Nucleon Structure Studies Through Elastic Electron Scattering: Electromagnetic Form Factors

- Introduction
- Experimental Status of EMFF
- Analysis and Interpretation
- Outlook
- Summary

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Nucleon Electro-Magnetic Form Factors

- Fundamental properties of the nucleon
  - ➙ Give information on the electric charge and magnetic moment distributions of the nucleon
  - ➙ Provide excellent testing ground for QCD and QCD-inspired models
  - ➙ Are not yet calculable from first principles
  - ➙ Cleanly probed through elastic electron-nucleon scattering
    - ➙ Wavelength of probe can be tuned by selecting momentum transfer Q:

      < 0.1 GeV^2 integral quantities (charge radius,...)

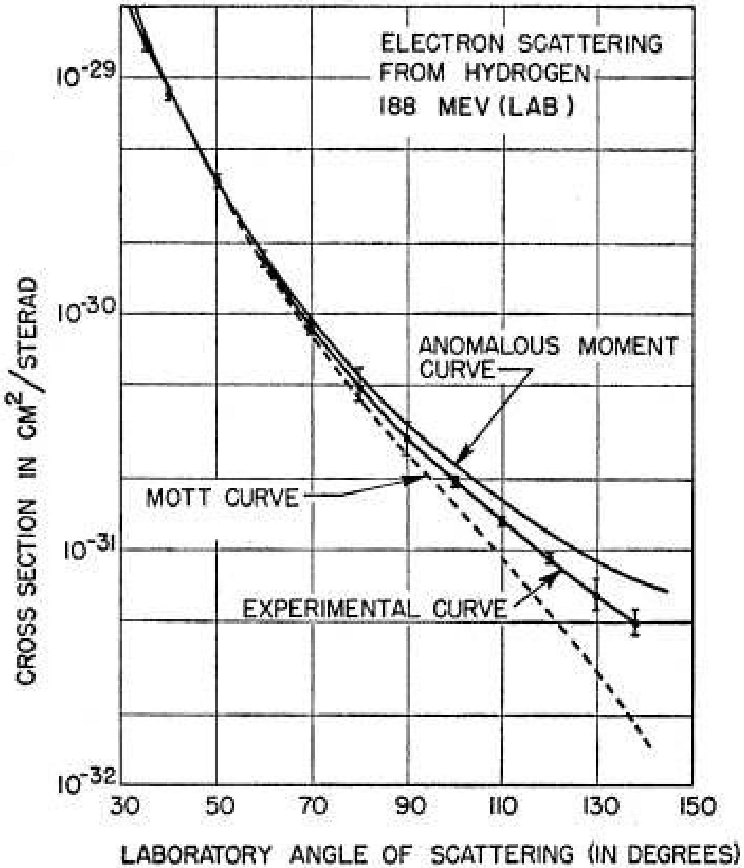
      0.1-10 GeV^2 internal structure of nucleon

      > 20 GeV^2 pQCD scaling

Caveat: If Q is several times the particle that the virtual photon is interacting with (~Compton wavelength), dynamical (relativistic) effects make a physical interpretation more difficult

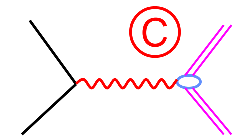
Historical Overview

- 1910s - Rutherford discovers positively charged core of atoms
- 1932 – James Chadwick discovers the neutron
- 1933 - Stern observes anomalous magnetic moment of proton deflection of a beam of hydrogen molecules in an inhomogeneous magnetic field
- 1955 - Hofstadter *et al.* at Stanford discovers protons have size through electron scattering, quotes an RMS charge radius of 0.74 ± 0.24 fm
- 1968 – nucleon constituents were established from scaling in deep inelastic scattering



Formalism

Dirac (non-spin-flip) F_1 and Pauli (spin-flip) F_2 Form Factors



$$\frac{d\sigma}{d\Omega}(E,\theta) = \frac{\alpha^2 E'\cos^2(\frac{\theta}{2})}{4E^3\sin^4(\frac{\theta}{2})}[(F_1^2 + \kappa^2\tau F_2^2) + 2\tau(F_1 + \kappa F_2)^2\tan^2(\frac{\theta}{2})]$$

with E (E') incoming (outgoing) energy, θ scattering angle,
κ anomalous magnetic moment and τ = $Q^2/4M^2$

Alternatively, Sachs Form Factors G_E and G_M can be used

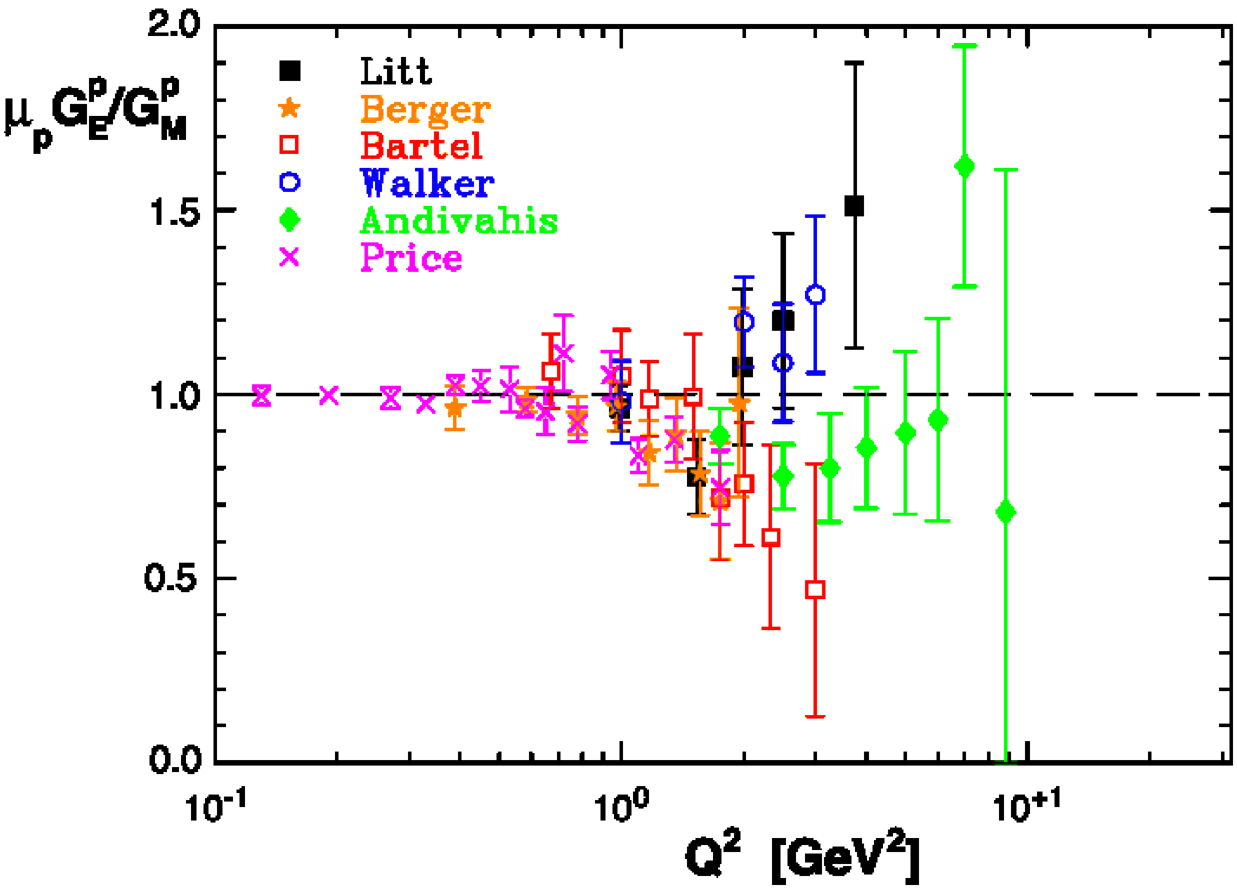
$$F_1 = G_E + \tau G_M \qquad F_2 = \frac{G_M - G_E}{\kappa(1+\tau)} \qquad \tau = \frac{Q^2}{4M^2}$$

$$\frac{d\sigma}{d\Omega}(E,\theta) = \sigma_M[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2\tan^2(\frac{\theta}{2})]$$

$$\sigma_M = \frac{\alpha^2 E'\cos^2(\frac{\theta}{2})}{4E^3\sin^4(\frac{\theta}{2})}$$

Separate the two Sachs FFs by measuring the cross section at one Q^2-value for various Θ-values (Rosenbluth separation).
In the Breit (centre-of-mass) frame the Sachs FF can be written as the Fourier transforms of the charge and magnetization radial density distributions

World Data Set on G_E^p by mid 1990s



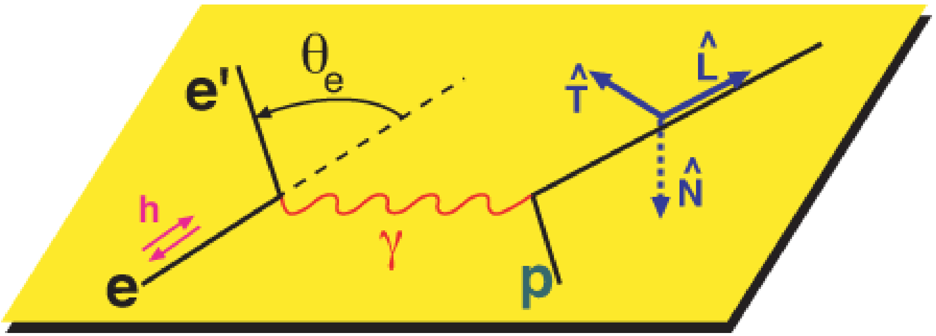
- Relied on Rosenbluth separation
- General assumption that $G_E^p/G_M^p \approx 1$
- Although data showed large scatter

