

# Determining the $(D^0, \bar{D}^0) \rightarrow K_{S,L} K^+ K^-$ strong phase difference and the impact on the CKM angle $\gamma$

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On behalf of the CLEO-c and LHCb collaborations



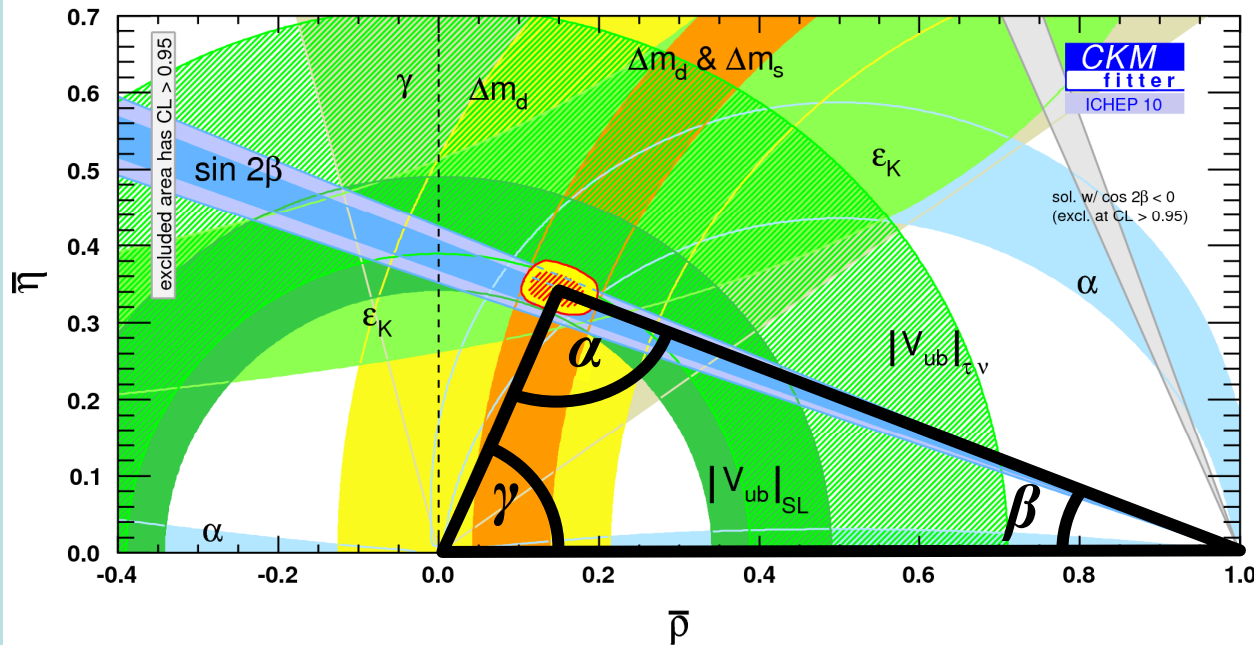
# Outline

- Determining  $\gamma$  using  $B^\pm \rightarrow (D^0, \bar{D}^0) K^\pm$
- The CLEO-c detector
- Measuring the strong phase difference between  $D^0$  and  $\bar{D}^0$  decays to  $K_{S,L} K^+ K^-$
- Impact on future  $\gamma$  measurements

# Determining $\gamma$ using $B^\pm \rightarrow (D^0, \bar{D}^0) K^\pm$

# Unitarity triangle

- The  $b$ - $d$  unitarity triangle of the CKM matrix



$$\alpha = (89.0^{+4.4}_{-4.2})^\circ$$

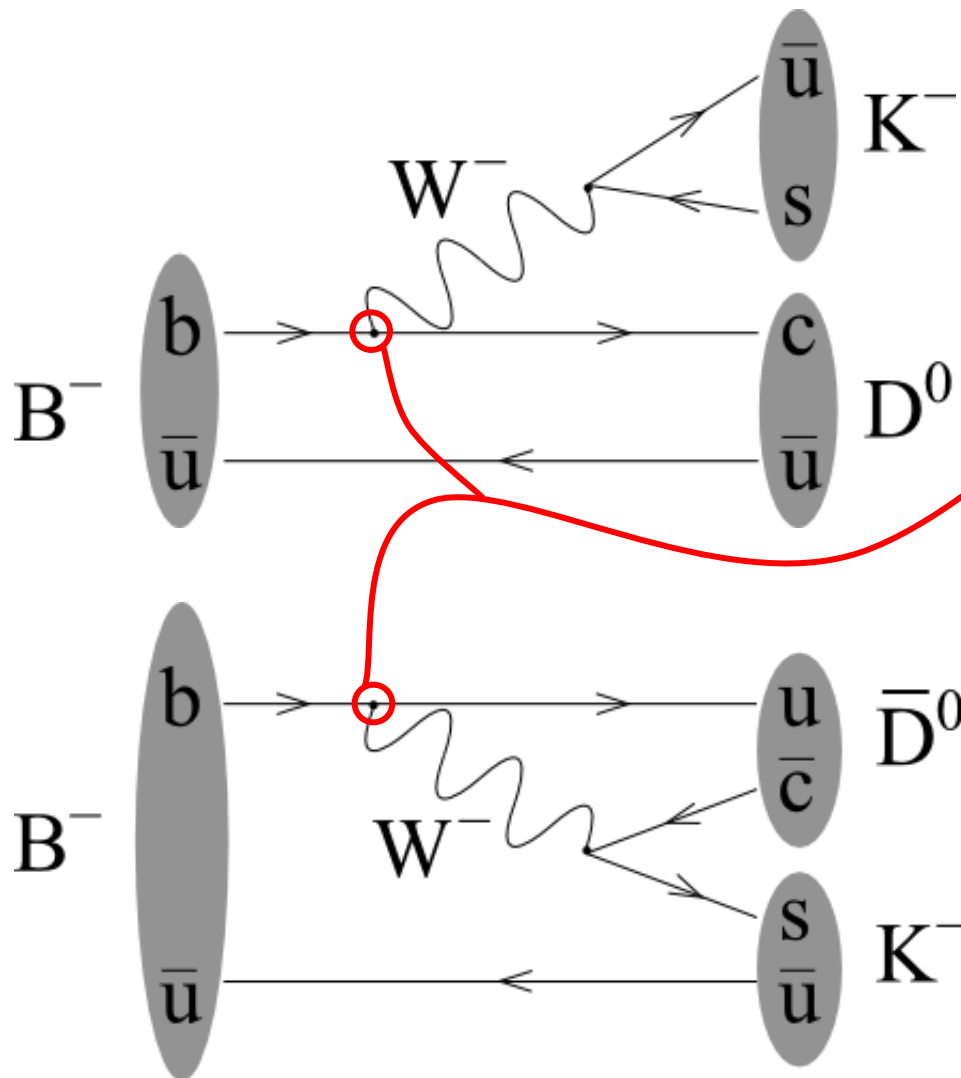
$$\beta = (21.15^{+0.90}_{-0.88})^\circ$$

$$\gamma = (71^{+21}_{-25})^\circ$$

CKMfitter: <http://ckmfitter.in2p3.fr>

- Results shown are **direct** angle measurements

# Determining $\gamma$ from $B^\pm \rightarrow (D^0, \bar{D}^0) K^\pm$



$$\gamma \equiv \arg \left( -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right)$$

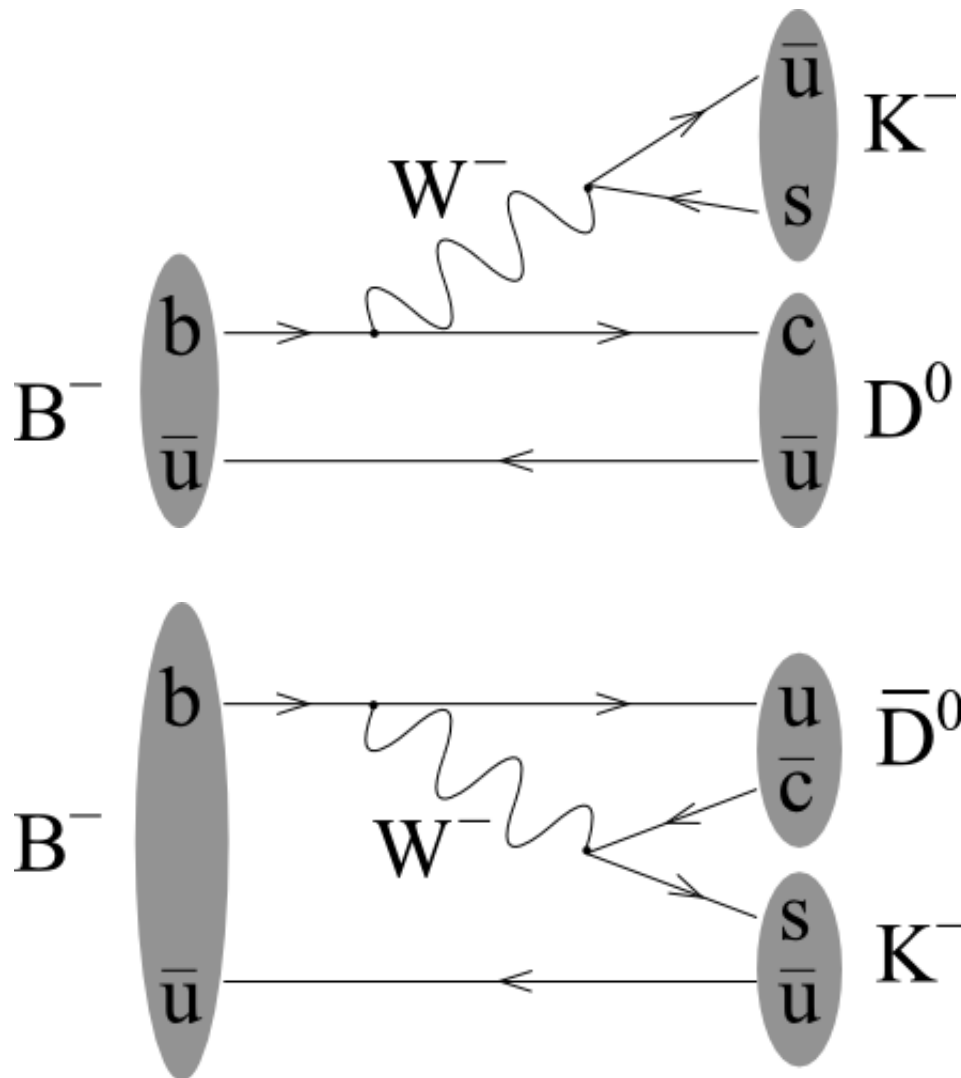
Amplitude ratio  
between  $b \rightarrow \bar{u}cs$  and  
 $b \rightarrow u\bar{c}s$  transitions:

