Particle Accelerators

A BRIEF (HYSTORICAL) INTRODUCTION

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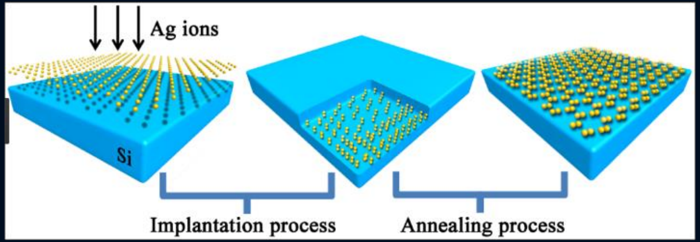
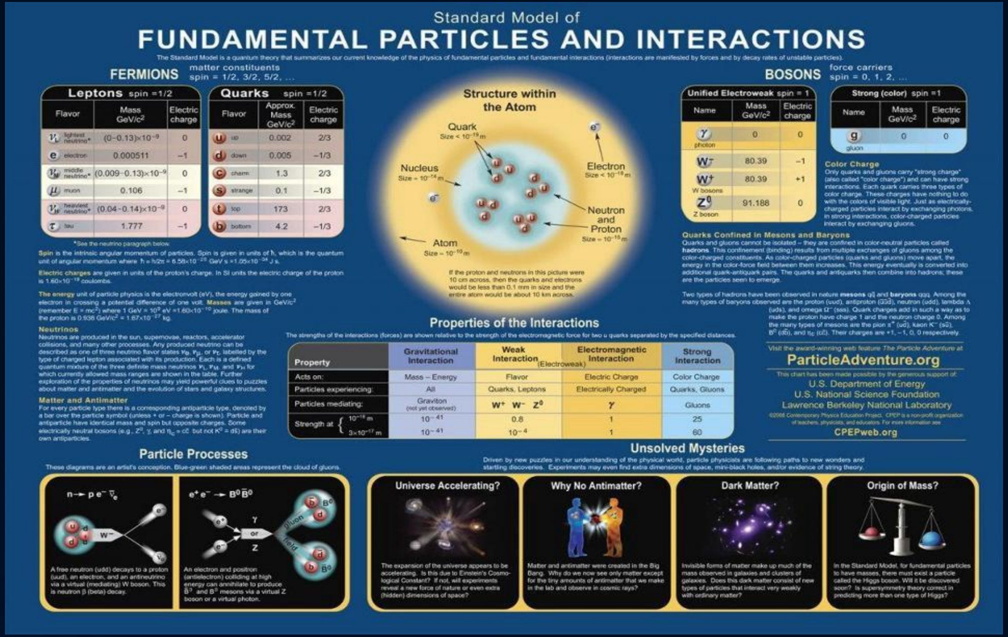
Particle Accelerators

- What for?
- How can we observe such small particles?
- Let’s try to build an accelerator

What for?

- 30000 accelerators in use world-wide:
    - 44% radiotherapy
    - 41% ion implantation
    - 9% industrial applications
    - 4% low energy research
    - 1% medical isotope production
    - <1% fundamental research





Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model is a quantum theory that summarizes our current knowledge of the physics of fundamental particles and fundamental interactions (interactions are manifested by forces and by decay rates of unstable particles).

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c²	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	−1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	−1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	−1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.002	2/3
d down	0.005	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	173	2/3
b bottom	4.2	−1/3

*See the neutrino paragraph below.

Spin is the intrinsic angular momentum of particles. Spin is given in units of $\hbar$, which is the quantum unit of angular momentum where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s $=1.05\times10^{-34}$ J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The energy unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c² (remember $E = mc^2$) where 1 GeV = 10^9 eV $=1.60\times10^{-10}$ joule. The mass of the proton is 0.938 GeV/c² = 1.67×10^{-27} kg.

Neutrinos
Neutrinos are produced in the sun, supernovae, reactors, accelerator collisions, and many other processes. Any produced neutrino can be described as one of three neutrino flavor states ν_e, ν_μ, or ν_τ, labelled by the type of charged lepton associated with its production. Each is a defined quantum mixture of the three definite mass neutrinos ν_L, ν_M, and ν_H for which currently allowed mass ranges are shown in the table. Further exploration of the properties of neutrinos may yield powerful clues to puzzles about matter and antimatter and the evolution of stars and galaxy structures.

Matter and Antimatter
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or − charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$ but not $K^0 = d\bar{s}$) are their own antiparticles.

Structure within the Atom

Quark Size < 10^{-19} m
Nucleus Size ≈ 10^{-14} m
Electron Size < 10^{-18} m
Neutron and Proton Size ≈ 10^{-15} m
Atom Size ≈ 10^{-10} m

If the proton and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.39	−1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c²	Electric charge
g gluon	0	0

Color Charge
Only quarks and gluons carry "strong charge" (also called "color charge") and can have strong interactions. Each quark carries three types of color charge. These charges have nothing to do with the colors of visible light. Just as electrically-charged particles interact by exchanging photons, in strong interactions, color-charged particles interact by exchanging gluons.

Quarks Confined in Mesons and Baryons
Quarks and gluons cannot be isolated – they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs. The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge.

Two types of hadrons have been observed in nature **mesons** $q\bar{q}$ and **baryons** qqq. Among the many types of baryons observed are the proton (uud), antiproton ($\bar{u}\bar{u}\bar{d}$), neutron (udd), lambda Λ (uds), and omega Ω^- (sss). Quark charges add in such a way as to make the proton have charge 1 and the neutron charge 0. Among the many types of mesons are the pion π^+ ($u\bar{d}$), kaon K^- ($s\bar{u}$), B^0 ($d\bar{b}$), and η_c ($c\bar{c}$). Their charges are +1, −1, 0, 0 respectively.

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property		Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:		Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:		All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:		Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at	10^{-18} m	10^{-41}	0.8	1	25
	3×10^{-17} m	10^{-41}	10^{-4}	1	60

Visit the award-winning web feature *The Particle Adventure* at

ParticleAdventure.org

This chart has been made possible by the generous support of:
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Particle Processes

These diagrams are an artist's conception. Blue-green shaded areas represent the cloud of gluons.

$n \rightarrow p\, e^-\, \bar{\nu}_e$

A free neutron (udd) decays to a proton (uud), an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β (beta) decay.

$e^+ e^- \rightarrow B^0 \bar{B}^0$

An electron and positron (antielectron) colliding at high energy can annihilate to produce $\bar{B}^0$ and B^0 mesons via a virtual Z boson or a virtual photon.

Unsolved Mysteries

Driven by new puzzles in our understanding of the physical world, particle physicists are following paths to new wonders and startling discoveries. Experiments may even find extra dimensions of space, mini-black holes, and/or evidence of string theory.

Universe Accelerating?
The expansion of the universe appears to be accelerating. Is this due to Einstein's Cosmological Constant? If not, will experiments reveal a new force of nature or even extra (hidden) dimensions of space?

Why No Antimatter?
Matter and antimatter were created in the Big Bang. Why do we now see only matter except for the tiny amounts of antimatter that we make in the lab and observe in cosmic rays?

Dark Matter?
Invisible forms of matter make up much of the mass observed in galaxies and clusters of galaxies. Does this dark matter consist of new types of particles that interact very weakly with ordinary matter?

Origin of Mass?
In the Standard Model, for fundamental particles to have masses, there must exist a particle called the Higgs boson. Will it be discovered soon? Is supersymmetry theory correct in predicting more than one type of Higgs?

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The micro-world ➔ the atoms

- In a typical beach there are tens of thousands of millions of millions of sand grains
- But … within a single grain of sand, there are as many atoms!

The micro-world ➔ atoms’ constituents

- The atom nucleus weights more than 99% of the atom mass
- If the atom was as big as the “Stade de France”
- … the nucleus would be smaller than the foot ball

How can we observe such small particles?

The structures under research are EXTRAORDINARILY SMALL (~ < 10^{-15} m)

- ➔ probes with correspondingly high spatial resolution are needed. Visible light is inadequate: size ~ 5 10^{-7} m
- ➔ what could we use instead?

mm = 10^{-3} m
µm = 10^{-6} m
nm = 10^{-9} m
light
pm = 10^{-12} m ➔ R_{atoms} ~ 30 – 300 pm (0.03 – 3 Å)
fm = 10^{-15} m ➔ $R_{nucleus}$ ~ 1 – 10 fm
am =10^{-18} m ➔ Quarks - leptons

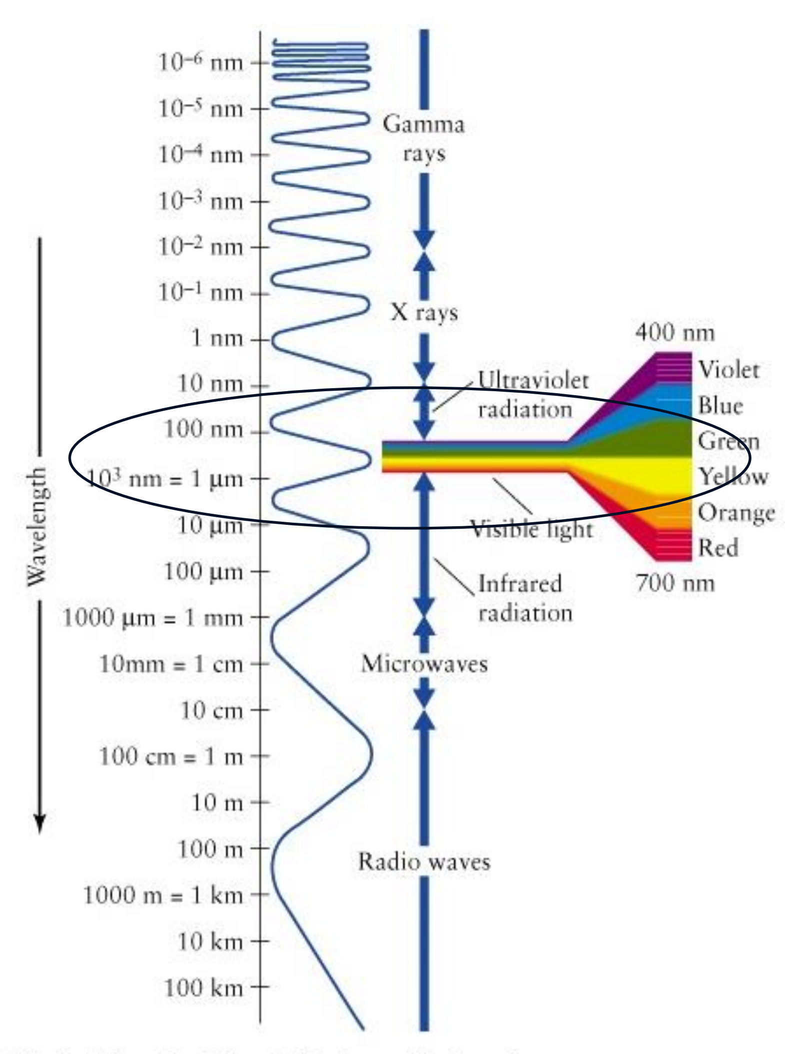
How can we observe such small particles?

Aggregate of molecules: cell/bacteria

Size: $\lambda = 10^{-5} - 10^{-7}$ m

➔ 10 micro – 100 nano

$$E = \frac{hc}{\lambda\beta}$$ ➔ 0.1 eV – 10 eV



(Wavelength image from *Universe* by Freedman and Kaufmann.)

Optical microscope

Planck constant:

$h=E/\nu=6.6\ 10^{-34}$ Js

$$\hbar = \frac{h}{2\pi}$$

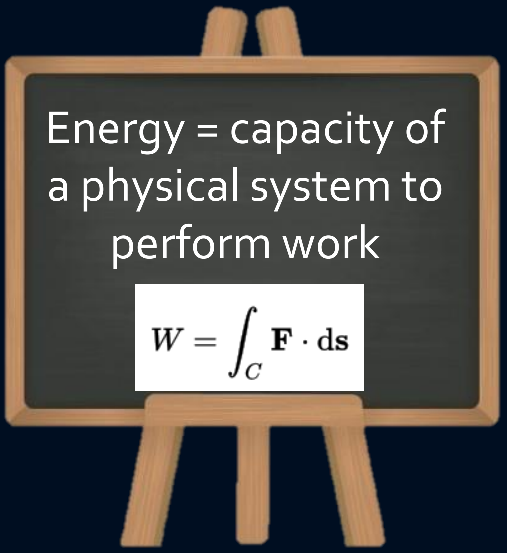
$\beta = v/c$

How can we observe such small particles?

	Size (m)	Size	Beam energy	Instrument
Aggregate of molecules: cell/bacteria	10^{-5}	10 micro meter	0.1 eV	Optical microscope
	10^{-7}	100 nano meter	10 eV	
Aggregate of atoms: molecules	10^{-9}	1 nano meter	1 keV	Electron microscope
Atoms: nucleus+electrons	10^{-10}	0.1 nano meter	10 keV	Synchrotron radiation
Nucleus (Oxygen: 8p+8n)	10^{-14}	0.01 pico meter	>100 MeV	Low energy e- or p+ accelerator
Aggregate of quarks: hadrons	10^{-15}	1 femto meter	> 1 GeV	High energy p+ accelerator
Quarks+leptons	10^{-18}	1 atto meter	> 1 TeV	High energy e- or p+ collider

LHC 27 km circumference
7 TeV beam energy

A little parenthesis about Energy Units



Energy = capacity of a physical system to perform work

$$W = \int_C \mathbf{F} \cdot d\mathbf{s}$$

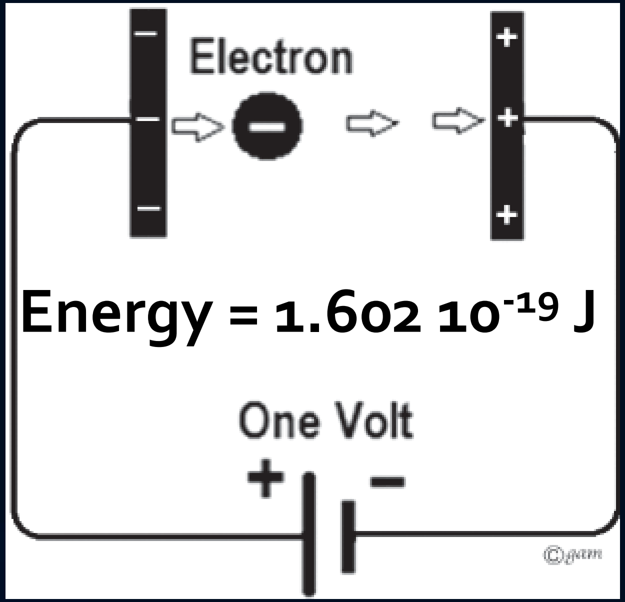
- In physics, energy is usually measured in Joule (J)

1 Joule = energy expended (or work done) in applying a force of one newton through a distance of one metre (SI).

- Joule is not convenient when describing particle beams because the energy is very small, e.g.,

- Therefore a new unit was invented ➔ eV ➔ kinetic energy gained by a particle of elementary charge 1.602 10^{-19} C as it crosses a potential difference of 1 V.

