Precise prediction for the W-boson mass in U(1) extensions of the standard model

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Workshop on
Standard Model and Beyond
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Corfu

[2305.11931]

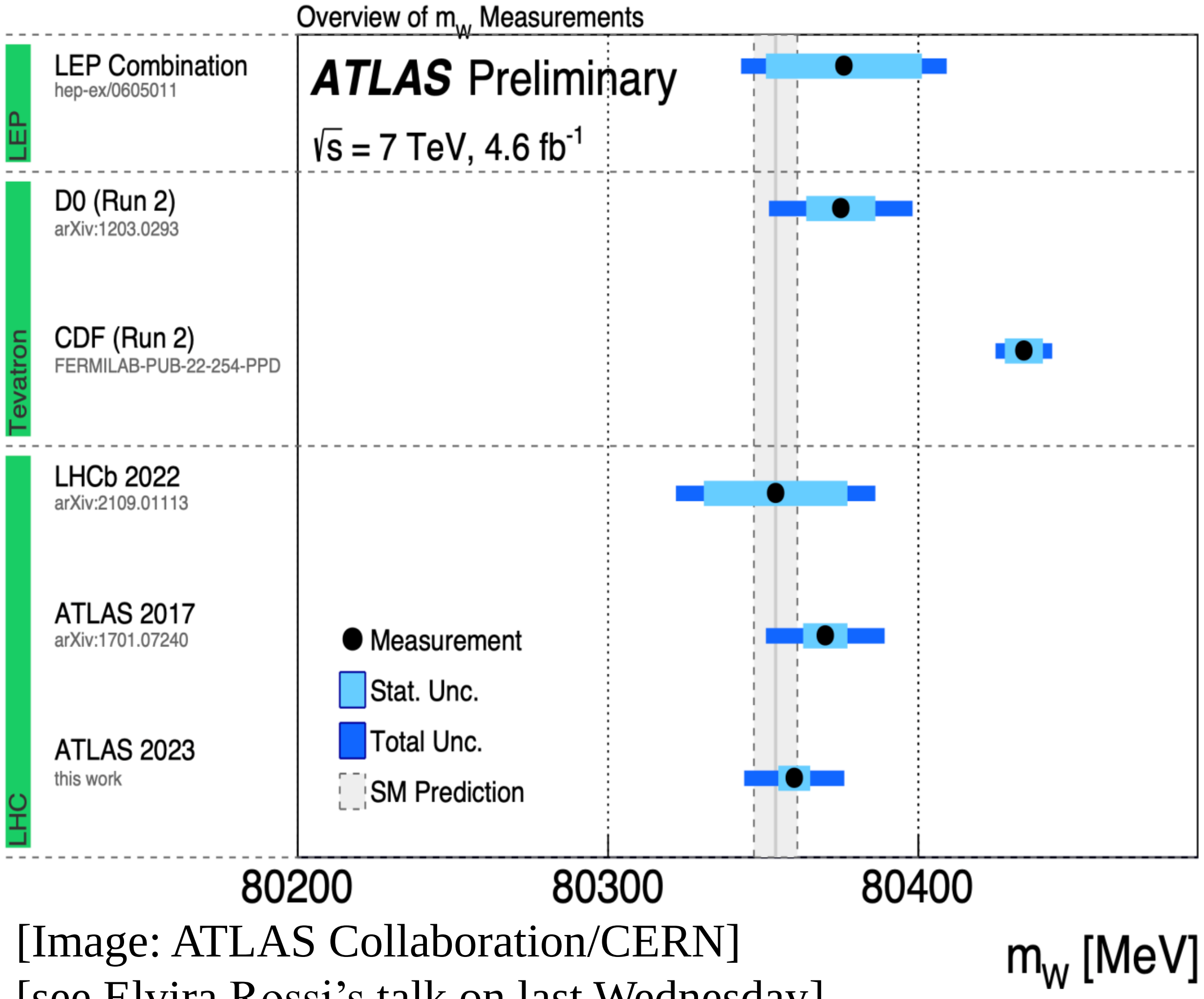
Outline

- W-boson mass and BSM physics
- U(1) extensions of the standard model
- On the effect of the full 1-loop correction to the W-mass

BSM and M_W

- Small uncertainty (~10 MeV) in theory and experiment

Prediction & measurement of M_W



[Image: ATLAS Collaboration/CERN]
[see Elvira Rossi's talk on last Wednesday]