$$r_B e^{i(\delta_B - \gamma)} \equiv \frac{\mathcal{A}(B^- \rightarrow \bar{D}^0 K^-)}{\mathcal{A}(B^- \rightarrow D^0 K^-)}$$

CKM and colour

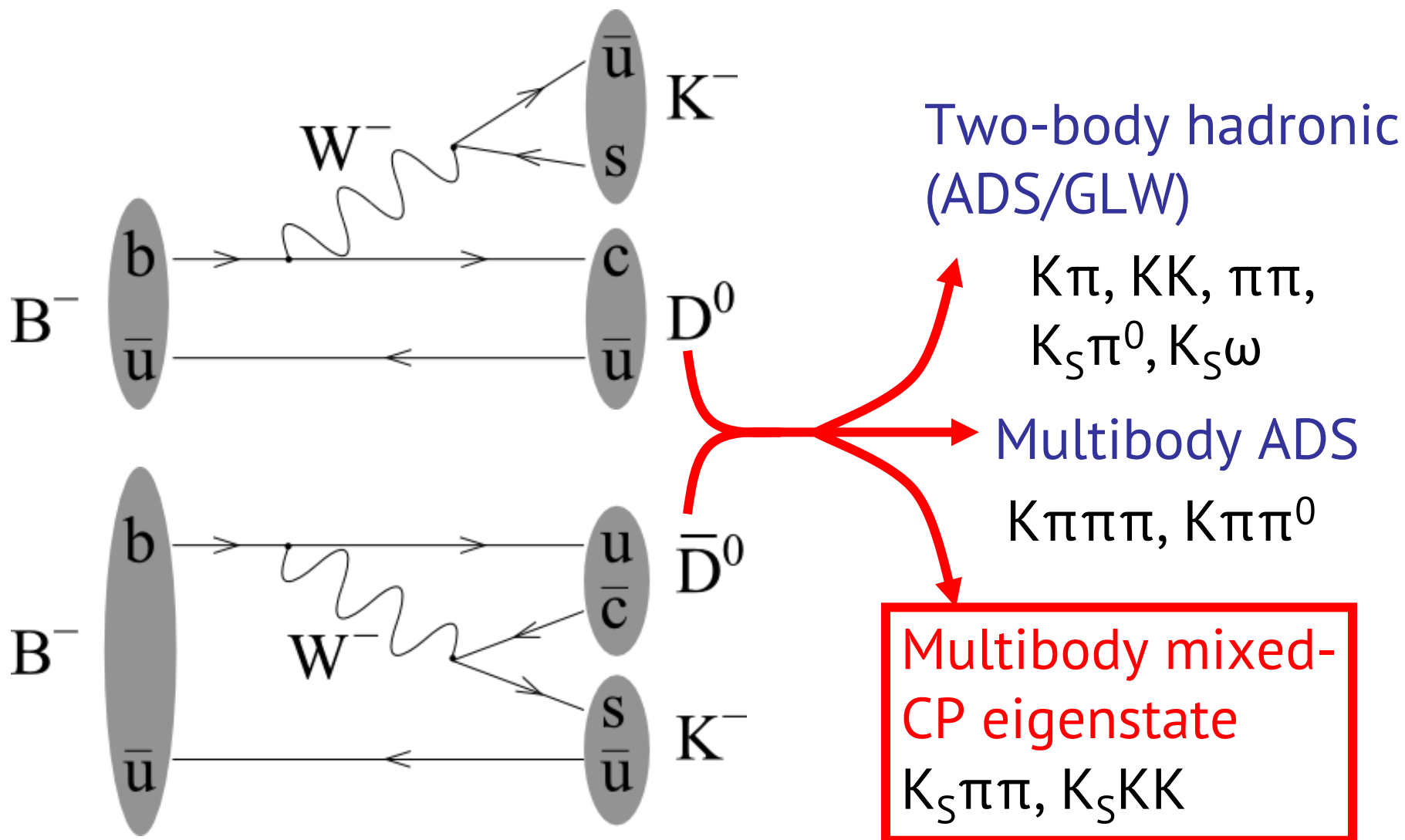
suppression:  $r_B \sim 0.1$  <sup>5</sup>

# Determining $\gamma$ from $B^\pm \rightarrow (D^0, \bar{D}^0) K^\pm$

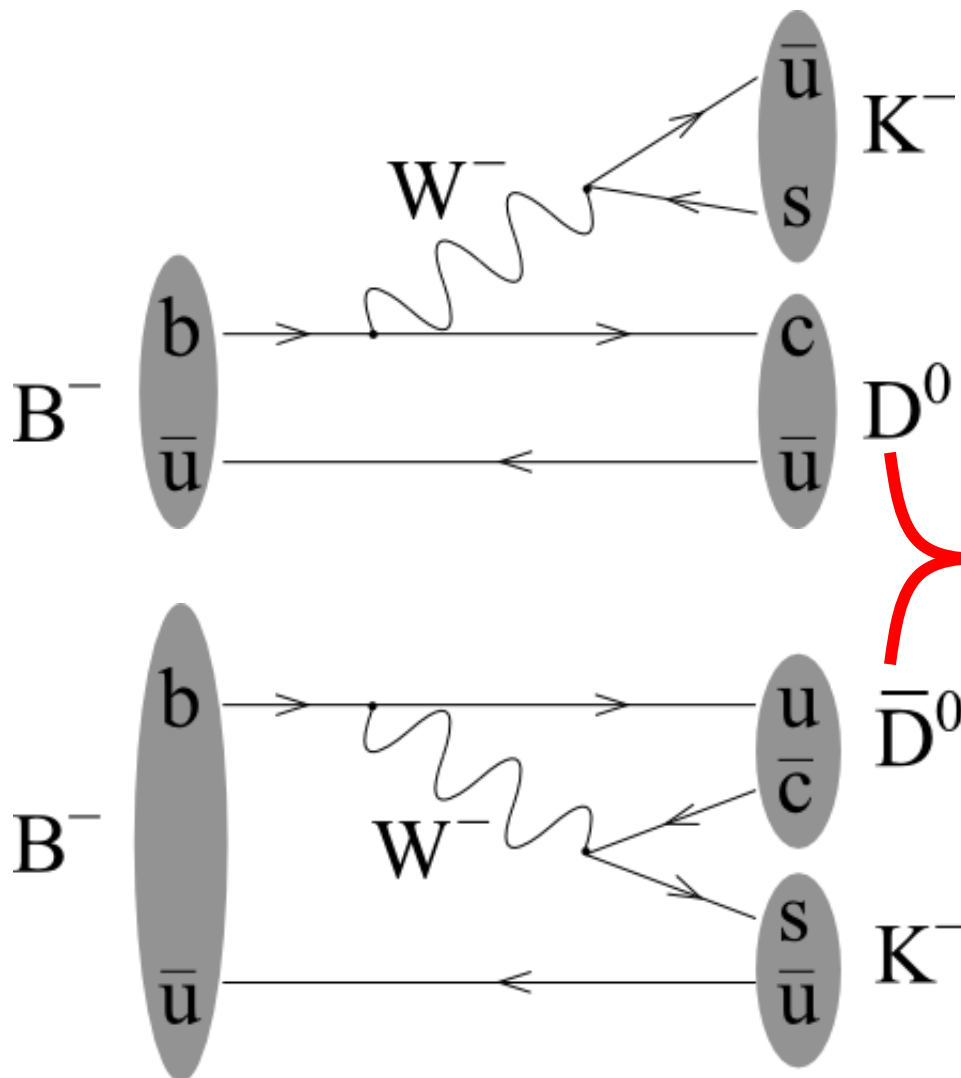


$D^0$  and  $\bar{D}^0$  must decay to common final state to provide interference and hence sensitivity to  $\gamma$

# $D^0$ final states



# Importance of charm physics



$D^0$  decay parameters must be precisely determined: charm physics input is vital

