Unitarity Relations in the Presence of Vector-Like Quarks

G. C. Branco
CFTP/IST, U. Lisboa

Workshop on the Standard Model and Beyond

Corfu, September 2023

Work done in collaboration with Francisco Albergaria
arXiv: 2307.13073 [hep-ph]

As well as with J. Alves, José Filipe Bastos, Francisco Botella, G. C. Branco, A. L. Cherchiglia, C. C. Nishi, J. T. Penedo, Pedro M. F. Pereira, J. I. Silva-Marcos

Work Partially supported by:

FCT Fundação para a Ciência e a Tecnologia

MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

europa.eu

European Union

QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL

PORTUGAL 2007.2013

COMPETE

PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE

FCT

Fundação para a Ciência e a Tecnologia

MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Physics BSM with vector-like quarks (VLQs)

Quarks with L and R components transforming in the same way under the SM gauge group

Bare mass terms in the Lagrangian are allowed

VLQs may populate the desert between the EW and the GUT scale without worsening the hierarchy problem

P. Ramond, 1981

Mixing of the new quarks with the SM-like quarks gives rise to:

Deviations from unitarity of the VCKM

Z mediated Flavour-Changing-Neutral-Currents

Higgs mediated Flavour-Changing-Neutral-Currents

These new phenomena are suppressed by the ratio of electroweak scale and the masses of the new heavy quarks

Rich variety of new Physics

Possible motivations to introduce isosinglet vector-like quarks

Vector-like fermions arise for instance in grand unified models

Naturally small violation of 3x3 unitarity of the VCKM and non-vanishing but naturally suppressed flavour-changing neutral currents (FCNC)

This opens up many interesting possibilities for rare K and B decays as well as CP asymmetries in neutral B decays

Adding isosinglet quarks to the SM leads to new sources of CP violation

In particular one may achieve spontaneous CP violation in this framework with the addition of a complex scalar singlet to the Higgs sector

Possibility of solving the strong CP problem a la Barr and Nelson

Bento, Branco, Parada, 1991

Possibility of having a Common Origin for all CP Violations

Branco, Parada, MNR, 2003

Changes in the unitarity relations in the presence of VLQs

Moduli differences:

In the SM, 3x3 unitarity of the CKM matrix leads to an “asymmetry” defined as:

$$\mathbf{a} \equiv |V_{31}|^2 - |V_{13}|^2 = |V_{23}|^2 - |V_{32}|^2 = |V_{12}|^2 - |V_{21}|^2$$

In an SM extension with one up-type VLQ the quark mixing matrix consists of the first three columns of a 4x4 unitary matrix:

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{pmatrix}$$

Changes in the unitarity relations in the presence of VLQs

From unitarity of first row and first column of V, one derives:

$$a_{12,13} \equiv \left(|V_{12}|^2 - |V_{21}|^2\right) - \left(|V_{31}|^2 - |V_{13}|^2\right) = |V_{41}|^2 - |V_{14}|^2$$

Using unitarity of other rows and columns of V one obtains:

$$a_{12,32} \equiv \left(|V_{12}|^2 - |V_{21}|^2\right) - \left(|V_{23}|^2 - |V_{32}|^2\right) = |V_{24}|^2 - |V_{42}|^2,$$
$$a_{13,23} \equiv \left(|V_{13}|^2 - |V_{31}|^2\right) - \left(|V_{32}|^2 - |V_{23}|^2\right) = |V_{34}|^2 - |V_{43}|^2.$$

From $D_0 - \overline{D_0}$ mixing, we know that, in models with one up-type VLQ, we have

$$|V_{14}|^2 |V_{24}|^2 < (2.1 \pm 1.2) \times 10^{-8}.$$

Differences between the imaginary parts of the quartets

In the SM, one can show that all imaginary parts of rephasing invariant quartets:

$$V_{us} V_{cb} V_{ub}^{*} V_{cs}^{*} = Q_{uscb}$$

$$V_{cd} V_{ts} V_{td}^{*} V_{cs}^{*} = Q_{cdts}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

have the same modulus

In the presence of VLQs one obtains a different result, for exam

Im Q_2112 - Im Q_1132 = Im Q_1142

Fundamental properties of the CKM matrix

G. C Branco, L. Lavoura, J. P. Silva "CP Violation" Oxford University Press 1999

$$\mathscr{L}_{\mathrm{CC}} = \begin{pmatrix} \overline{u} & \overline{c} & \overline{t} \end{pmatrix}_L \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \mathrm{H.c.}$$

The CKM matrix is complex but not all its phases have physical meaning

$$u_\alpha = e^{i\varphi_\alpha} u'_\alpha , \qquad d_k = e^{i\varphi_k} d'_k$$

There is freedom to rephase the mass eigenstate quark fields. As a result:

$$V'_{\alpha k} = e^{i(\varphi_k - \varphi_\alpha)} V_{\alpha k}$$

Only rephasing invariant quantities have physical meaning.
The simplest rephasing invariants of the CKM matrix are moduli and "quartets"

$$|V_{\alpha k}| \qquad Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i} \qquad \text{with } \alpha \neq \beta \text{ and } i \neq j.$$

Higher order Invariants can in general be written in terms of these .

