Norway grants

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Quantum information and CP measurement in $H \to \tau^+\tau^-$ at future lepton colliders

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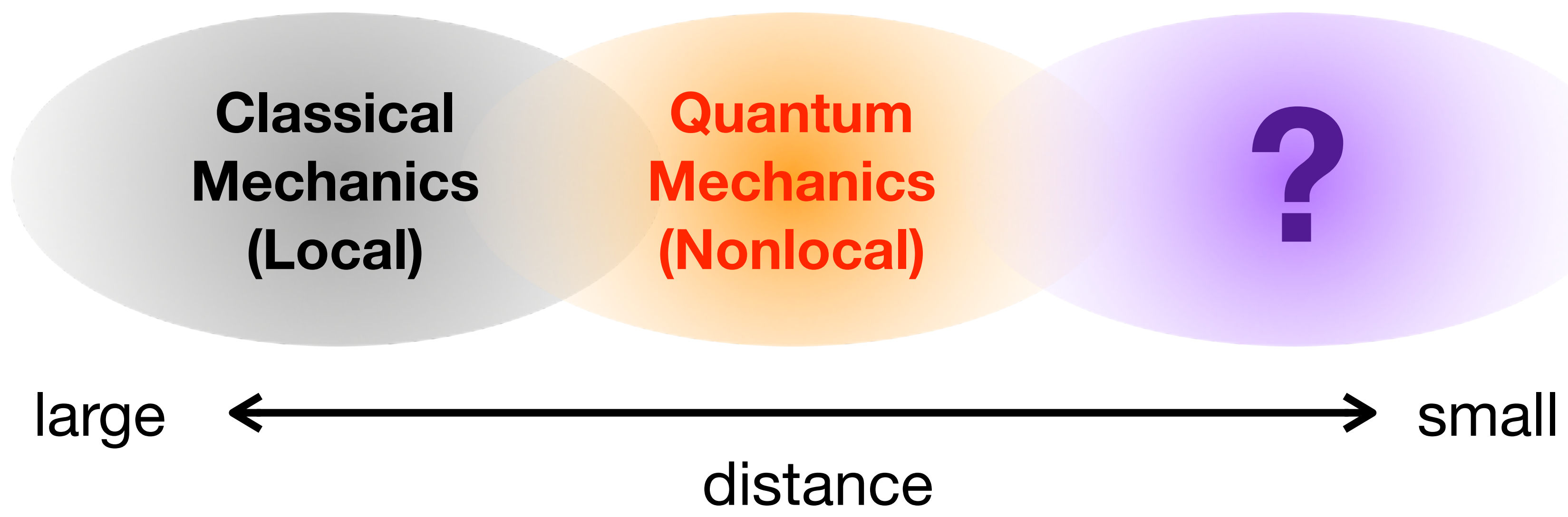
25/09/2023, ECFA HTE mini-workshop on e+e- physics at 240-350 GeV

Testing QM at high energy colliders



Motivation

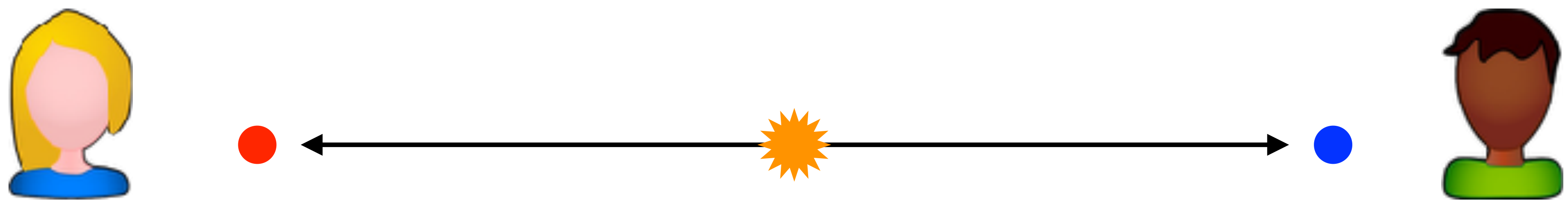
✤ Bell inequalities/Entanglement have not been tested at the TeV energy scale:

➡ LHC (and $FCC_{ee/hh}$) provides the unique opportunity for this test

✤ Detection of Entanglement/Bell violation requires a detailed analysis of spin correlation:

➡ provides a very good test for the Standard Model (sensitive to BSM)

Entanglement at Colliders



$$|A\rangle_L = \sum_{m_a} c_{m_a} |m_a\rangle$$

Alice's local spin state

$$|B\rangle_L = \sum_{m_b} c_{m_b} |m_b\rangle$$

Bob's local spin state

Entanglement

$$|\Psi\rangle \neq |A\rangle_L \otimes |B\rangle_L$$

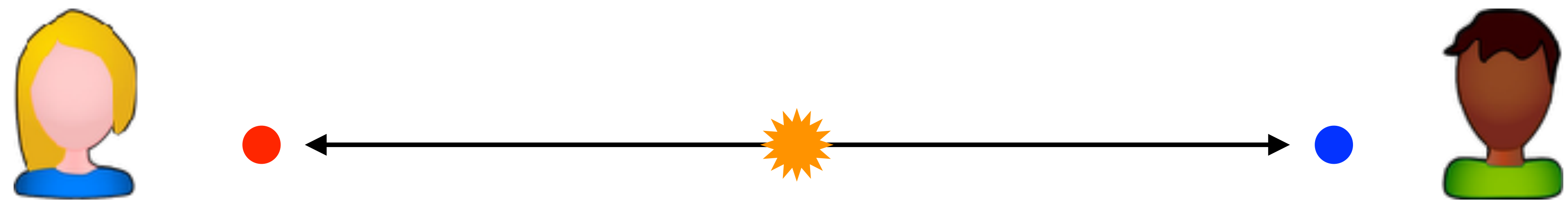
Total quantum state

e.g.) $$|\Psi^{(0,0)}\rangle \doteq \frac{|+_z\rangle_A \otimes |-_z\rangle_B - |-_z\rangle_A \otimes |+_z\rangle_B}{\sqrt{2}}$$

Entanglement at Colliders

Spin measurement

S_i^A



Spin measurement

S_j^B

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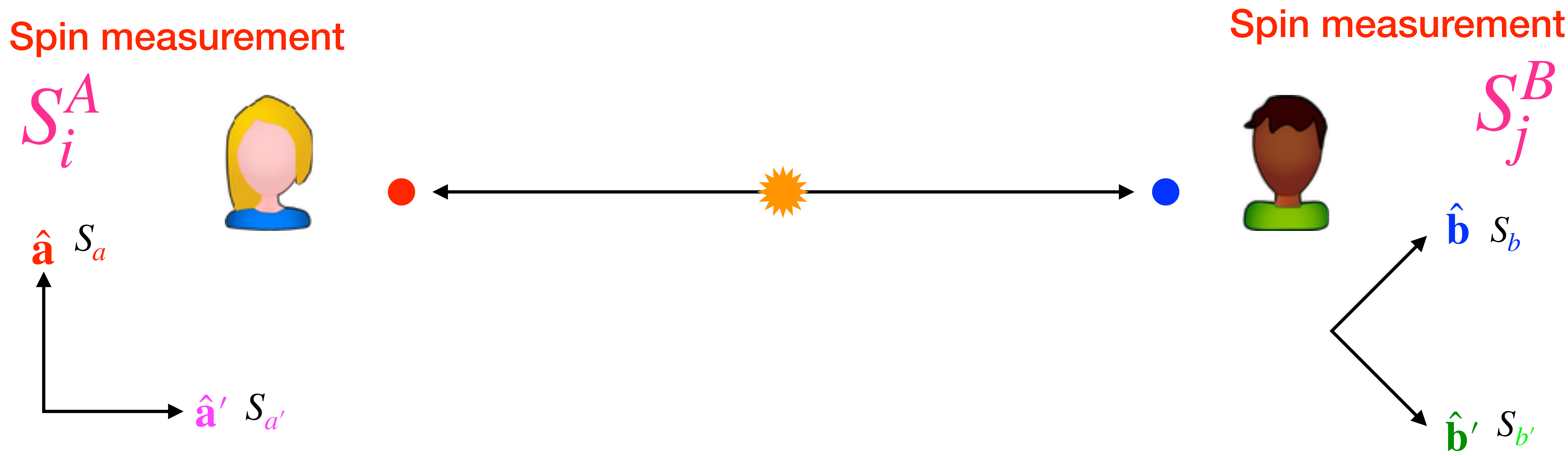
Spin correlation $C_{ij} = \langle S_i^A S_j^B \rangle$ **contains the information of entanglement**

Entanglement at Colliders



$$R_{\rm CHSH} \equiv \frac{1}{2}\left| \langle S_a^A S_b^B \rangle - \langle S_a^A S_{b'}^B \rangle + \langle S_{a'}^A S_b^B \rangle + \langle S_{a'}^A S_{b'}^B \rangle \right|$$

Local Hidden Variable

$$\langle S_i^A \cdot S_j^B \rangle = \sum_\lambda P(\lambda)\, S_i^A(\lambda)\, S_j^B(\lambda)$$

$\Rightarrow$ $R_{\rm CHSH} \leq 1$ **(Bell-inequality)** [CHSH(1969)]

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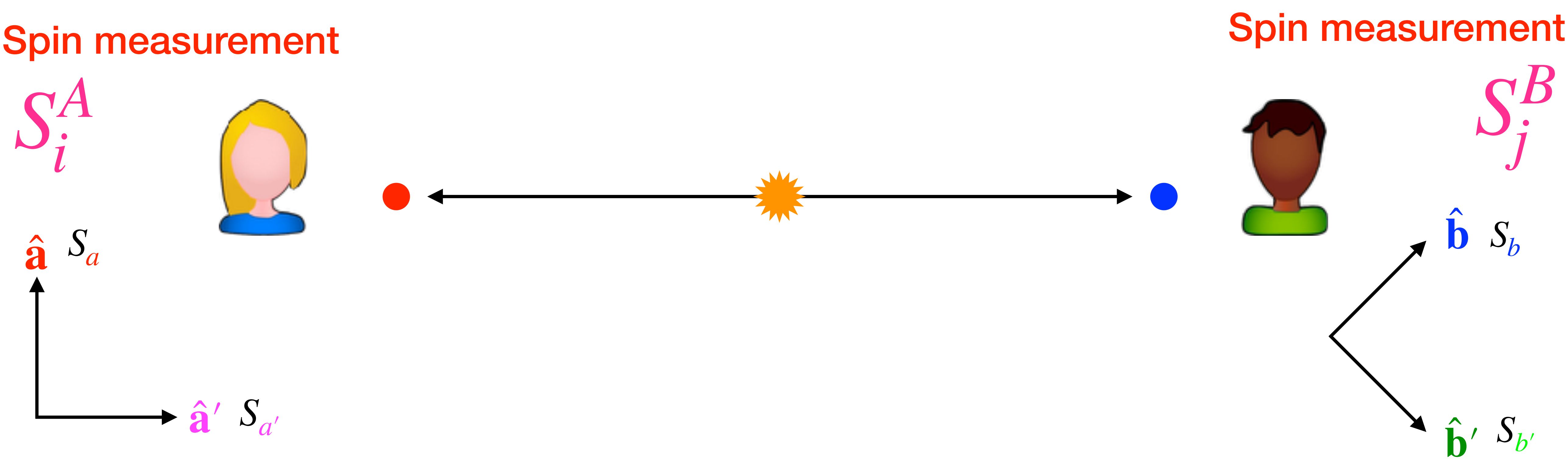
Entanglement at Colliders