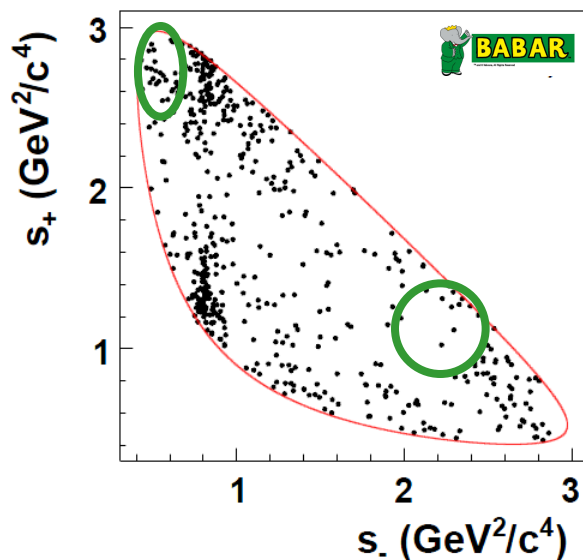
Multibody mixed-CP eigenstate  
 $K_S \pi \pi$ ,  $K_S K K$



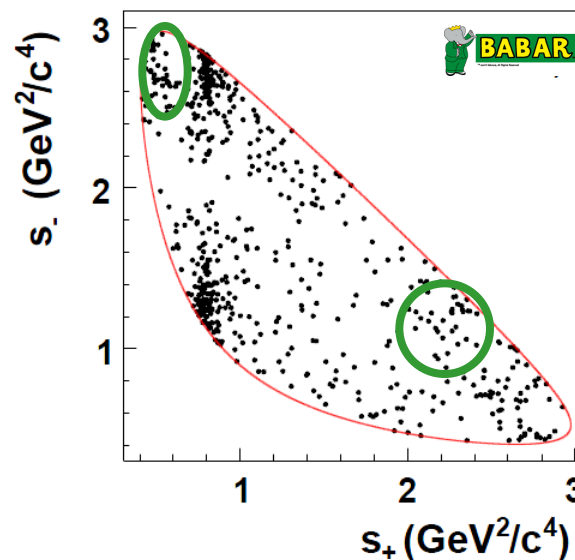
# Dalitz analysis of $B^\pm \rightarrow (K_S hh)_D K^\pm$

# $B^\pm \rightarrow (K_S hh)_D K^\pm$

- Three body final state – analyse  $K_S hh$  Dalitz plot
- CP violation is manifested as an **asymmetry** in event **distributions** across the plot



$B^- \rightarrow D(K_S \pi \pi) K^-$



$B^+ \rightarrow D(K_S \pi \pi) K^+$

Phys. Rev D **78**, 034023 (2008)

# Modelling B decays

- Define the Dalitz plot coordinates  $(x, y)$ :

$$x \equiv m_{K_S^0 h^+}^2 \quad y \equiv m_{K_S^0 h^-}^2$$

- Define the  $D^0 \rightarrow K_S h h$  decay amplitude:

$$f_{D^0}(x, y) \equiv A_{xy} e^{i\delta_{xy}} = \mathcal{A}(D^0 \rightarrow K_S^0 h^+ h^-)(x, y)$$

- Then the  $B^-$  decay amplitude is:

$$\mathcal{A}(B^- \rightarrow (K_S^0 h^+ h^-)_D K^-) \propto f_{D^0}(x, y) + r_B e^{i(\delta_B - \gamma)} f_{D^0}(y, x)$$

$\gamma$  dependence

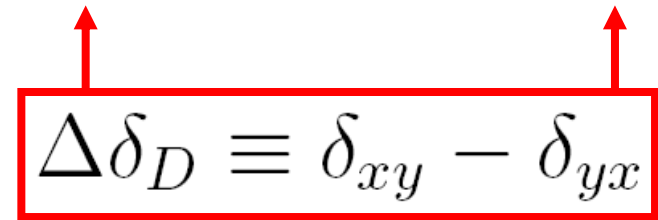


# Modelling B decays

- The corresponding decay rate is:

$$\Gamma(B^- \rightarrow (K_S^0 h^+ h^-)_D K^-) \propto A_{xy}^2 + r_B^2 A_{yx}^2 + 2r_B A_{xy} A_{yx} [\cos(\delta_B - \gamma) \cdot \cos(\Delta\delta_D) - \sin(\delta_B - \gamma) \cdot \sin(\Delta\delta_D)]$$

- Strong phase difference

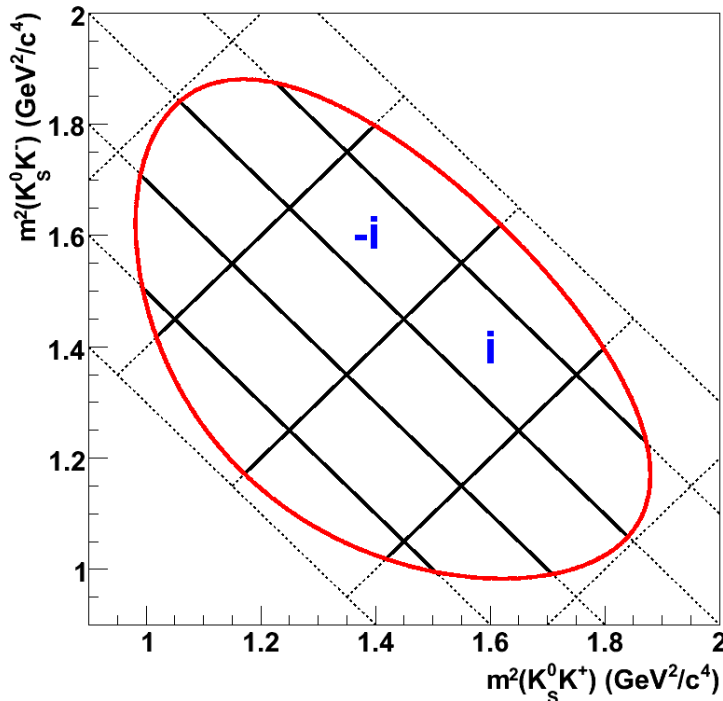

$$\Delta\delta_D \equiv \delta_{xy} - \delta_{yx}$$

- To measure this rate, can:
  - Determine all parameters from  $B^\pm$  decays
  - Fit a **model** to a flavour-tagged  $D^0$  decay sample (unbinned)
  - Determine  $D^0$  decay parameters in **bins** across Dalitz plane
- An example of how **charm physics input** is important

# Binning the Dalitz plot

Phys. Rev. D **68**, 054018 (2003)

- Instead of using a model of the  $D^0$  decays, divide the Dalitz plot into **bins** and measure yields in each bin



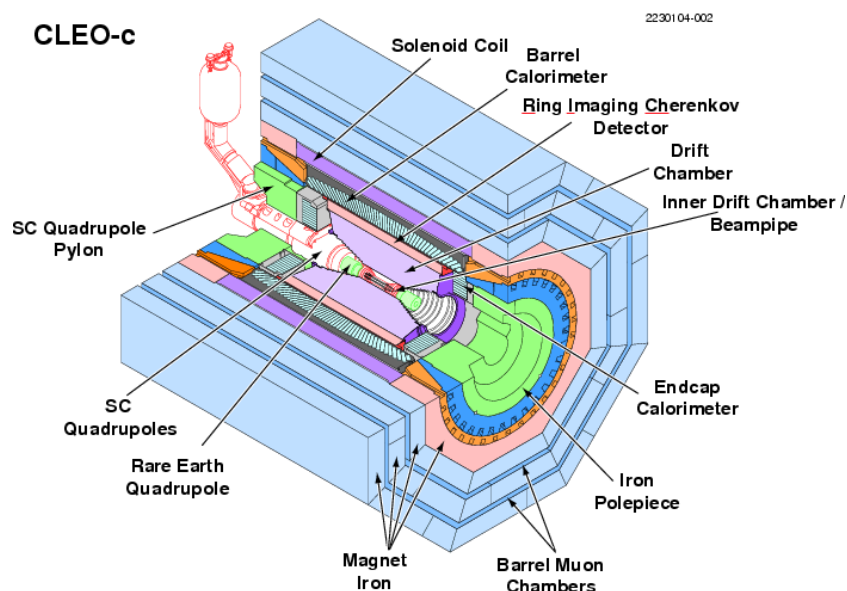
- Number of events in  $i^{\text{th}}$  bin,  $N_i$ , depends on weighted average **cosine** ( $c_i$ ) and **sine** ( $s_i$ ) of strong phase difference  $\Delta\delta_D$
- Example: rectangular partitioning of  $K_S K K$  Dalitz plot
- **No model error** but increased statistical error
- Measure at CLEO-c using **quantum correlated** events

# CLEO-c analysis of $D^0 \rightarrow K_S hh$

Phys. Rev. D **82**, 112006 (2010)

# Brief overview of CLEO-c

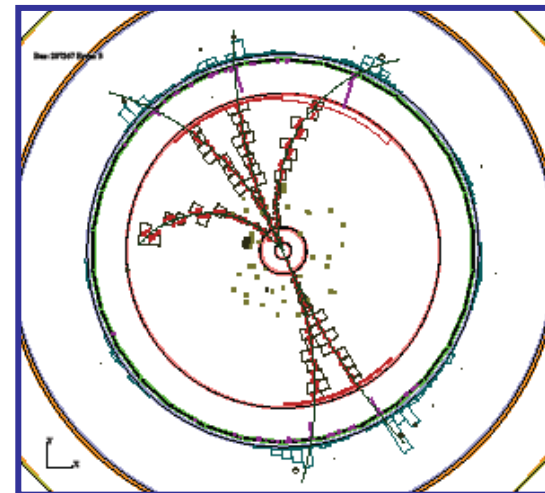
- Symmetric general-purpose detector located at Cornell Electron Storage Ring (CESR)



- $e^+e^-$  collider
- 93% coverage
- Excellent tracking and PID
- $818\text{pb}^{-1}$  @  $E_{\text{CoM}} = 3.77\text{GeV}$ 
  - $\psi(3770)$  resonance