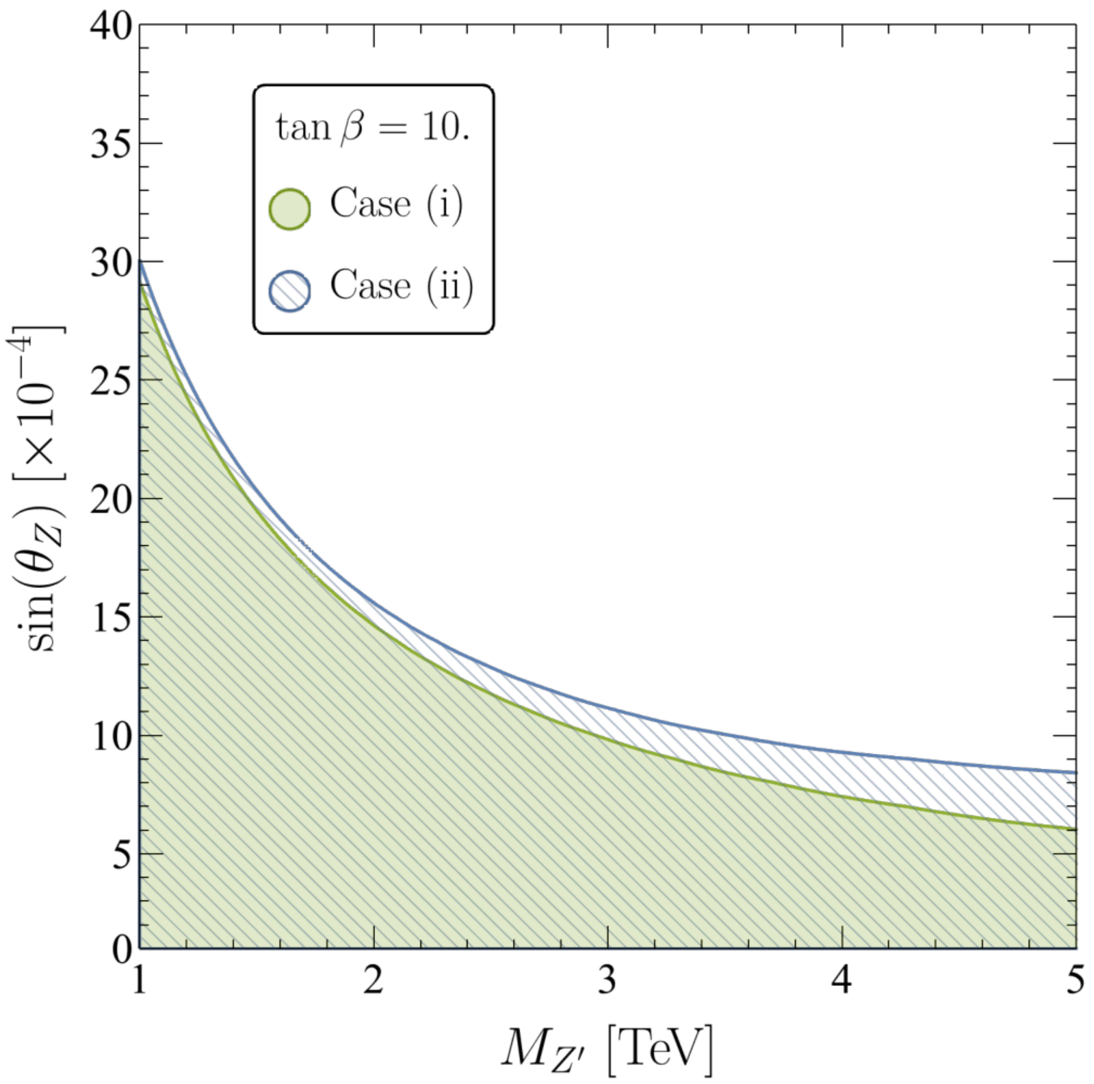
- **Theory** (SM, MS-bar [1411.7040]):
  $M_W^{\text{theo}} = 80353 \pm 9$ MeV (with PDG 2022 inputs)
- **Experiment** (PDG 2022 world. avg.):
  $M_W^{\text{exp.}} = 80377 \pm 12$MeV

BSM and M_W

- Small uncertainty (~10 MeV) in theory and experiment
- Small BSM effects on M_W can be exposed
- U(1) extensions affect M_W at tree level so precision is important (1-loop)

- Precise predictions in BSM models are important
- Full **Δr** at 1-loop **in U(1) extensions** is computed
- **Full Δr** (Case i.) **may** become **important for heavy $M_{Z'}$**
- …compared to the available predictions (Case ii.)
- Fig. shows **region where :**

$$|M_W^{\text{exp.}} - M_W| < 2\sigma$$



Take home message

What's new in a U(1) extension?