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Quantum Mechanics

$\Rightarrow$ $R_{\rm CHSH} \leq \sqrt{2}$ [Tsirelson(1987)]

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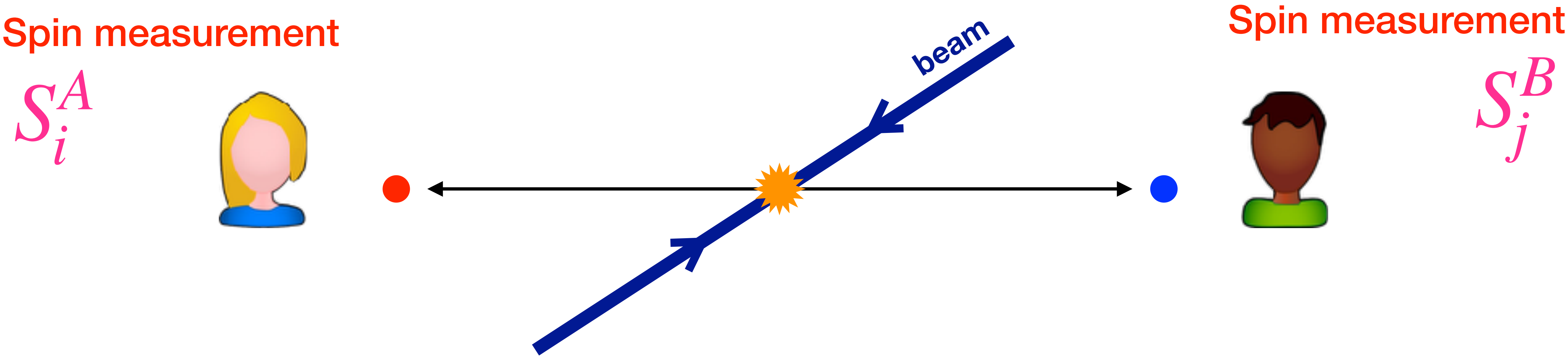
Entanglement at Colliders



How can we measure spins?

Entanglement at Colliders

Spin measurement

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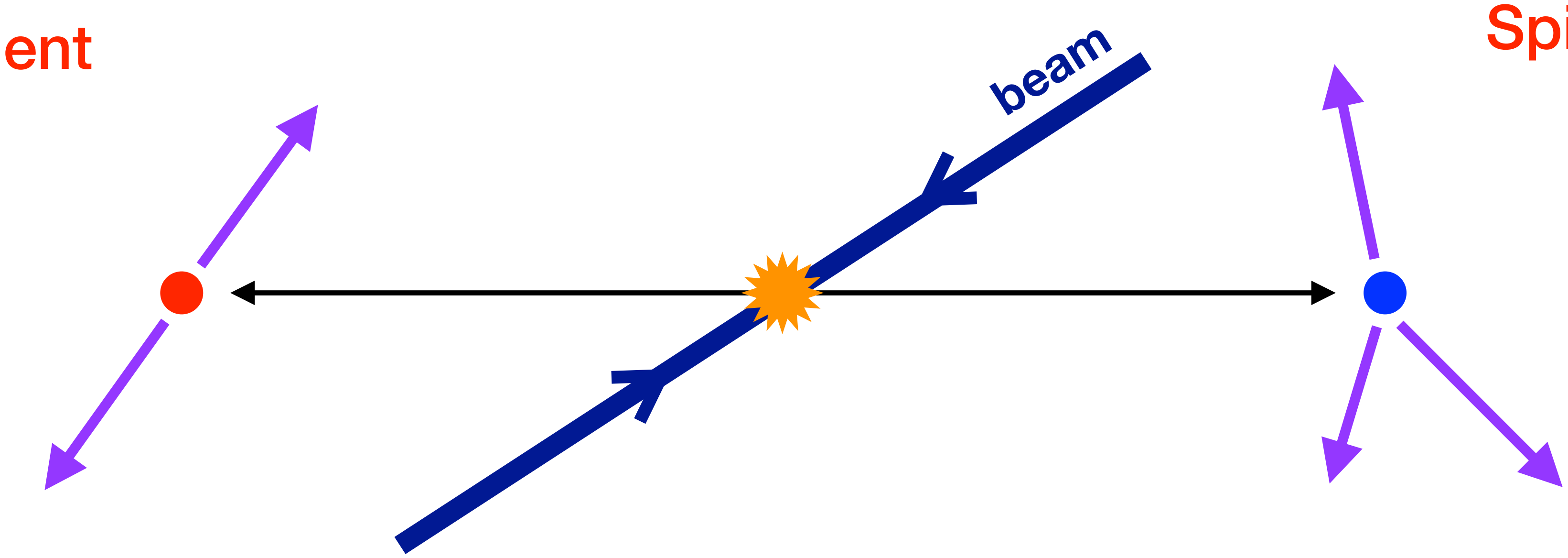


Spin measurement

S_j^B

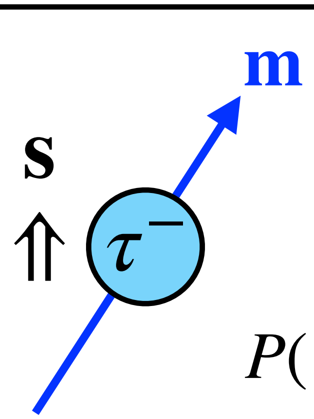
spin analyzing power $\alpha \in [-1,1]$

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha \cdot (\hat{\mathbf{u}} \cdot \hat{\mathbf{S}})$$

direction of a decay product

How can we measure spins?

$\Rightarrow$ Spin can only be inferred from the decay



$$P(+\,|\,\mathbf{m}) = |\langle +_{\mathbf{m}} \,|\, +_{\mathbf{s}} \rangle|^2$$

$$= \frac{1 + (\mathbf{m} \cdot \mathbf{s})}{2}$$

Entanglement at Colliders

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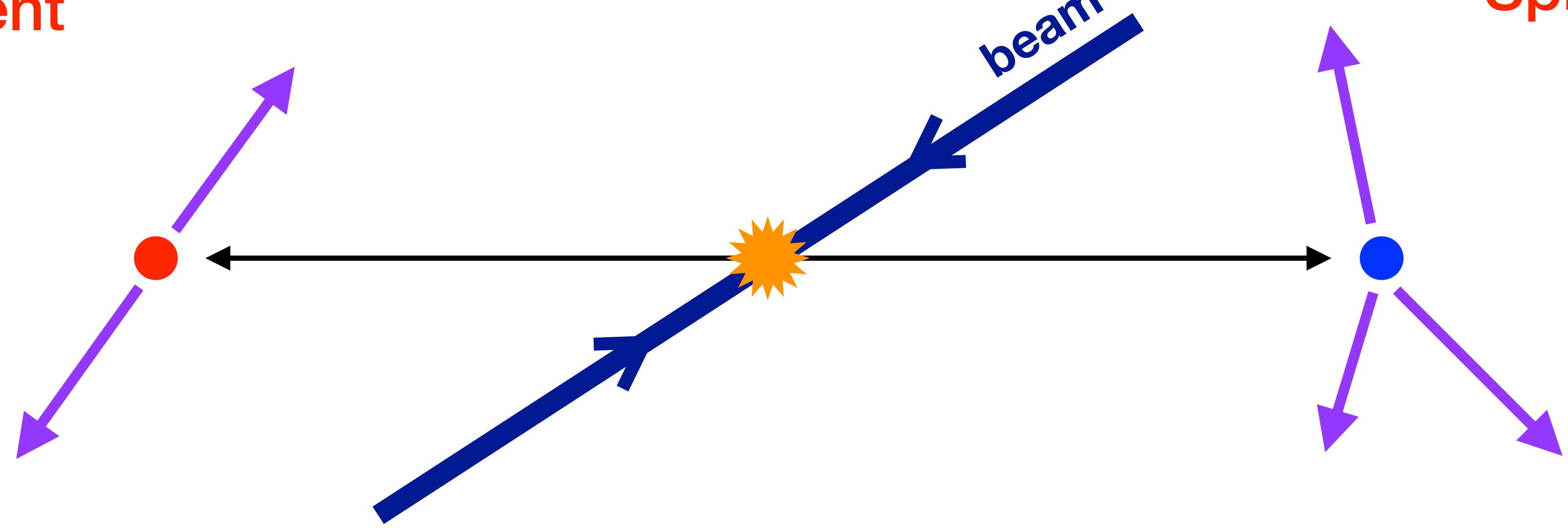

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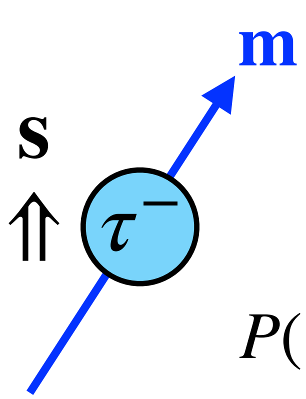

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How can we measure spins?