# Brief overview of CLEO-c

- Threshold decay at  $\psi(3770)$ :
  - Very **clean** environment
  - High reconstruction efficiency
  - Can reconstruct 'missing' particles ( $K_L$ ,  $\nu$ )
- $\psi(3770)$  decays to **quantum-correlated**  $D\bar{D}$  pair
- Determine quantum numbers (i.e. CP) of one D: have complete knowledge of quantum numbers of the other
- $CP(\psi(3770)) = -1$
- **CP tagging**





# CLEO-c $K_S hh$ event selections

- Selected  $K_S hh$  tagged with:

- CP eigenstates (e.g.  $KK$ ,  $\pi\pi$ ,  $K_S\pi^0$ )
- Flavour states (e.g.  $K\pi$ ,  $K\pi\pi$ ,  $K\pi\pi\pi$ )
- Mixed-CP eigenstates ( $K_S hh$ )

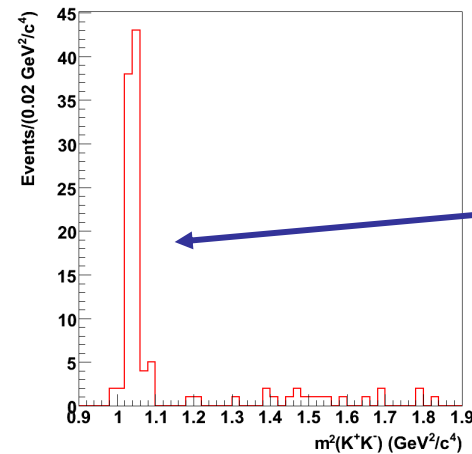
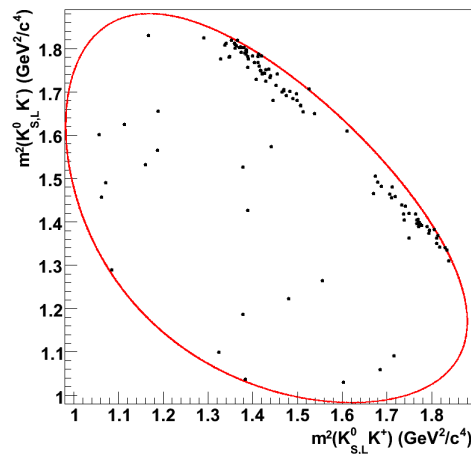
$K_S KK$	$K_S \pi\pi$
96	789
864	7634
200	473

- $BR(K_S\pi\pi)/BR(K_S KK) \sim 6.3$
- In addition to  $K_S hh$ , also select  $K_L hh$ 
  - Excellent detector & clean environment enables this
  - $CP(K_L hh) \sim -CP(K_S hh)$
  - More than doubles the dataset

# Quantum correlations at CLEO-c

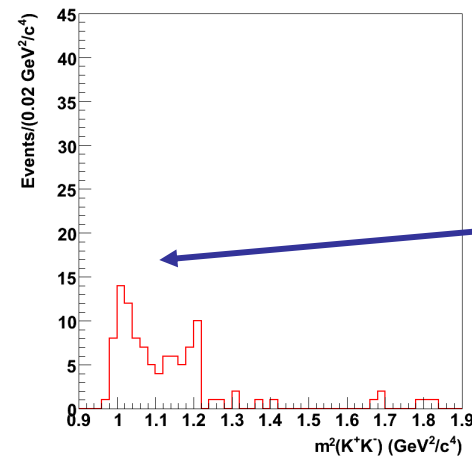
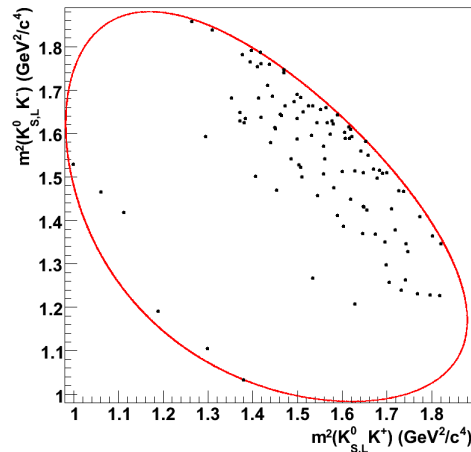
- Quantum correlations result in visible differences between different Dalitz plots
- Very striking difference seen in CP-tagged  $K_S KK$

$K_S KK$  vs CP+  
 $K_L KK$  vs CP-



Phys. Rev. D **82**, 112006 (2010)

$K_S KK$  vs CP-  
 $K_L KK$  vs CP+



4 April 2011

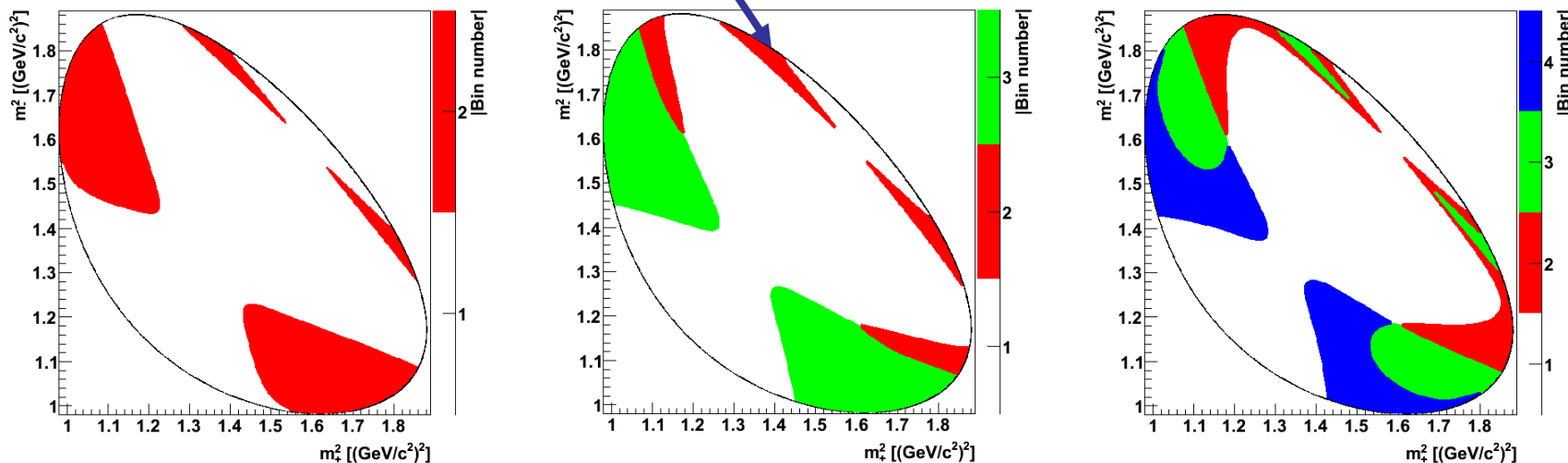
# Binning the Dalitz plane

- The Dalitz plane can be binned in any way desired
- Binning in regions of **similar strong phase difference** (predicted by the model) provides improved statistical precision Eur. Phys. J. C **47**, 347 (2006)
- Studied several binnings for both  $K_S\pi\pi$  and  $K_S K\bar{K}$

# Binning the $K_S K K$ Dalitz plane

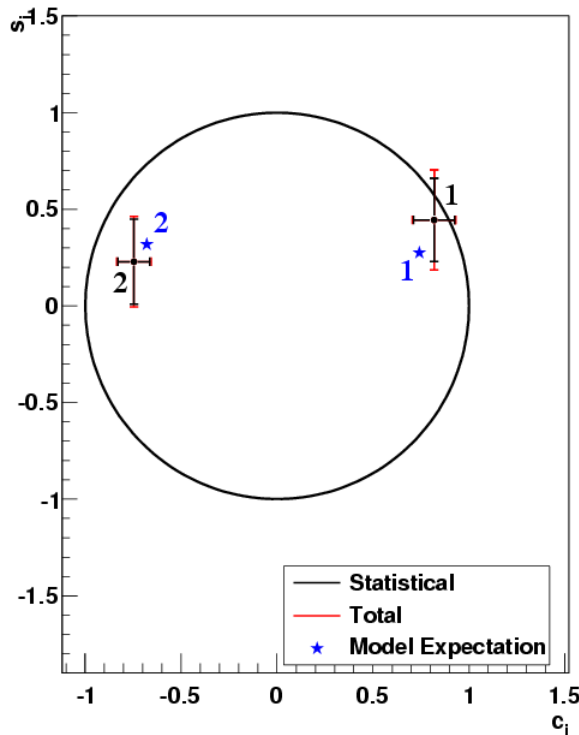
- $K_S K K$  binnings considered: 2, 3 and 4 bins
  - Relatively small number of events limits number of bins
  - Studying several binnings provides future flexibility