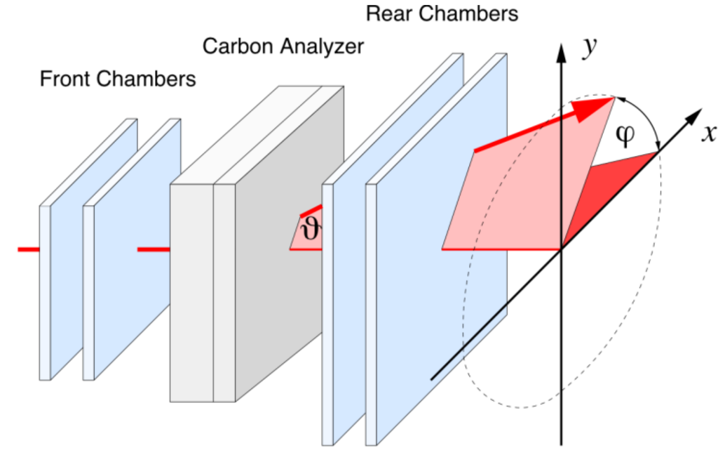
$$\tau = \frac{Q^2}{4M^2} = \frac{EE'\sin^2(\theta/2)}{M^2}$$

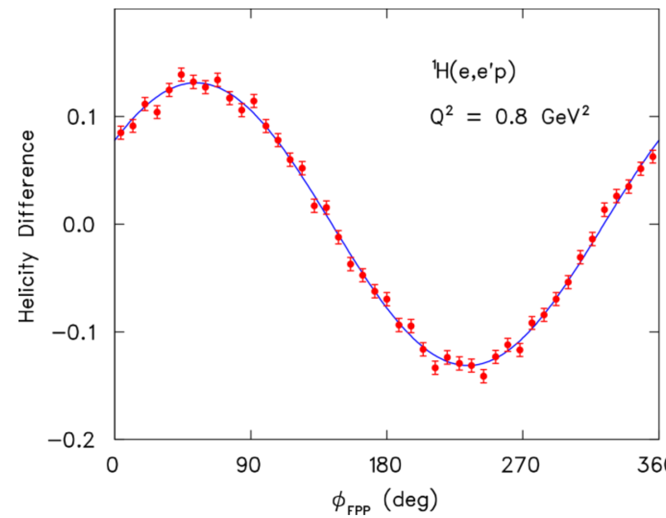
$$\sigma_R(Q^2,\varepsilon) = \varepsilon\left(1+\frac{1}{\tau}\right)\frac{E}{E'}\frac{\sigma(E,\theta)}{\sigma_{Mott}} = (G_M^p)^2(Q^2) + \frac{\varepsilon}{\tau}(G_E^p)^2(Q^2)$$

$$\varepsilon = \frac{1}{1+2(1+\tau)\tan^2(\theta/2)}$$

Alternative: Spin Transfer Reaction $^1H(\vec{e},e'\vec{p})$







$$P_n = 0$$

$$\pm hP_t = \mp h\, 2\sqrt{\tau(1+\tau)}\, G_E^p G_M^p \tan\left(\frac{\theta_e}{2}\right) / I_0$$

$$\pm hP_l = \pm h\left(E_e + E_{e'}\right)\left(G_M^p\right)^2 \sqrt{\tau(1+\tau)} \tan^2\left(\frac{\theta_e}{2}\right) / M / I_0$$

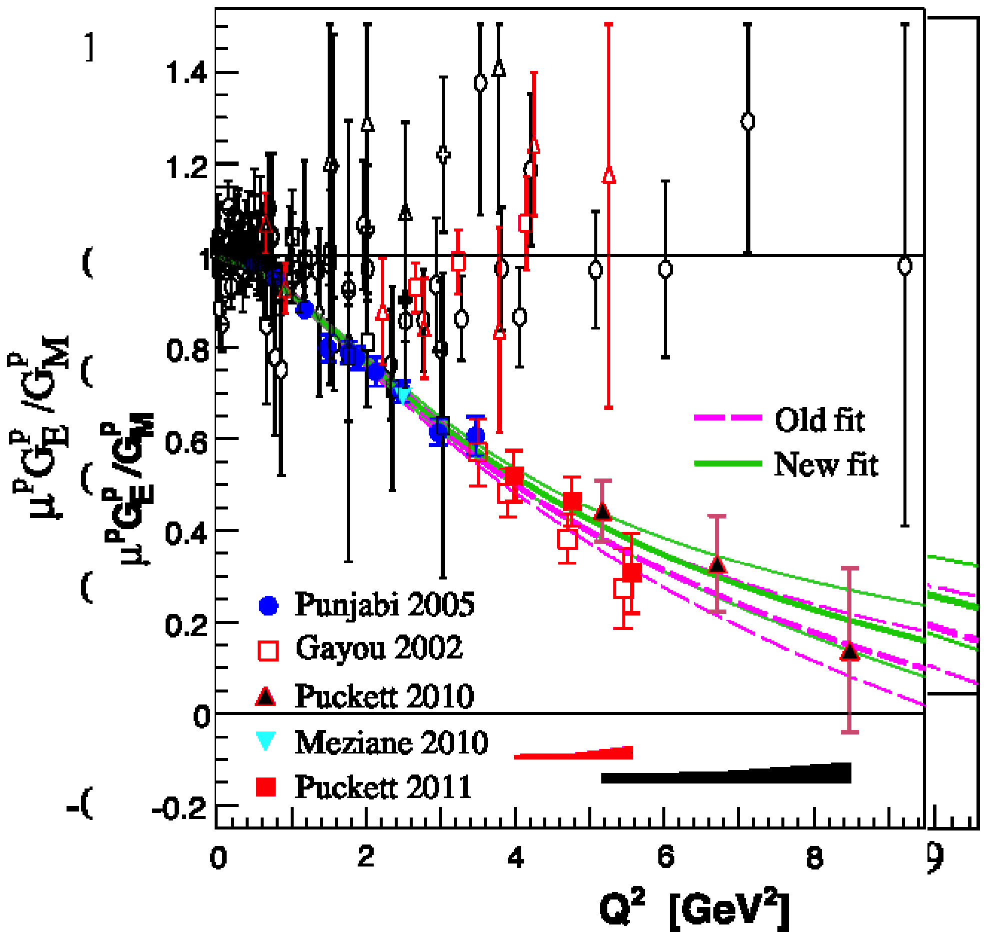
$$I_0 = \left\{G_E^p\left(Q^2\right)\right\}^2 + \tau\left\{G_M^p\left(Q^2\right)\right\}^2\left[1 + 2(1+\tau)\tan^2\left(\frac{\theta_e}{2}\right)\right]$$

$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l}\frac{E_e + E_{e'}}{2M}\tan\left(\frac{\theta_e}{2}\right)$$

Akhiezer et al., Sov. Phys. JETP 6, 588 (1958)

No error contributions from
- analyzing power
- beam polarimetry

World Data Set on G_E^p ten years later



- Large new data set based on polarization transfer show linear decrease of G_E^p/G_M^p with Q^2
- In contrast with Rosenbluth data

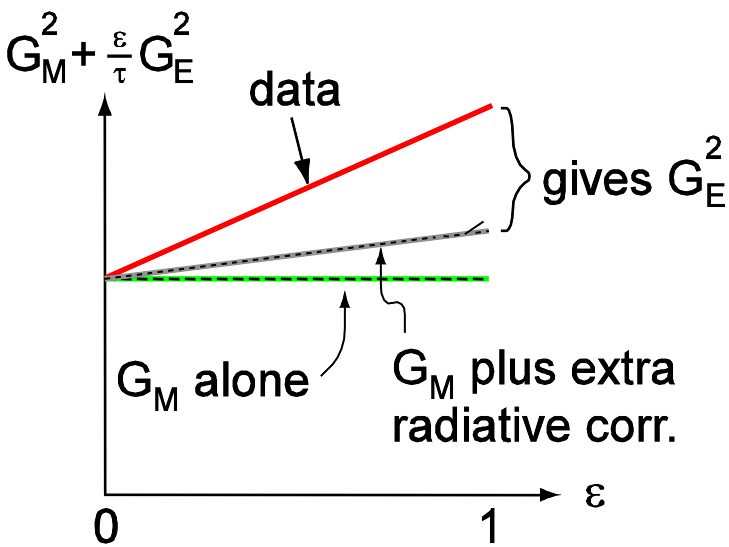
A. Puckett et al., arXiv: 1102.5737

- Detailed reanalysis of SLAC data resulted in acceptable scatter of data
- JLab Rosenbluth data (open red symbols) in agreement with SLAC data
- No reason to doubt quality of either Rosenbluth or polarization transfer data
- Investigate possible theoretical sources for discrepancy

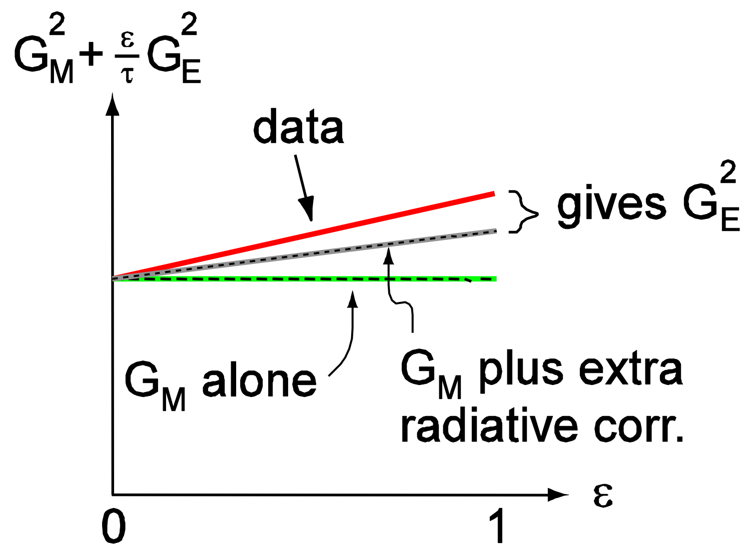
Speculation : missing radiative corrections

Speculation : The large discrepancy in the ratio G_E^p/G_M^p observed between Rosenbluth and polarization transfer techniques are expected to be explained by two-photon-exchange (2γ) effects

missing correction : linear in ε, but with no strong Q^2-dependence



Low τ (Low Q^2)



High τ (High Q^2)