⇒ Spin can only be inferred from the decay

→ Condition on produced particles:

- have non zero spin
- their decay must be analyzable

$$W^{\pm},\ Z^0,\ t,\ \tau$$

Entanglement at Colliders

Spin measurement

S_i^A

Spin measurement

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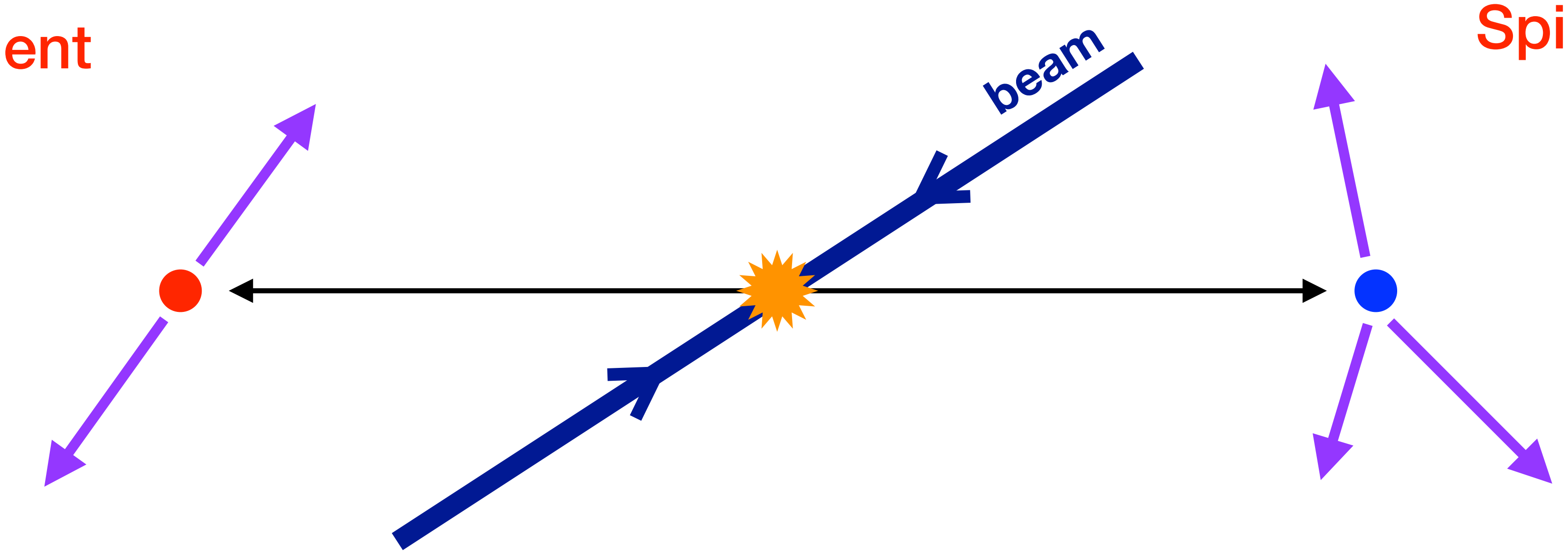

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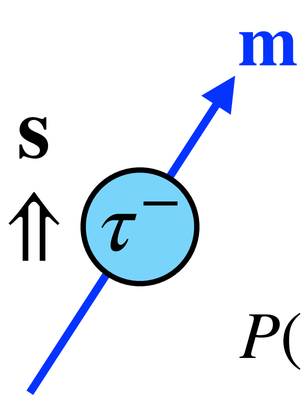

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$W^\pm,\ Z^0,\ t,\ \tau$

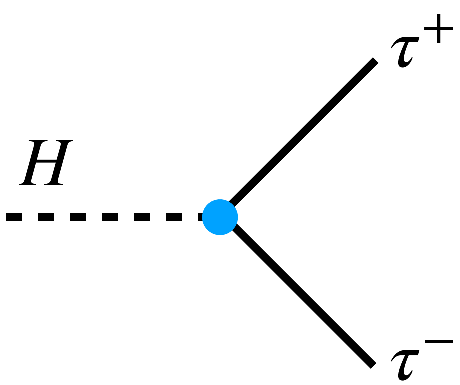
tau is special

$m_\tau \ll M_{\text{weak}}$ → **experimental challenge**

$H \to \tau^+\tau^-$

SM: $(\kappa, \delta) = (1,0)$

$$\mathscr{L}_{\rm int} = -\frac{m_\tau}{v_{\rm SM}} \kappa H \bar{\psi}_\tau (\cos\delta + i\gamma_5 \sin\delta)\, \psi_\tau$$



$$\mathcal{M}^{m\bar{m}} = c\, \bar{u}^m(p)(\cos\delta + i\gamma_5 \sin\delta) v^{\bar{m}}(\bar{p})$$

density matrix:

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}} |\mathcal{M}^{m\bar{m}}|^2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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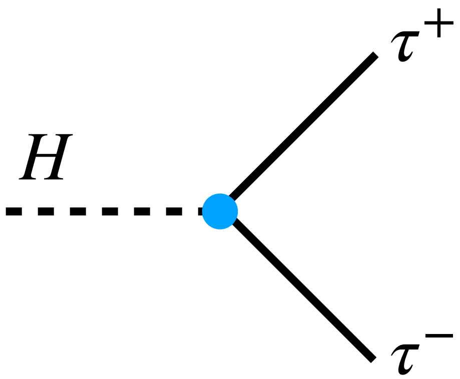

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$$|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle$$

$\delta = 0$
(CP even)

$$|+-\rangle + |-+\rangle$$

$\delta = \pi/2$ (CP odd)

$$|+-\rangle - |-+\rangle$$

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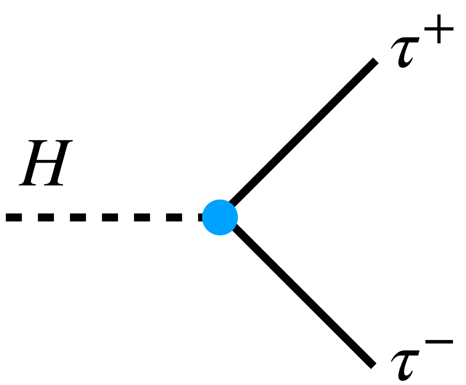

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$$|\Psi_{H\to\tau\tau}(\delta)\rangle \propto |+-\rangle + e^{i2\delta}|-+\rangle$$

$\delta = 0$ (CP even)

$$|\Psi^{(s=1,m)}\rangle \propto \begin{pmatrix} |++\rangle \\ |+-\rangle + |-+\rangle \\ |--\rangle \end{pmatrix}$$

$\delta = \pi/2$ (CP odd)

$$|\Psi^{(0,0)}\rangle \propto |+-\rangle - |-+\rangle$$

Parity: $P = (\eta_f \eta_{\bar{f}}) \cdot (-1)^l$ with $\eta_f \eta_{\bar{f}} = -1$:

$$J^P = \begin{cases} 0^+ \implies & l = s = 1 \\ 0^- \implies & l = s = 0 \end{cases}$$

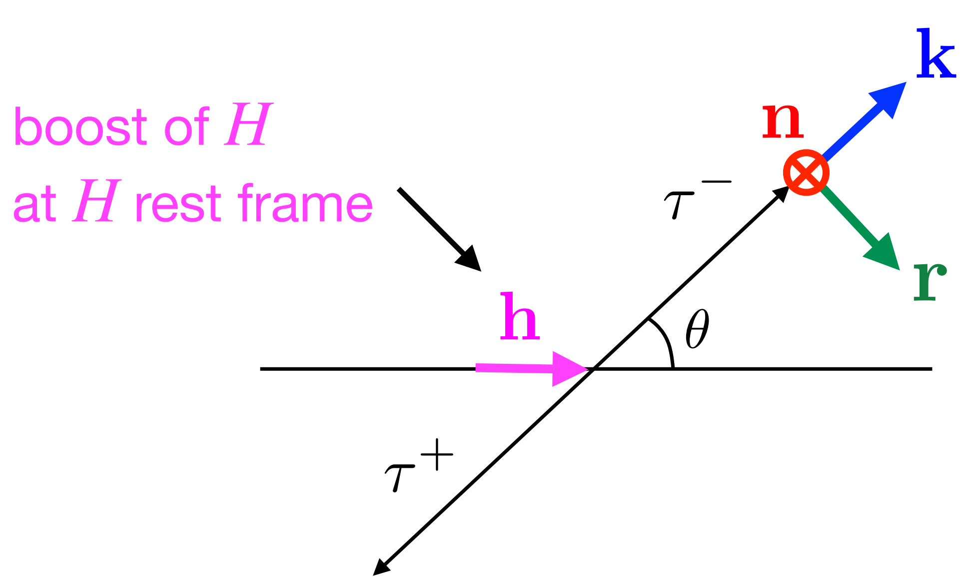
density matrix:

$$\rho_{mn,\bar{m}\bar{n}} = \frac{\mathcal{M}^{*n\bar{n}}\mathcal{M}^{m\bar{m}}}{\sum_{m\bar{m}}|\mathcal{M}^{m\bar{m}}|^2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho = \frac{1}{4}\left[\mathbf{1}_4 + B_i \cdot (\sigma_i \otimes \mathbf{1}) + \overline{B}_i \cdot (\mathbf{1} \otimes \sigma_i) + C_{ij} \cdot (\sigma_i \otimes \sigma_j)\right]$$

$$\langle \hat{S}_i^{\tau^-} \rangle = \mathrm{Tr}[(\sigma_i \otimes \mathbf{1})\rho] = B_i = 0 \qquad \langle \hat{S}_i^{\tau^+} \rangle = \mathrm{Tr}[(\mathbf{1} \otimes \sigma_i)\rho] = \overline{B}_i = 0$$

$$\langle \hat{S}_i^{\tau^-} \hat{S}_j^{\tau^+} \rangle = \mathrm{Tr}[(\sigma_i \otimes \sigma_j)\rho] = C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



$i, j = 1, 2, 3$

↕ ↕ ↕

helicity basis $(\hat{\mathbf{r}}, \hat{\mathbf{n}}, \hat{\mathbf{k}})$

Once $C_{ij} = \langle \hat{S}_i^{\tau^-} \hat{S}_j^{\tau^+} \rangle$ are measured/computed, it is straightforward to obtain:

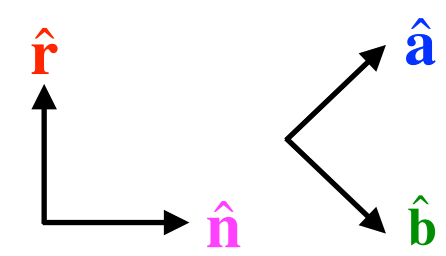
❖ Entanglement (Concurrence):

$$\mathcal{C}[\rho] = \max\left[0,\ \frac{D_+ + C_{kk} - 1}{2},\ \frac{D_- - C_{kk} - 1}{2}\right] \quad > 0$$

$$D_\pm \equiv \sqrt{(C_{rn} \pm C_{nr})^2 + (C_{rr} \mp C_{nn})^2}$$

❖ Bell non-locality

$$R_{\text{CHSH}} \equiv \frac{1}{2}\left| \langle \hat{S}_r^{\tau^-} \hat{S}_a^{\tau^+} \rangle - \langle \hat{S}_r^{\tau^-} \hat{S}_b^{\tau^+} \rangle + \langle \hat{S}_n^{\tau^-} \hat{S}_a^{\tau^+} \rangle + \langle \hat{S}_n^{\tau^-} \hat{S}_b^{\tau^+} \rangle \right| \quad > 1$$