$K_S \phi$  resonance

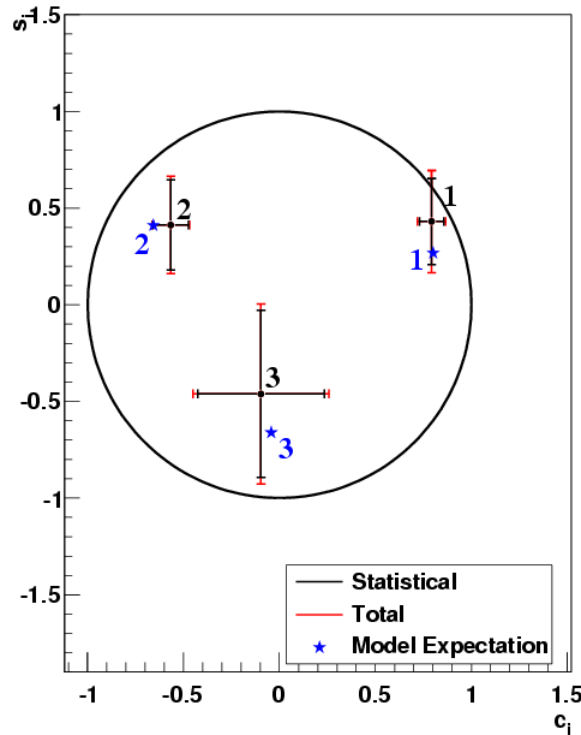


# $K_S K K$ ( $c_i, s_i$ ) results

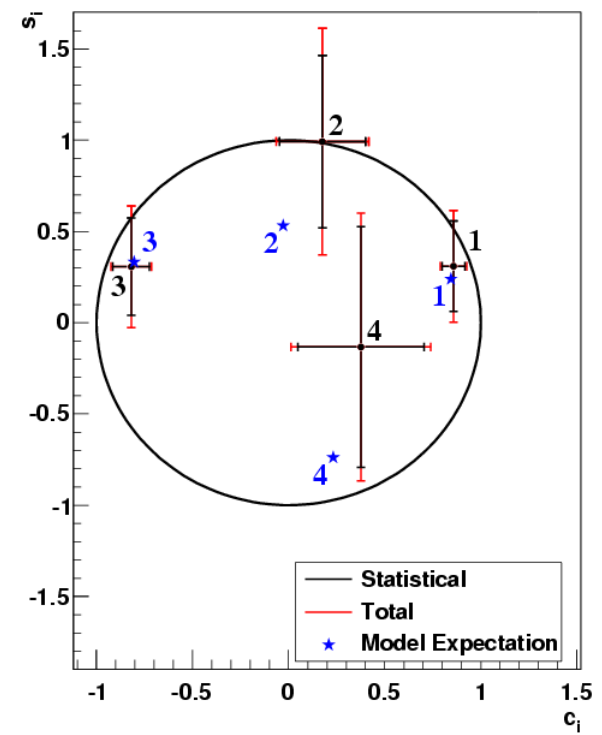
2 bins



3 bins



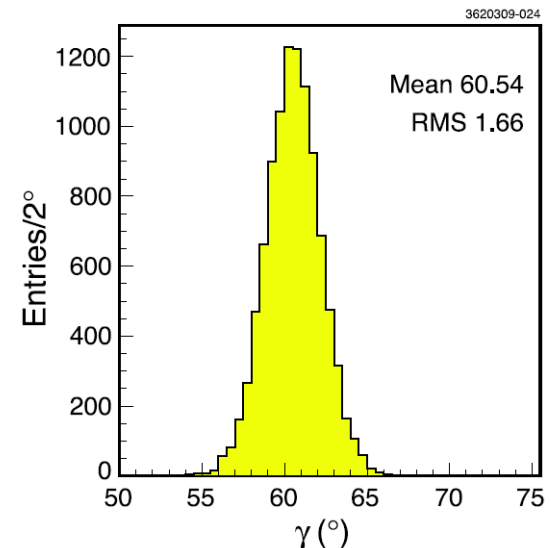
4 bins



- Good agreement between fit values and model
- Main systematic uncertainty from background estimation

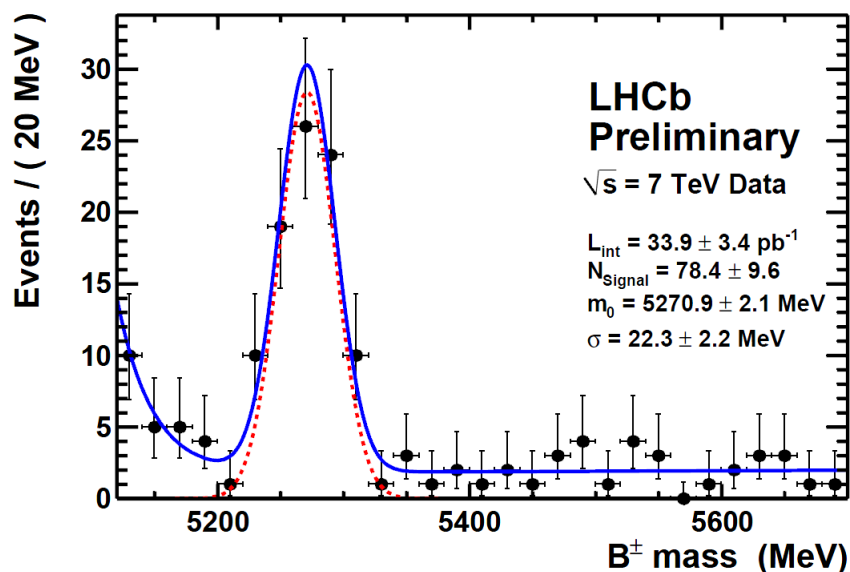
# Consequences for $\gamma$ measurement

- Toy MC experiments: generate 5M  $B \rightarrow DK$  events and determine impact of  $(c_i, s_i)$  on  $\gamma$  precision for different binnings
- Error on  $\gamma$  due to CLEO-c statistical uncertainty:
  - $K_S K K$ : **3.2–3.9°**
  - $K_S \pi \pi$ : **1.7–3.9°**
- Compare to 3–9° model uncertainty from B-factories
- **BES-III** will be able to reduce this even further

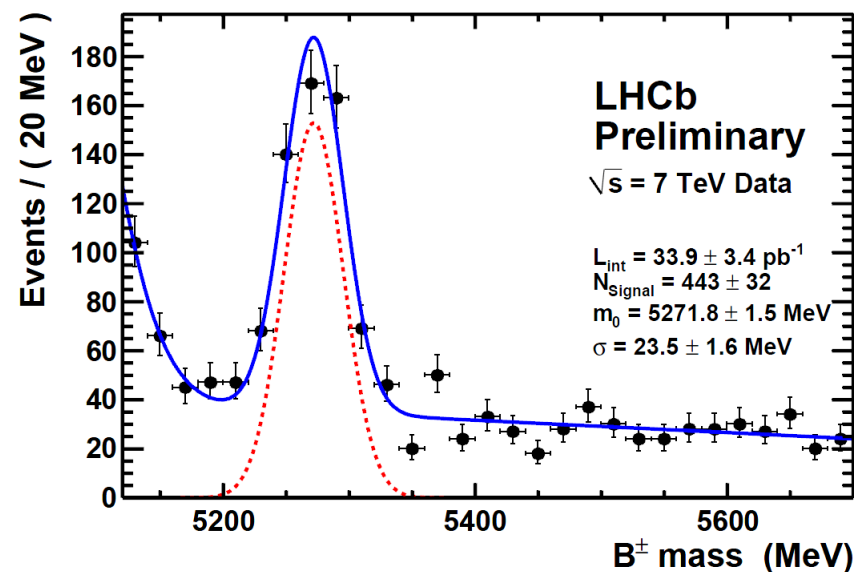


# LHCb selection of $B \rightarrow D(K_S hh)h$

- LHCb has observed  $B \rightarrow D(K_S hh)\pi$  in  $34\text{pb}^{-1}$
- This is a control mode for  $B \rightarrow DK$



$B \rightarrow D(K_S KK)\pi$



$B \rightarrow D(K_S \pi\pi)\pi$

- Excellent prospects for  $B \rightarrow D(K_S hh)K$  measurements

# Conclusions

- $B \rightarrow DK$  is a very promising way to determine  $\gamma$
- Input from charm physics is very important in order to precisely determine  $D^0$  decay parameters
- Recent model-independent CLEO-c analysis of  $D^0 \rightarrow K_S hh$  strong phase difference, including first measurement for  $K_S KK$  final state
- Projected consequences for  $\gamma$  measurement:  $1.7-3.9^\circ$  for  $K_S \pi\pi$ ,  $3.2-3.9^\circ$  for  $K_S KK$
- LHCb has selected  $B \rightarrow D(K_S hh)h$  events: very promising

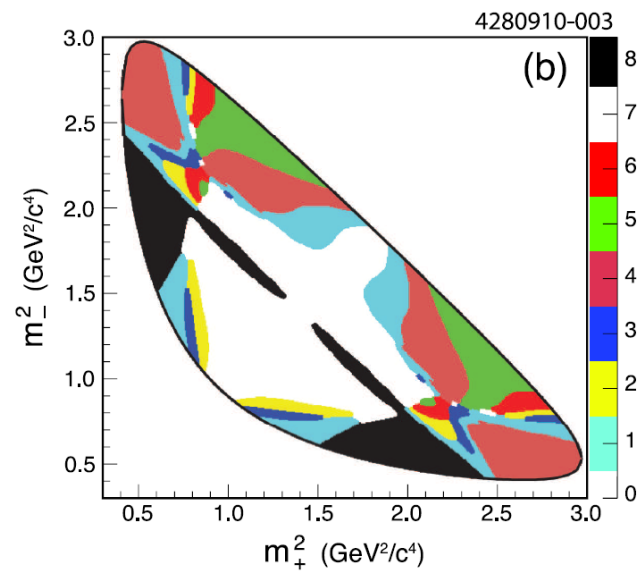
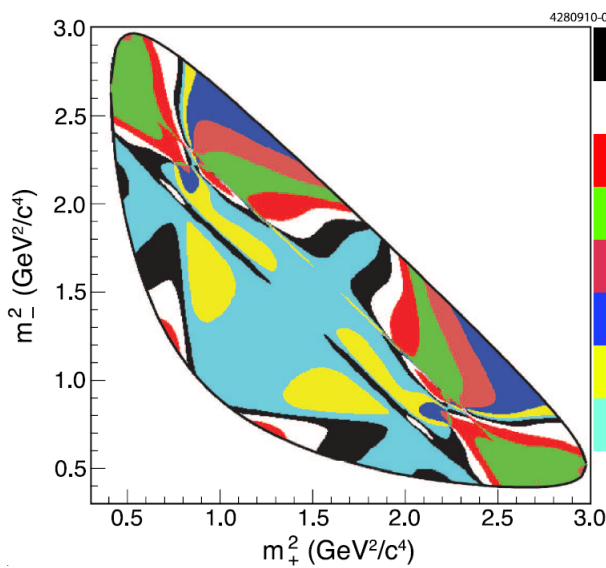


