Real effective potentials for phase transitions in models with extended scalar sectors

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Table of contents

Based on:
K. Seller, Z. Szép and Z. Trócsanyi, "Real effective potentials for phase transitions in models with extended scalar sectors," JHEP **04** (2023), 096 [arXiv:2301.07961 [hep-ph]].

Introduction & Motivation

Effective Potential

- Widely used phenomenological tool to study phase transitions:
  1. approximate the critical temperature(s) and
  2. estimate the order of phase transition(s).
- The effective potential defined via Legendre transformation is:
  1. the effective action evaluated for homogeneous field configurations,
  2. a real and convex function of the field expectation value(s) v_k.
- Perturbative expansion depends on the model, at one-loop:

$$V_{\mathrm{eff}}^{[1]}(\{v_k\}, T) = V_{\mathrm{cl}}(\{v_k\}) + \sum_{i \in \mathbf{P}} \left[V_i^{(1)}(\{v_k\}) + V_{\mathrm{T},i}^{(1)}(\{v_k\}, T) \right]$$

→ **P** denotes the set of particles coupled to the scalar field(s)

Effective Potential in SM at $T = 0$

- Fully renormalized potential at one-loop, our result from [arXiv:2301.07961]:

$$V_{\text{eff}}^{[1]}(v) = \frac{\lambda}{4}(v^2 - v_0^2)^2 + \sum_{i \neq G} \frac{\Delta_{\bar{\Pi},i}\lambda}{4}(v^2 - v_0^2)^2 + \sum_{i \neq G} \frac{s_i n_i}{64\pi^2}\left[m_i^4(v)\left(\ln\frac{m_i^2(v)}{m_i^2(v_0)} - \frac{3}{2}\right) + 2m_i^2(v_0)m_i^2(v)\right] + \frac{n_G m_G^4(v)}{64\pi^2}\left(\ln\frac{m_G^2(v)}{m_h^2(v_0)} + \frac{3}{2}\right)$$

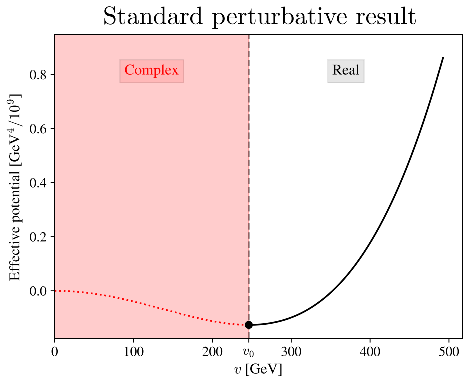
→ $\Delta_{\bar{\Pi},i}\lambda$ are finite constants
→ Has no free parameters
→ Independent of regularization scale → Improvement compared to [hep-ph/9206235]

The complex nature of the perturbative result

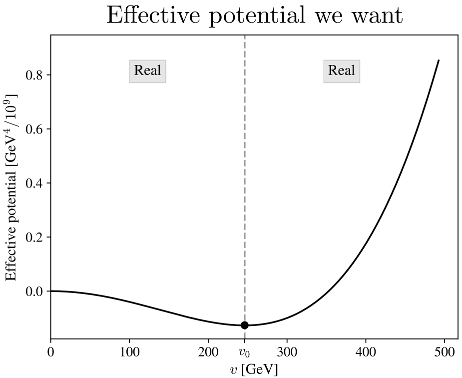
$$V_{\text{eff}}^{[1]}(v) \supset \frac{n_h}{64\pi^2}\left[m_h^4(v)\left(\ln\frac{m_h^2(v)}{M_h^2}-\frac{3}{2}\right)+2M_h^2 m_h^2(v)\right]+\frac{n_G m_G^4(v)}{64\pi^2}\left(\ln\frac{m_G^2(v)}{M_h^2}+\frac{3}{2}\right)$$

- $\mu^2 < 0 \to m_h^2(v) = \mu^2 + 3\lambda v^2$ and $m_G^2(v) = \mu^2 + \lambda v^2$ can be negative
  - $\to$ In particular $m_G^2(v) < 0$ for $v < v_0 \equiv |\mu^2|/\lambda$
  - $\to$ $V_{\text{eff}}^{[1]}(v < v_0)$ is complex!
- The perturbative expansion implicitly assumes a convex classical potential
- For non-convex potentials that have multiple saddle points:
  - $\to$ One has to take into account contributions from multiple saddle points
  - $\to$ [R. J. Rivers, *Z. Phys.* *C* **22** (1984) 137]
- Non-convexity and complexity are a consequence of loop expansion

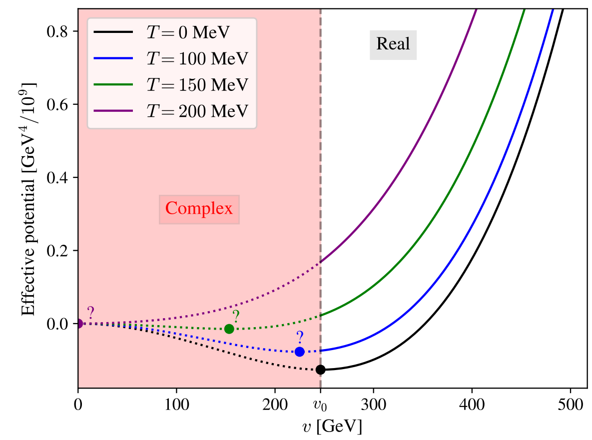
The complex effective potential



How can we get this?