❖ Steerability

$$\mathcal{S}[\rho] \equiv \frac{1}{2\pi} \int d\Omega_{\hat{\mathbf{n}}} \sqrt{\hat{\mathbf{n}}^T C^T C \hat{\mathbf{n}}} \quad > 1$$

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$H \to \tau^+ \tau^-$

$$\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{-i2\delta} & 0 \\ 0 & e^{i2\delta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

prediction

❖ **Entanglement (Concurrence):**

$$\mathcal{C}[\rho] = \max\left[0, \frac{D_+ + C_{kk} - 1}{2}, \frac{D_- - C_{kk} - 1}{2}\right] = 1 > 0$$

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❖ **Bell non-locality**

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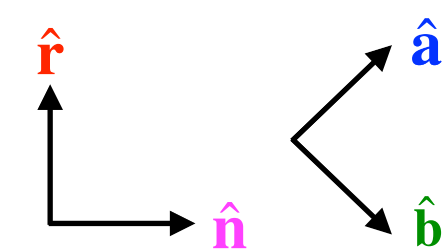


❖ **Steerability**

$$\mathcal{S}[\rho] \equiv \frac{1}{2\pi} \int d\Omega_{\hat{\mathbf{n}}} \sqrt{\hat{\mathbf{n}}^T C^T C \hat{\mathbf{n}}} = 2 > 1$$

independent of CP phase δ

How can we measure spin correlation?

$$\frac{d\Gamma}{d\Omega} \propto 1 + \alpha \cdot (\hat{\mathbf{u}}^{\pi^{\pm}} \cdot \hat{\mathbf{S}}^{\tau^{\pm}})$$

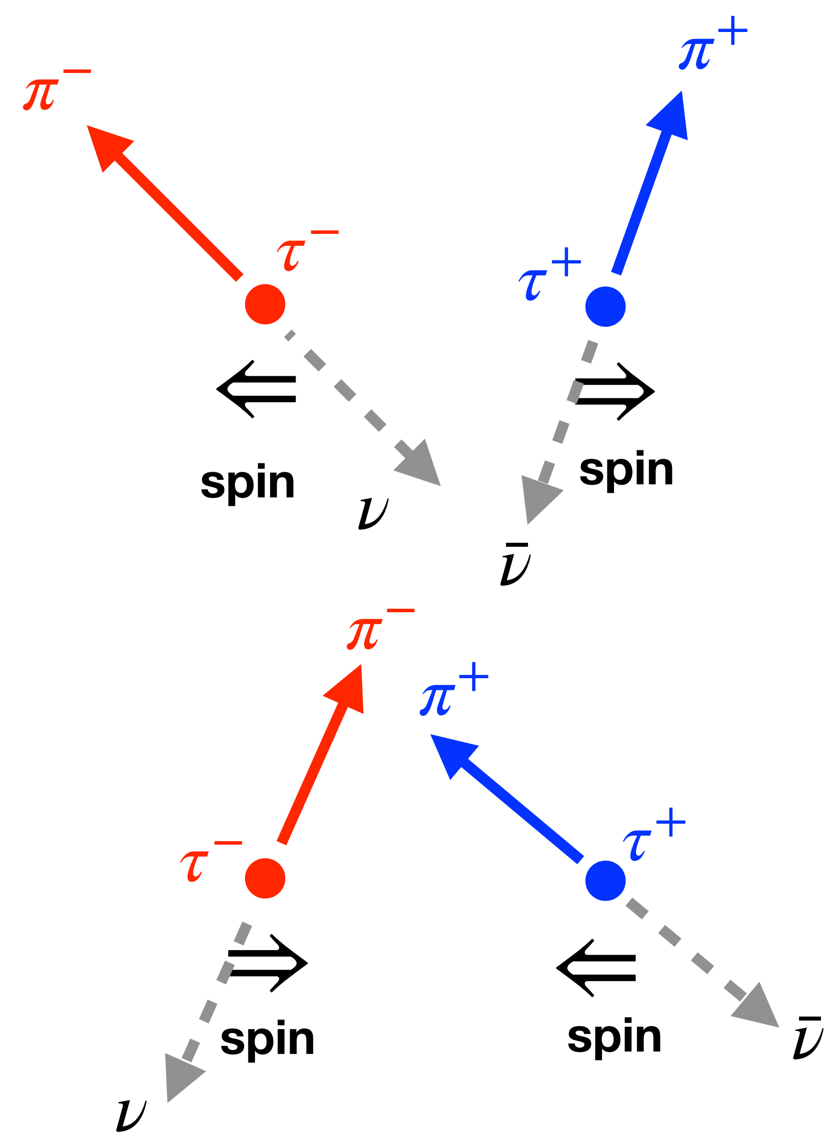
$\uparrow$

$\alpha = 1$ for $\tau^- \to \pi^- \nu$

- *At the rest frame* of $\tau^{\pm}$, the spin can be inferred from the $\pi^{\pm}$ direction

$$\Rightarrow \langle \hat{S}_i^{\tau^-} \hat{S}_j^{\tau^+} \rangle = C_{ij} = -9 \cdot \langle (\hat{\mathbf{u}}^{\pi^-} \cdot \hat{\mathbf{e}}_i)(\hat{\mathbf{u}}^{\pi^+} \cdot \hat{\mathbf{e}}_j) \rangle$$

→ **Reconstruction of the tau rest frames (i.e. neutrino reconstruction) is necessary**



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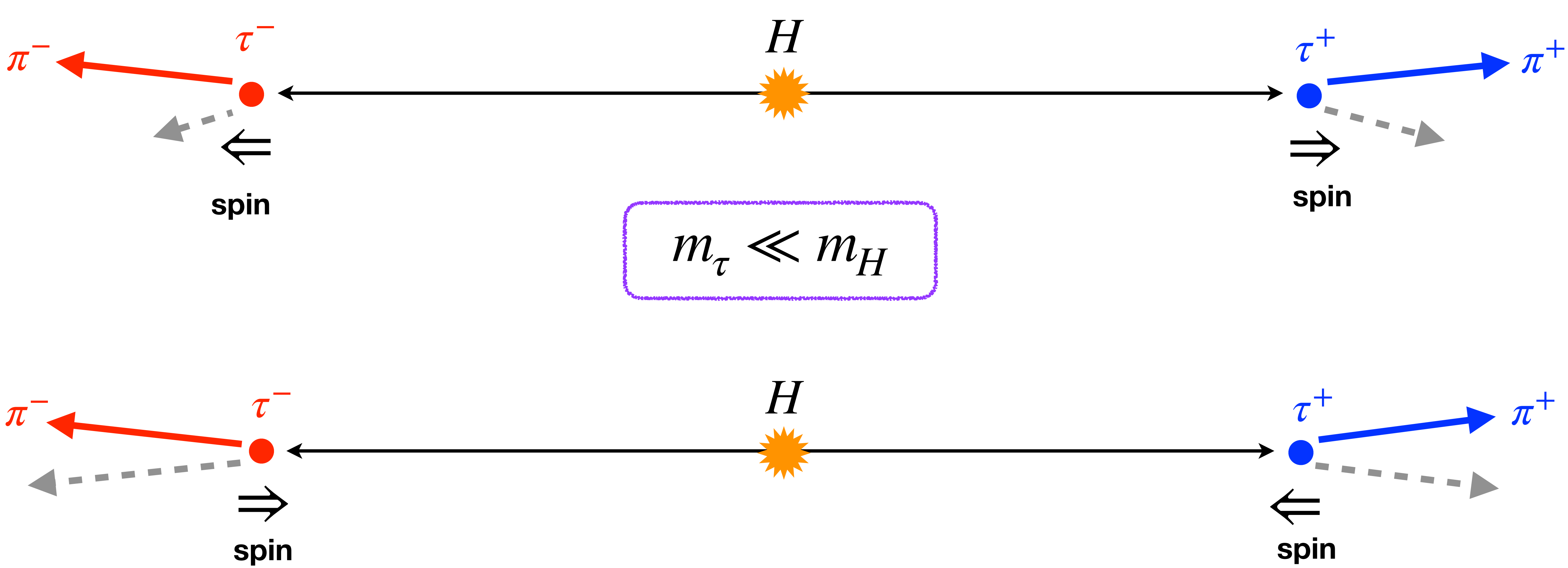
→ **Reconstruction of the tau rest frames (i.e. neutrino reconstruction) is necessary**