G_E term is proportionally smaller at large Q^2

effect more visible at large Q^2

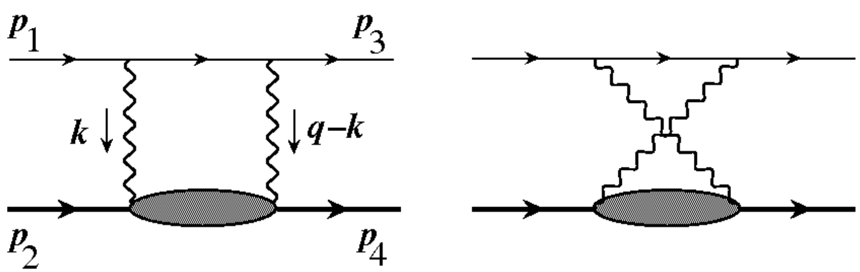
$Q^2 = 6\ \mathrm{GeV}^2$

$$\frac{G_E^2}{\tau\, G_M^2} = \frac{4\, M^2}{Q^2\, \mu_p^2} = 7.5\%$$

if both FF scale in same way

John Arrington

Calculations of TPE effects



$d\sigma = d\sigma_0 \, (1 + \delta)$

$$\delta = 2f(Q^2,\varepsilon) + \frac{2\Re\{\mathfrak{M}_0^\dagger \mathfrak{M}_1\}}{|\mathfrak{M}_0|^2} \longrightarrow \delta_{2\gamma} = \frac{2\Re\left\{M_\gamma^\dagger M_{2\gamma}\right\}}{|M_\gamma|^2}$$

$f(Q^2, \varepsilon)$ is the standard Mo & Tsai correction (soft photon exchange), which has some Σ-dependence and is IR divergent

IR divergent terms are canceled by soft-photon emission terms

Two methods of calculating $\delta_{2\gamma}$:

Hadronic
Use nucleon-pole diagrams with on-shell form factors in photon-nucleon vertices
Blunden, Melnitchouk,Tjon (BMT), PRC 72, 034612 (2005)

Partonic
Factorize TPE amplitude into hard process of e-q scattering and a soft process described by GPDs

Effect on L-T Extractions

Arrington, Melnitchouk, Tjon
PRC 76, 035205 (2007)

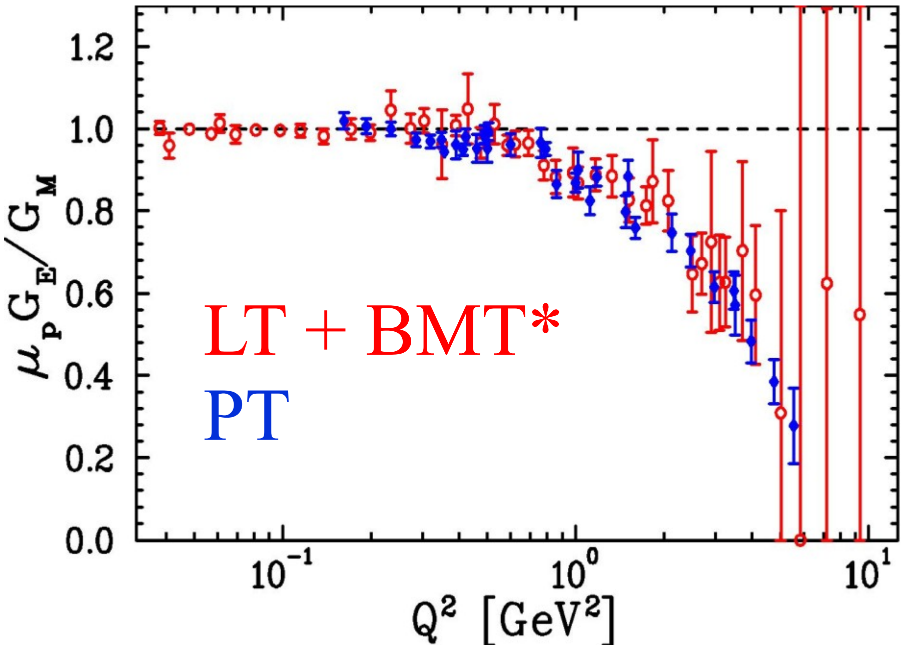
full reanalysis of data, incorporating BMT calculations, but adding extra (small) phenomenological correction above $Q^2 = 1$ GeV^2

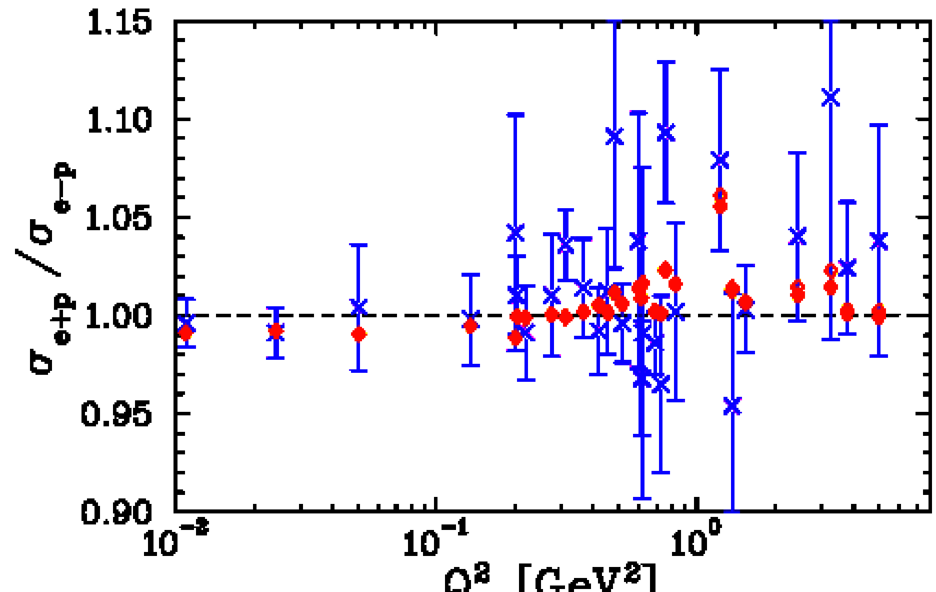
$$\delta_{2\gamma} = 0.01[\varepsilon - 1]\frac{\ln Q^2}{\ln 2.2}$$

~1% at 2 GeV^2, 2% at 5 GeV^2

- Apply 100% of the extra correction as an uncertainty (affects G_M^p uncertainty)

- Corrections hardly visible in e^+/e^- ratio





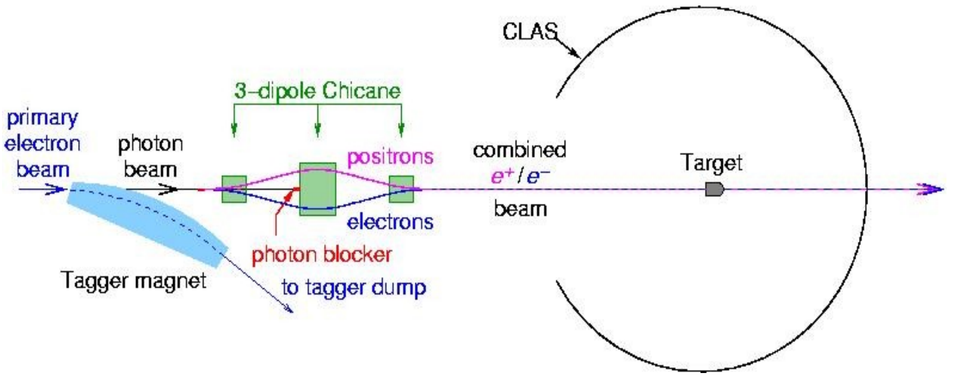
TPE – 2© effects in ep scattering

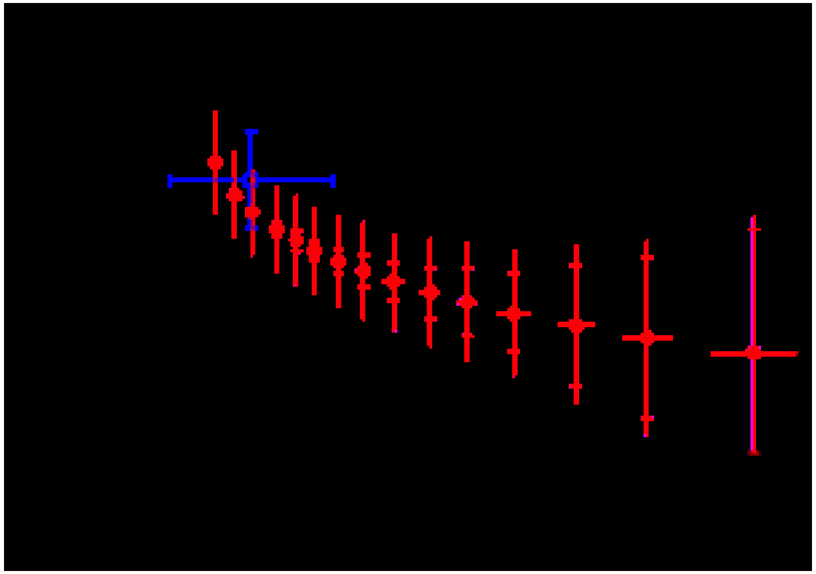
- The ratio of e^-p and e^+p elastic scattering cross sections measures the real part of the 2γ amplitude. The 2γ/1γ interference term $\delta_{2\gamma}$ has opposite sign for e^+ and e^- and is expected to vary from 1 to 10%

$$\frac{\sigma(e^+)}{\sigma(e^-)} \approx 1 - \delta_{2\gamma}$$

- New e^+/e^- data expected soon: BINP (data), DESY (2012), CLAS (data)
- The CLAS experiment has just been completed. It created an intense photon beam and then converted it to a simultaneous mixed identical e^+ and e^- beam directed onto a lH2 target. The scattered leptons and protons are detected in the CLAS detector.
- Other processes sensitive to TPE:
  - Non-linearity of Σ- dependence
  - Target Single-Spin Asymmetries