- 1 keV = 10^3 eV, 1 MeV = 10^6 eV, 1 GeV = 10^9 eV, 1 TeV = 10^{12} eV



How can we accelerate charged particles?

- Simplest particle accelerators use a constant electric field (DC accelerators) between two electrodes, produced by a high energy voltage generator

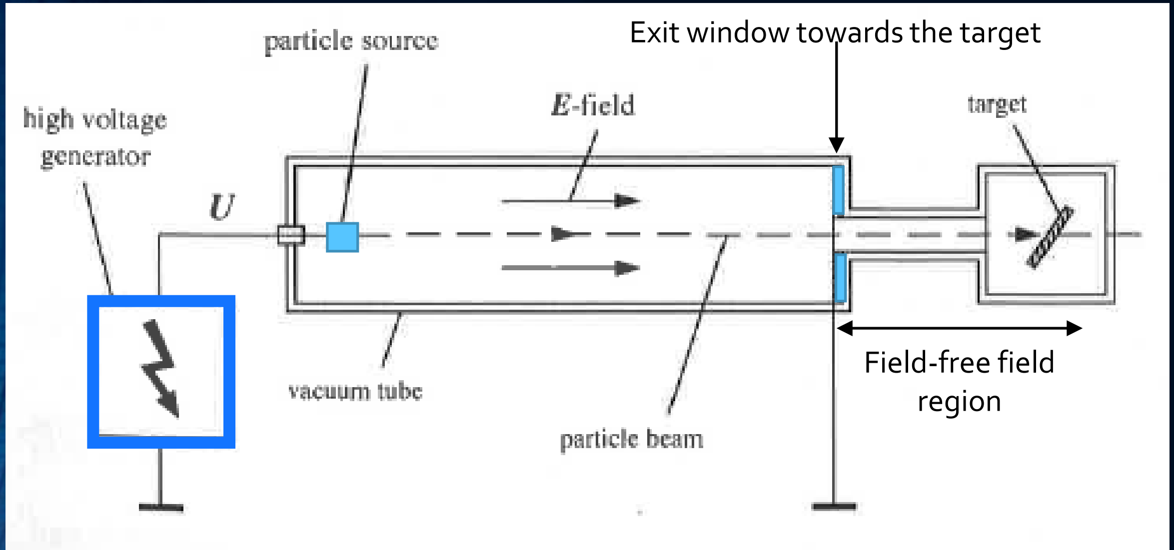


Fig. 1.3 General principle of the electrostatic accelerator.

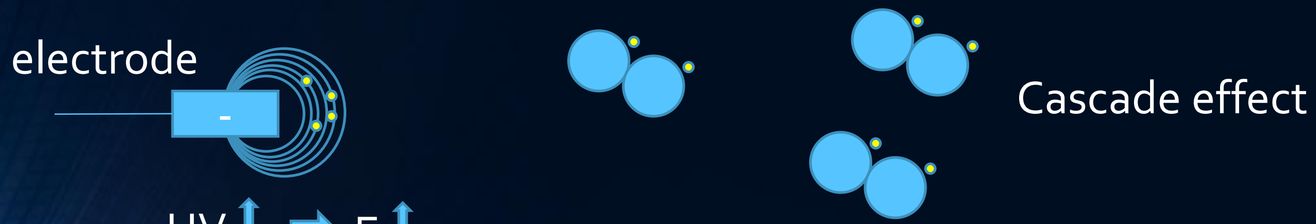
- One of the electrodes has the particle source
- If e- beams: particle source is a thermionic cathode (widely used in vacuum technology)
- In the accelerating region there is good vacuum to avoid beam-gas collisions

- Limited achievable particle energies
- Depends on the maximum voltage that can be given by the generator

How can we accelerate charged particles?

What is the energy limit in DC accelerators?

- CORONA FORMATION is the actual energy limit



HV ↑ ➡ E ↑

⬇

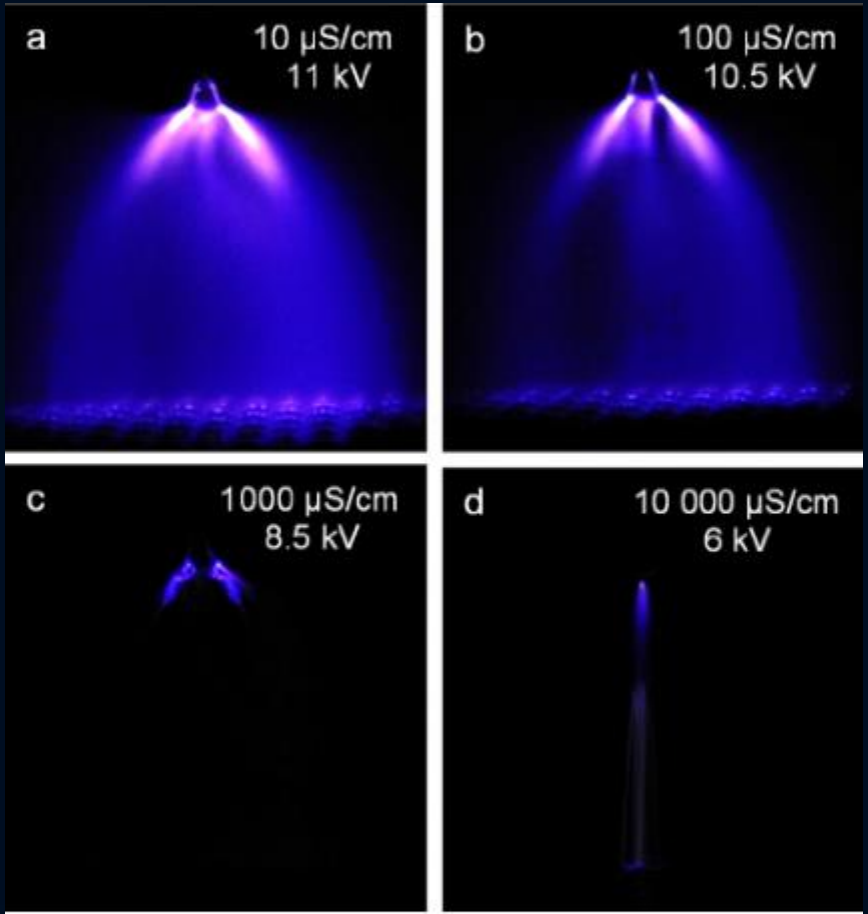
e- & ions acceleration ↑

⬇

Ionization of residual gas molecules in cascade

⬇

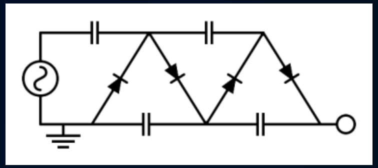
Spark discharge & HV breakdown

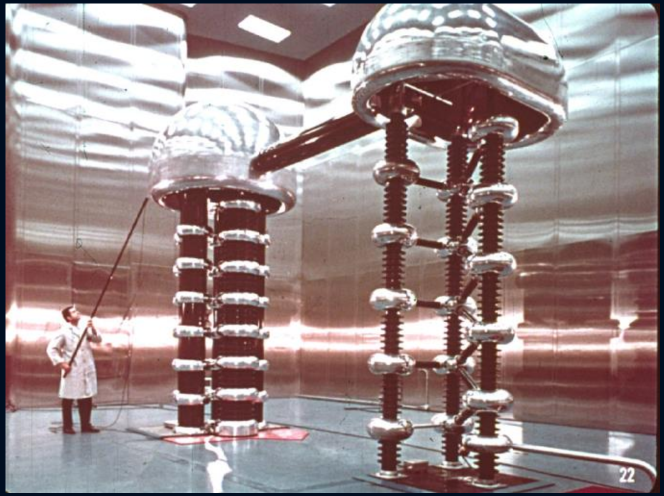


HV of ~ MV ➔ particle energy ~ few MeV

Examples of electrostatic accelerators

- Cockroft-Walton (1030's)



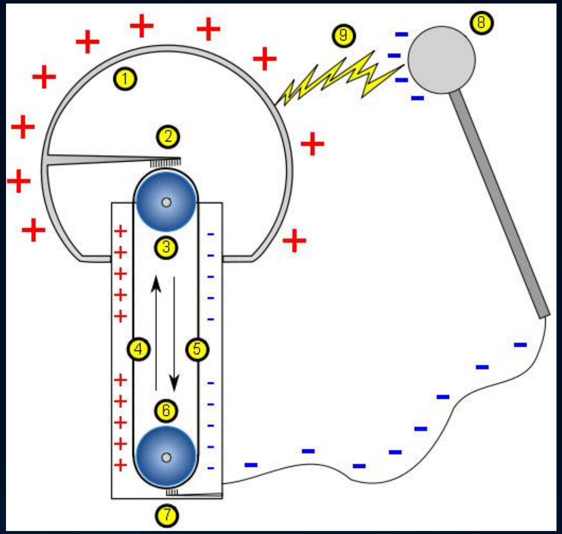


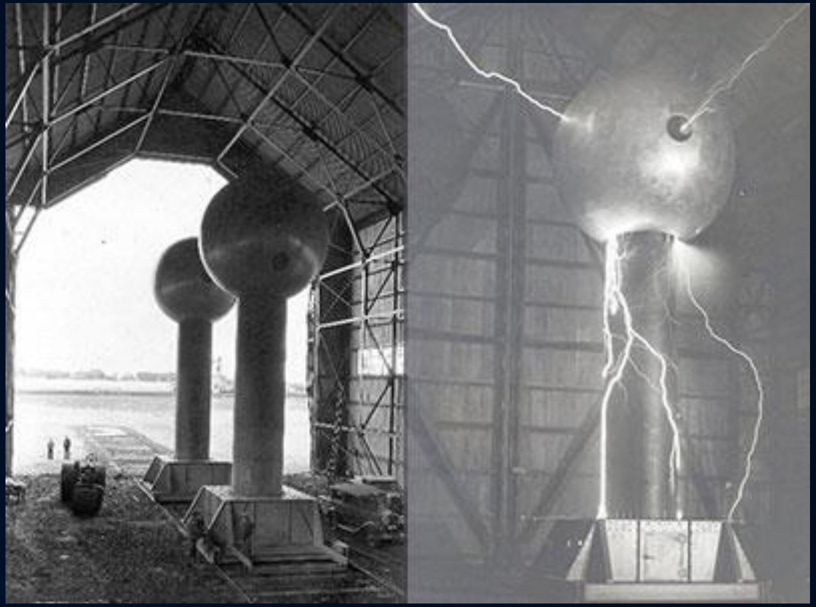
We had one at CERN to accelerate protons up to 750 keV

HV ~ 4 MV

- Van de Graaff (1030's)

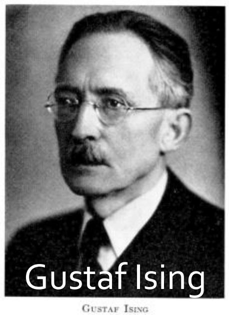
HV ~ 2 MV – 10 MV

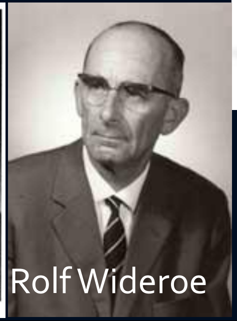


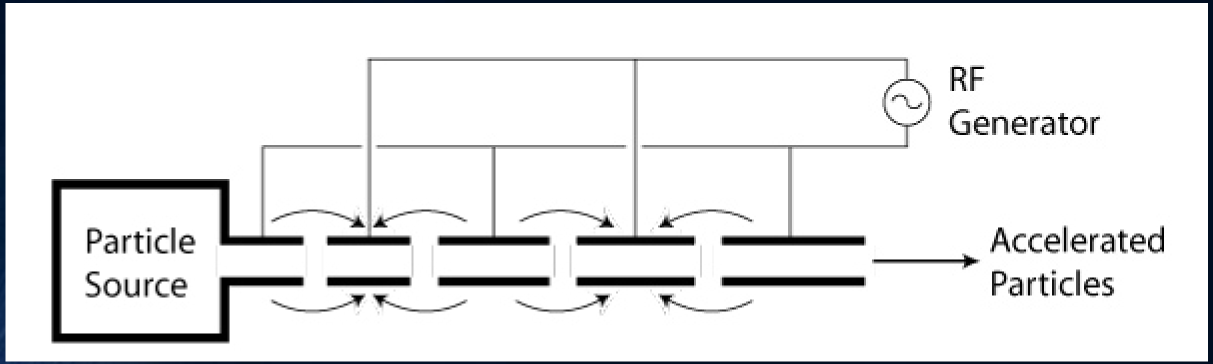


How could we overcome the corona formation energy limit and go beyond few MeV regime?

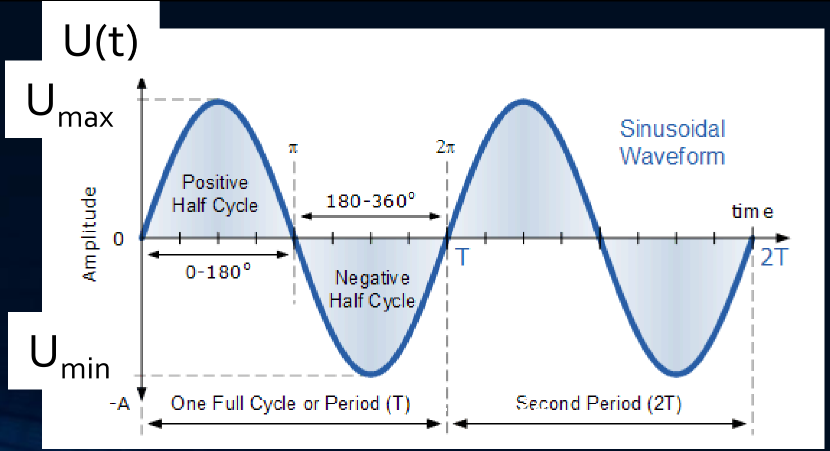
- Ising 1925 → AC voltage!!
- Wideroe 1928 → first successful test of AC accelerator


Gustaf Ising


Rolf Wideroe



RF generator voltage: $U(t) = U_{max} sin\omega t$



AC Linear Accelerators

- RF generator voltage: $V(t) = V_{max} sin\omega t$
- Energy reached by the particle per crossed gap:

$$\Delta Energy = \int_{s1}^{s2} F\, ds \longrightarrow F_{electric} = qE_{electric}$$

$$\Delta Energy = \int_{s1}^{s2} qE_{electric}\, ds$$

- $E_{electric}$ is cte between s1 and s2 when the particle crosses the gap, therefore, q and $E_{electric}$ come out from the integral.
- We are left with the integral of ds between s1 and s2 ➔ s2-s1 = Δs

$$\Delta Energy = q\underbrace{E_{electric}\Delta s}_{= V}$$

$$\Delta Energy = qV = qV_{max} sin\varphi_0$$

Average phase of the RF voltage the particle sees as it crosses the gap

AC Linear Accelerators

- Energy gained by the particle after passing the *n-th* gap:

$$\Delta Energy = nqV = nqV_{max} sin\varphi_0$$

- Energy gain is proportional to the number of stages/gaps traversed by the particle
- However, the largest voltage in the entire system is never greater than U_{max}

No corona discharge

- At CERN we have linear accelerators for the first acceleration steps: LINAC2, LINAC3, LINAC4

Pulse of beam ~100 µs long every 1.2 s

What is the limitation of linear accelerators?