- SM gauge group + an extra U(1) adds a **new interaction**
- May add new scalar field(s), can stabilize the EW vacuum
- May add right-handed (sterile) neutrinos: neutrino mass generation via see-saw, dark matter

See Zoltán Trócsányi's talk on the superweak model on Tuesday!

New parameters: 5 (2 gauge + 3 scalar)

Gauge sector:

- M'_Z: mass of the new gauge boson Z'
- s_Z : new gauge mixing angle, rotation of gauge eigenstates to mass eigenstates:

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \\ B'_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W & 0 \\ s_W & c_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_Z & -s_Z \\ 0 & s_Z & c_Z \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

Scalar sector:

- $\tan\beta = \frac{w}{v}$: ratio of new VEV to BEH VEV
- M_s : mass of the new scalar boson
- s_S : new scalar mixing angle to mass eigenstates

$$\begin{pmatrix} \phi^0 \\ \chi \end{pmatrix} = \begin{pmatrix} c_S & -s_S \\ s_S & c_S \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}$$

Gauge boson masses

Concise relation:

$$\frac{M_W^2}{c_W^2} = c_Z^2 M_Z^2 + s_Z^2 M_{Z'}^2$$

Express predictions with Lagrangian couplings or **pheno parameters** e.g.:

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2} = 1 - s_Z^2 \left(1 - \frac{M_{Z'}^2}{M_Z^2} \right)$$

See also: [2306.01836]

Gauge boson masses

Concise relation:

$$\frac{M_W^2}{c_W^2} = c_Z^2 M_Z^2 + s_Z^2 M_{Z'}^2,$$

Express predictions with Lagrangian couplings or **pheno parameters** e.g.:

$$M_W^2 = \frac{\rho M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}\, G_F \rho M_Z^2}(1 + \Delta r)}\right)$$

Renormalization

- Split bare parameters into $g^{(0)} \to g + \delta g$
- The Weinberg angle changes at tree level:

$$M_W^2 \frac{\delta c_W^2}{c_W^2} = \delta M_W^2 - c_W^2 \left(c_Z^2\, \delta M_Z^2 + s_Z^2\, \delta M_{Z'}^2 - 2 s_Z \left(M_Z^2 - M_{Z'}^2\right) \delta s_Z\right)$$

Renormalization

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Δr receives completely new corrections:

Δr *collectes the radiative corrections to the* ***μ-decay*** *and hence to* M_W

$$\boldsymbol{\Delta r} = \left(\text{formally } \Delta r^{\text{SM}} \text{ with BSM loops}\right) - $$

$$- s_Z^2 \frac{c_W^2}{s_W^2} \frac{c_W^2}{M_W^2} \left(\mathrm{Re}\Pi_{ZZ}(M_Z^2) - \mathrm{Re}\Pi_{Z'Z'}(M_{Z'}^2) + 2\left(M_Z^2 - M_{Z'}^2\right) \frac{\delta s_Z}{s_Z} \right)$$

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\,\alpha}{2\,M_W^2\,s_W^2}(1+\Delta r)$$

$$\Delta r^{\mathrm{SM}} = \underbrace{\frac{2\delta e}{e}}_{\text{Renormalization of the electric charge, formula known all order}} + \underbrace{\left(\frac{\mathrm{Re}\Pi_{WW}(M_W^2) - \Pi_{WW}(0)}{M_W^2}\right) + \delta_{\mathrm{BV}}}_{\text{Diagrammatic corrections to the muon decay graph: W-propagator and box and vertex diagrams}}$$

$$+ \underbrace{\frac{c_W^2}{s_W^2}\left(\frac{\mathrm{Re}\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\mathrm{Re}\Pi_{WW}(M_W^2)}{M_W^2}\right)}_{\text{Renormalization of } s_W}$$