Details about Rephasing invariant quantities

Example :

$$Q = V_{us} V_{cb} V^*_{cs} V^*_{ub}$$

$$\mathrm{Im}\ Q \simeq \lambda^6 \sin(\arg Q)$$

λ is essentially the sine of the Cabibbo angle and it is a parameter appearing in the Wolfenstein parametrisation of the CKM matrix

|Im Q| has the same value for all quartets and measures the strength of CP violation in the SM.

Identification of the Small numbers in VCKM: 3

$$|V_{ub}| \simeq 3.6 \times 10^{-3}$$

$$|\mathrm{Im}\, Q| \simeq 3 \times 10^{-5}$$

Q → Rephasing invariant quartet of VCKM

In the SM, $|\mathrm{Im}\, Q|$ has the same value for all quartets and gives the strength of CP violation in the SM

A surprising result: In the 3×3 up corner of a V^{CKM} matrix of arbitrary size one has:

$9-5=4$ rephasing invariant phases

The following phase convention may be chosen, in general

$$\arg V^{3\times 3} = \begin{pmatrix} 0 & \beta_K & \gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\beta_s & 0 \end{pmatrix}$$

The phases γ, β, β_s, β_K are arguments of rephasing invariant quartets:

$$\gamma = \arg\left(-V_{ud} V_{cb} V_{ub}^{*} V_{cd}^{*}\right)$$

$$\beta = \arg\left(-V_{cd} V_{tb} V_{cb}^{*} V_{td}^{*}\right)$$

$$\beta_s = \arg\left(-V_{cb} V_{ts} V_{cs}^{*} V_{tb}^{*}\right)$$

$$\beta_K = \arg\left(-V_{us} V_{cd} V_{ud}^{*} V_{cs}^{*}\right)$$

Sometimes one also introduces $\alpha = \arg\left(-V_{td} V_{ub} V_{ud}^{*} V_{tb}^{*}\right)$ which is unnecessary, because

$$\alpha \equiv \pi - \beta - \gamma$$

By definition!!!

Within the SM, 3x3 unitarity implies some exact relations among rephasing invariant quantities:

$$\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\gamma} \frac{|V_{tb}|}{|V_{ud}|}$$

$$\sin\beta_s = \frac{|V_{td}|}{|V_{ts}|} \frac{|V_{cd}|}{|V_{cs}|} \sin\beta = O(\lambda^2)$$

$$\sin\beta_K = \frac{|V_{ub}|}{V_{us}} \frac{|V_{cb}|}{|V_{cs}|} \sin\gamma = O(\lambda^4)$$

Conjecture: The small numbers in V^{CKM} arise from New Physics

The conjecture implies that within the SM,

$$|V_{ub}| = 0$$

$$\mathrm{Im}\, Q = 0$$

A simple realization of the Conjecture can be constructed within SM + VLQs

A crucial question:

What can VLQs do for you?

(i) They provide a simple alternative solution to the Strong CP problem without axions. Barr and Nelson
Bento, G.C.B and Parada

(ii) They provide the simplest extension of the SM with Spontaneous CP Violation in a model consistent with experiment.

Requirements to have a viable model of Spontaneous CP Violation:

- Lagrangian should be CP invariant but CP invariance should be broken by the vacuum.
  One has to be careful. Often a "geometrical" vacuum phase does not violate CP
- The vacuum phase should be able to generate a complex CKM matrix
  Experimentally $\underline{\gamma} \neq 0, \pi$

(iii) Provide a simple framework where there are (New Physics (NP) contributions to $B_d - \overline{B}_d$ mixing, $B_s - \overline{B}_s$ mixing and/or $\overline{D}^0 - D^0$ mixing; Also new contri-butions to

$$t \to c\, Z_\mu$$

may receive tree-level contributions in models with up-type VLQs

IV VLQs may populate the desert between v and some higher scale (M_{GUT}?) without worsening the hierarchy problem

To my knowledge, this was first emphasized in a paper by Pierre Ramond.