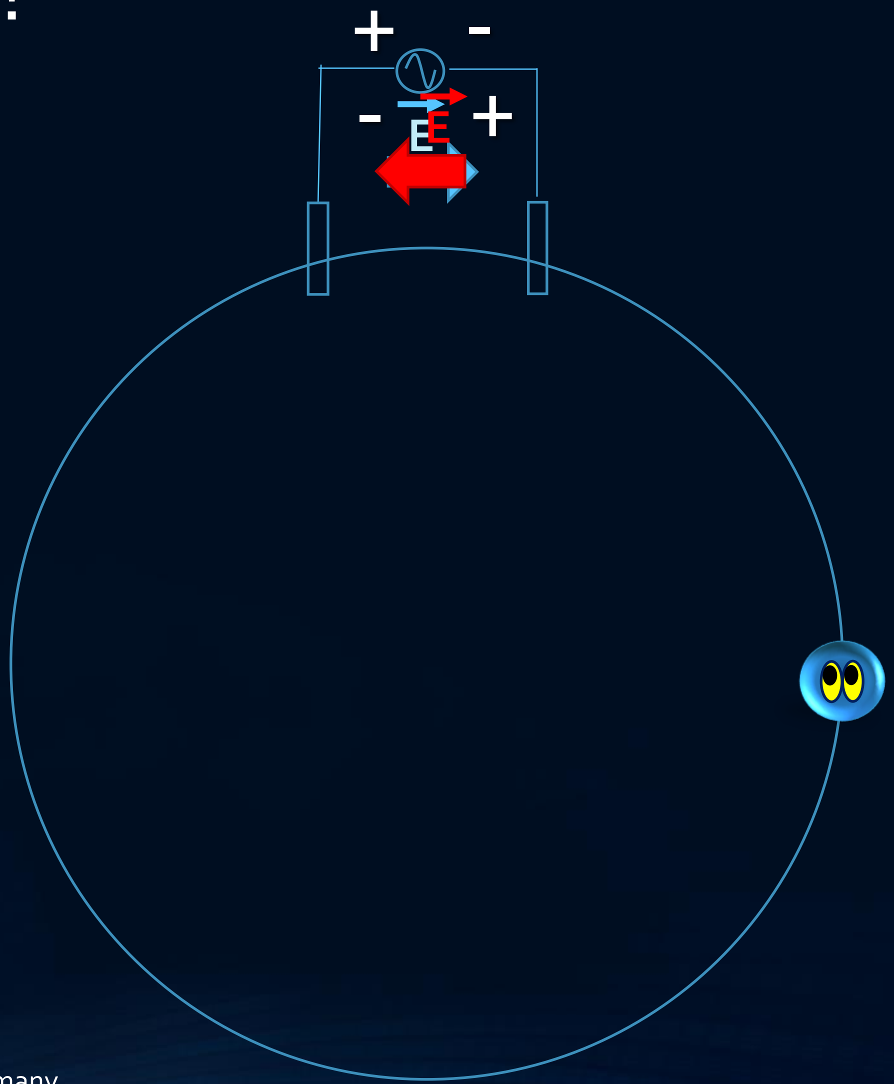
radiofrequency (RF) structures and a two-beam concept to produce accelerating fields as high as 100 MV per meter to reach a nominal total energy of 3 TeV

≈ 50 km

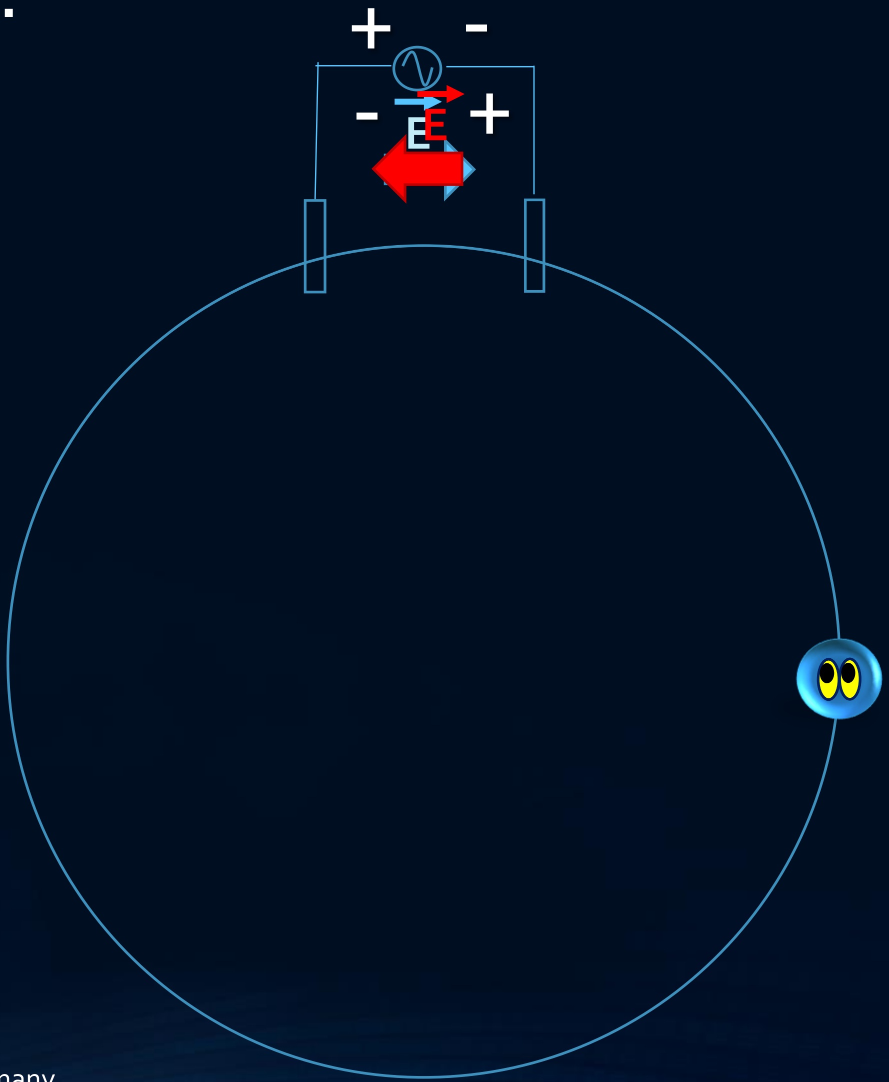
Size & cost could be a problem since it grows with energy

How can we overcome the limitation of linear accelerators to increase the energy without increasing the size?

How can we overcome the limitation of linear accelerators to increase the energy without increasing the size?

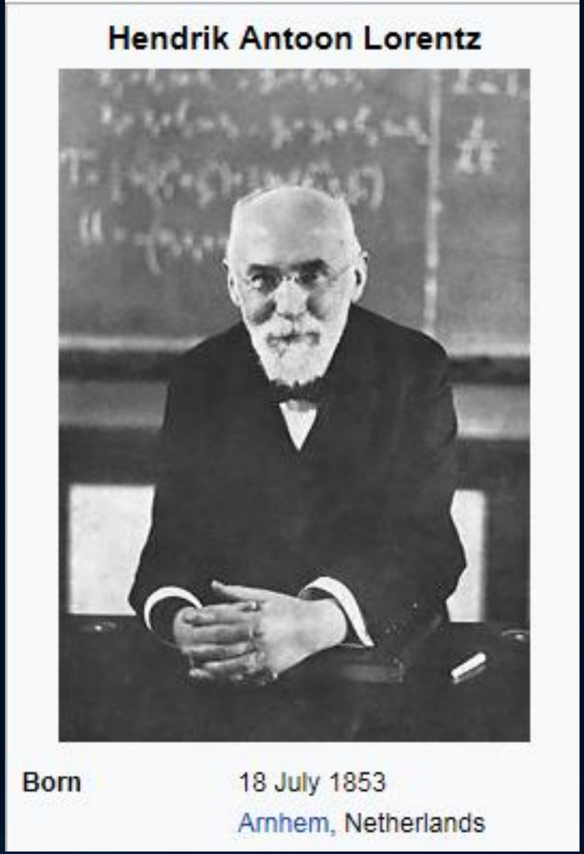


How can we overcome the limitation of linear accelerators to increase the energy without increasing the size?



But how can we keep a charged particle running in circles?

Let’s ask Lorentz



Hendrik Antoon Lorentz

Born 18 July 1853
Arnhem, Netherlands

But how can we keep a charged particle running in circles?

- We need a magnetic field

LORENTZ FORCE

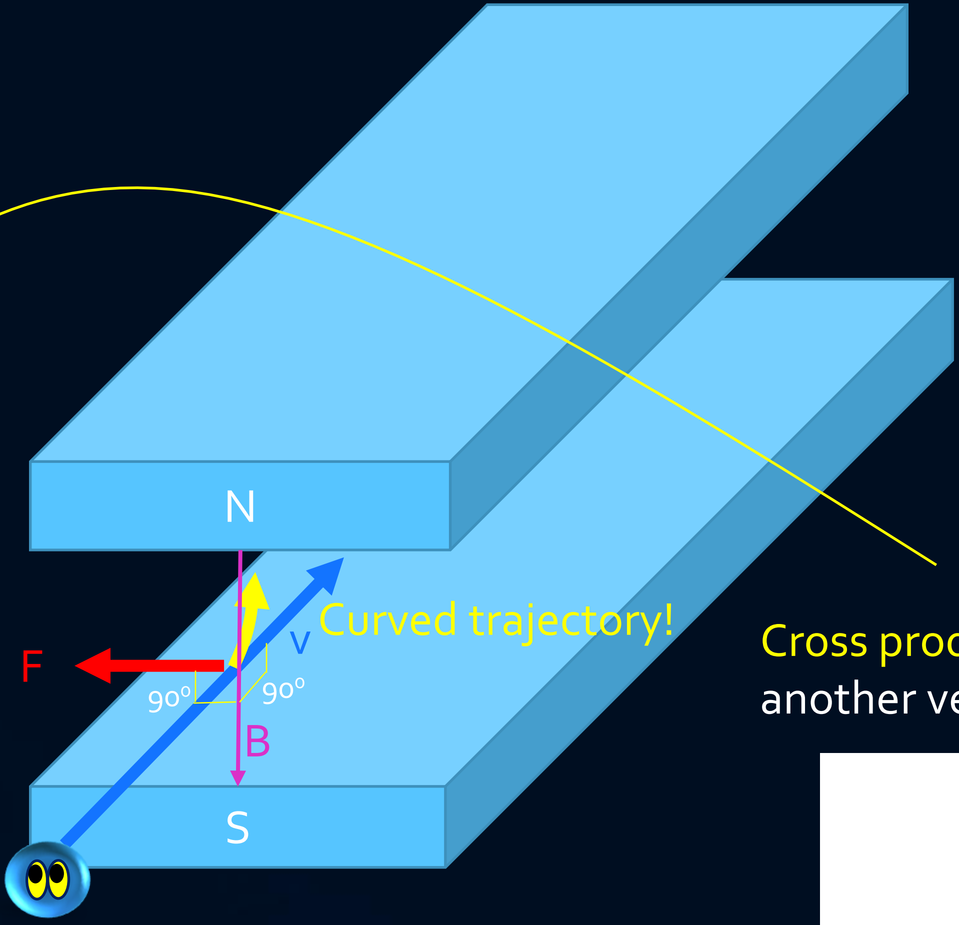
$$\vec{F} = q \cdot \vec{E} + q \cdot (\vec{v}\ x\ \vec{B})$$

If an electric field is present

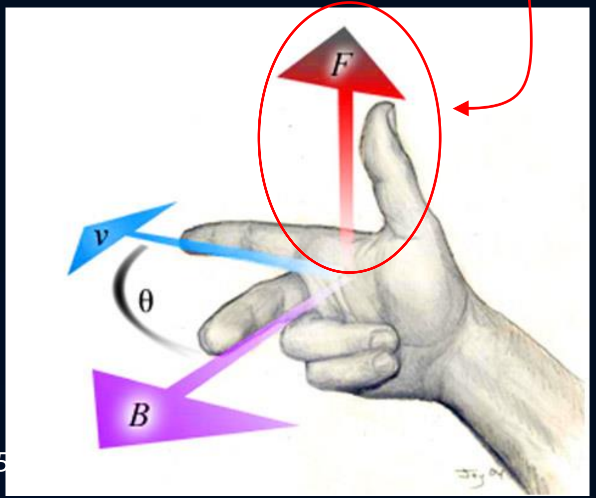
If a magnetic field is present

q: particle charge

Can we accelerate and bend neutral particles?



Cross product of two vectors is another vector orthogonal to them



Before we continue, first we should understand the beam rigidity

- ➢ What is the condition for a circular orbit in the presence of a uniform magnetic field?

Lorentz force = centrifugal force

$$F_{Lorentz} = q \cdot v \cdot B \quad = \quad F_{centrifugal} = \frac{m \cdot v^2}{\rho}$$

ρ: curvature radius
m: particle mass
v: particle velocity

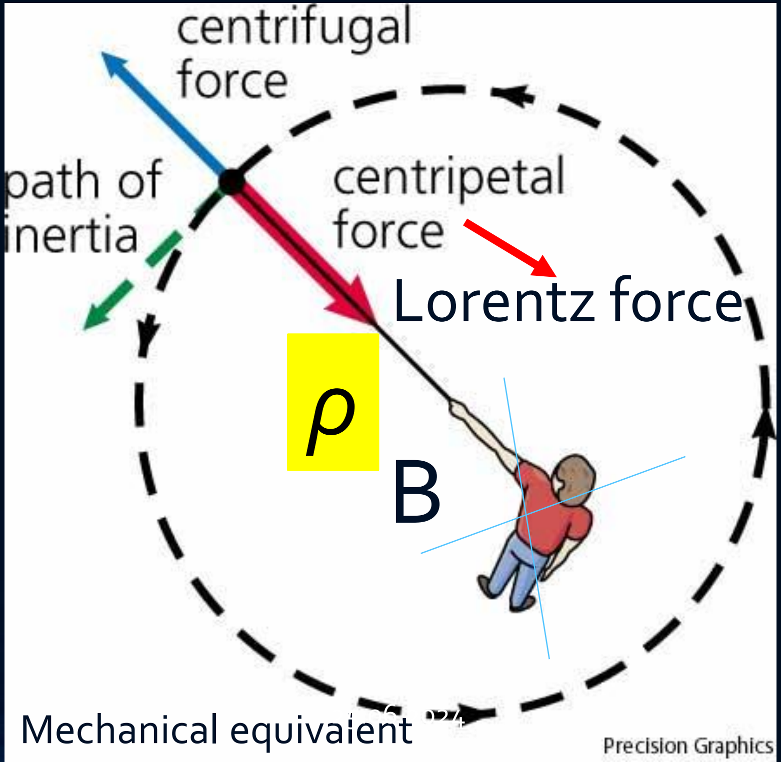
$$q \cdot v \cdot B = \frac{m \cdot v^2}{\rho}$$

$$p = m \cdot v$$

Particle momentum

$$\boxed{B\rho = \frac{p}{q}}$$

Beam rigidity formula



Mechanical equivalent

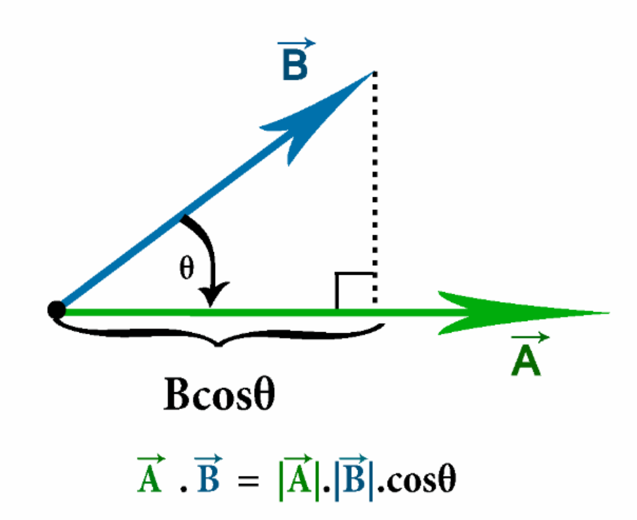
Let’s build our first circular accelerator!!