Schematic e^+/e^- beamline





Proton cross-section data from MAMI

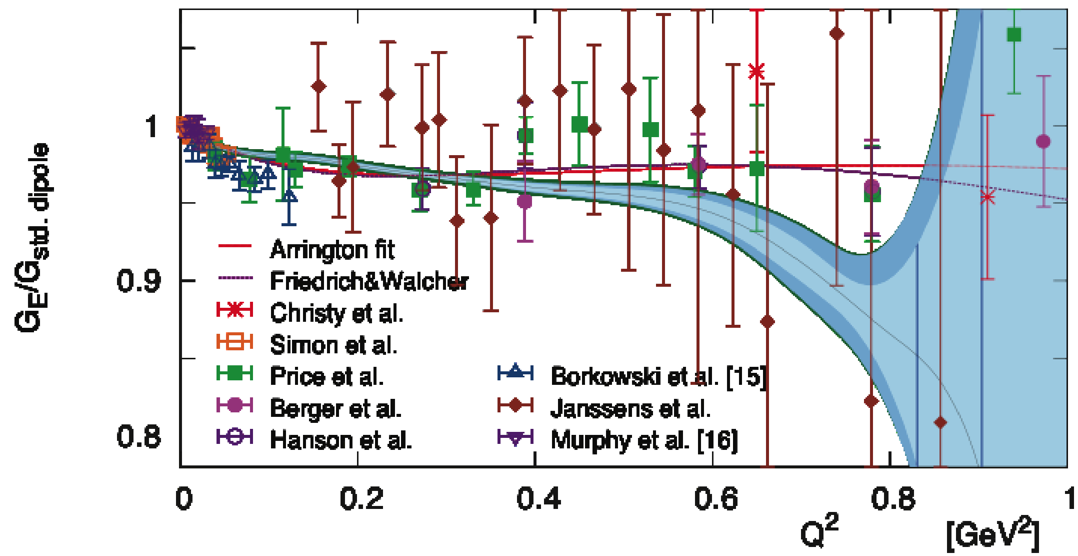
- Bernauer et al. (PRL 105, 242001 (2010)) collected a large data set (1400 data at six beam energies, each with a free normalization) using all three A1 spectrometers
- The ≲1% accuracy allowed an L/T separation in a Q^2-range of 0.02 to 0.5 GeV^2, error bands shown are of fits to complete data set, not representative of individual errors
- Results for G_E^p/G_M^p in reasonable agreement with JLab data, but G_M^p data 2-3% larger than world data set

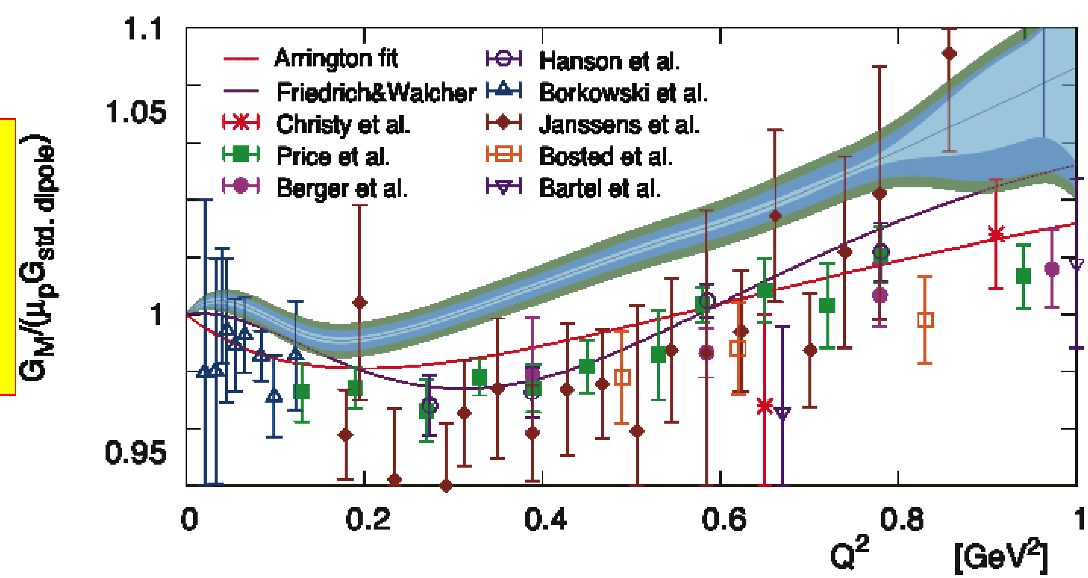
$\langle r^2\rangle_E^{1/2}$ = 0.879(8)±5±4±2±4 fm

$\langle r^2\rangle_M^{1/2}$ = 0.777(18)±13±9±5±2 fm

stat;syst;model

CODATA (dominated by electronic Lamb Shift)
$\langle r^2\rangle_E^{1/2}$ = 0.879±7 fm





Polarization Transfer at low Q^2-values

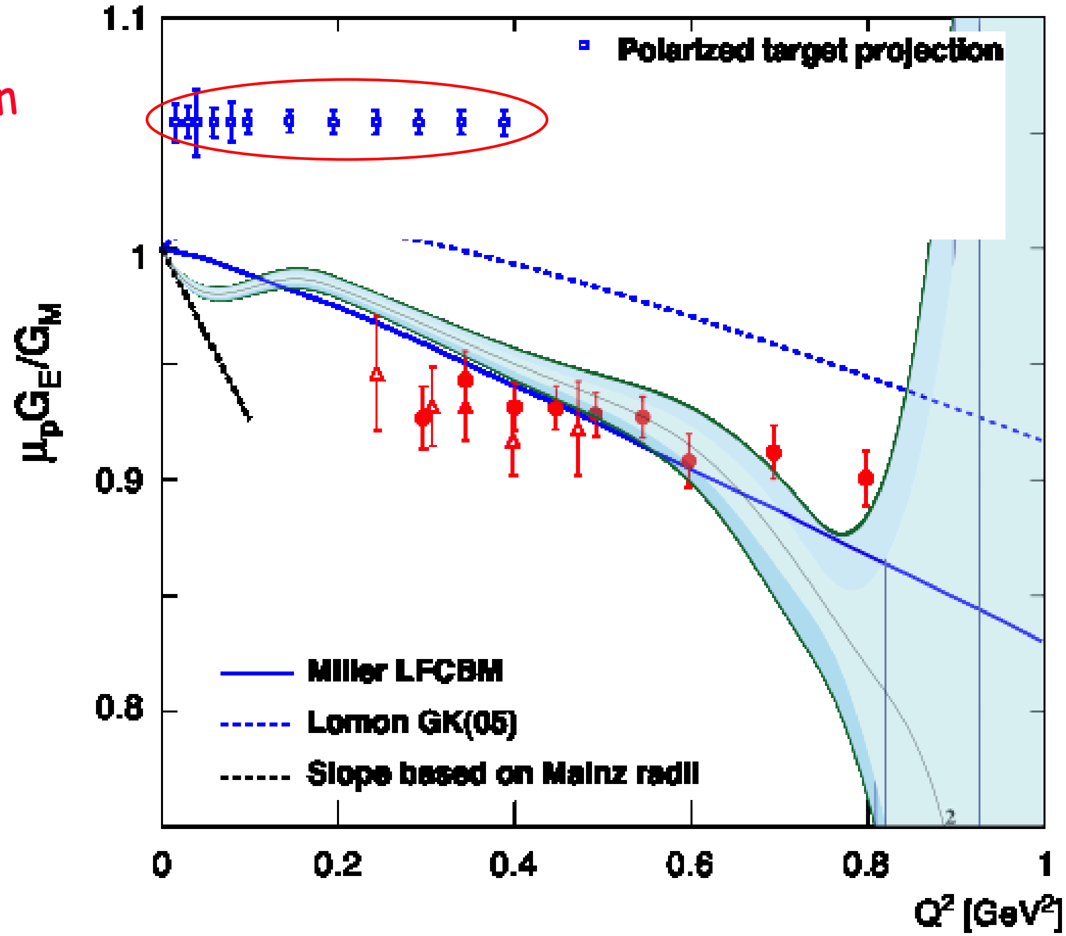
Detailed understanding of Hall A HRS spectrometer optics and availability of BigBite spectrometer has made possible polarization transfer measurements with a ~1% accuracy in a Q^2-range from 0.3 – 0.7 GeV^2

Results agree with Bernauer et al. but the magnetic radius is significant larger

$\langle r^2 \rangle_E^{1/2}$ = 0.875(10)±8±6fm

$\langle r^2 \rangle_M^{1/2}$ = 0.867(20)±9±18 fm

X. Zhan et al., arXiv: 1102.0318



These new data analyzed together with the new data set from MAMI will allow to set sensitive limits onTPE effects at low Q^2

Further data at Q^2-values down to 0.01 GeV^2 are scheduled for late 2011 with a DNP target

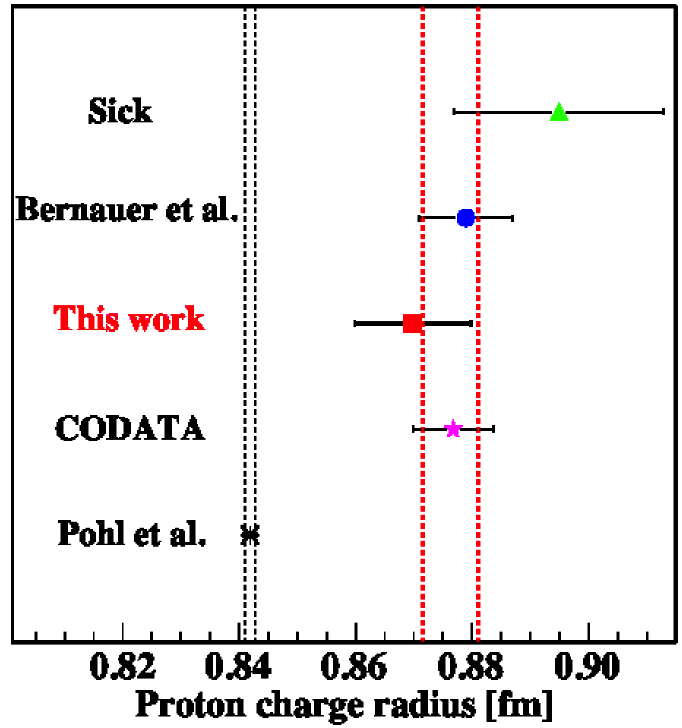
The proton charge radius

CODATA (electronic Lamb shift)

$\langle r^2 \rangle_E^{1/2} = 0.8768(69)$ fm

PSI (muonic Lamb shift) Nature 466, 213 (2010)