"Fermions in the Desert"

(talk given at Erice)

Appears in Spires

V VLQs may play an important rôle in providing an explanation for the V^{CKM} unitarity problem.

$$|V_{us}|^2 + |V_{ud}|^2 + |V_{ub}|^2 < 1$$

at the level of 2,3 standard deviations

See J. T. Penedo, Pedro Pereira, M.N. Rebelo, GCB
published in JHEP

See also nice work by Belfatto and Berezhiani

The generation of |V_ub| and ImQ

from New Physics

We propose that the CKM matrix is generated from three different contributions

$$V_{\rm CKM}^{\rm eff} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\rm up} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{12} \end{pmatrix}_{\rm NP} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\rm dow}$$

In order to implement the structure we assume that there is a basis where the down and up quark matrices take the form:

$$M_d = \begin{pmatrix} m_{11}^d & m_{12}^d & 0 \\ m_{21}^d & m_{22}^d & 0 \\ 0 & 0 & m_{33}^d \end{pmatrix} \qquad M_u = \begin{pmatrix} m_{11}^u & 0 & 0 \\ 0 & m_{22}^u & m_{23}^u \\ 0 & m_{32}^u & m_{33}^u \end{pmatrix}$$

It can be shown that one can obtain these patterns through the introduction of a Z_4 symmetry at the Lagrangian level

Without the introduction of New Physics, one simply obtains a simplified and reduced CKM mixing, where

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\rm up} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\rm dow} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{23}c_{12} & -s_{23} \\ -s_{23}s_{12} & s_{23}c_{12} & c_{23} \end{pmatrix}$$

At this level one has: $|V_{31}| = |V_{12}|\,|V_{23}|$ and $V_{13} = 0$

Our conjecture offers an explanation why:

|V_31| > |V_13| !!!

$V_{13} = 0$ also leads to vanishing CP violation

Introduce an up-type VLQ and assume the 4×4 up-type quark mass matrix:

$$M_u = \begin{bmatrix} 0 & 0 & 0 & m_{14} \\ 0 & m_{22} & m_{23} & 0 \\ 0 & m_{32}e^{i\alpha} & m_{33} & 0 \\ m_{41} & 0 & m_{43} & M \end{bmatrix}$$

Then one can generate

$$\left(V^{CKM}_{eff}\right)_{13} \neq 0 \qquad \left|\mathrm{Im}\, Q_{eff}\right| \neq 0$$

Numerical Example

Mass matrices in GeV at M_Z scale

$$M_d = \begin{bmatrix} 0.0029 & -1.35 \times 10^{-2} & 0 \\ 6.73 \times 10^{-4} & 0.058 & 0 \\ 0 & 0 & 2.9 \end{bmatrix}$$

$m_d = 0.003$; $m_s = 0.06$; $m_b = 2.9$

$$M_u = \begin{bmatrix} 0 & 0 & 0 & 53.73 \\ 0 & 0.59 & -6.91 & 1.25\, e^{-0.285 i} \\ 0 & -0.024 & 172.8 & 0 \\ 0.016 & 0 & 14.88\, e^{-1.89 i} & 1250 \end{bmatrix}$$

$m_u = 0.02 \quad m_c = 0.60 \quad m_t = 173 \quad m_T = 1251$

$$\begin{bmatrix}\bar{u} & \bar{c} & \bar{t} & \bar{T}\end{bmatrix}\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{ts} & V_{Tb} \end{bmatrix}\begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

The CKM matrix is the 4x3 left submatrix of the following 4x4 unitary matrix

$$\Upsilon = \begin{pmatrix} 0.9735 & 0.2244 & 0.0037 & 0.0423 \\ 0.224 & 0.9736 & 0.0399 & 0.00099 \\ 0.00834 & 0.0393 & 0.999 & 0.00151 \\ 0.04163 & 0.0105 & 0.001674 & 0.999 \end{pmatrix}.$$

These mass matrices lead to:

$\gamma \approx 68^\circ$

$\sin 2\beta \approx .746$

$\beta_s \cong 0.02$

$$I^{CP} \equiv |\mathrm{Im}\, Q| \approx 3\times 10^{-5}$$

Conclusions

- VLQs are one of the simplest extension of the SM, with a large number of phenomenological implications
- VLQs are "cousins" of ν_R which provide through seesaw the most plausible explanation of the smallness of neutrino masses.
- The effects of VLQs may have been seen already in deviations of unitarity in the first line of V_{CKM}.

- Weak point : No firm prediction for the scale of VLQs.

This is a universal weak point in all (so far) proposed New Physics proposals... !!

The SM was an notable exception.

Before gauge interactions the sugestion was IVB with ≈ 2 GeV !

intermediate vector boson...