- We need a magnetic field perpendicular to the particle trajectory to bend the particles
- We need an electric field to give energy to the particles ➔ magnetic fields do not change the energy of the particles, why?

$$\Delta Energy = \int_{s1}^{s2} \vec{F} d\vec{s}$$

Those are vectors! They have direction and magnitude

This is the scalar product:



If A and B are parallel ➔ θ = 0°
➔ cosθ = 1

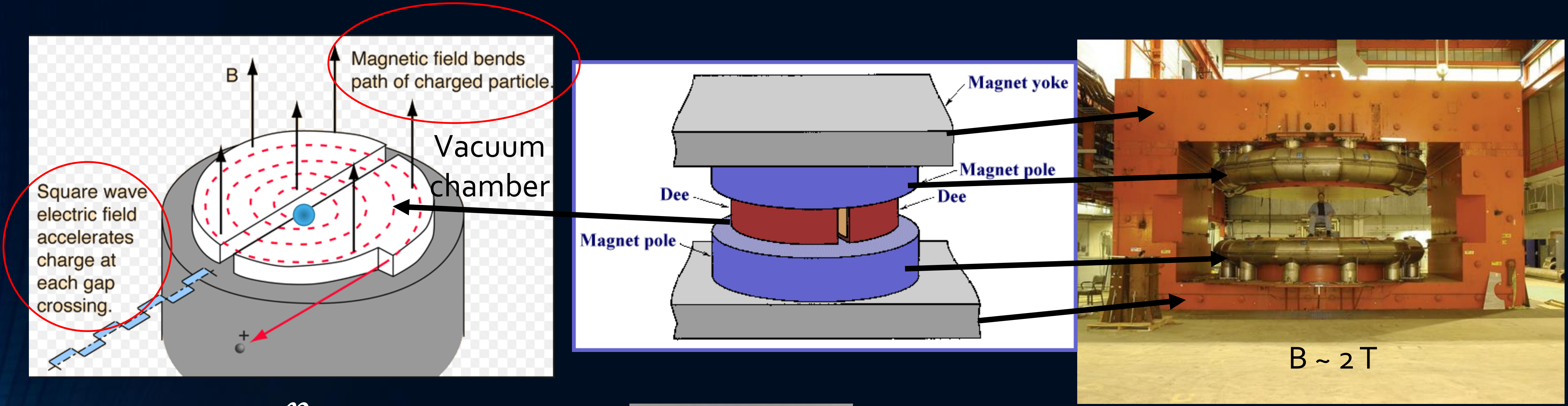
The force gives the maximum energy increase

If A and B are orthogonal ➔ θ = 90°
➔ cosθ = 0

$\Delta Energy$ = 0

Since the magnetic field is orthogonal to the particle trajectory $\Delta Energy$ = 0

This is our first circular accelerator ➔ cyclotron



$$\rho = \frac{p}{qB}$$

If B is a constant uniform magnetic field
➔ ρ increases as the particle momentum increases

The vacuum chamber has to be big enough to accommodate the full particle trajectory before extraction


E. O. Lawrence

The first circular accelerator was developed by E. O. Lawrence at Univ. California in 1930.
In 1932 Lawrence and Livingston built the first cyclotron suitable for experiments with 1.2 MeV peak energy.

A little parenthesis about Relativity

- For over 200 years Newton's equations of motion were believed to describe nature correctly. But in 1905 Einstein discovered an error in these laws and proposed a solution.

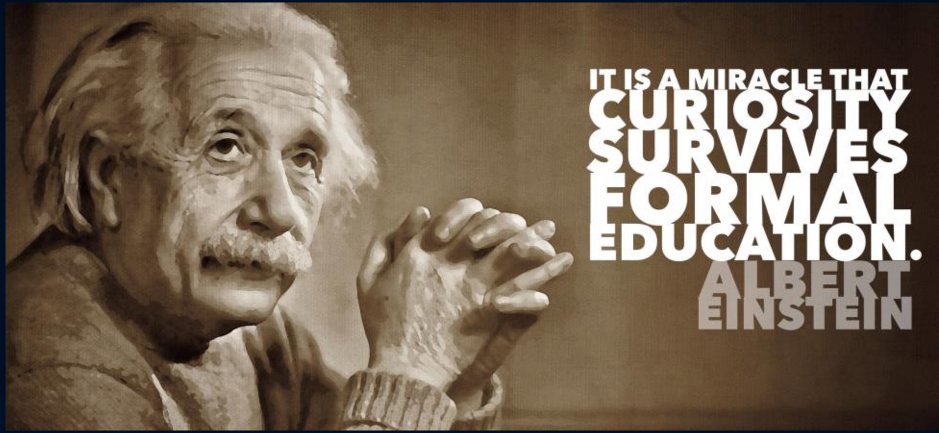
$$F = \frac{d(m \cdot v)}{dt} = \mathrm{m}\frac{dv}{dt} = m \cdot a$$

Newton assumes m is constant

- But Einstein, based on experimental observations, realised that the mass of a body increases with velocity!!

$$\boxed{m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

m_0 is the rest mass, the mass of a not-moving body
c: speed of light ($3x10^5$ km/s)



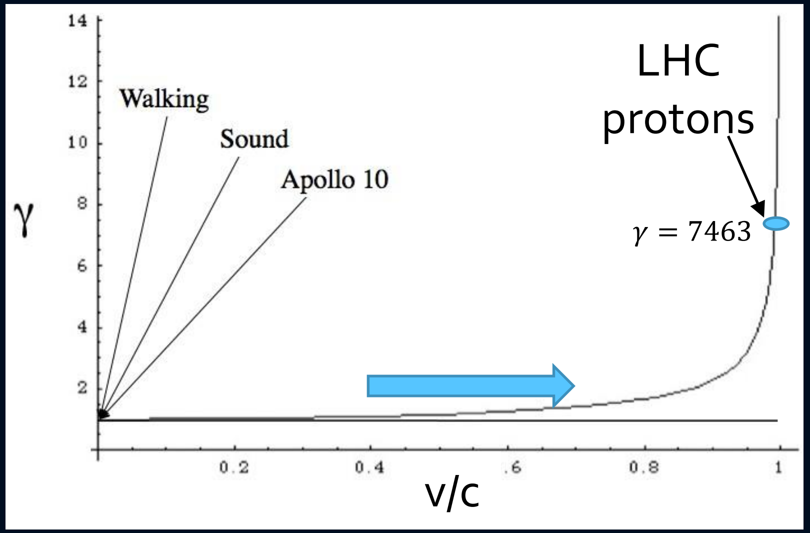
A little parenthesis about Relativity

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \boxed{\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} \Rightarrow \boxed{m = \gamma \cdot m_0} \Rightarrow \boxed{m \cdot c^2 = \gamma \cdot m_0 \cdot c^2}$$

Relativistic gamma factor

$$\Downarrow$$

$$\boxed{E = m \cdot c^2 = \gamma \cdot m_0 \cdot c^2}$$



- As the velocity of the particle gets closer to c, the mass m is greater and greater
- The body inertia increases and increases and the force applied to move the particle is less and less efficient, so the velocity increases more and more slowly and asymptotically approaches c
- But it will never be equal to c because the mass grows exponentially

e.g. LHC $$\gamma = \frac{E}{E_0} = \frac{m \cdot c^2}{m_0 \cdot c^2} = \frac{7000\ GeV}{0.938\ GeV} = 7463 \Rightarrow \frac{v}{c} = 0.9999 = \beta$$

Relativistic beta factor

We are doing well so far, but MeV it is not enough, how can we reach GeV energies?

	Size (m)	Size	Beam energy	Instrument
Aggregate of molecules: cell/bacteria	10^{-5}	10 micro meter	0.1 eV	Optical microscope
	10^{-7}	100 nano meter	10 eV	
Aggregate of atoms: molecules	10^{-9}	1 nano meter	1 keV	Electron microscope
Atoms: nucleus+electrons	10^{-10}	0.1 nano meter	10 keV	Synchrotron radiation
Nucleus (Oxygen: 8p+8n)	10^{-14}	0.01 pico meter	>100 MeV	Low energy e- or p+ accelerator
Aggregate of quarks: hadrons	10^{-15}	1 femto meter	> 1 GeV	High energy p+ accelerator
Quarks+leptons	10^{-18}	1 atto meter	> 1 TeV	High energy e- or p+ collider

What is the limitation of the cyclotrons?

- If B is a constant uniform magnetic field → ρ increases as the particle momentum increases → we get a spiral orbit → cyclotron

$$\rho = \frac{p}{qB}$$

- But there is a limitation to the B field

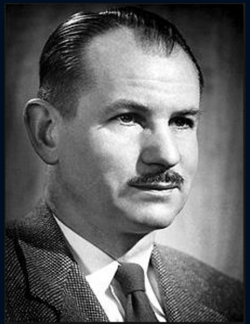
- In the end the spiral gets bigger and bigger → the size (= cost) of the cyclotron has to increase!

What can we do to keep the radius of the accelerator cte?

Synchrotrons

- If B increases synchronously with the particle momentum such the ratio p/B remains cte, then the accelerator radius is cte.

- Synchrotron principle developed almost simultaneously by E. M. McMillan (California University) & V. Veksler (Soviet Union) in 1945.



E. M. McMillan

V. Veksler

- 1949: Cosmotron @BNL ➔ proton synchrotron of 3.3 GeV, C ~ 57 m

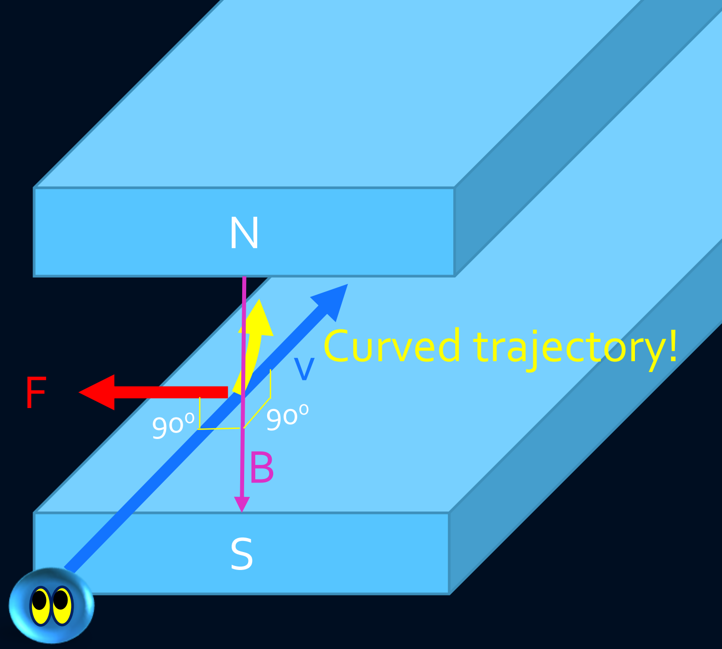
- 2008: LHC ➔ proton synchrotron of 7000 GeV, C = 27 km

- There is a technical limit to the value of B, ~ 1.5 Tesla for normal conducting magnets and ~ 8 Tesla for superconducting magnets

Let’s build a synchrotron

We need dipole magnets

- A dipole with a uniform field deviates a particle by an angle θ
- The bending angle θ depends on:
  - the length L
  - the magnetic field B
  - the particle momentum

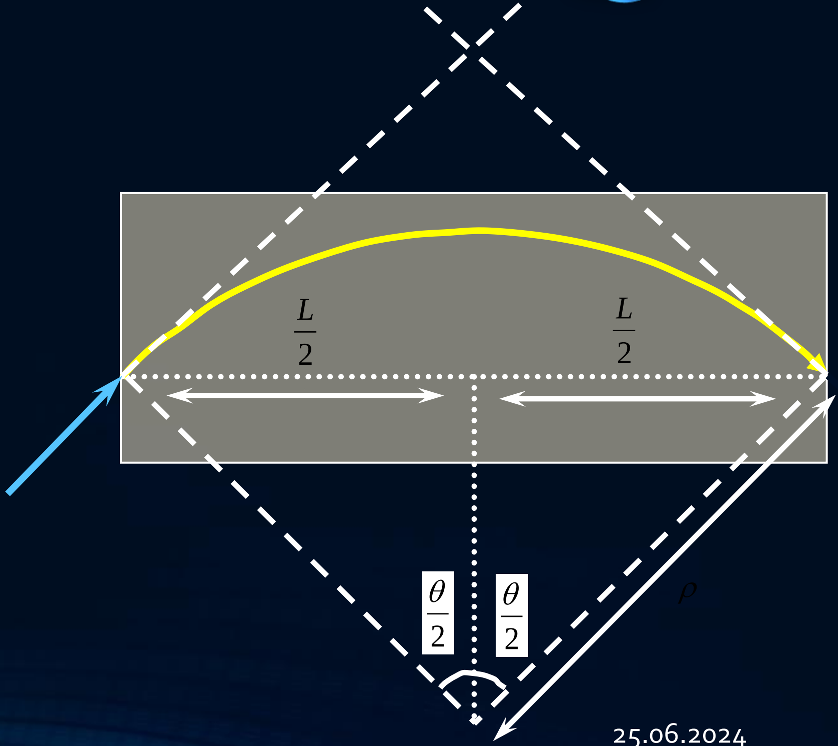


$$arc \approx angle \cdot radius$$

$$arc = L \qquad angle = \theta \qquad radius = \rho$$

$$L = \theta \cdot \rho$$

$$\theta = \frac{L}{\rho} \cdot \frac{B}{B} \qquad \theta = \frac{LB}{B\rho} = \frac{LB}{\frac{p}{q}}$$