# Backups

# Binning the $K_S\pi\pi$ Dalitz plane

- Also updated  $K_S\pi\pi$  analysis (previously studied in 2008) by considering several different binnings
- Compare and contrast models from BaBar and Belle
- Also **optimised** binning by considering sensitivity given finite B-statistics *Eur. Phys. J. C* **55**, 51-56 (2008)
- Optimised binning for LHCb background expectation

Equal strong  
phase difference  
using BaBar  
model

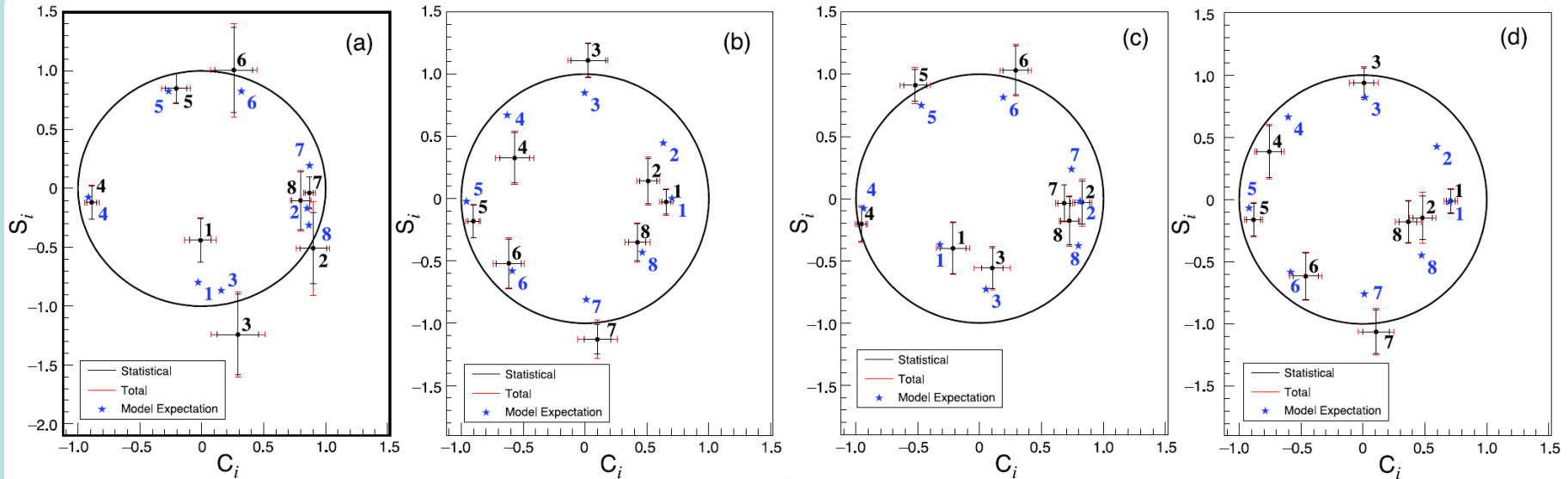


Optimised for  
finite B  
statistics

4 April 2011

# $K_S\pi\pi$ results

- Results for different binnings



**BaBar model,  
optimised for  
B-stats**

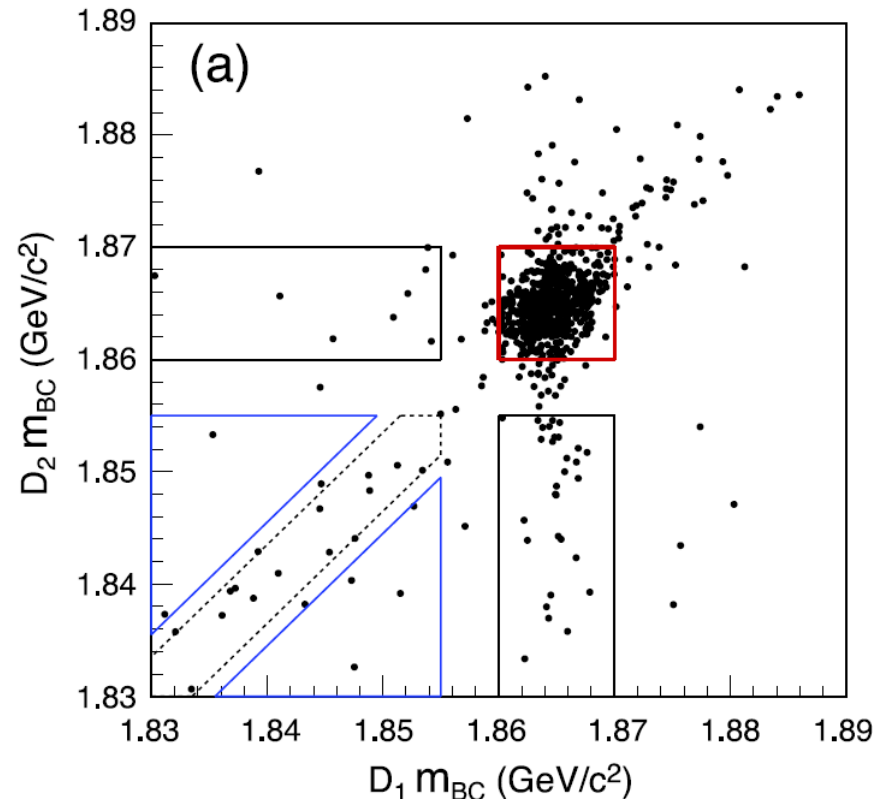
**BaBar model,  
equal  $\Delta\delta_D$**

**BaBar model,  
optimised for  
background**

**Belle model,  
equal  $\Delta\delta_D$**

# CLEO-c event selections

- $K_S K K$  vs flavour tags ( $K\pi$ ,  $K3\pi$ ,  $K\pi\pi^0$ ,  $K\pi\nu$ )