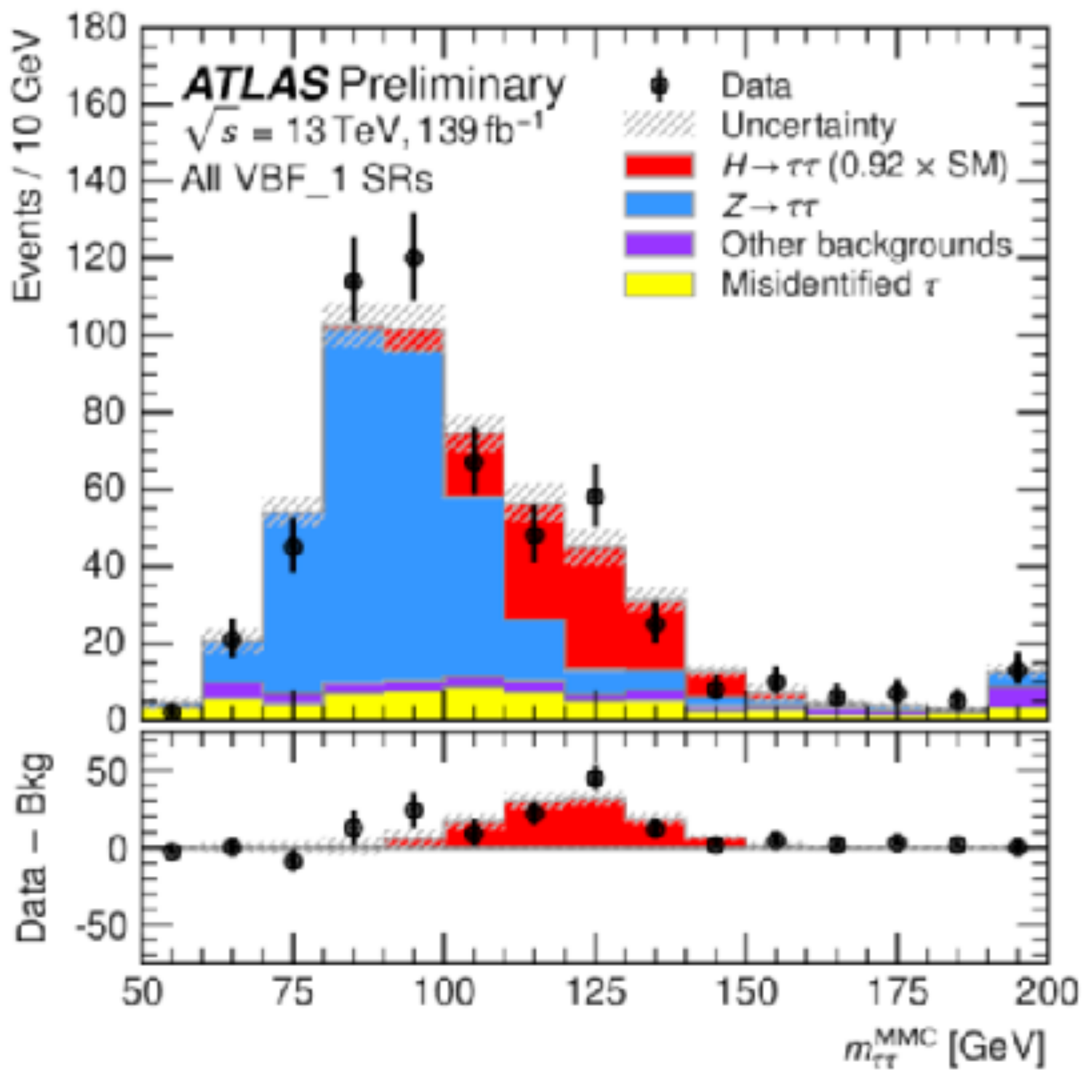
taus are highly boosted → Very accurate event reconstruction is required



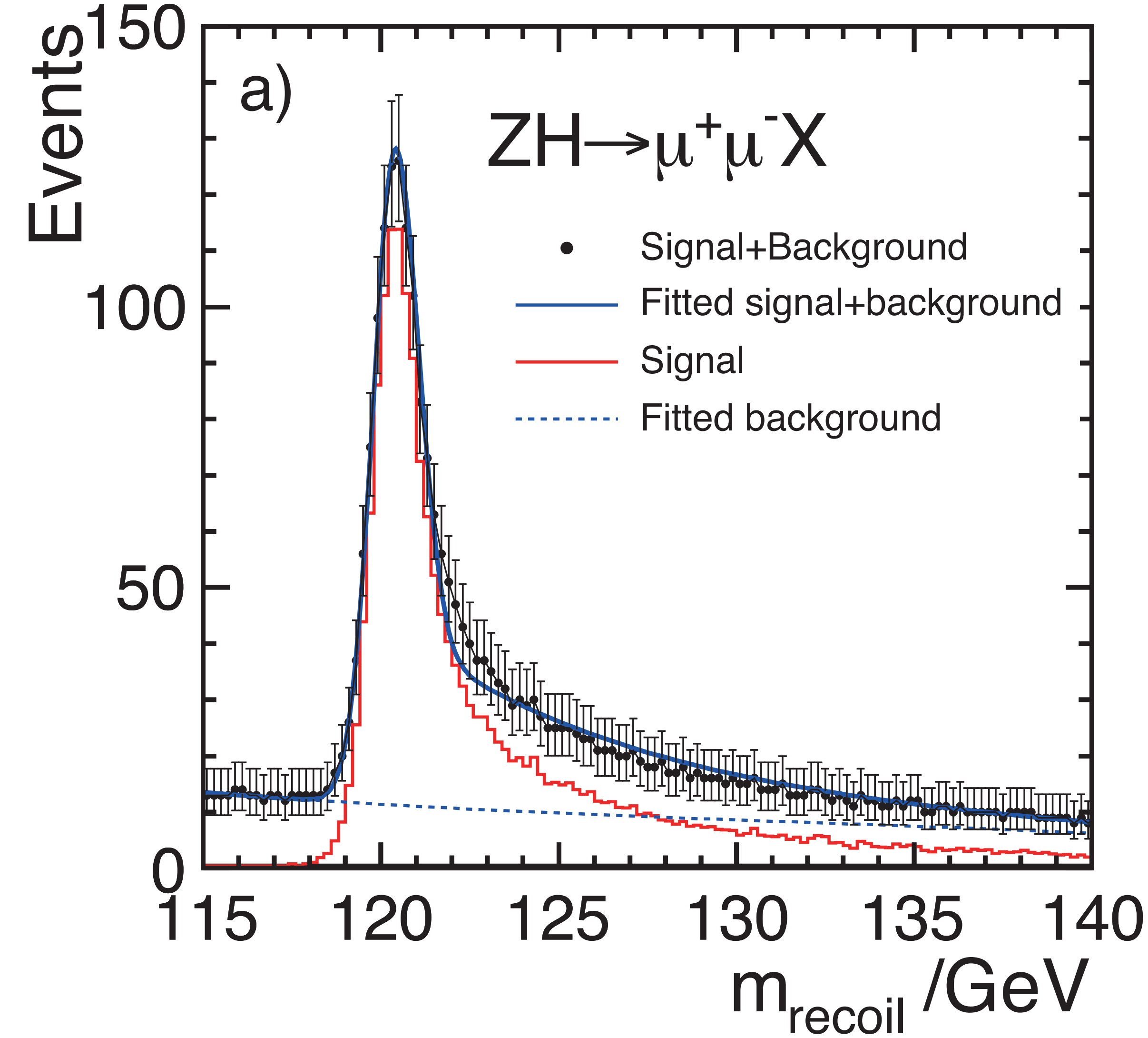
$H \to \tau^+\tau^-$ @ lepton colliders

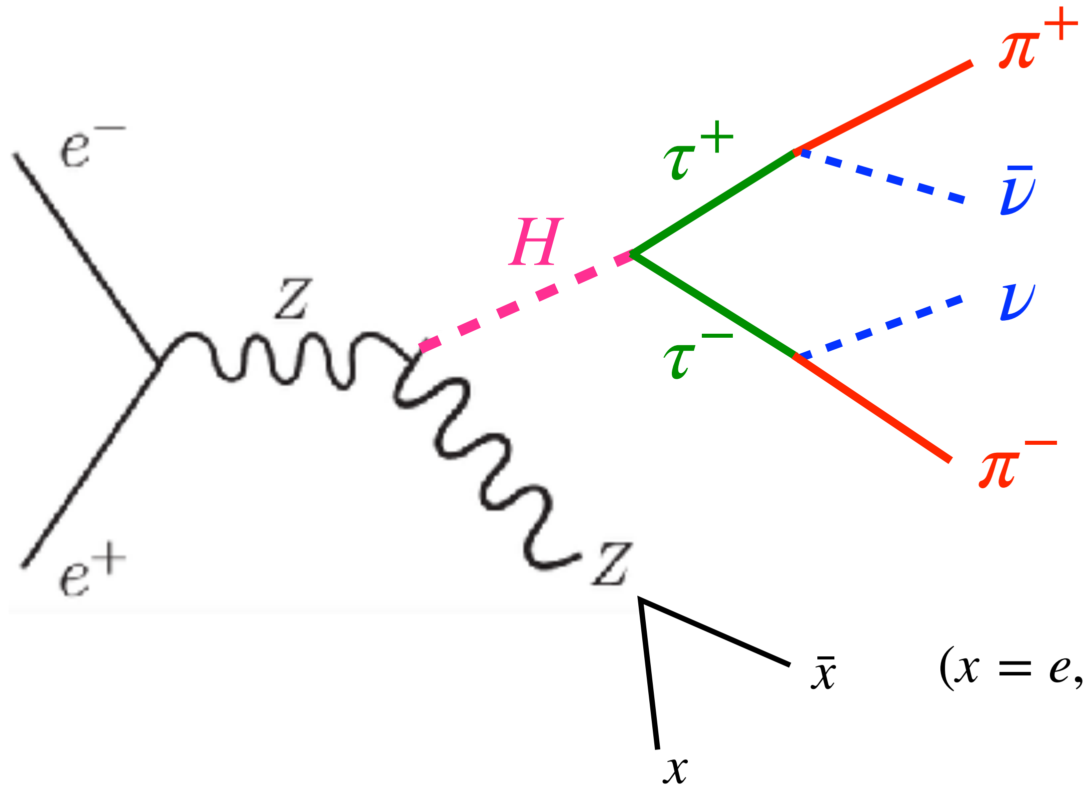
- For precise event reconstruction and for much smaller background, we consider lepton colliders.