Renormalization of the electric charge, formula known all order

Diagrammatic corrections to the muon decay graph: W-propagator and box and vertex diagrams

Renormalization of s_W

[Sirlin, 1980]

Checks

- The ε poles cancel in $\boldsymbol{\Delta r}$ in R_ξ-gauge with general z-charge assignment
- For several benchmark points $\boldsymbol{\Delta r}$ is independent of the gauge parameters ξ_i, with $i = W, A, Z, Z'$
- Compare $\boldsymbol{\Delta r}$ in two cases:

Case I. :

$$\boldsymbol{\Delta r} = \left(\text{formally } \Delta r^{\mathrm{SM}} \text{ with BSM loops}\right) -$$

$$-s_Z^2 \frac{c_W^2}{s_W^2} \frac{c_W^2}{M_W^2} \left(\mathrm{Re}\Pi_{ZZ}(M_Z^2) - \mathrm{Re}\Pi_{Z'Z'}(M_{Z'}^2) + 2\left(M_Z^2 - M_{Z'}^2\right) \frac{\delta s_Z}{s_Z} \right)$$

When is it safe to neglect the **new** terms?

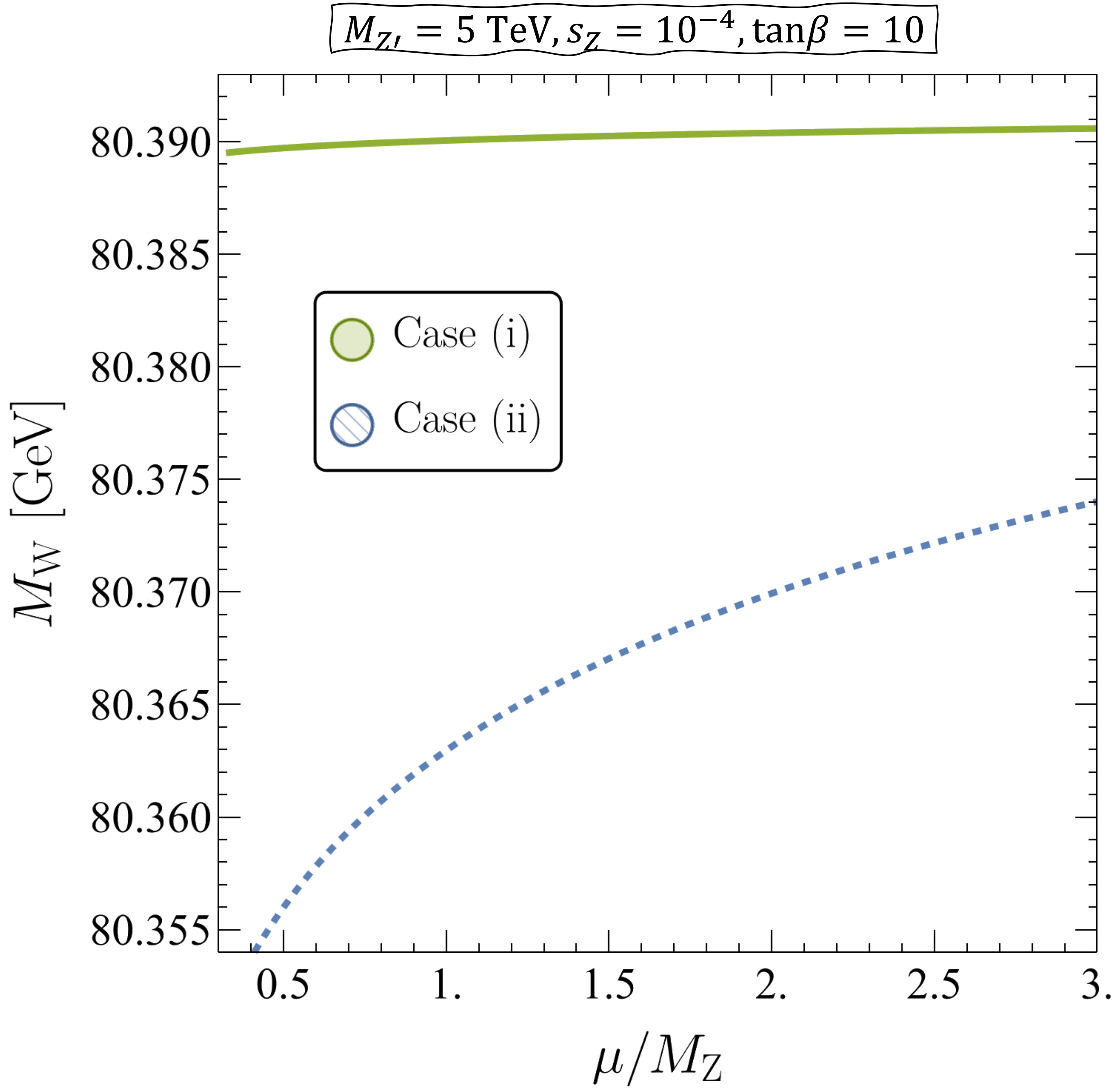
Case II. :