Data

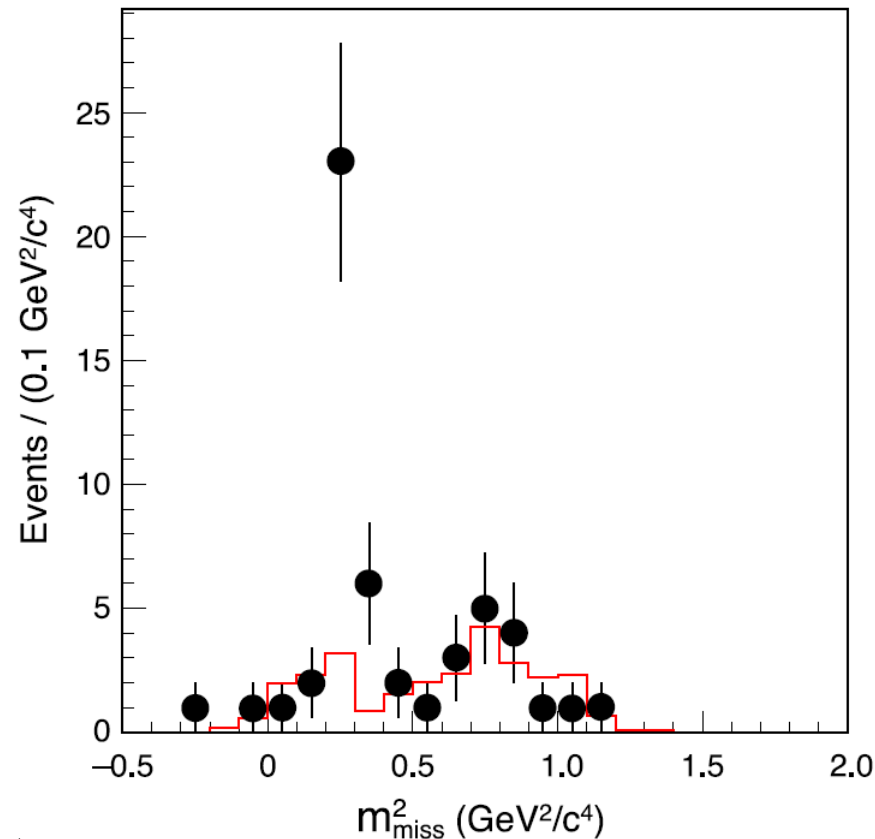
Signal region

Sidebands

- $m_{BC} = \text{beam constrained mass} = \sqrt{E^2 - p_D^2}$

# CLEO-c event selections

- $K_L K K$  vs CP tags



Data

Background

- $m^2_{\text{miss}}$  = squared missing mass

# CLEO-c $K_S hh$ event yields

Mode	ST yield	DT yields			
		$K_S^0 \pi^+ \pi^-$	$K_L^0 \pi^+ \pi^-$	$K_S^0 K^+ K^-$	$K_L^0 K^+ K^-$
Flavor tags					
$K^- \pi^+$	$144563 \pm 403$	1444	2857	168	302
$K^- \pi^+ \pi^0$	$258938 \pm 581$	2759	5133	330	585
$K^- \pi^+ \pi^+ \pi^-$	$220831 \pm 541$	2240	4100	248	287
$K^- e^+ \nu$		1191		100	
$CP$ -even tags					
$K^+ K^-$	$13349 \pm 128$	124	357	12	32
$\pi^+ \pi^-$	$6177 \pm 114$	61	184	4	13
$K_S^0 \pi^0 \pi^0$	$6838 \pm 134$	56		7	14
$K_L^0 \pi^0$		237		17	
$K_L^0 \eta(\gamma\gamma)$				4	
$K_L^0 \eta(\pi^+ \pi^- \pi^0)$				1	
$K_L^0 \omega$				4	
$K_L^0 \eta'$				1	
$CP$ -odd tags					
$K_S^0 \pi^0$	$19753 \pm 153$	189	288	18	43
$K_S^0 \eta(\gamma\gamma)$	$2886 \pm 71$	39	43	4	6
$K_S^0 \eta(\pi^+ \pi^- \pi^0)$				2	1
$K_S^0 \omega$	$8830 \pm 110$	83		14	10
$K_S^0 \eta'$				3	4
$K_L^0 \pi^0 \pi^0$				5	
$K_S^0 \pi^+ \pi^-$		473	1201	56	126
$K_L^0 \pi^+ \pi^-$				140	
$K_S^0 K^+ K^-$				4	9

# Dalitz plot yields

- Number of events in  $i^{\text{th}}$  bin,  $N_i$ , depends on weighted average **cosine** and **sine** of strong phase difference as well as **amplitude<sup>2</sup>** in that bin

$$N_i(B^\pm \rightarrow (K_S^0 h^+ h^-)_D K^\pm) \propto T_i + r_B^2 T_{-i} + 2r_B \sqrt{T_i T_{-i}} [\cos(\delta_B \pm \gamma) c_i + \sin(\delta_B \pm \gamma) s_i]$$

$$T_i \equiv \int_i |f_{D^0}(x, y)|^2 dx dy$$

$$c_i \equiv \frac{1}{\sqrt{T_i T_{-i}}} \int_i |f_{D^0}(x, y)| |f_{D^0}(y, x)| \cos(\Delta\delta_D(x, y)) dx dy$$

$$s_i \equiv \frac{1}{\sqrt{T_i T_{-i}}} \int_i |f_{D^0}(x, y)| |f_{D^0}(y, x)| \sin(\Delta\delta_D(x, y)) dx dy$$

# Determining $(c_i, s_i)$

- $c_i$  is determined using CP-tagged  $K_S hh$

$$\frac{M_i^\pm}{S_\mp} = \frac{K_i \pm 2c_i \sqrt{K_i K_{-i}} + K_{-i}}{2S_f}$$

Diagram labels and arrows:

- Double CP tag yield  $\rightarrow M_i^\pm$
- Single CP tag yield  $\rightarrow S_\mp$
- Double flavour tag yield  $\rightarrow 2c_i \sqrt{K_i K_{-i}}$
- Single flavour tag yield  $\rightarrow S_f$

- $c_i$  and  $s_i$  are determined using  $K_S hh$  tagged with  $K_S h' h'$

$$M_{ij} = \frac{N_{D, \bar{D}}}{2S_f^2} (K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-j} K_{-i} K_j} (c_i c_j + s_i s_j))$$

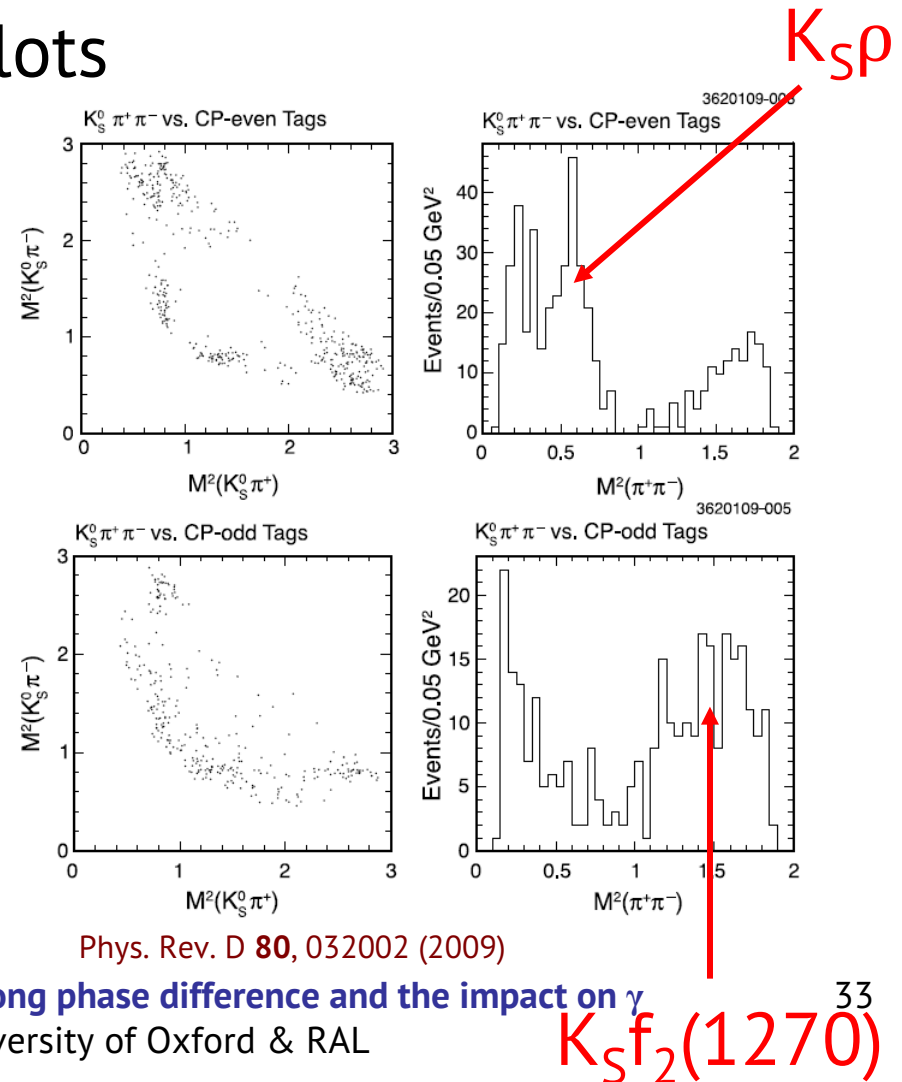
Diagram labels and arrows:

- Double  $K_{S,L} hh$  tag yield  $\rightarrow N_{D, \bar{D}}$
- Double flavour tag yield  $\rightarrow (c_i c_j + s_i s_j)$
- Single flavour tag yield  $\rightarrow S_f^2$



# Quantum correlations at CLEO-c

- Quantum correlations result in visible differences between different Dalitz plots
- Decays to  $K_S \pi \pi$  proceed via different intermediate resonances of specific CP depending on opposite-side final state (tag)



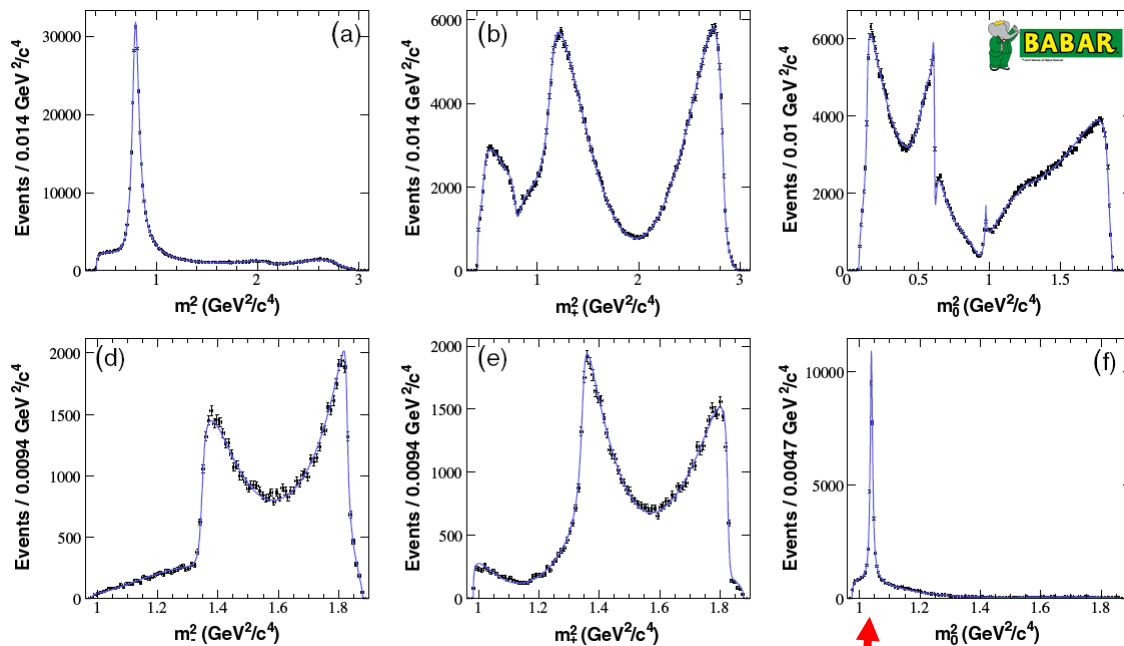
Phys. Rev. D **80**, 032002 (2009)