LHC



ILC





$(x = e, \mu, j)$

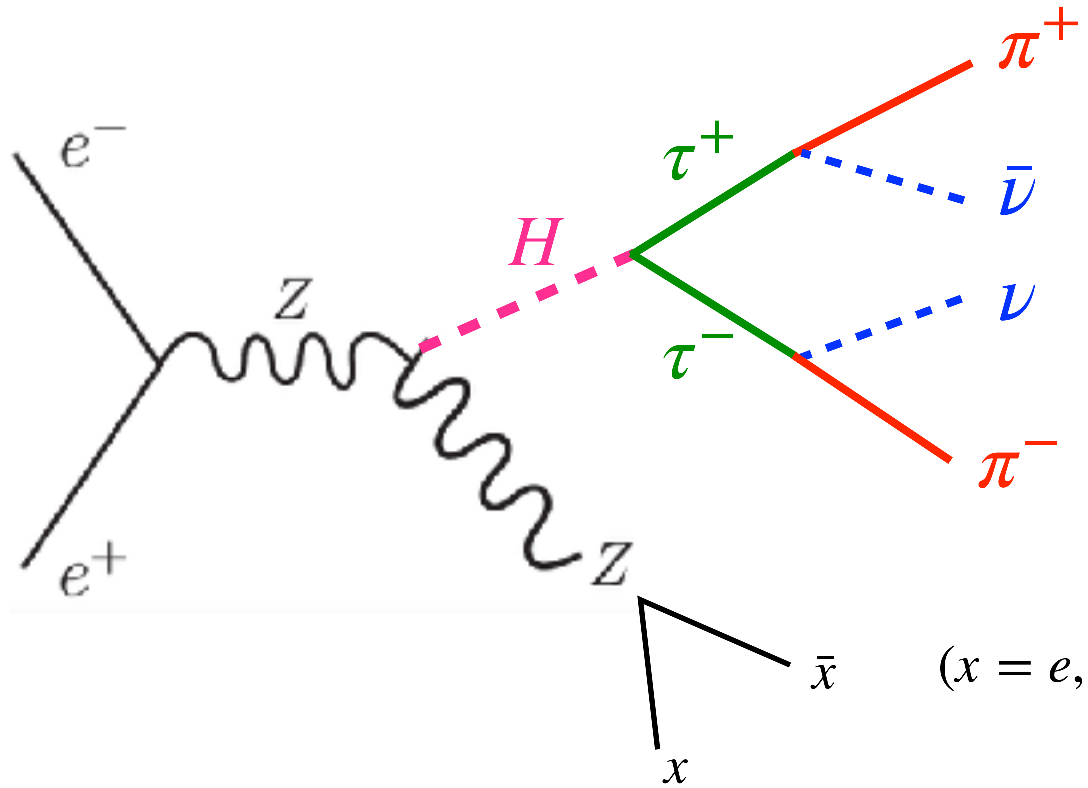
$$(P_H^{\text{reco}})^\mu \equiv P^\mu_{e^+e^-} - P^\mu_{Z\to x\bar{x}} \qquad M^2_{\text{recoil}} \equiv (P_H^{\text{reco}})^2$$

Event selection: $|M_{\text{recoil}} - 125\,\text{GeV}| < 5\,\text{GeV}$

$$e^+e^- \to Z + (Z^*/\gamma^*) \to f\bar{f} + \tau^+\tau^-$$

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	0.83×10^{-4}
beam resolution e^- (%)	0.27	0.83×10^{-4}
$\sigma(e^+e^- \to HZ)$ (fb)	240.1	240.3
# of signal ($\sigma \cdot \text{BR} \cdot L \cdot \epsilon$)	385	663
# of background ($\sigma \cdot \text{BR} \cdot L \cdot \epsilon$)	20	36

- Generate the SM events $(\kappa, \delta) = (1, 0)$ with **MadGraph5**.
- **100 pseudo-experiments** to estimate the statistical uncertainties



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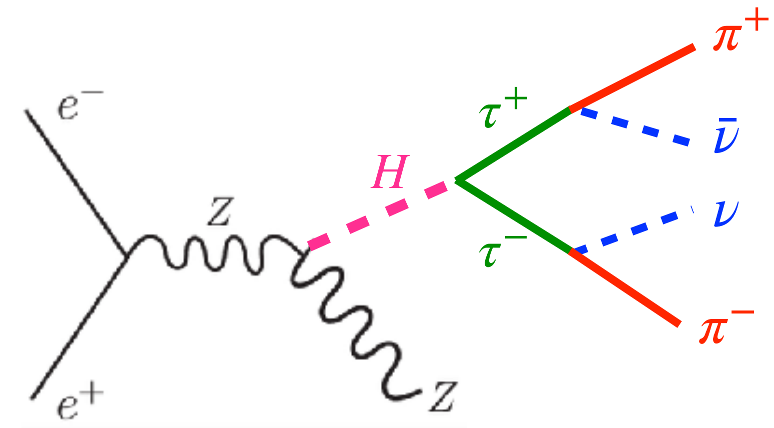
- **100 pseudo-experiments** to estimate the statistical uncertainties

- To determine the tau momenta, we have to reconstruct the unobserved neutrino momenta $(p_x^\nu, p_y^\nu, p_z^\nu)$, $(p_x^{\bar\nu}, p_y^{\bar\nu}, p_z^{\bar\nu})$.



- **6** unknowns can be constrained by **2** mass-shell conditions and **4** energy-momentum conservation.

$$m_\tau^2 = (p_{\tau^+})^2 = (p_{\pi^+} + p_{\bar\nu})^2$$

$$m_\tau^2 = (p_{\tau^-})^2 = (p_{\pi^-} + p_{\nu})^2$$

$$(p_{ee} - p_Z)^\mu = p_H^\mu = \left[(p_{\pi^-} + p_\nu) + (p_{\pi^+} + p_{\bar\nu})\right]^\mu$$

=> 2-fold solutions.

$$C_{ij}^{\rm SM} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

$$\mathscr{C}_{\rm SM}[\rho] = 1$$

$$\mathscr{S}_{\rm SM}[\rho] = 2$$

$$R_{\rm CHSH}^{\rm SM} = \sqrt{2}$$

reproduced very accurately in the simulation

→ we found that false solutions also give the correct correlations! (?)

Effect of momentum mismeasurement

$$E_i^{\text{true}} \to E_i^{\text{obs}} = (1 + \sigma_E \cdot \omega) \cdot E_i^{\text{true}} \qquad \sigma_E = 0.03 \qquad (i = \pi^\pm, e^\pm, \mu^\pm, j)$$

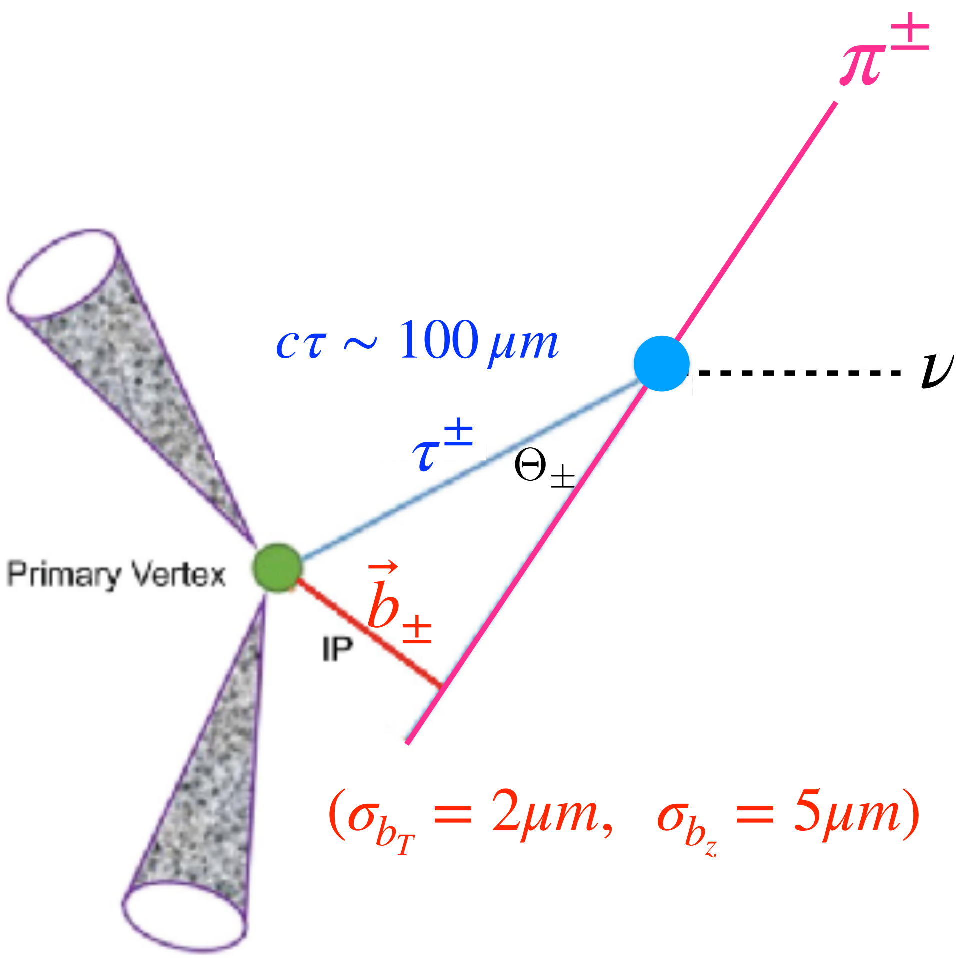
ω: random number drawn from the normal distribution

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} -0.600 \pm 0.210 & 0.003 \pm 0.125 & 0.020 \pm 0.149 \\ 0.003 \pm 0.125 & -0.494 \pm 0.190 & 0.007 \pm 0.128 \\ 0.048 \pm 0.174 & 0.0007 \pm 0.156 & 0.487 \pm 0.193 \end{pmatrix}$	$\begin{pmatrix} -0.559 \pm 0.143 & -0.010 \pm 0.095 & -0.014 \pm 0.122 \\ -0.010 \pm 0.095 & -0.494 \pm 0.139 & -0.002 \pm 0.111 \\ 0.012 \pm 0.124 & 0.020 \pm 0.105 & 0.434 \pm 0.134 \end{pmatrix}$
E_k	-1.057 ± 0.385	-0.977 ± 0.264
$\mathcal{C}[\rho]$	0.030 ± 0.071	0.005 ± 0.023
$\mathcal{S}[\rho]$	1.148 ± 0.210	1.046 ± 0.163
R^*_{CHSH}	0.769 ± 0.189	0.703 ± 0.134

$$C_{ij}^{\text{SM}} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad \mathscr{C}_{\text{SM}}[\rho] = 1 \qquad \mathscr{S}_{\text{SM}}[\rho] = 2 \qquad R_{\text{CHSH}}^{\text{SM}} = \sqrt{2}$$

Momentum smearing spoils the previous good result…

Use impact parameter information



Goal:

$$E_i^{\rm true} \rightarrow E_i^{\rm obs} \rightarrow E_i^{\rm true} \quad (i = \pi^{\pm}, e^{\pm}, \mu^{\pm}, j)$$

What we do:

- modify $E_i^{\rm obs}$ for some amount by δ

$$E_i^{\rm obs} \rightarrow E_i(\delta_i) = (1 + \delta_i\,\sigma_E) \cdot E_i^{\rm obs}$$