Dipole seen from the top

Two particles in a dipole field

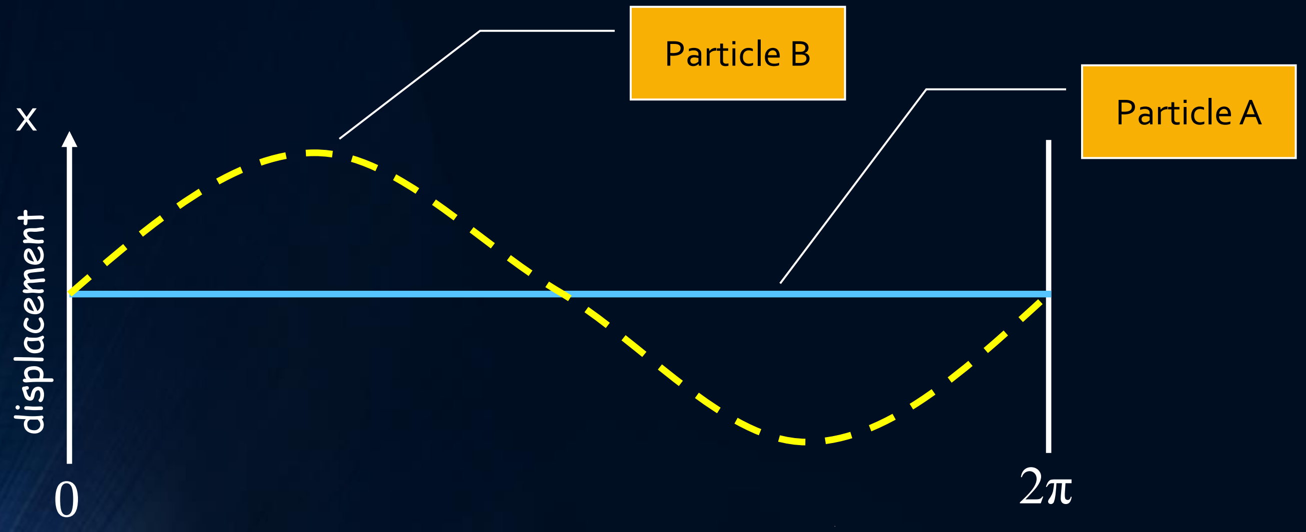
- ➢ What happens with two particles that travel in a dipole field with different initial angles, but with equal initial position and equal momentum ?

- ✓ Assume that Bρ is the same for both particles.
- ✓ Lets unfold these circles……

The 2 trajectories unfolded

Remember, this is the horizontal plane

- The horizontal displacement of particle B with respect to particle A.



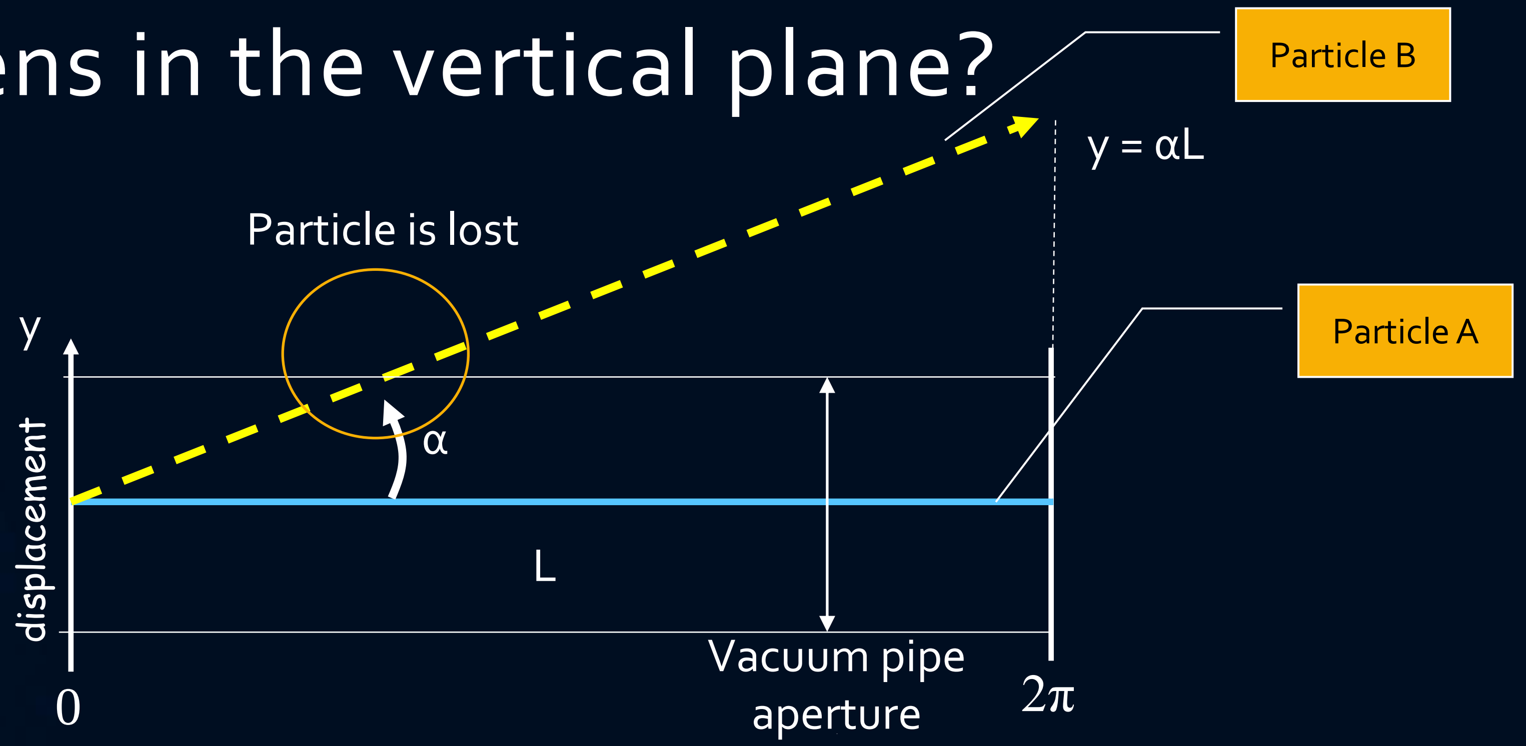
- Particle B oscillates around particle A ➔ a dipole magnet bends the particle in the horizontal plane and has a focusing effect proportional to 1/ρ (1/bending radius)
- Strength of the dipole field normalized to the particle momentum/charge:

Remember two slides ago: $L = \theta \cdot \rho$

$$\frac{B}{p/q} = \frac{1}{\rho}$$

- This type of oscillation forms the basis of all transverse motion in an accelerator.
- It is called **Betatron Oscillation**

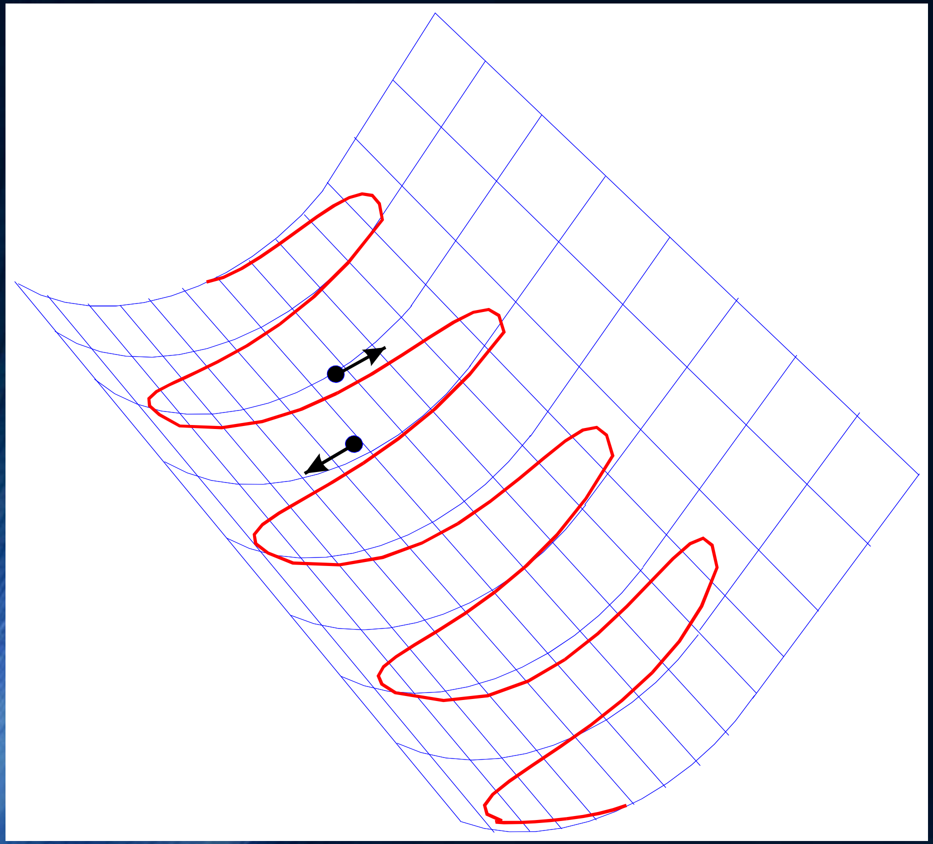
What happens in the vertical plane?



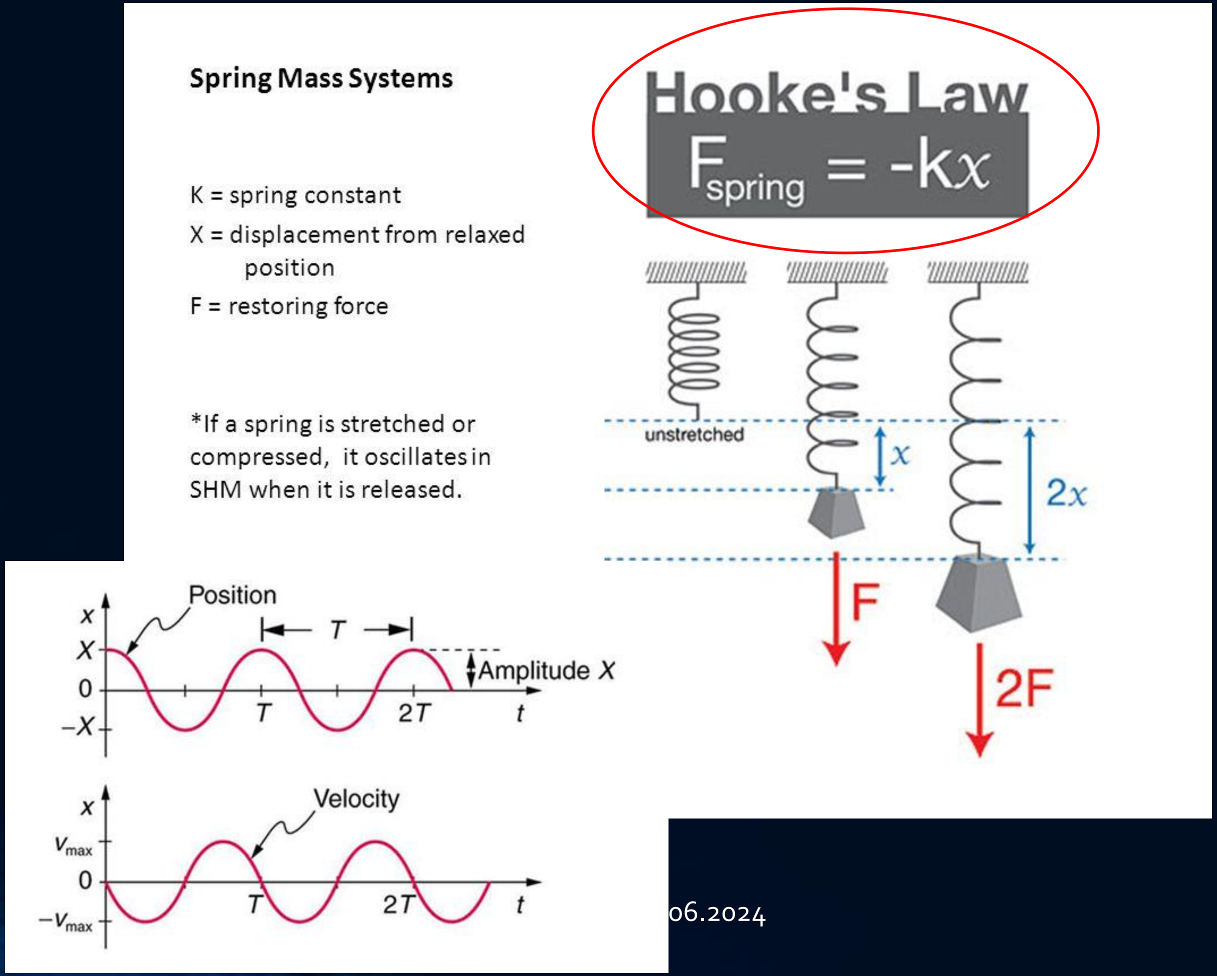
- What can we do to keep the particle inside the vacuum pipe?
- We need something that when the particle deviates from the reference trajectory by an amount y, there is a force applied to the particle proportional to y that brings the particle back on track
- Do you know a force of this kind?