# $D^0 \rightarrow K_S K K$ systematic errors

Uncertainty	$c_1$	$c_2$	$c_3$	$s_1$	$s_2$	$s_3$
(Pseudo-)flavor statistics	0.005	0.007	0.055	0.015	0.013	0.039
Momentum resolution	0.002	0.004	0.012	0.018	0.025	0.032
Mode-to-mode normalization	0.004	0.008	0.017	0.001	0.010	0.004
Multiple-candidate selection	0.004	0.003	0.015	0.004	0.008	0.002
DCS correction	0.001	0.001	0.003	0.002	0.005	0.002
$K_{S,L}^0 \pi^+ \pi^-$ ( $c_i^{(\prime)}, s_i^{(\prime)}$ )	0.006	0.011	0.036	0.132	0.063	0.135
Fitter assumptions	0.008	0.001	0.013	0.013	0.003	0.043
MC statistics for determining $\mathbf{U}$	0.005	0.007	0.057	0.024	0.051	0.048
Parameterization of non- $K_L^0$ final state background	0.001	0.001	0.006	0.000	0.008	0.003
Parameterization of $K_L^0$ final state background	0.034	0.020	0.061	0.038	0.015	0.071
Background Dalitz space distribution	0.006	0.015	0.062	0.005	0.029	0.022
Assumed background $\mathcal{B}$	0.004	0.014	0.032	0.001	0.007	0.009
Total systematic	0.038	0.034	0.131	0.142	0.094	0.175
Statistical plus $K_L^0 K^+ K^-$ model	0.063	0.092	0.329	0.222	0.234	0.432
$K_L^0 K^+ K^-$ model alone	0.000	0.000	0.136	0.007	0.000	0.039
Total	0.073	0.098	0.354	0.264	0.252	0.466

# Modelling B decays

- B-factories use unbinned model approach
- Use high-statistics sample of  $D^{*\pm} \rightarrow D^0 \pi^\pm$ 
  - Pion acts as a tag to determine  $D^0$  flavour



$D^0 \rightarrow K_S \pi \pi$

$D^0 \rightarrow K_S K K$

$K_S \phi$  resonance

Phys. Rev D **78**, 034023 (2008)

# B-factory $\gamma$ measurements

- BaBar (468M  $B\bar{B}$ , both  $K_S\pi\pi$  and  $K_SKK$ )

$$\gamma = 68^\circ \pm 14^\circ(\text{stat}) \pm 4^\circ(\text{syst}) \pm 3^\circ(\text{model})$$

Phys. Rev. Lett. **105**, 121801 (2010)

- Belle (657M  $B\bar{B}$ ,  $K_S\pi\pi$ )

$$\gamma = 78.4^\circ_{-11.6^\circ}^{+10.8^\circ}(\text{stat}) \pm 3.6^\circ(\text{syst}) \pm 8.9^\circ(\text{model})$$

Phys. Rev. D **81**, 112002 (2010)

- The **model error** will be a limiting factor in future  $\gamma$  determination

# CKM matrix

- Quantifies weak mixing between quark generations

$$V_{\text{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

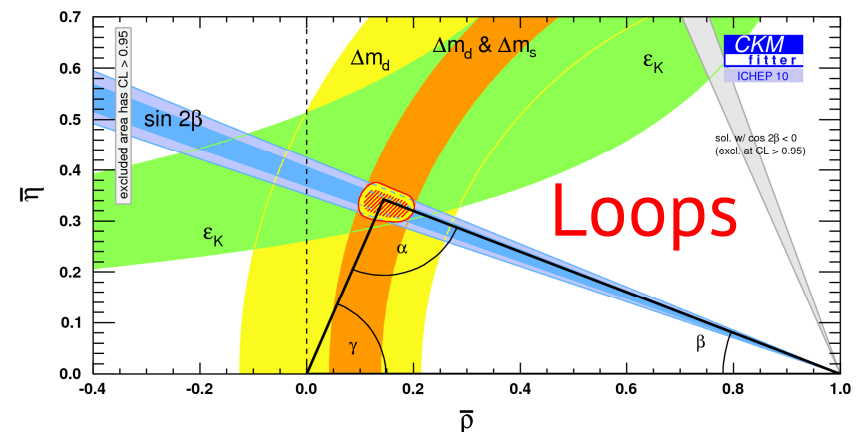
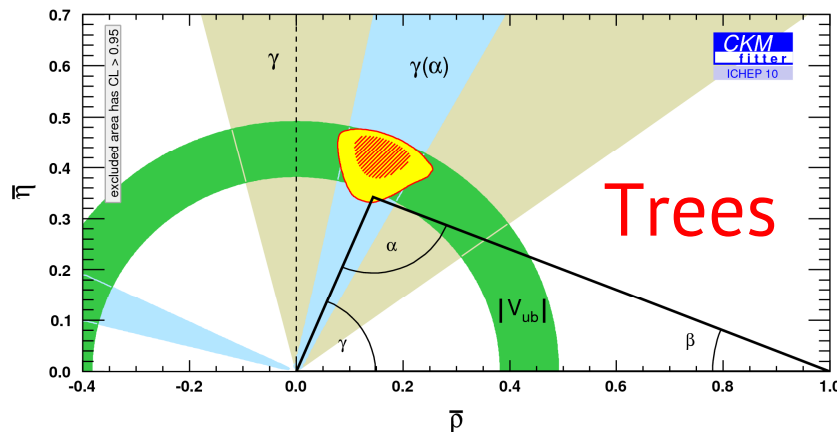
- Unitary:  $V_{\text{CKM}}^\dagger V_{\text{CKM}} = \mathbb{I}$
- Six relations such as the following:

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

- Each represents a triangle in the complex plane

# Trees vs loops

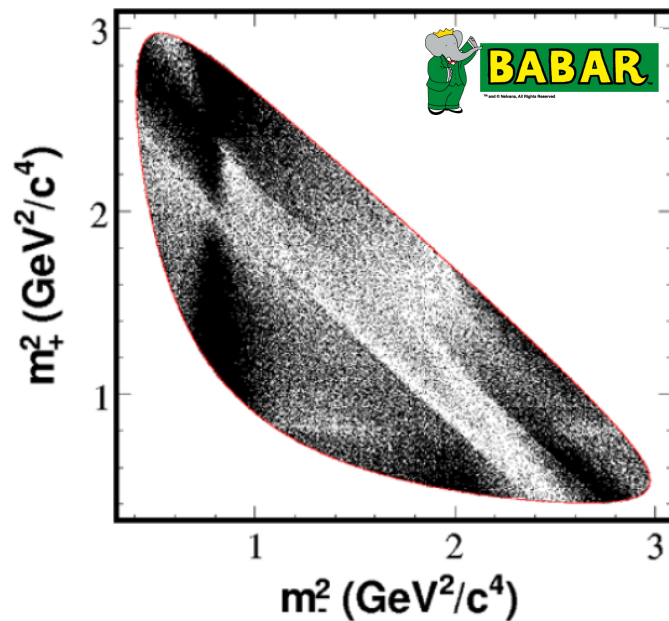
- Tension between the values of  $\gamma$  obtained from **tree**-level and **loop**-level measurements will be a strong indication of **new physics**



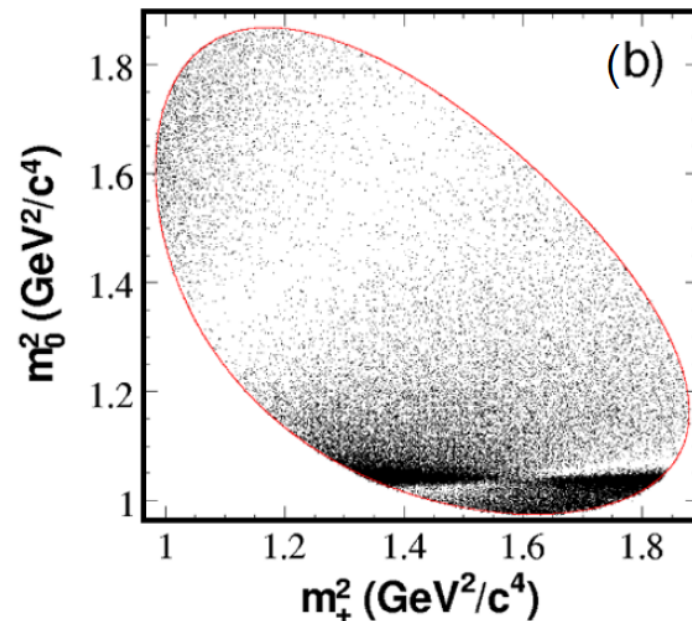
- In this talk concentrate on tree-level determinations, in particular **time-independent** measurements in the  $B^\pm \rightarrow D^0 K^\pm$  system

# Dalitz plots

- Representation of the structure of a decay in terms of (invariant mass)<sup>2</sup> of pairs of the final state particles
- Reveals interference and intermediate resonances



$K_S \pi \pi$



$K_S K K$

# DCS contribution to $K_L hh$

- $K_{S,L} hh$  contain both CF and DCS decay amplitudes
- Amplitudes are related:
- $A(K_L hh) = A(K_S hh) - \sqrt{2} \cdot A(K^0 hh)$
- Correction term  $\sim \tan^2 \theta_c \sim 1/20$
- Estimated using decay model
  - different resonances behave differently under CP transformation
- Small systematic error to account for this