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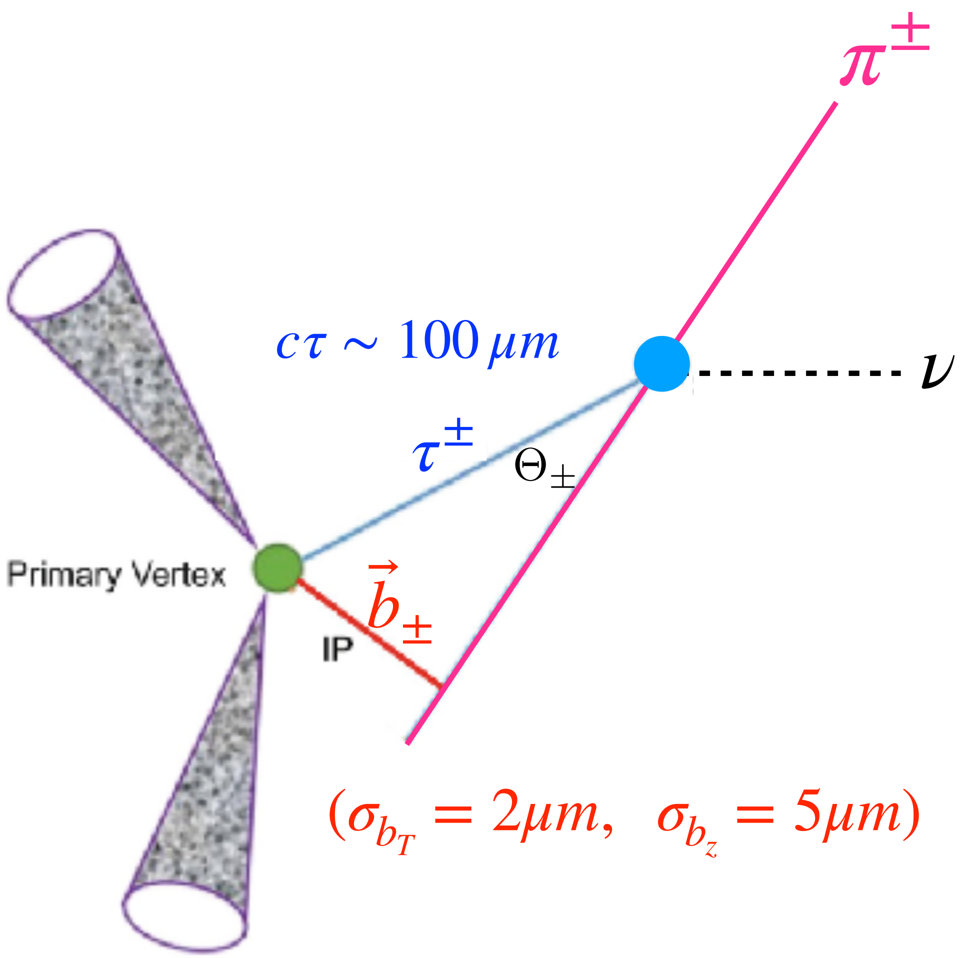

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What we do:

- modify E_i^{obs} for some amount by δ

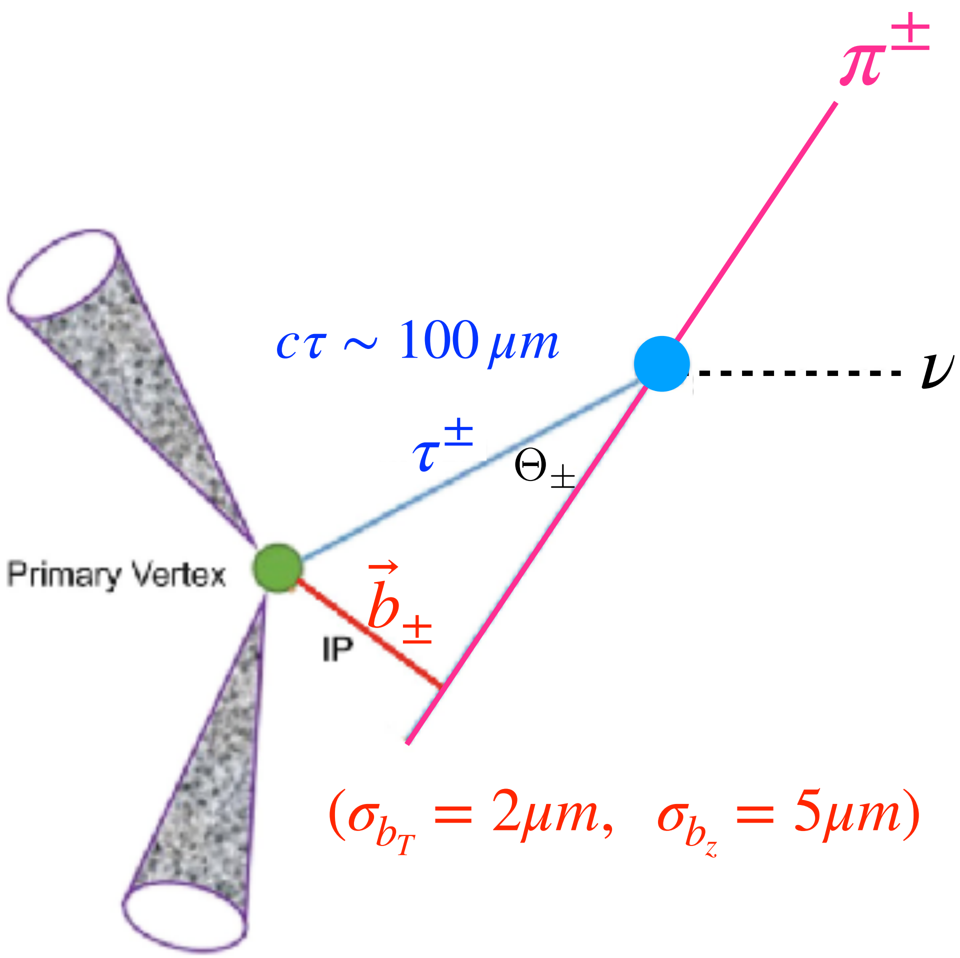
$$E_i^{\text{obs}} \to E_i(\delta_i) = (1 + \delta_i\, \sigma_E) \cdot E_i^{\text{obs}}$$

- solve tau direction $\mathbf{e}_{\tau^\pm}(\boldsymbol{\delta})$

→ lets us calculate $\vec{b}_\pm$ as functions of δ

$$\vec{b}_\pm^{\text{reco}}\,(\mathbf{e}_{\tau^\pm}) \;=\; |\vec{b}_\pm| \cdot \left[\mathbf{e}_{\tau^\pm} \cdot \sin^{-1}\Theta_\pm \;-\; \mathbf{e}_{\pi^\pm} \cdot \tan^{-1}\Theta_\pm\right]$$

Use impact parameter information



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  → lets us calculate $\vec{b}_\pm$ as functions of δ

$$\vec{b}^{\rm reco}_\pm(\mathbf{e}_{\tau^\pm}) = |\vec{b}_\pm| \cdot [\mathbf{e}_{\tau^\pm} \cdot \sin^{-1}\Theta_\pm - \mathbf{e}_{\pi^\pm} \cdot \tan^{-1}\Theta_\pm]$$

- compare the calculated $\vec{b}^{\rm reco}_\pm(\boldsymbol{\delta})$ and measured $\vec{b}^{\rm obs}_\pm$ and construct the likelihood function

2 fold solutions: $i_s = 1,2$

$$\vec{\Delta}^{i_s}_{b_\pm}(\boldsymbol{\delta}) \equiv \vec{b}_\pm - \vec{b}^{\rm reco}_\pm\left(\mathbf{e}^{i_s}_{\tau^+}(\boldsymbol{\delta})\right)$$

$$L^{i_s}_\pm(\boldsymbol{\delta}) = \frac{[\Delta^{i_s}_{b_\pm}(\boldsymbol{\delta})]^2_x + [\Delta^{i_s}_{b_\pm}(\boldsymbol{\delta})]^2_y}{\sigma^2_{b_T}} + \frac{[\Delta^{i_s}_{b_\pm}(\boldsymbol{\delta})]^2_z}{\sigma^2_{b_z}} + \delta^2_{\pi^+} + \delta^2_{\pi^-} + \delta^2_x + \delta^2_{\bar{x}}\,.$$

minimizing $L^{i_s}(\boldsymbol{\delta})$ would give us the correct set of $\boldsymbol{\delta}s$ and solution i_s

Result

2211.10513

	ILC	FCC-ee
C_{ij}	$\begin{pmatrix} 0.830 \pm 0.176 & 0.020 \pm 0.146 & -0.019 \pm 0.159 \\ -0.034 \pm 0.160 & 0.981 \pm 0.1527 & -0.029 \pm 0.156 \\ -0.001 \pm 0.158 & -0.021 \pm 0.155 & -0.729 \pm 0.140 \end{pmatrix}$	$\begin{pmatrix} 0.925 \pm 0.109 & -0.011 \pm 0.110 & 0.038 \pm 0.095 \\ -0.009 \pm 0.110 & 0.929 \pm 0.113 & 0.001 \pm 0.115 \\ -0.026 \pm 0.122 & -0.019 \pm 0.110 & -0.879 \pm 0.098 \end{pmatrix}$
E_k	2.567 ± 0.279	2.696 ± 0.215
$\mathcal{C}[\rho]$	0.778 ± 0.126	0.871 ± 0.084
$\mathcal{S}[\rho]$	1.760 ± 0.161	1.851 ± 0.111
$R^*_{\rm CHSH}$	1.103 ± 0.163	1.276 ± 0.094

$$C_{ij}^{\rm SM} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad \mathscr{C}_{\rm SM}[\rho] = 1 \qquad \mathscr{S}_{\rm SM}[\rho] = 2 \qquad R^{\rm SM}_{\rm CHSH} = \sqrt{2}$$

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E_k	2.567 ± 0.279	$\sim 5\sigma$	2.696 ± 0.215	$\gg 5\sigma$
$\mathcal{C}[\rho]$	0.778 ± 0.126	$\sim 5\sigma$	0.871 ± 0.084	$\gg 5\sigma$
$\mathcal{S}[\rho]$	1.760 ± 0.161	$\sim 3\sigma$	1.851 ± 0.111	$\sim 5\sigma$
R^*_{CHSH}	1.103 ± 0.163		1.276 ± 0.094	$\sim 3\sigma$

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Superiority of FCC-ee over ILC is due to a better beam resolution

	ILC	FCC-ee
energy (GeV)	250	240
luminosity (ab^{-1})	3	5
beam resolution e^+ (%)	0.18	$0.83 \cdot 10^{-4}$
beam resolution e^- (%)	0.27	$0.83 \cdot 10^{-4}$

CP measurement

- Under CP, the spin correlation matrix transforms: $C \xrightarrow{CP} C^T$
- This can be used for a *model-independent* test of CP violation. We define:

$$A \equiv (C_{rn} - C_{nr})^2 + (C_{nk} - C_{kn})^2 + (C_{kr} - C_{rk})^2 \ \geq 0$$

- Observation of $A \neq 0$ immediately confirms CP violation.

