 0.1



0.0045
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
2
3
4
5
6
7
8
9
10
11
T
l
T_C

Conductivity

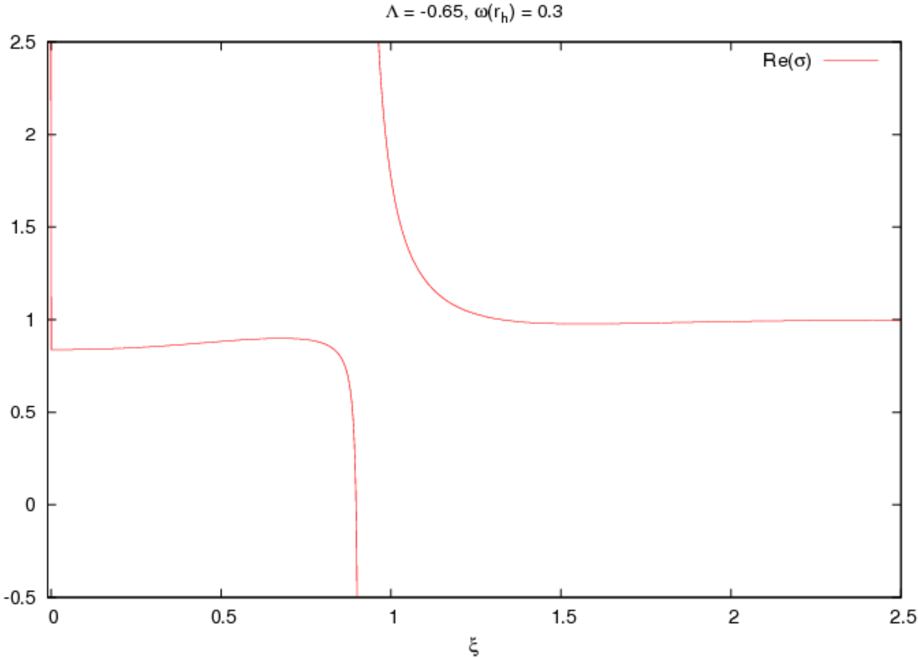
- We apply a perturbation to the gauge field with time dependence $e^{-i\xi t}$, of the form

$$\delta A = \frac{1}{g} e^{-i\xi t} \sum_{k=1}^{N-1} \left(\frac{i}{2} \left(\delta u_k F_k + \delta v_k G_k \right) dt + \delta h_{1,k} H_k dx + \delta h_{2,k} H_k dy \right)$$

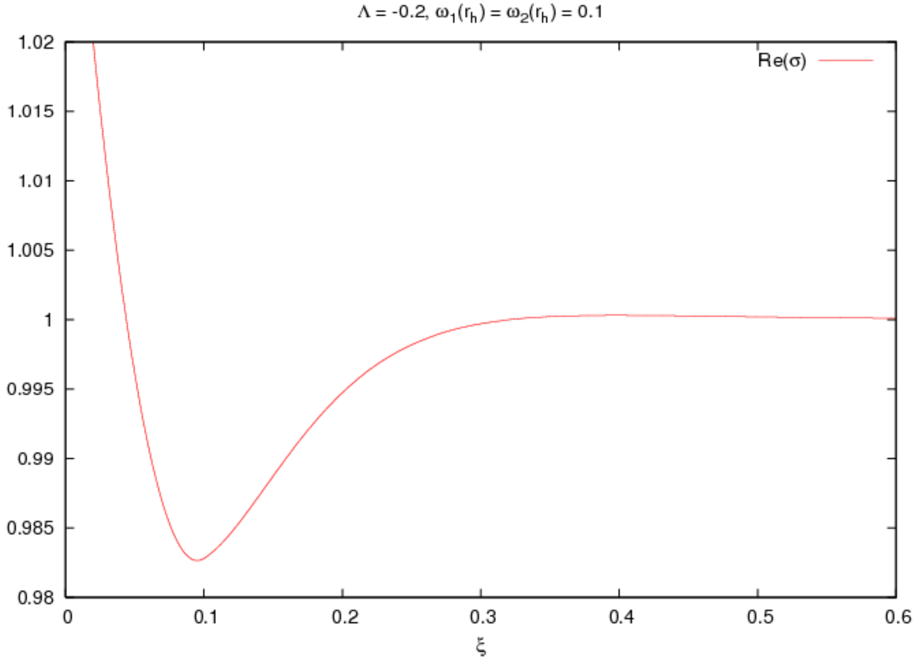
- In the $\mathfrak{su}(2)$ case there is a set of gauge transformations that preserve the structure of δA, so we define gauge invariant quantities

$$\delta \hat{h}_1 = \delta h_1 + \frac{\omega \left(i\xi \delta v + h \delta u \right)}{h^2 - \xi^2} \qquad \delta \hat{h}_2 = \delta h_2 + \frac{\omega \left(i\xi \delta u - h \delta v \right)}{h^2 - \xi^2}$$

- The conductivity is determined from the behaviour of these perturbations at large r.



Λ = -0.65, ω(r_h) = 0.3
Re(σ)
2.5
2
1.5
1
0.5
0
-0.5
0
0.5
1
1.5
2
2.5
ξ



$\Lambda = -0.2,\ \omega_1(r_h) = \omega_2(r_h) = 0.1$
Re(σ)
1.02
1.015
1.01
1.005
1
0.995
0.99
0.985
0.98
0
0.1
0.2
0.3
0.4
0.5
0.6
ξ

Conclusions

Black hole solutions with non-zero $\mathfrak{su}(N)$ gauge fields ω_k:

- Exist at low temperatures
- Are thermodynamically favourable over Reissner-Nordström solutions below T_C
- Have infinite DC conductivity below T_C
- Exhibit a gap in the conductivity at non-zero frequency