$\langle r^2 \rangle_E^{1/2} = 0.84184(67)$ fm

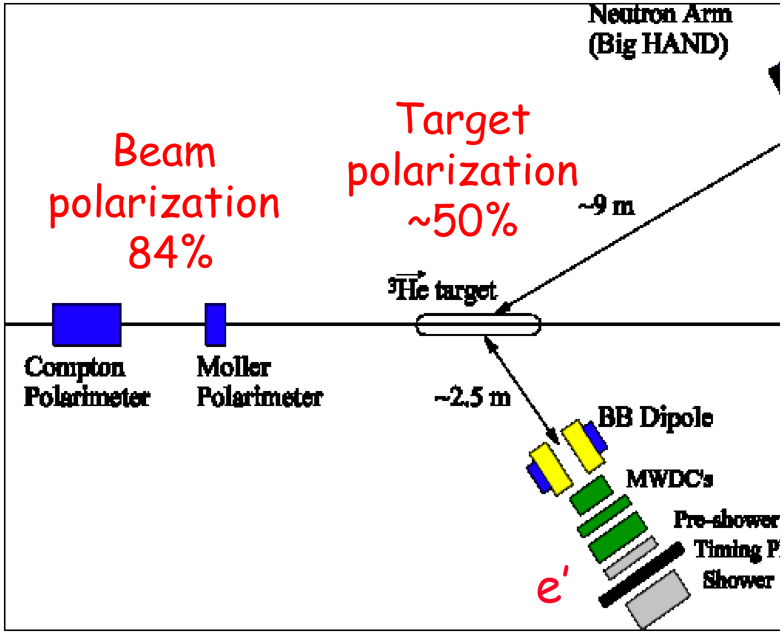


- What is reason for this 5σ discrepancy?
- Electron scattering and electronic Lamb shift agree
- Unknown interaction between μ and p?
- Muonic hydrogen much smaller than atomic hydrogen, more sensitive to off-shell effects?
- Leading theoretical uncertainty in HFS of hydrogen ground state dominated by low-Q^2 behaviour in Zemach radius:

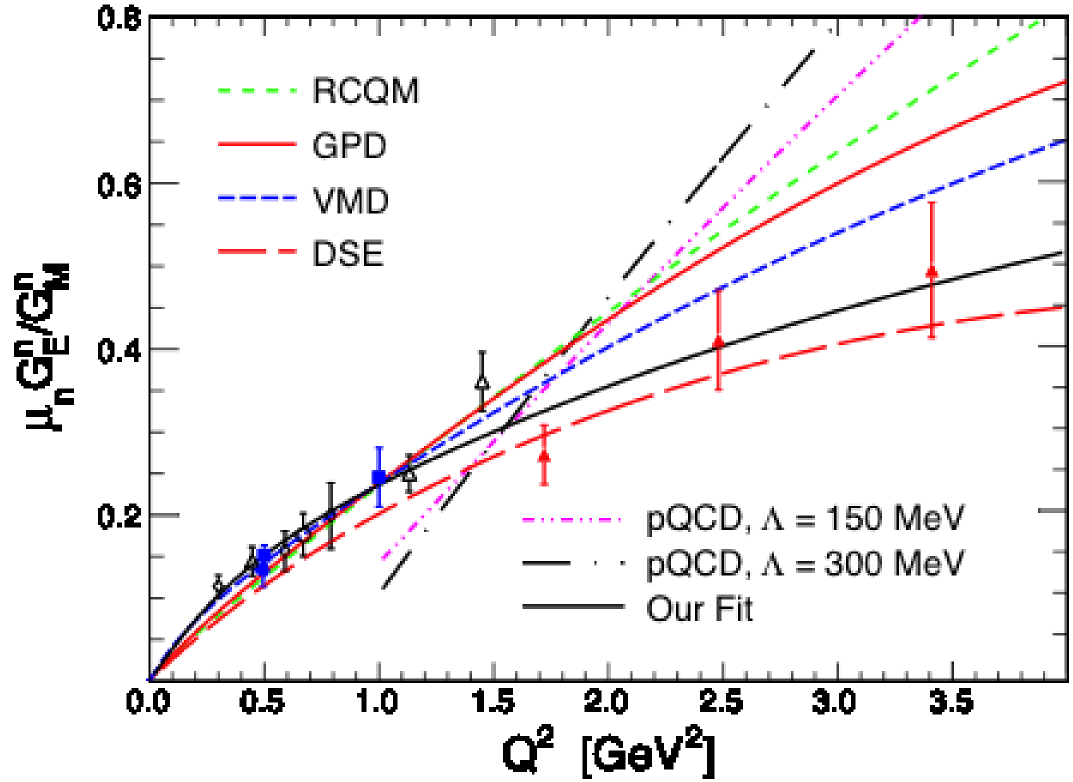
$$r_Z = -\frac{4}{\pi}\int \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{1+\kappa_p} - 1 \right]$$

De Rujula, PLB 693, 555 (2010); PLB 697 26 (2010)
Bernauer et al. PLB 696, 343 (2011)
Cloet & Miller, PRC 83, 012201 (2011)
Jentschura, EPJD 61, 7 (2011)
Barger et al., arXiv: 1011.3519
Tucker-Smith and Yavin, arXiv: 1011.4922
Miller et al., arXiv: 1101.407

G_E^n from polarized ^3He target: $^3\overrightarrow{He}(\vec{e},e'n)$



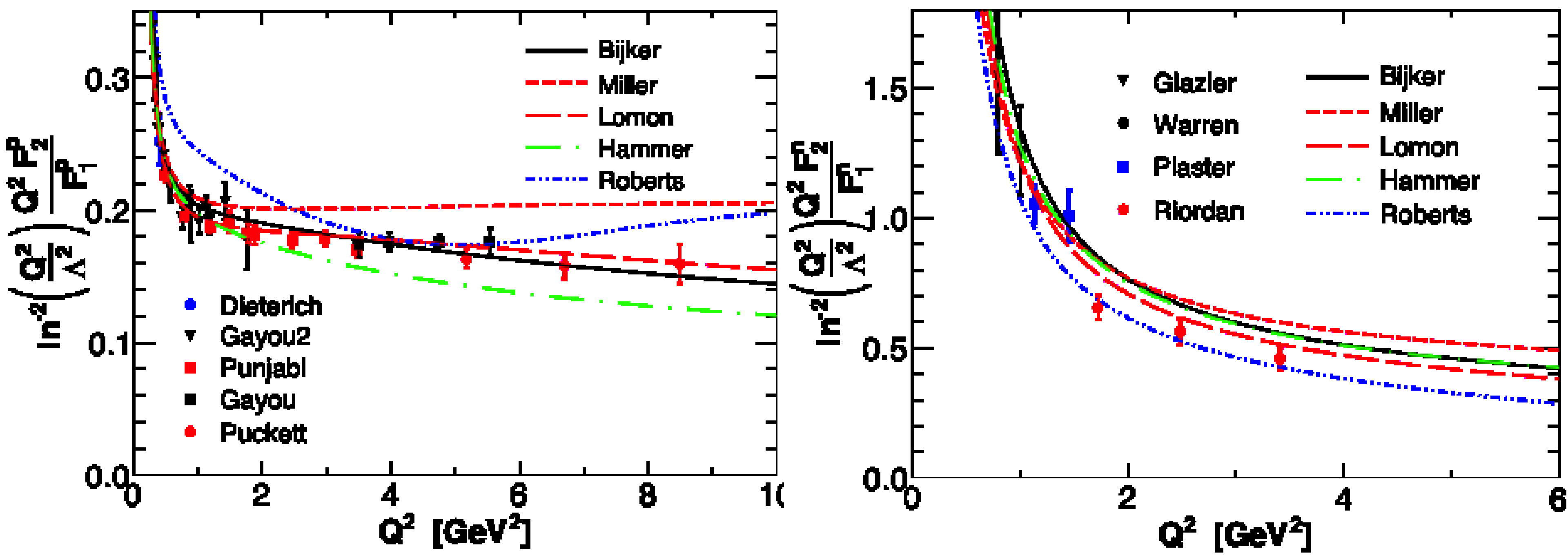
S. Riordan et al., PRL 105, 262302 (2(



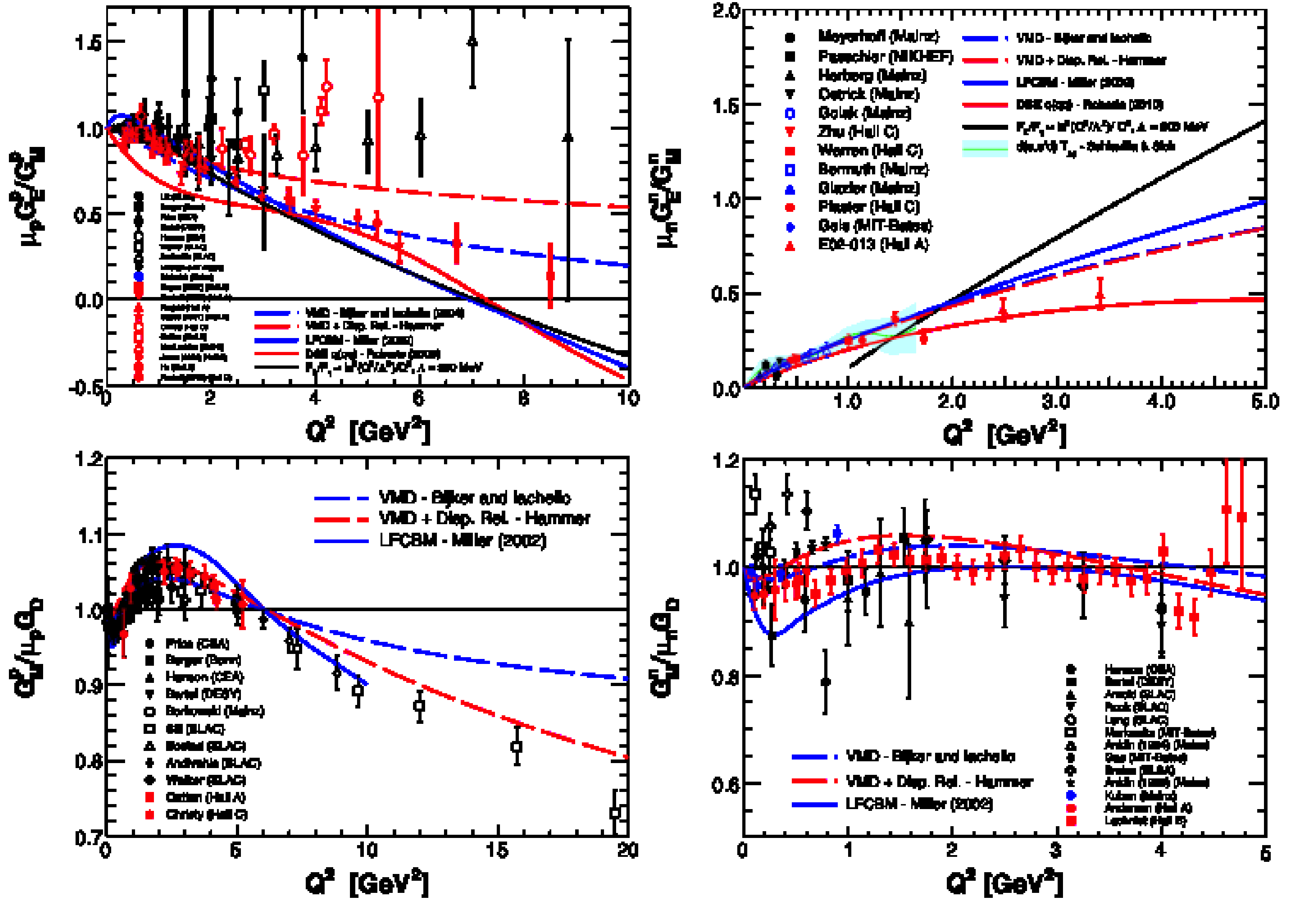
- New data more than double the Q^2-range of the world data set
- Roberts' dressed quark-diquark model using the Dyson-Schwinger and Faddeev equations in good agreement, better than Miller's CQM prediction
- Belitsky/Ji logarithmic scaling does not hold for the neutron in the Q^2-region where it was validated by the proton data
- New data will add significant constraints to GPD modeling