- From our simulation, we observe

$$A = \begin{cases} 0.204 \pm 0.173 & \text{(ILC)} \\ 0.112 \pm 0.085 & \text{(FCC-ee)} \end{cases}$$

← **consistent with absence of CPV**

- This model independent bounds can be translated to the constraint on the CP-phase δ

$$\mathscr{L}_{\text{int}} \propto H \bar{\psi}_\tau (\cos\delta + i\gamma_5 \sin\delta)\, \psi_\tau \;\Rightarrow\; C_{ij} = \begin{pmatrix} \cos 2\delta & \sin 2\delta & 0 \\ -\sin 2\delta & \cos 2\delta & 0 \\ 0 & 0 & -1 \end{pmatrix} \;\Rightarrow\; A(\delta) = 4\sin^2 2\delta$$

CP measurement

- Focusing on the region near $|\delta| = 0$, we find the 1-σ bounds:

$$|\delta| < \begin{cases} 8.9^o & \text{(ILC)} \\ 6.4^o & \text{(FCC-ee)} \end{cases}$$

- Other studies:

$$\Delta\delta \sim 11.5^o \quad \text{(HL-LHC)}$$ [Hagiwara, Ma, Mori 2016]

$$\Delta\delta \sim 4.3^o \quad \text{(ILC)}$$ [Jeans and G. W. Wilson 2018]

Summary

- The quantum state of $H \to \tau^+\tau^-$ is simple but measuring quantum properties is challenging even at lepton colliders since taus are highly boosted.
- Very accurate event reconstruction is required, which can be achieved by using the impact parameters.
- ILC and FCC-ee are able to see entanglement, steering and violation of BI.
- Spin correlation is sensitive to CP-phase and we can measure the CP-phase as byproduct of the quantum property measurement.

	Entanglement	Steering	Bell-inquality	CP-phase
ILC	$\sim 5\sigma$	$\sim 3\sigma$		8.9^o
FCC-ee	$\gg 5\sigma$	$\sim 5\sigma$	$\sim 3\sigma$	6.4^o

Norway grants

NATIONAL SCIENCE CENTRE
POLAND

The research leading to the results presented in this talk has received funding from the Norwegian Financial Mechanism for years 2014-2021, grant nr 2019/34/H/ST2/00707

Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen

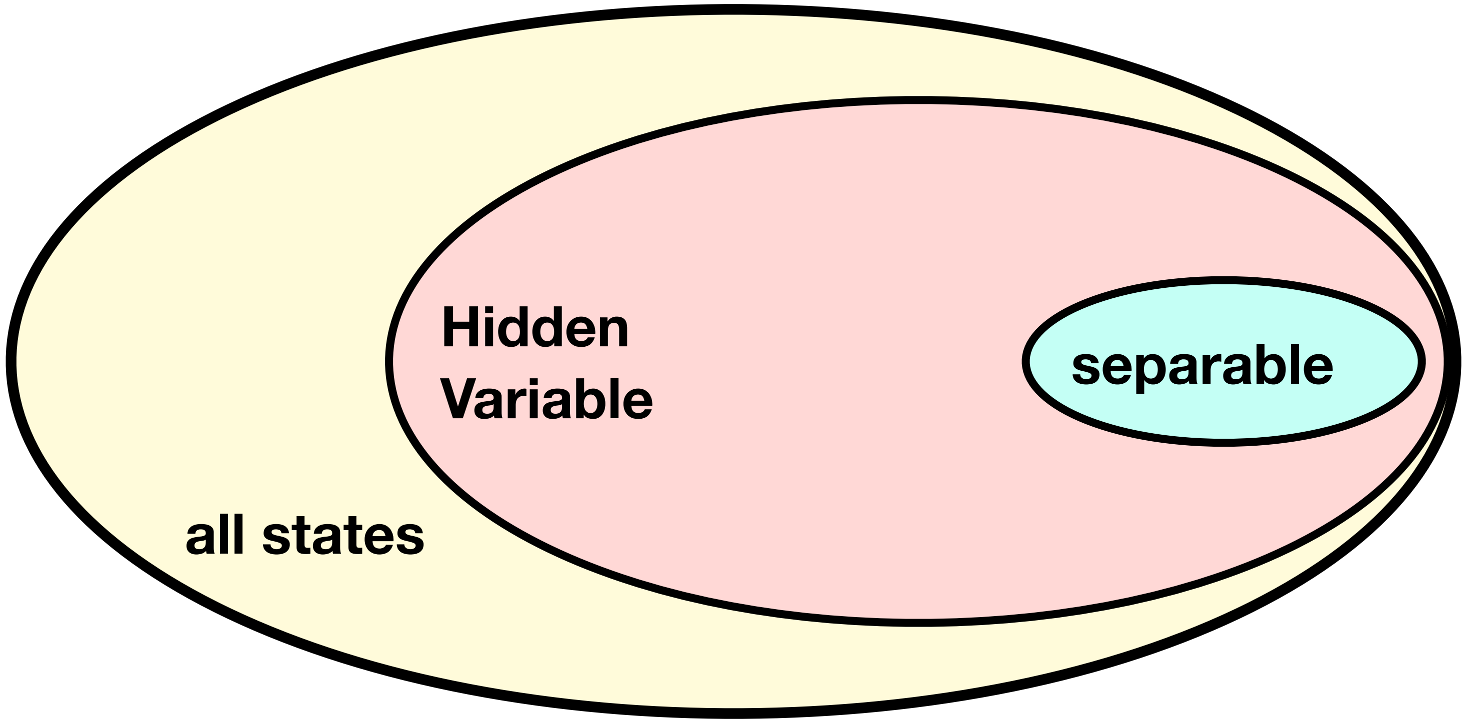
Separable state (compliment of entangled state):

$$P(a,b|A,B) = \sum_\lambda p_\lambda \langle a|\rho_\lambda^\alpha|a\rangle \cdot \langle b|\rho_\lambda^\beta|b\rangle \quad \longleftarrow \quad \rho = \sum_\lambda p_\lambda \rho_k^\alpha \otimes \rho_\lambda^\beta$$

Hidden Variable state (complement of Bell nonlocal state):

$$P(a,b|A,B) = \sum_\lambda p_\lambda P_\alpha(a|A,\lambda) \cdot P_\beta(b|B,\lambda)$$

arbitrary conditional probabilities



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Un-steerable state (not-steerable by Alice):

[Jones, Wiseman, Doherty 2007]

$$P(a,b\,|\,A,B) = \sum_\lambda p_\lambda P_\alpha(a\,|\,A,\lambda) \cdot \langle b\,|\,\rho_\lambda^\beta\,|\,b\rangle \quad \longleftarrow$$

If this description is possible, Alice cannot influence (`steer") Bob's local state

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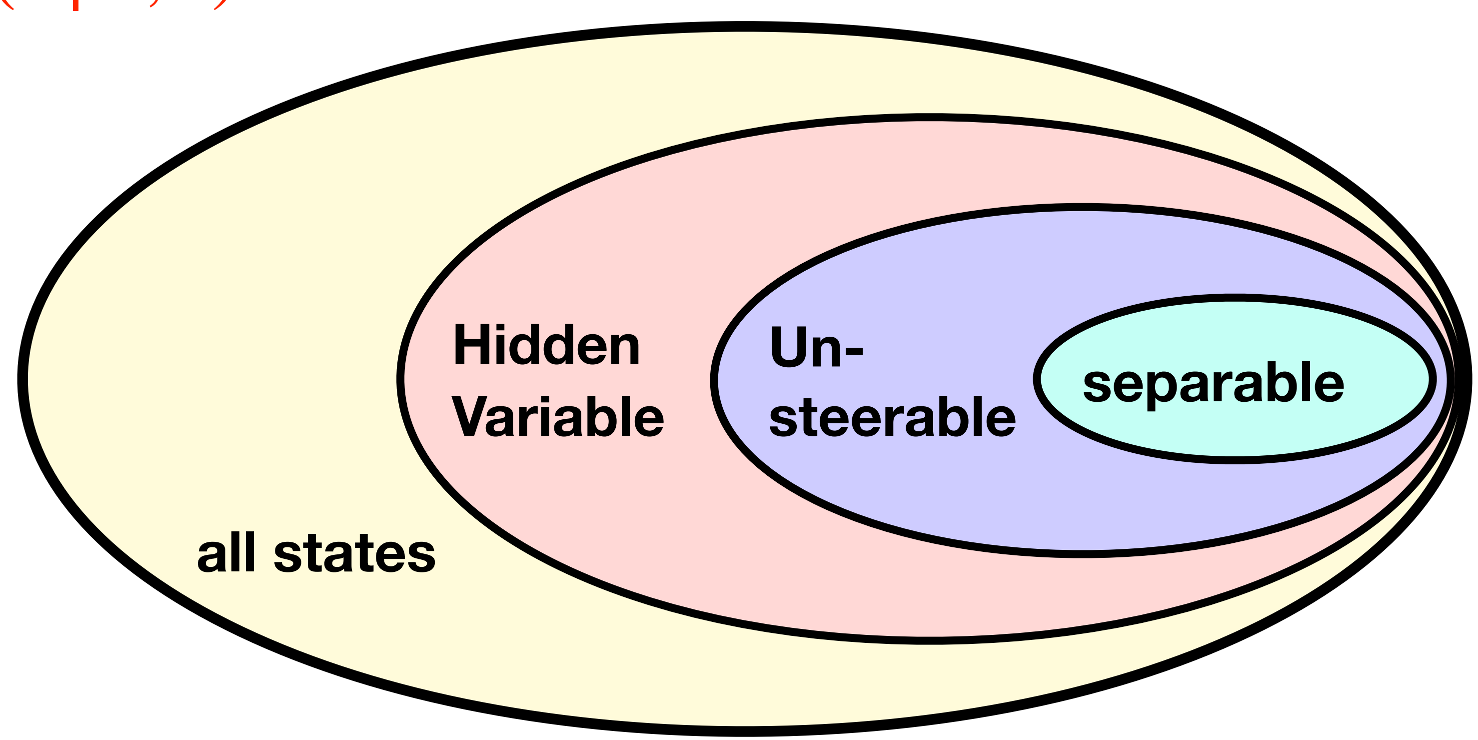
↑ ↑ arbitrary conditional probabilities



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$R_{\mathrm{CHSH}} > 1$

Bell nonlocal $\subset$ **Steerable** $\subset$ **Entangled**