$$\Delta r = \left(\text{formally } \Delta r^{\mathrm{SM}} \text{ with BSM loops}\right)$$

[2204.05285, 1309.7223]

Checks

- The ε poles cancel in $\boldsymbol{\Delta r}$ in R_ξ-gauge with general z-charge assignment
- For several benchmark points $\boldsymbol{\Delta r}$ is independent of the gauge parameters ξ_i, with $i = W, A, Z, Z'$
- Weak dependence on the renormalization scale μ at fixed benchmark points



Benchmarks: $M_W - M_{W,\mathrm{SM}}$ [MeV]

SMALL $M_{Z'}$ $= 50$ MeV
and $s_S = 0.1$
Irrelevant

s_Z	$5\cdot10^{-4}$			
M_S / $\tan\beta$	0.5 TeV (i)	0.5 TeV (ii)	5 TeV (i)	5 TeV (ii)
0.1	−1	−1	−2	−2
1	−1	−1	−2	−2
10	−1	−1	−2	−2

BSM corrections to the SM prediction for M_W in MeV units

LARGE $M_{Z'}$ $= 5$ TeV
and $s_S = 0.1$
Potentially relevant

s_Z	$5\cdot10^{-4}$				$7\cdot10^{-4}$			
M_S / $\tan\beta$	0.5 TeV (i)	0.5 TeV (ii)	5 TeV (i)	5 TeV (ii)	0.5 TeV (i)	0.5 TeV (ii)	5 TeV (i)	5 TeV (ii)
10	37	10	35	13	75	29	73	36
20	39	34	35	34	81	76	74	79
30	40	38	35	37	83	85	75	85

Conclusions

- Precise predictions in BSM models are important
- Full **Δr** at 1-loop **in U(1) extensions** is computed
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- Fig. shows **region where :**