(Logarithmic) Scaling

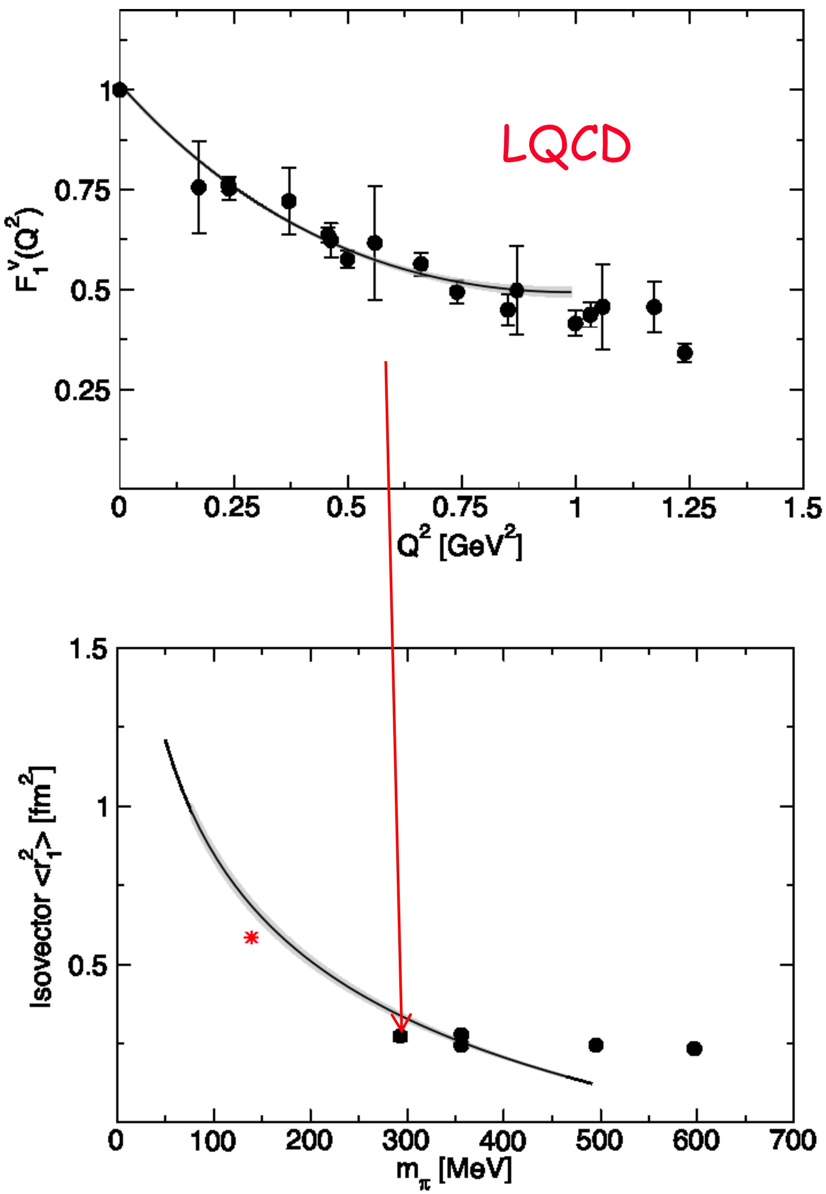
- Basic pQCD scaling predicts $F_1 \propto 1/Q^4$; $F_2 \propto 1/Q^6 \rightarrow F_2/F_1 \propto 1/Q^2$
- Data clearly do not follow this trend (yet?)
- The introduction of a quark orbital angular momentum component results in
  - $F_2/F_1 \propto 1/Q$
- Belitsky et al. have included logarithmic corrections in pQCD limit
- Proton data appear to follow this scaling behaviour, but new neutron data do not

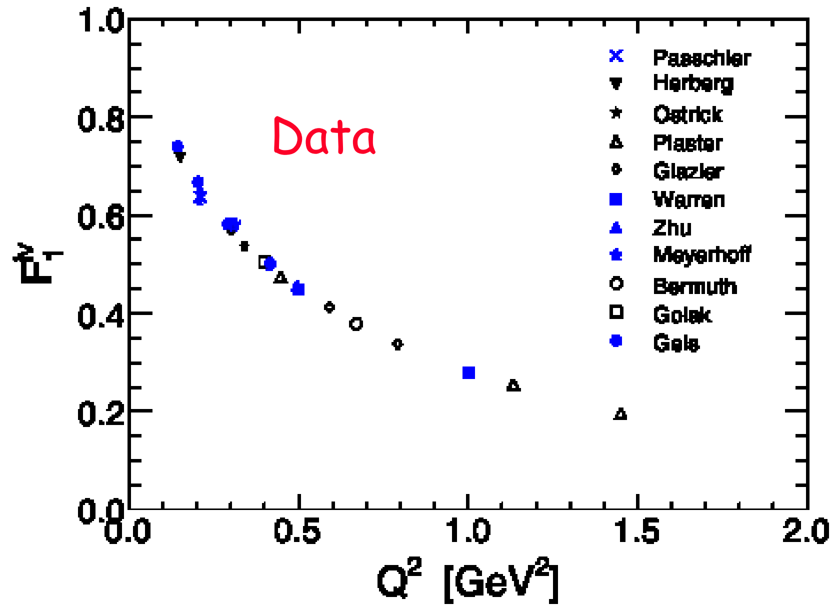


Comparison with Theory



Status of Lattice QCD





Significant progress in LQCD, but still limited to $m_\pi \geq 300$ MeV and neglect of disconnected diagrams, resulting in large underestimates of e.g. isovector charge radius

Bratt et al., arXiv: 1001.3620

Nucleon densities and relativity

$$\rho(r) = \frac{2}{\pi}\int_0^\infty dk\, k^2\, j_0(k\,r)\,\tilde{\rho}(k)$$

rest frame

rest frame density

intrinsic FF

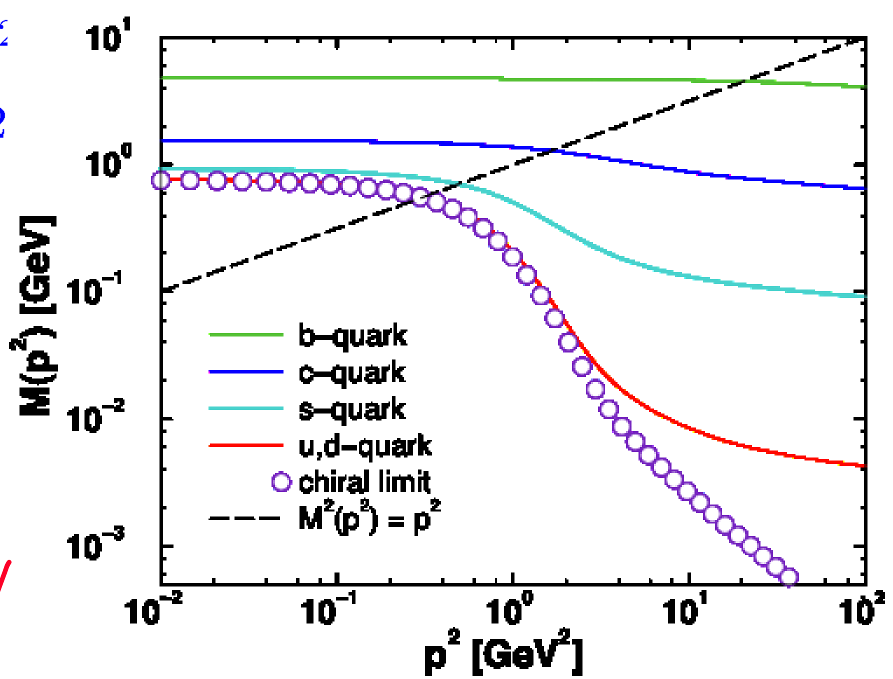
Q²-evolution of quark mass (nucl-th/9812063)

- non-relativistic limit: $\tilde{\rho}(k) = G(Q^2)$
- importance of relativity (with increasing Q^2) : Lorentz contraction of spatial distributions in Breit frame

$$k^2 = Q^2/(1+\tau) \qquad \tau = Q^2/(4M^2)$$

$$\tilde{\rho}_{E,M}(k) = G_{E,M}(Q^2)(1+\tau)^2$$

- limit : k = 2 M (Compton wavelength) Thus, Fourier transform remains valid for $\delta r > r_{min} \approx 0.3$ fm