The mechanical equivalent

- The gutter below illustrates how the particles in our accelerator should be focused



- The force we are looking for is of the type:



Spring Mass Systems

Hooke's Law

$$F_{spring} = -kx$$

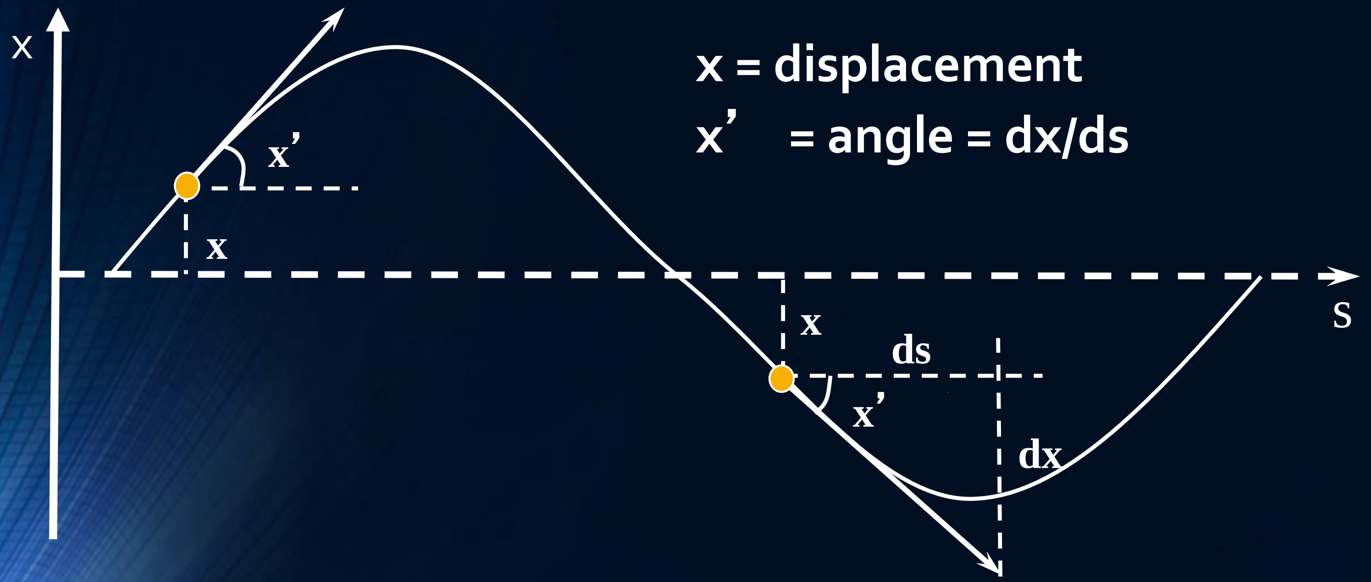
K = spring constant

X = displacement from relaxed position

F = restoring force

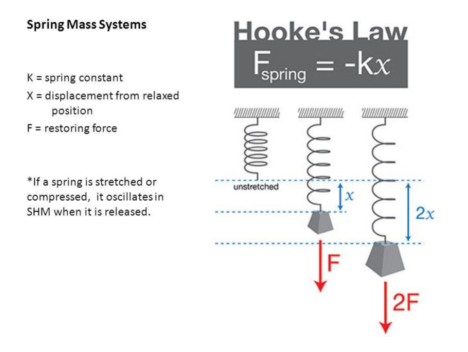
*If a spring is stretched or compressed, it oscillates in SHM when it is released.

- A particle during its transverse motion in our accelerator is characterized by:
  - **<u>Position</u>** or displacement from the central orbit
  - **<u>Angle</u>** with respect to the central orbit



x = displacement

x' = angle = dx/ds


Spring Mass Systems

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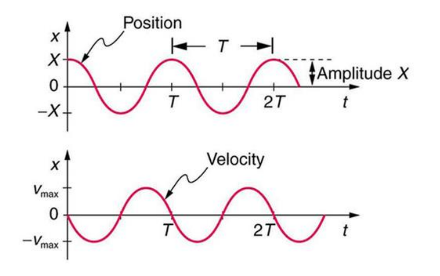


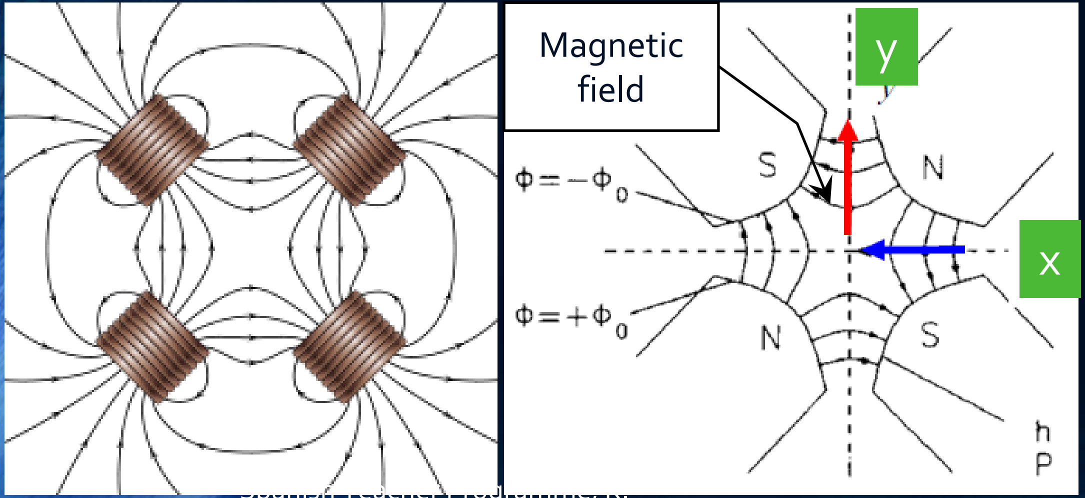
- This is a motion with a **<u>linear restoring force =f(position)</u>**, like the **<u>pendulum or spring mass system</u>**

Quadrupole fields ↔ Hook's law

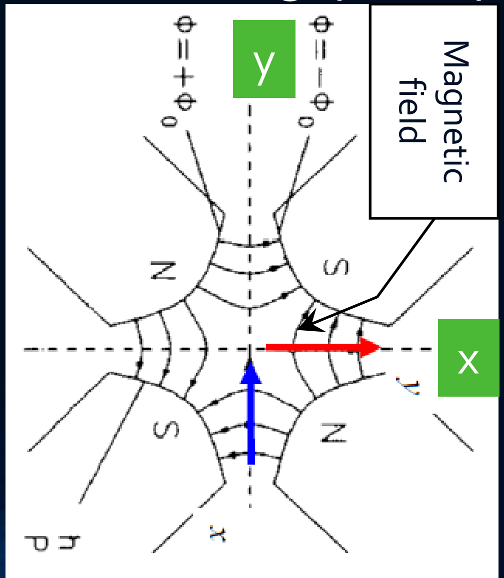
- A Quadrupole has 4 poles, 2 north and 2 south
- They are symmetrically arranged around the centre of the magnet
- There is no magnetic field along the central axis

- On the x-axis (horizontal) the field is vertical and given by $B_y \propto x$
- On the y-axis (vertical) the field is horizontal and given by $B_x \propto y$
- Field gradient, K: $\frac{d(B_y)}{dx}$ (Tm^{-1})
- Normalised gradient, k: $\frac{K}{(B\rho)}(m^{-2})$

Horizontal focusing quadrupole



Vertical focusing quadrupole



Focusing and Stable motion

- Using a combination of focusing (QF) and defocusing (QD) quadrupoles solves our problem of 'unstable' vertical motion.

- Remember that the focusing strength of a dipole magnet goes with

$$\frac{B}{p/q} = \frac{1}{\rho}$$

- The bigger the accelerator radius, ρ, the smaller the strength of the dipole field

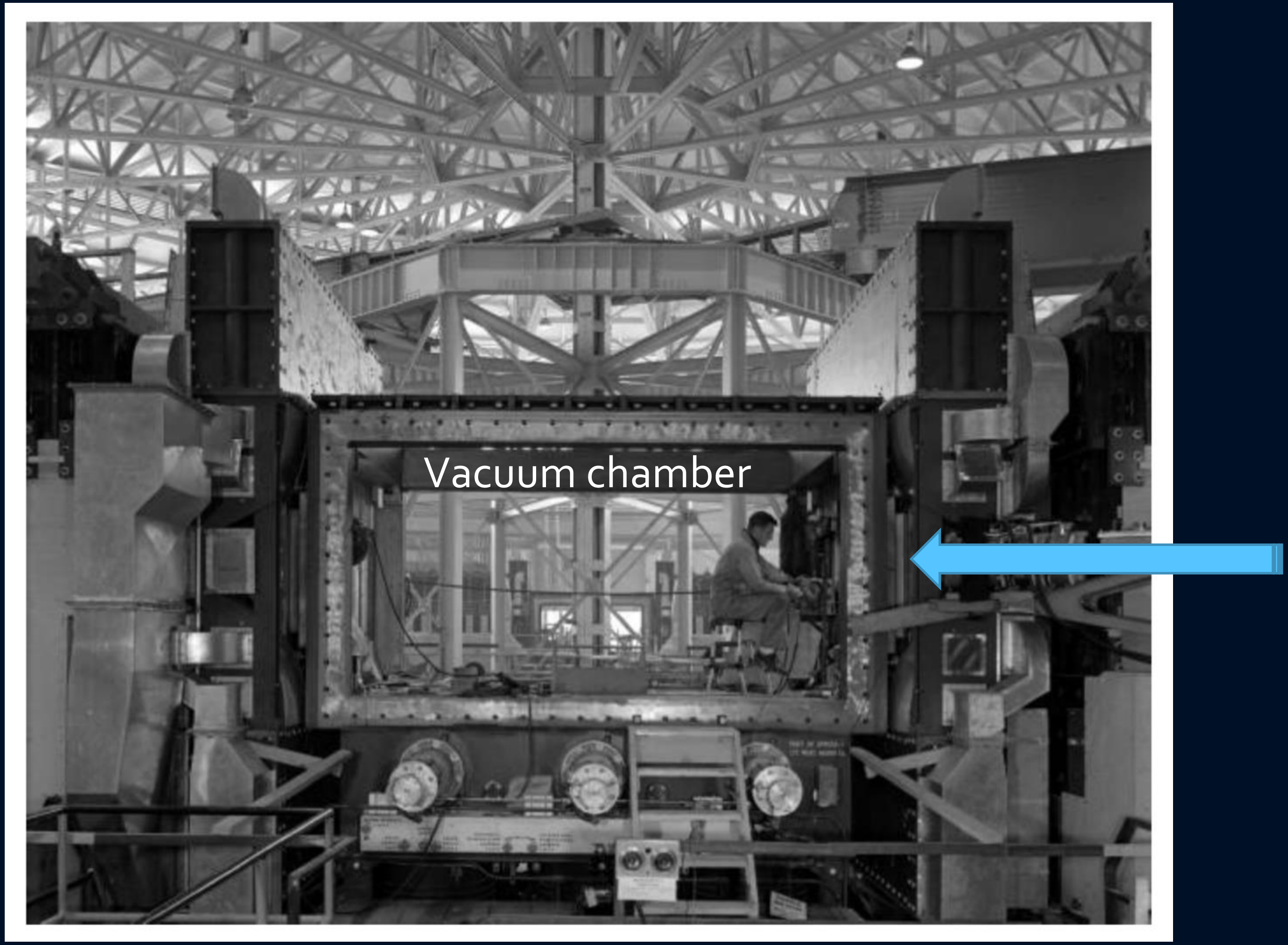
- So the focusing effect in the horizontal plane works for small radius accelerators, but would it work for LHC?

$$\rho \approx \frac{26658.9\ m}{2\pi} \cdot 66\% \approx 2800\ m \qquad \frac{1}{\rho} = \frac{1}{2800} = 0.0004\ m^{-1}$$

- Even for smaller radius accelerators would not work ➡

Does not matter!! We'll profit from the focusing effect of the quadrupoles because it works in both planes!!!!

- COSMOTRON (1949) & BEVATRON (1954) used weak focusing (strong focusing not yet invented) ➔ beam area in BEVATRON ~ 120 cm2 ➔ huge vacuum chambers



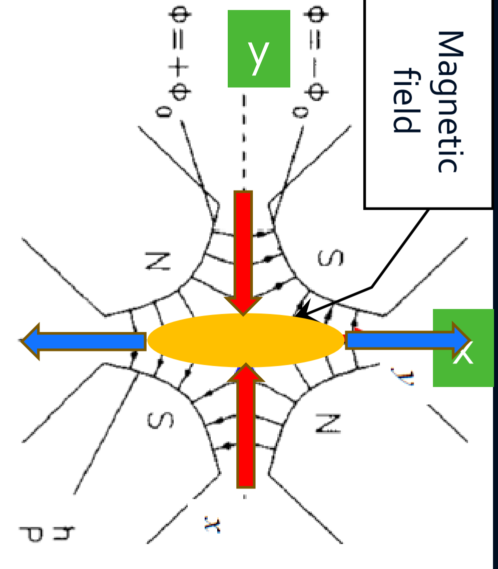
Beam direction

Focusing and Stable motion

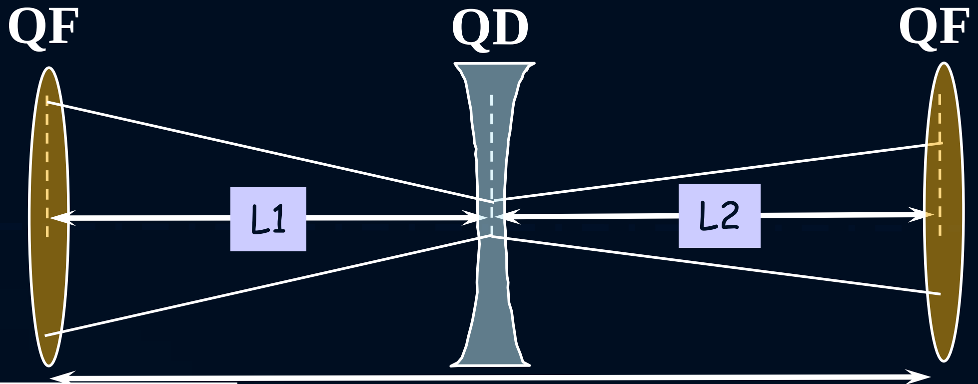
- Quadrupoles will keep the beams focused in **both planes** when the position in the accelerator, type and strength of the quadrupoles are well chosen
- By now our accelerator is composed of:
  - **Dipoles**, constrain the beam to some closed path (orbit)
  - **Focusing and Defocusing Quadrupoles**, provide horizontal and vertical focusing in order to constrain the beam in transverse directions
- A combination of focusing and defocusing sections that is very often used is the so called: **FODO lattice**
- This is a configuration of magnets where focusing and defocusing magnets alternate and are separated by non-focusing drift spaces

FODO cell

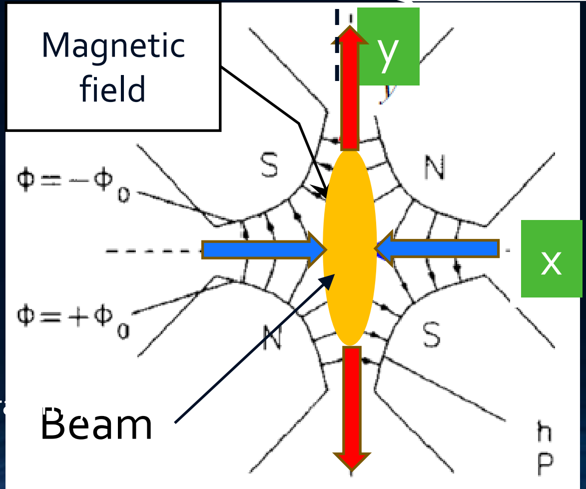
- The FODO cell is defined as follows:



Defocusing quadrupole (QD) focuses in vertical and defocuses in horizontal



'FODO' cell



Focusing quadrupole (QF) focuses in horizontal and defocuses in vertical

Circular accelerator with a FODO structure

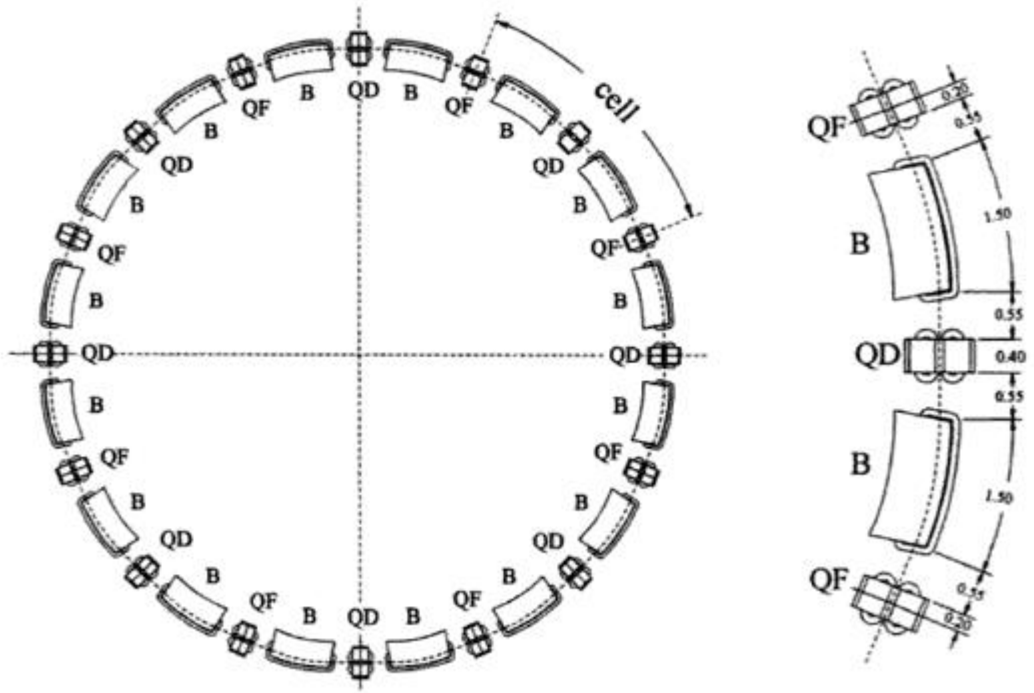


Fig. 3.31 Example of a circular accelerator employing a FODO structure. The ring consists of a number of identical cells, each consisting of two bending magnets, with quadrupoles arranged with alternating polarity between them. (by K. Wille)

$$\boldsymbol{X}_{\mathrm{E}} = \mathbf{M}_{\mathrm{D5}} \cdot \mathbf{M}_{\mathrm{Q4}} \cdot \mathbf{M}_{\mathrm{D4}} \cdot \mathbf{M}_{\mathrm{Q3}} \cdot \mathbf{M}_{\mathrm{D3}} \cdot \mathbf{M}_{\mathrm{Q2}} \cdot \mathbf{M}_{\mathrm{D2}} \cdot \mathbf{M}_{\mathrm{Q1}} \cdot \mathbf{M}_{\mathrm{D1}} \cdot \boldsymbol{X}_0 . \quad (3.90)$$

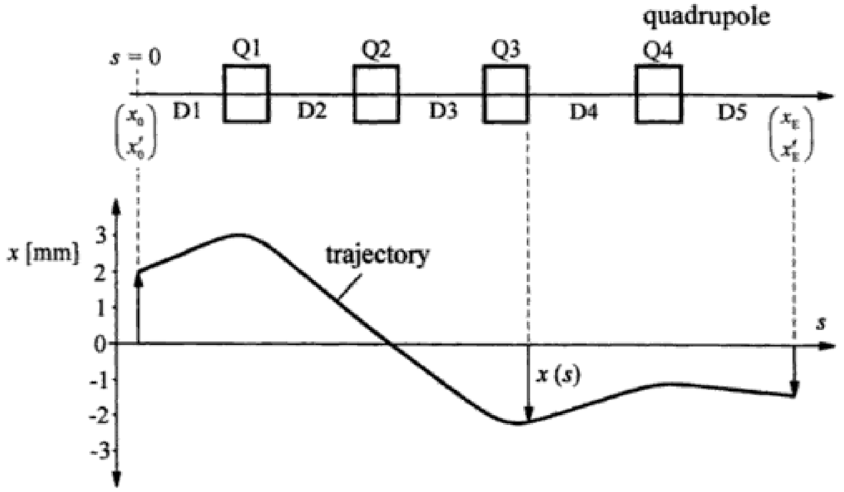
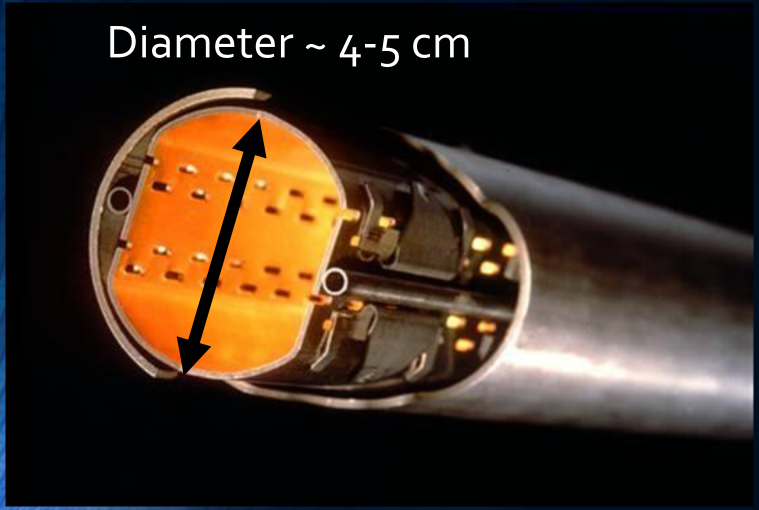


Fig. 3.21 Calculation of particle motion through a structure of multiple beam steering elements.

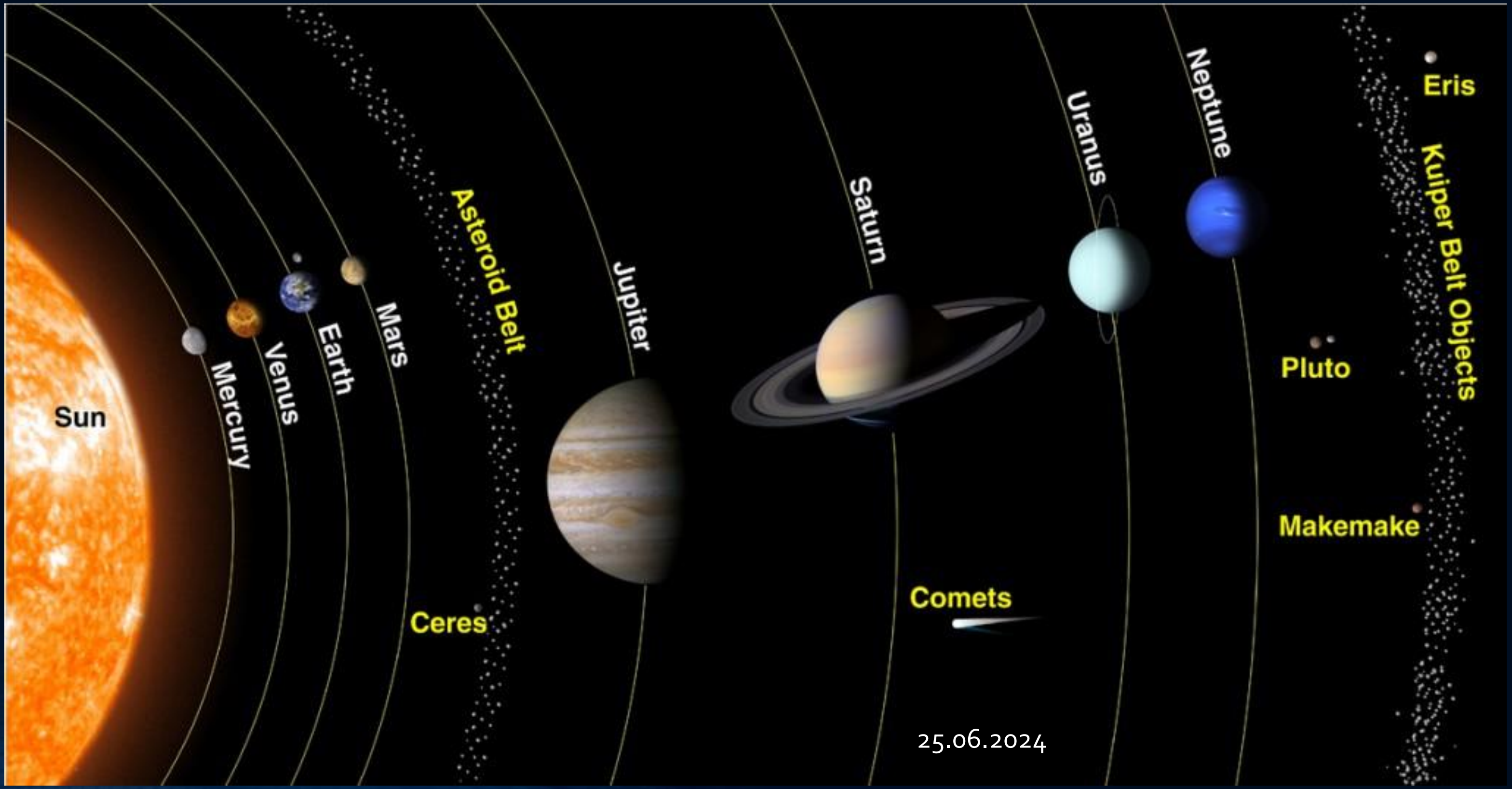
(by K. Wille)

What do we know by now?

- We know how to guide the particles on a well defined design orbit
- We know how to focus the particles to keep each single particle trajectory with in the vacuum chamber of the accelerator close to the design orbit
- In this way particles are accelerated and stored for several hours (~ 12 hours or more) travelling at about v~c ➔ L= 10^{10}-10^{11} km several times the distance Sun-Pluto and back



LHC vacuum pipe Ultra high vacuum: 10^{-10} mbar, like at 1000 km over see level



At which energy can we accelerate the particles now? Let's take LHC as example

Golden formula (you should know by heart) ➔ $B\rho = \frac{p}{q}$

Circumference ➔ FIXED!!! by LEP: 26658.9 m ➔ $\rho \approx \frac{26658.9\ m}{2\pi} \cdot \underset{\uparrow}{66\%} \approx 2800\ m$

~ 66% of the lattice elements are dipoles

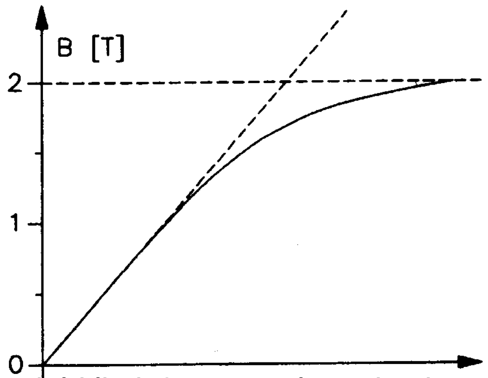
Magnetic field in the dipole magnets = 8 Tesla

We need SUPERCONDUCTING technology!!

$$p = 0.33 \cdot q \cdot B \cdot \rho \approx 0.33 \cdot 8\ T \cdot 2780\ m \approx 7\ \text{TeV}$$

	Size (m)	Size	Beam energy	Instrument
Aggregate of molecules: cell/bacteria	10^{-5}	10 micro meter	0.1 eV	Optical microscope
	10^{-7}	100 nano meter	10 eV	
Aggregate of atoms: molecules	10^{-9}	1 nano meter	1 keV	Electron microscope
Atoms: nucleus+electrons	10^{-10}	0.1 nano meter	10 keV	Synchrotron radiation
Nucleus (Oxygen: 8p+8n)	10^{-14}	0.01 pico meter	>100 MeV	Low energy e- or p+ accelerator
Aggregate of quarks: hadrons	10^{-15}	1 femto meter	> 1 GeV	High energy p+ accelerator
Quarks+leptons	10^{-18}	1 atto meter	> 1 TeV	High energy e- or p+ collider

Finally we can observe quarks and leptons!!



Field limit for normal conducting magnets due to saturation

WELL DONE !!!

Spares

Hill’s equation

- These betatron oscillations exist in both horizontal and vertical planes.
- The number of betatron oscillations per turn is called the betatron tune and is defined as Qx and Qy.
- Harmonic oscillator equation of motion:

$F = ma$ Newton’s second law

$F = -kx$ Hooke’s law

$$F = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{dx}{dt}\right) = \mathrm{m}\frac{d^2x}{dt^2}$$

$$\mathrm{m}\frac{d^2x}{dt^2} = -\mathrm{kx} \rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

K

$$\boxed{\frac{d^2x}{dt^2} + Kx = 0}$$

- If the restoring force, K is constant in ‘s’ then this is just a **<u>S</u>imple <u>H</u>armonic <u>M</u>otion**.
- In a real accelerator K = f(s), remember the FODO, there are different quadrupole magnets, each can have its own value for the restoring force.
- The equation of motion is in this case

$$\boxed{\frac{d^2x}{ds^2} + K(s)x = 0}$$

Hill’s equation, with K(s):

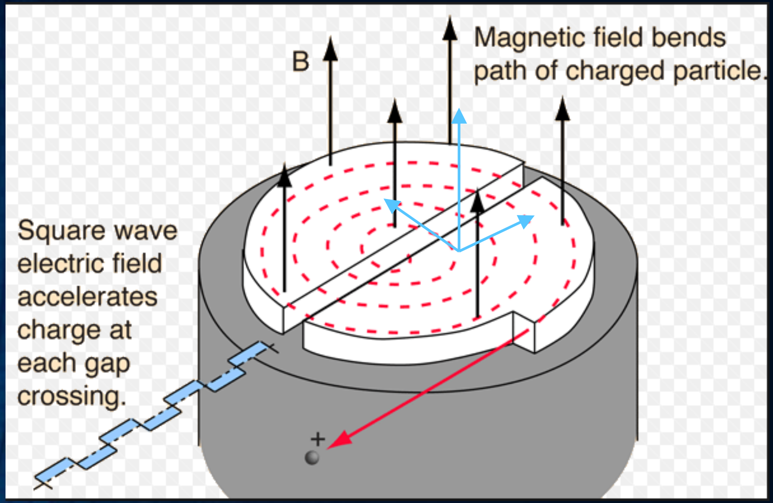
$$K(s) = \frac{1}{\rho(s)^2} - k(s)$$

Focusing effect: from dipoles

from quadrupoles

This is our first circular accelerator ➔ cyclotron

Not valid for relativistic particles



Equation of motion within the homogeneous magnetic field:

It is the Lorentz force without the electric field:

$$\vec{F} = \underbrace{q\vec{E}}_{\text{Electric force}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{Magnetic force}}$$

The particle follows a circular orbit with revolution frequency: $\omega_z = \frac{q}{m} B_z$ Cyclotron frequency

The RF frequency = cyclotron frequency

The higher the velocity the larger the radius of curvature (provided the mass remains cte) ➔ the particle gets more and more “rigid” and the magnetic field, which remains constant, has more and more difficulties to bend the particle.