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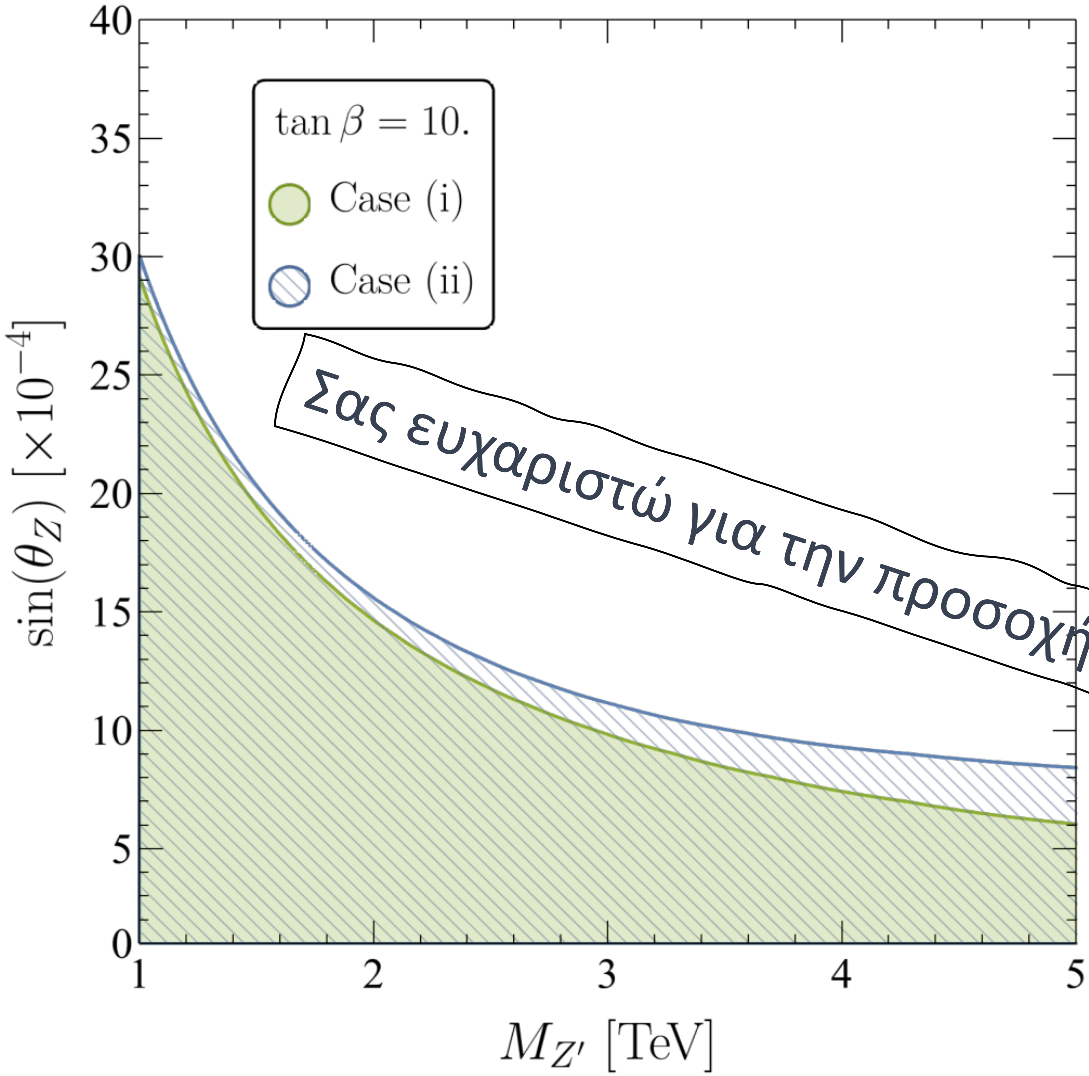
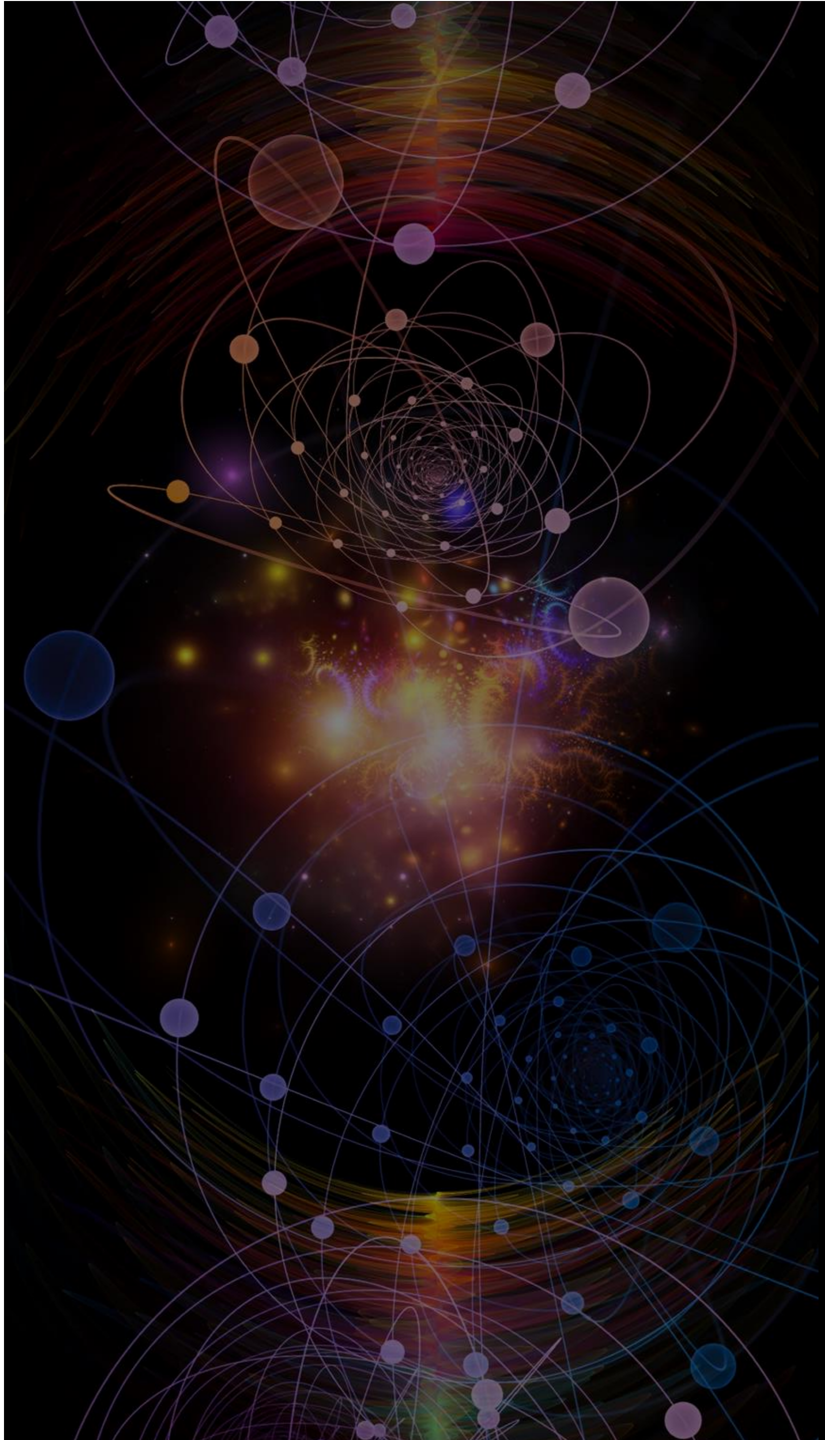