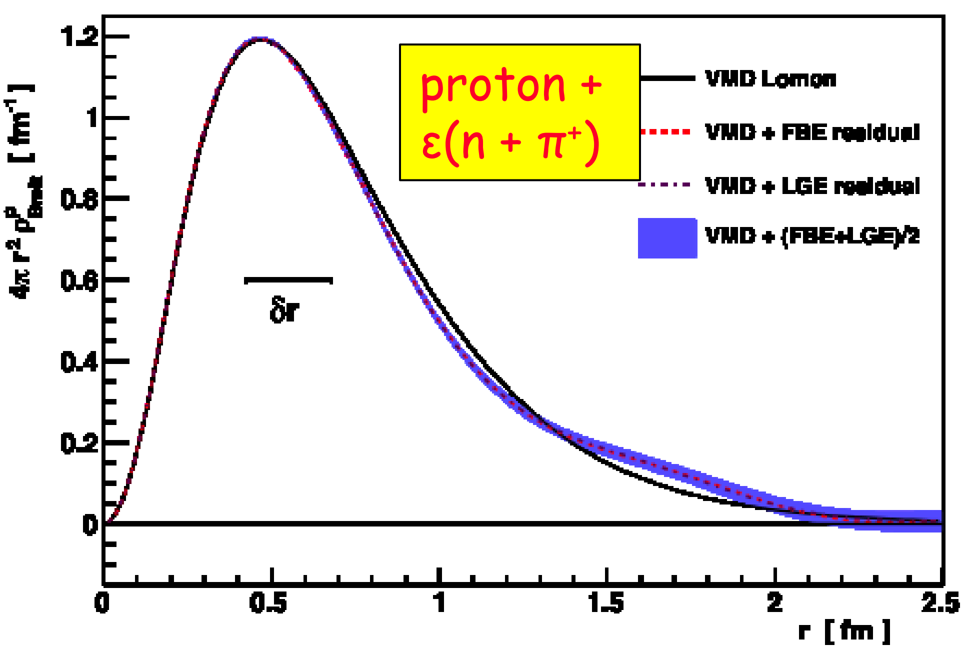


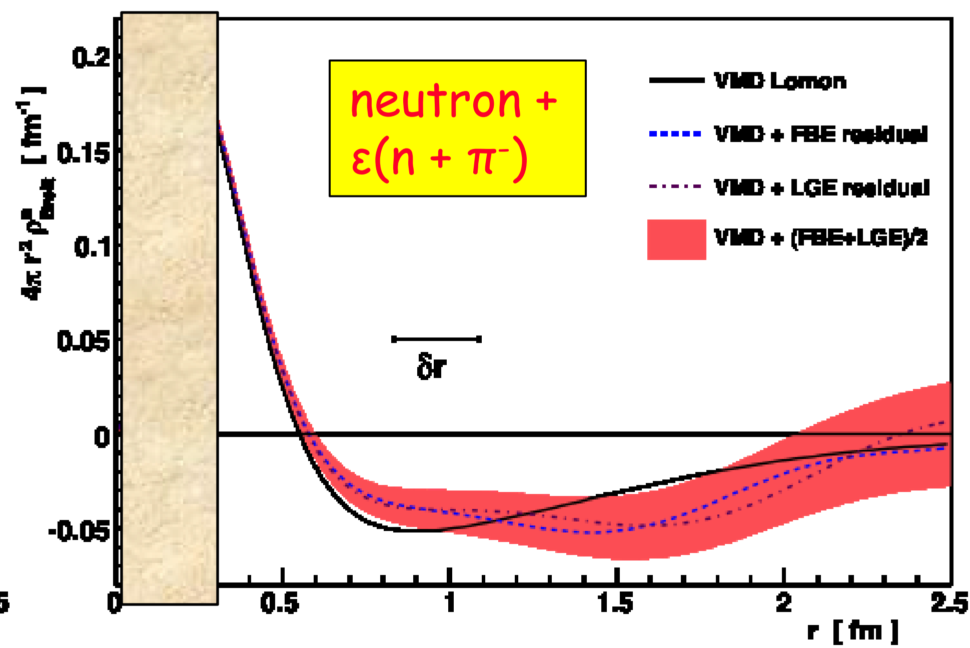
At Q ≈ 0.6 GeV (r ≈ 0.3 fm) $m_{u/d} \approx 0.3$ GeV

Pion Cloud

- Crawford et al. performed a global fit to all four EMFF within the framework of Lomon's VMD parametrization, including an estimate of the unmeasured high-Q^2 region. They observe a structure in the proton and neutron densities at 1-2 fm (which they assign to a pion cloud) in a straight-forward transformation to coordinate space (shown below)
- As shown in the previous slide relativistic effects obscure any radial fine structure at a scale smaller than ~0.3 fm-1, implying that no quantitative information can be extracted in the rest frame

C. Crawford et al., arXiv:1003.0903





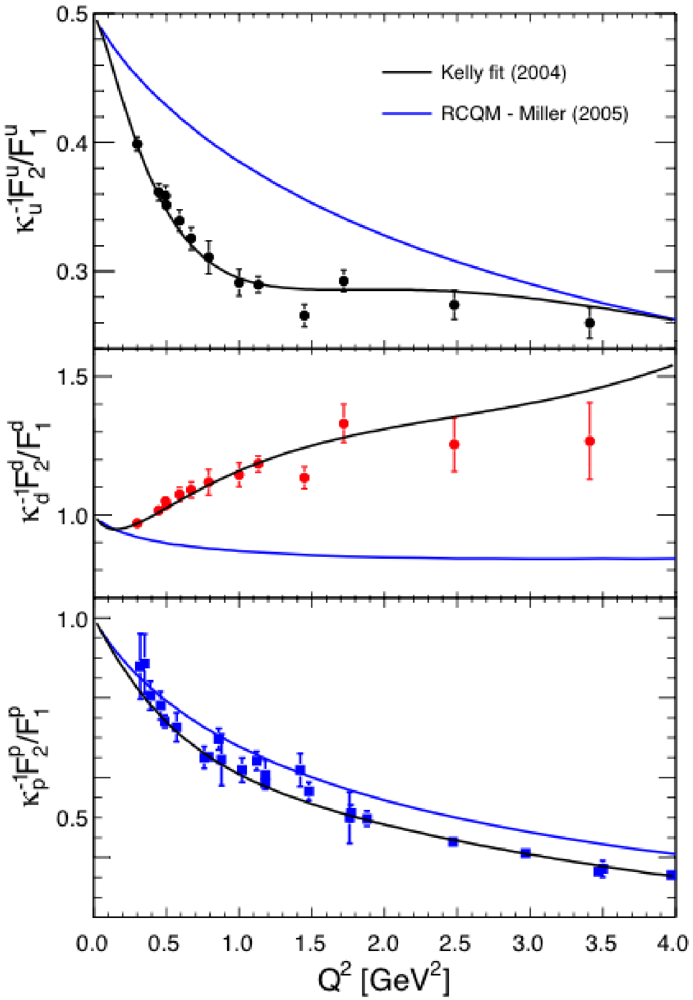
$F_{1,2}$ form-factor decomposition

$$F^{p}_{1,2} = \frac{2}{3}F^{u}_{1,2} - \frac{1}{3}F^{d}_{1,2}; \qquad F^{n}_{1,2} = \frac{2}{3}F^{d}_{1,2} - \frac{1}{3}F^{u}_{1,2}$$

assuming isospin symmetry: $F^{u,p}_{1,2} = F^{d,n}_{1,2}$ and $F^{d,p}_{1,2} = F^{u,n}_{1,2}$

G. Cates et al., arXiv: 1101.1808

- Assuming that the s-quark contribution is negligible (based on the PVe results)



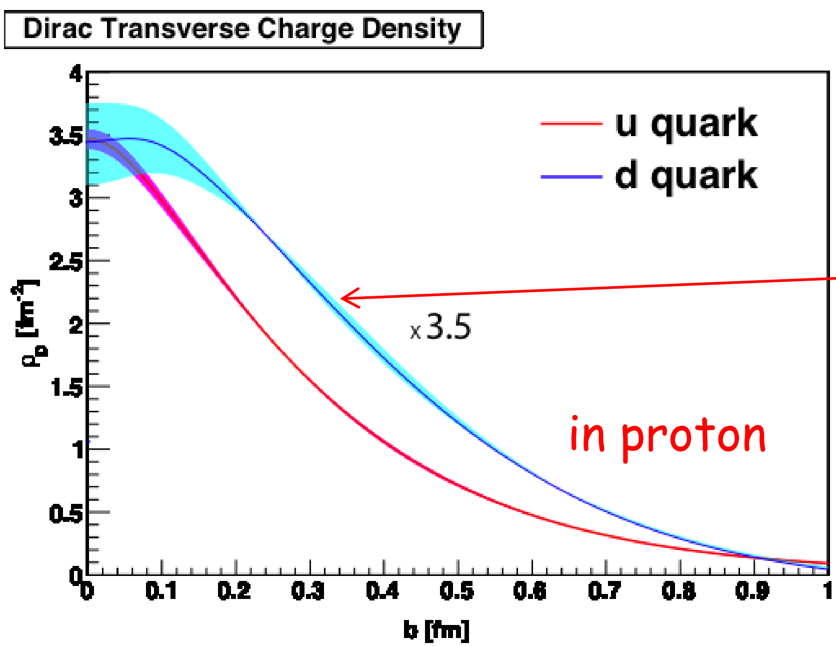
- Shown are the results for (u,d) in the proton
- The ratio F_2/F_1 appears to become constant for both constituents from ~ 1.5 GeV^2, in contrast even to the expectation for that ratio for each nucleon to scale with $1/Q^2$, at least in the pQCD limit (this scaling has not – yet – been observed)
- Constituent Quark Model is unable to describe this behaviour