$\omega_z \neq$ f(particle velocity)

This is our first circular accelerator ➔ (synchro)cyclotron

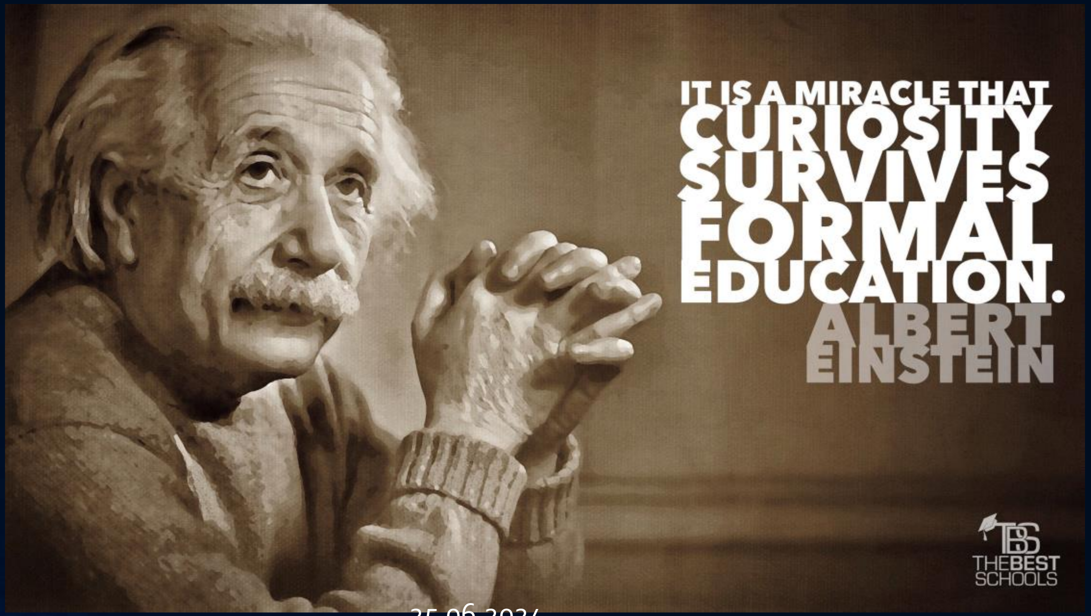
- Classical cyclotrons can accelerate protons, deuterons and alpha particles up to 22 MeV per charge.
- At this energies those particles are not relativistic (v ~ 0.15c), so the mass ~ cte, so the cyclotron frequency remains cte.

$$\omega_z = \frac{q}{m} B_z$$

- As the energy increases the particles are more and more relativistic and their mass is not cte, it increases:

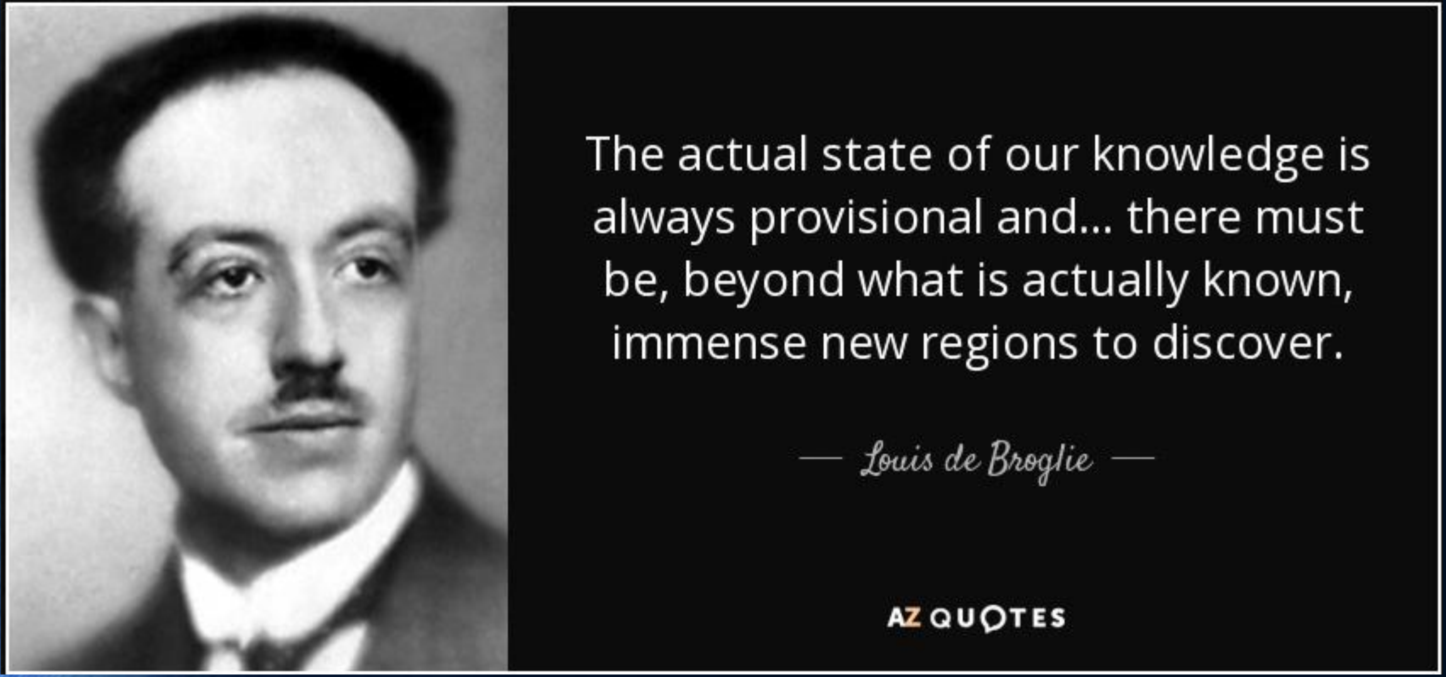
$$E = mc^2 = \gamma m_0 c^2$$

- Therefore, ω_z decreases with increasing m
- If ω_{RF} is decreased accordingly, higher energies can be reached.
- This is the synchrocyclotron principle



How can we observe such small particles?

Let’s ask De Broglie



The actual state of our knowledge is always provisional and... there must be, beyond what is actually known, immense new regions to discover.

— Louis de Broglie —

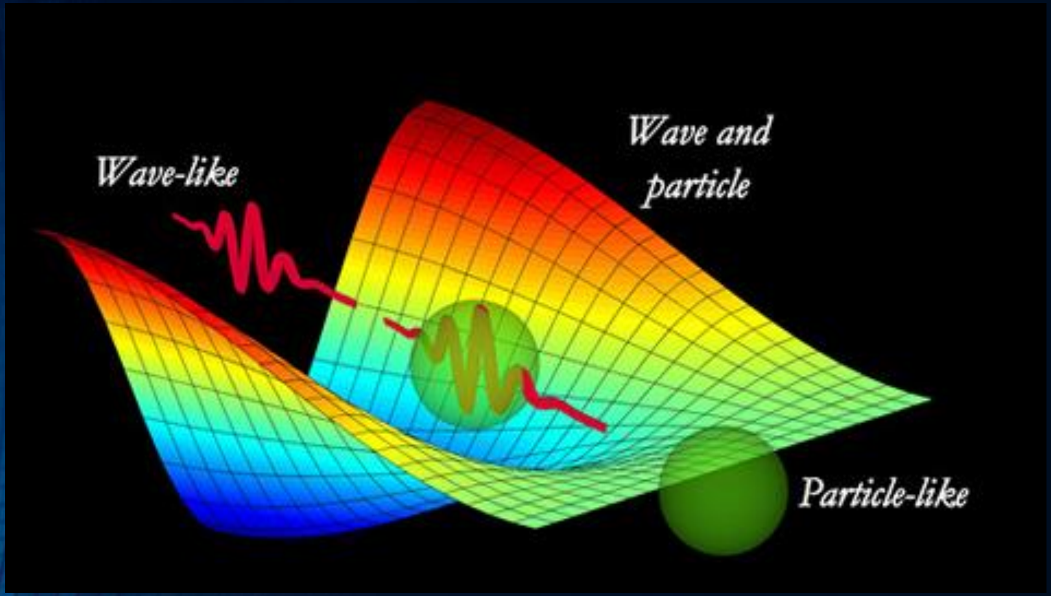
AZ QUOTES

In his 1924 PhD thesis suggested that “MOVING OBJECTS ACT LIKE WAVES”

A particle of mass m and speed v behaves like a wave with wavelength λ:

$$\lambda = \frac{h}{mv}$$

How can we observe such small particles?



De Broglie wavelength

$$\lambda = \frac{h}{mv}$$

- We just saw that photons are limited in size, what else we can use?
- Good candidates are the microscopic particles itself
- We just learnt they are waves as well
- Its De Broglie wavelength must be small compared to the size of the structure we want to observe

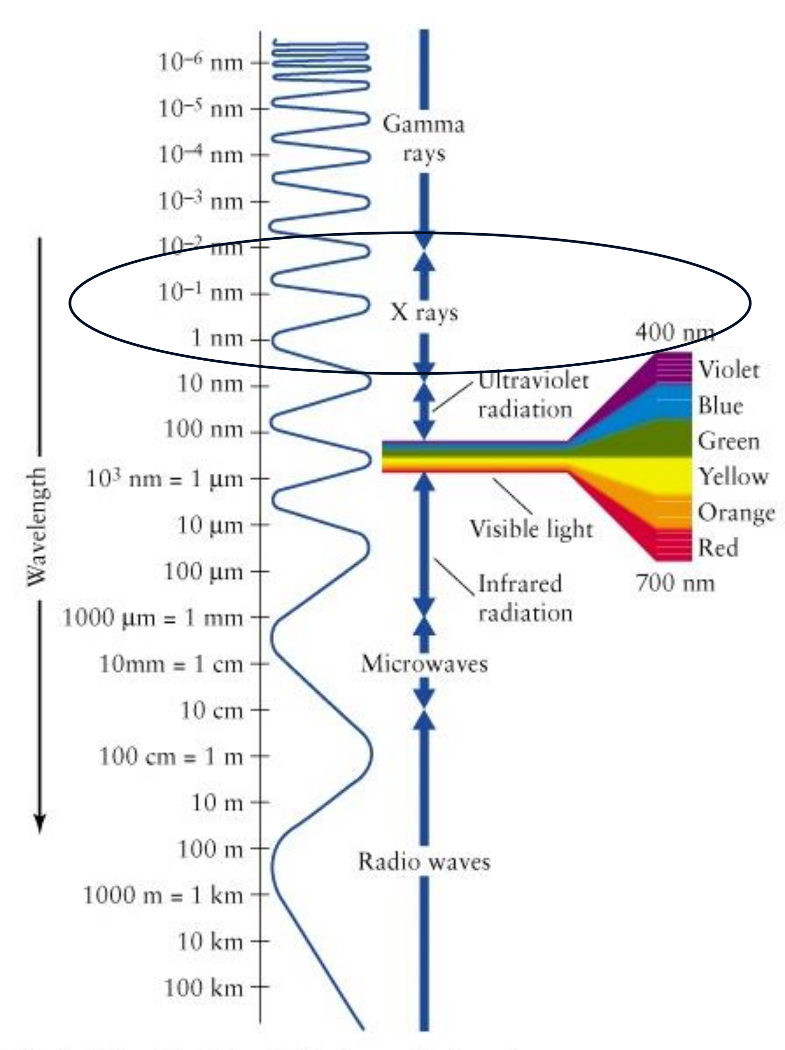
$$\lambda = \frac{h}{mv} = \frac{hc}{E\beta} \Rightarrow E = \frac{hc}{\lambda\beta}$$

How can we observe such small particles?

Aggregate of atoms:
molecules

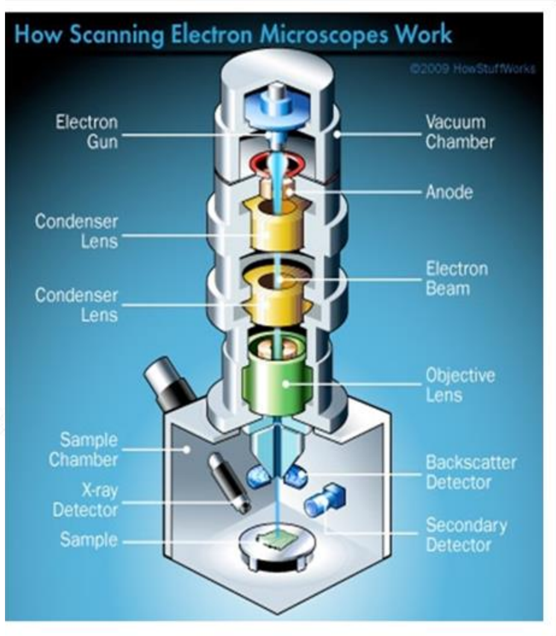
Size = 10^{-9} m → 1 nano

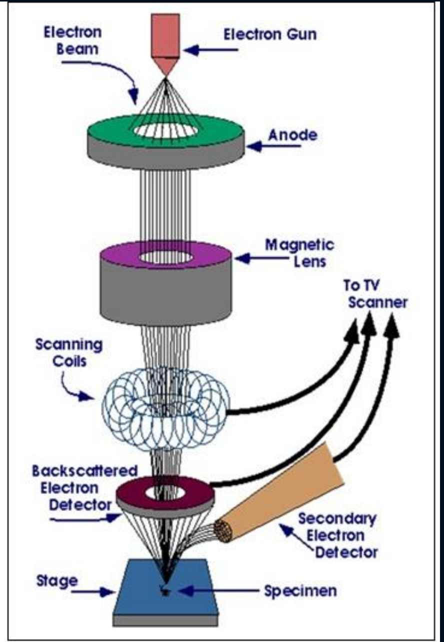
$$E = \frac{hc}{\lambda\beta}$$ → 1 keV



(Wavelength image from *Universe* by Freedman and Kaufmann.)

Electron microscope

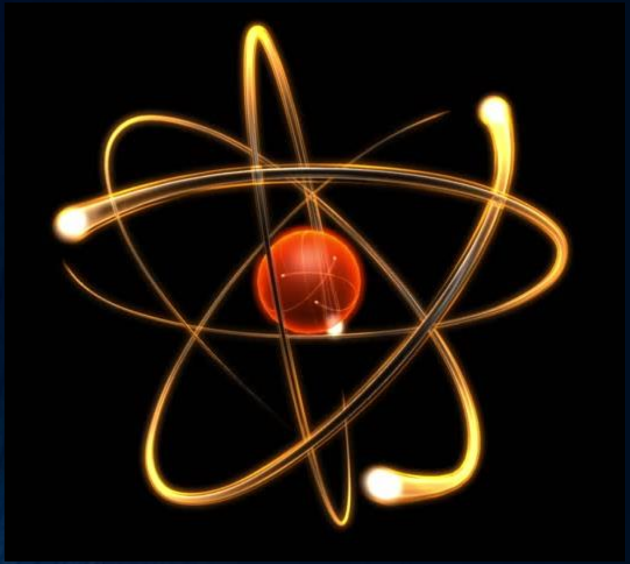




This is an accelerator!!

How can we observe such small particles?

Atoms: nucleus+electrons



Size = 10^{-10} m ➔ 0.1 nano

$$E = \frac{hc}{\lambda\beta}$$ ➔ 10 keV

Synchrotron radiation facility