Σας ευχαριστώ για την προσοχή σας!



Backup slides

How to obtain δs_Z I.

- Relate unrotated and rotated fields:

$$B_\mu^{(0)} = c_W^{(0)} A_\mu^{(0)} - s_W^{(0)} (c_Z^{(0)} Z_\mu^{(0)} - s_Z^{(0)} Z_\mu'^{(0)})$$

$$B_\mu'^{(0)} = s_Z^{(0)} Z_\mu^{(0)} + c_Z^{(0)} Z_\mu'^{(0)}$$

- Also true for renormalized fields:

$$B_\mu = c_W A_\mu - s_W (c_Z Z_\mu - s_Z Z_\mu')$$

$$B_\mu' = s_Z Z_\mu + c_Z Z_\mu'$$

- Unrotated fields are renormalized such that

$$B_\mu^{(0)} = \sqrt{Z_B} B_\mu \text{ and } B_\mu'^{(0)} = \sqrt{Z_{B'}} B_\mu'$$

- Rotated fields may mix:

$$\begin{pmatrix} A_\mu^{(0)} \\ Z_\mu^{(0)} \\ Z_\mu'^{(0)} \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{AA}} & \frac{1}{2} Z_{AZ} & \frac{1}{2} Z_{AZ'} \\ \frac{1}{2} Z_{ZA} & \sqrt{Z_{ZZ}} & \frac{1}{2} Z_{ZZ'} \\ \frac{1}{2} Z_{Z'A} & \frac{1}{2} Z_{Z'Z} & \sqrt{Z_{Z'Z'}} \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z_\mu' \end{pmatrix}$$

How to obtain δs_Z II.

- Express bare fields with renormalized ones and collect coefficients:

$$\sqrt{Z_B}\, c_W = c_W^{(0)} \sqrt{Z_{AA}} - \frac{1}{2} s_W^{(0)} \left(c_Z^{(0)} Z_{ZA} - s_Z^{(0)} Z_{Z'A} \right)$$

$$\sqrt{Z_{B'}} s_Z = s_Z^{(0)} \sqrt{Z_{ZZ}} + \frac{1}{2} c_Z^{(0)} Z_{Z'Z}$$

$$\sqrt{Z_{B'}} c_Z = \frac{1}{2} s_Z^{(0)} Z_{ZZ'} + c_Z^{(0)} \sqrt{Z_{Z'Z'}}$$

- First equation is used to derive δe

(U(1) Ward identity $\sqrt{Z_B}\, Z_{g_y} = 1$)

[hep-ph/0209084]

- 2nd and 3rd ones are divided to cancel $\sqrt{Z_{B'}}$ and express δs_Z