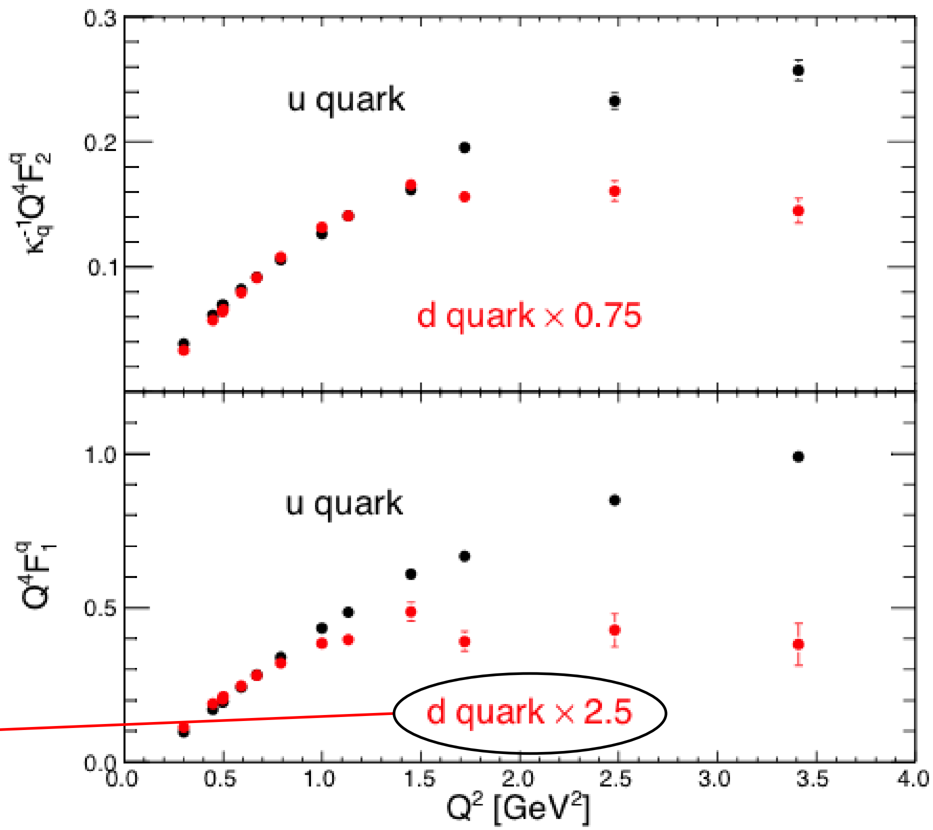
Mapping of nucleon constituents (in the proton)

Impact parameter *b* is defined relative to the transverse center of the quark's longitudinal momentum fractions

$$R = \sum x_i r_i$$

$$\rho_{Dirac}(b) = \int_0^\infty \frac{QdQ}{2\pi} J_o(bQ) F_1(Q^2)$$



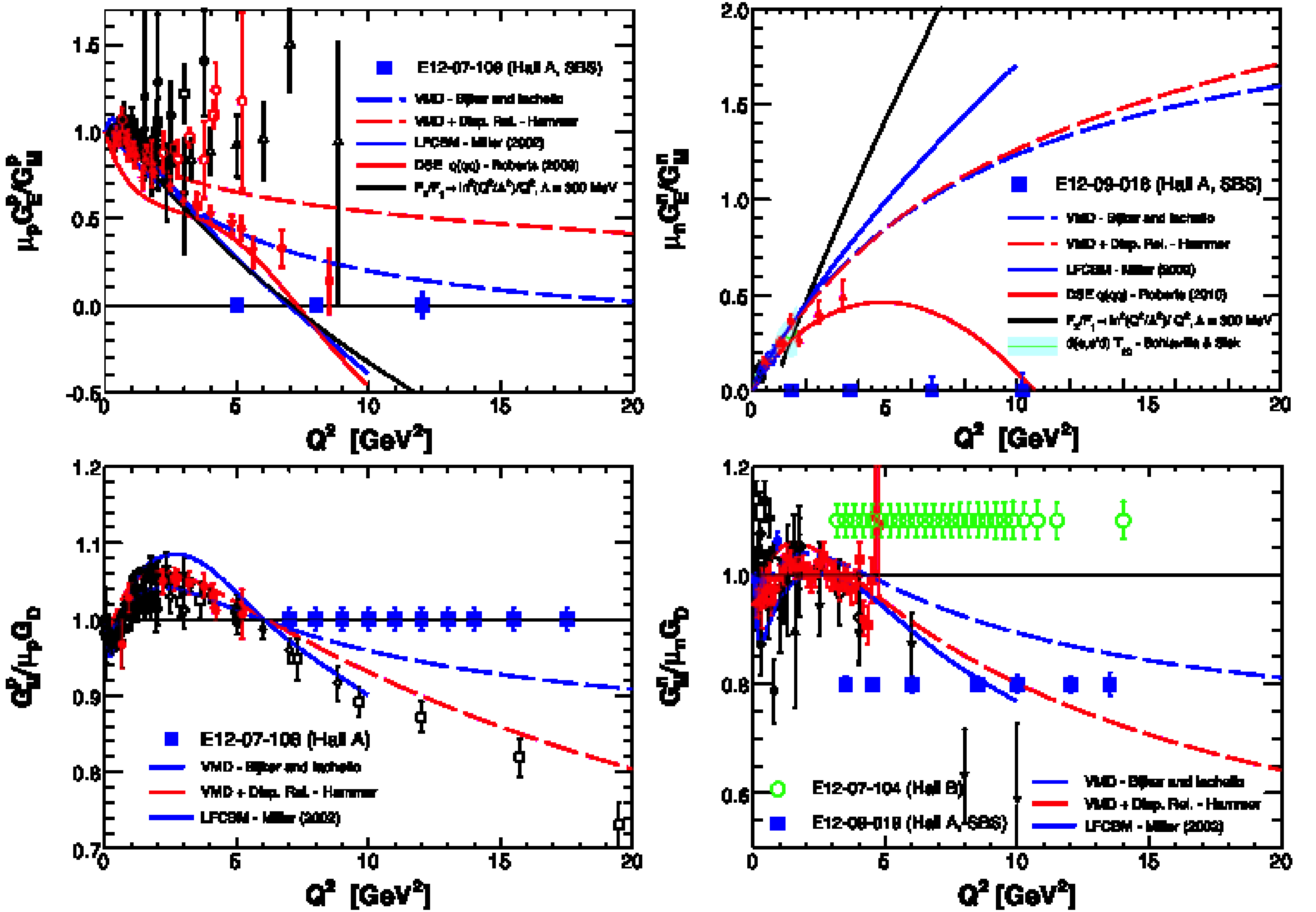


→ Why is the d-quark so much wider?

→ The individual behaviour of the FFs for each constituent differs strikingly

→ The flavor-separated F_1 and F_2 ratios were then used to extract the transverse densities for the u- and d-quark (in the proton)

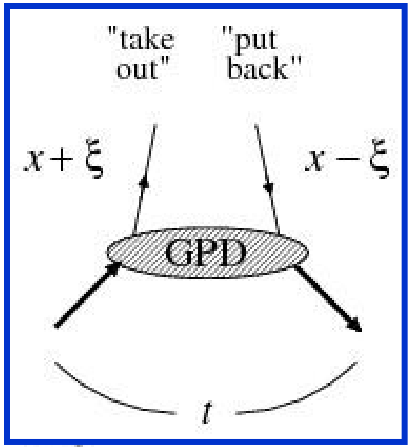
Projected EMFF data with SBS @ 12 GeV

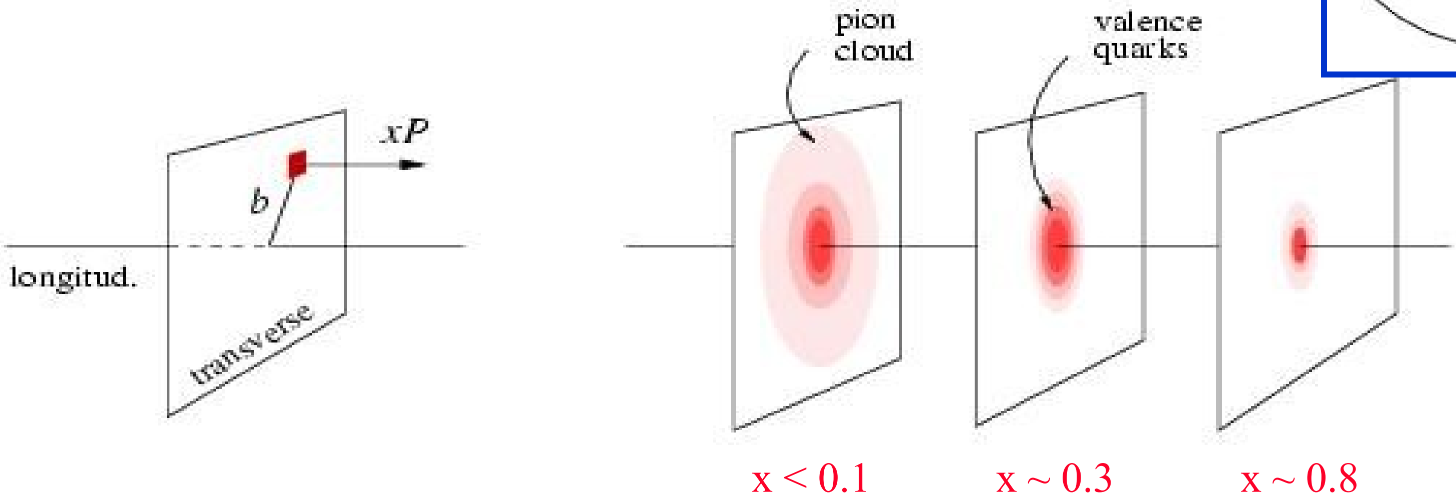


Impact of EMFF on GPDs

1. Allows for a unified description of
   form factors and parton distributions
2. Describes correlations of quarks/gluons
3. Allows for Transverse Imaging

Fourier transform in momentum transfer





gives transverse spatial distribution of quark (parton) with momentum fraction x and related to EMFFs through first moments

$$\sum_q e_q \int_{-1}^{1} dx H^q(x, \xi = 0, Q^2) = F_1(Q^2); \quad \sum_q \kappa_q \int_{-1}^{1} dx E^q(x, \xi = 0, Q^2) = F_2(Q^2)$$

4. Allows access to quark angular momentum (in model-dependent way)

Summary and Outlook

- Very active experimental program on nucleon electro-magnetic form factors thanks to development of polarized beam (> 100 μA, > 85 %), polarized targets and polarimeters with large analyzing powers at MAMI and JLab
  - → G_E^p discrepancy between Rosenbluth and polarization transfer not an experimental problem, but probably caused by TPE effects
  - → Broad ongoing program to obtain quantitative information on TPE
  - → Strong discrepancy with muonic result on proton charge radius
  - → G_E^n precise data up to Q^2 = 3.5 GeV^2 provides strong indication that OAM has different effect on neutron than on proton
  - → New G_E^n data set has allowed a flavor separation of F_1 and F_2
  - → The SuperBigBite project, to be implemented once the JLab 12 GeV upgrade has been completed, will extend the present knowledge of the nucleon EMFF G_E^p, G_E^n and G_M^p to double or triple the Q^2-range covered by existing data
  - → It is imperative that this experimental program is accompanied by a similar progress in our theoretical understanding of the nucleon

THANK YOU !

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