Complexity problem at finite temperature



Optimized perturbation theory

Objective

- Goal: Obtain a real $V_{\text{eff}}^{[1]}$ for any $v_k \longrightarrow$ Minimization possible at finite T

$$V_{\text{eff}}^{[1]}(\{v_k\}, T) = V_{\text{cl}}(\{v_k\}) + \sum_{i \in \mathbf{P}} \left[V_i^{(1)}(\{v_k\}) + V_{\text{T},i}^{(1)}(\{v_k\}, T) \right]$$

- All terms should be real separately!
  - → Classical potential is real
  - → One-loop $T = 0$ potential is complex for scalars at $v < v_0$
  - → Finite T potential is also complex due to the same problem with imaginary masses

- Renormalization in the Optimized Perturbation Theory scheme is shown in [arXiv:hep-ph/9803226]

Optimized perturbation theory approach

- The root of the problem is $\mu^2 < 0 \to$ Introduce a shifted mass parameter $m^2 > 0$

$$\mathcal{L} \supset \mathcal{V}_{\mathrm{OPT}}[\phi] = m^2|\phi|^2 + \lambda|\phi|^4 + \boxed{(\mu^2 - m^2)|\phi|^2}$$

[arXiv:hep-ph/9803226]

- Important to keep in mind:
  - Treat the last term as an interaction or finite part of counter-term
  - Tree level masses defined above are now shifted as $\mu^2 \to m^2$

$$V^{[1]}_{\mathrm{OPT}}(v;\mu^2,m^2) = \underbrace{V_{\mathrm{cl}}(v;m^2) + \overbrace{\frac{\mu^2 - m^2}{2}v^2}}_{V_{\mathrm{cl}}(v;\mu^2)}\overbrace{{} + V^{(1)}(v;m^2)}^{\text{one-loop}}$$

Parametrization conditions in SM at $T = 0$

- Need physical conditions → fix the values of parameters $\{\mu^2, m^2, \lambda\}$

Condition 1: $$\left.\frac{\partial V^{[1]}_{\mathrm{OPT}}(v;\mu^2,m^2)}{\partial v}\right|_{v=v_0} = 0$$ ← Position of minimum

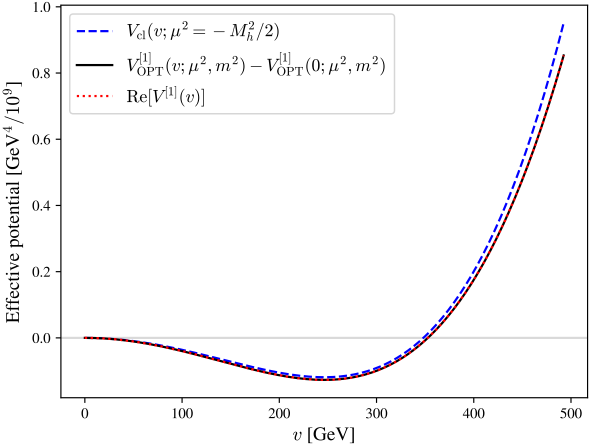
Condition 2: $$\left.\frac{\partial^2 V^{[1]}_{\mathrm{OPT}}(v;\mu^2,m^2)}{\partial v^2}\right|_{v=v_0} = M_h^2$$ ← Curvature of V is the Higgs mass

Condition 3: $$\left.\frac{\partial V^{[1]}_{\mathrm{OPT}}(v;\mu^2,m^2)}{\partial m^2}\right|_{v=v_0} = 0$$ ← Principle of minimum sensitivity

- System is solvable for SM with parameter values:

$$m^2 = 69\,094.6\ \mathrm{GeV}^2, \qquad \lambda = 0.12\,861, \qquad \mu^2 = -8\,847.85\ \mathrm{GeV}^2$$

SM effective potential at $T = 0$



Scalar potential in SM+singlet scalar models

- Adding a new singlet scalar to the potential:

$$\mathcal{V}[\phi,\chi] = \mu_\phi^2|\phi|^2 + \lambda_\phi|\phi|^4 + \mu_\chi^2|\chi|^2 + \lambda_\chi|\chi|^4 + \underbrace{\lambda'|\phi|^2|\chi|^2}_{\text{scalar mixing}}$$

$$\downarrow$$

$$\mathcal{V}_{\mathsf{OPT}}[\phi,\chi] = \mathcal{V}[\phi,\chi]\Big|_{\mu_\phi^2\to m_\phi^2,\,\mu_\chi^2\to m_\chi^2} + (\mu_\phi^2 - m_\phi^2)|\phi|^2 + (\mu_\chi^2 - m_\chi^2)|\chi|^2$$

- The one-loop effective potential:

$$V_{\mathsf{OPT}}^{[1]}(v,w;\mu_\phi^2,\mu_\chi^2,m_\phi^2,m_\chi^2) = V_{\mathsf{cl}}(v,w;\mu_\phi^2,\mu_\chi^2) + V^{(1)}(v,w;m_\phi^2,m_\chi^2)$$

Generalizing the OPT approach

- The previous parametrization conditions fix 6 out of 7 parameters

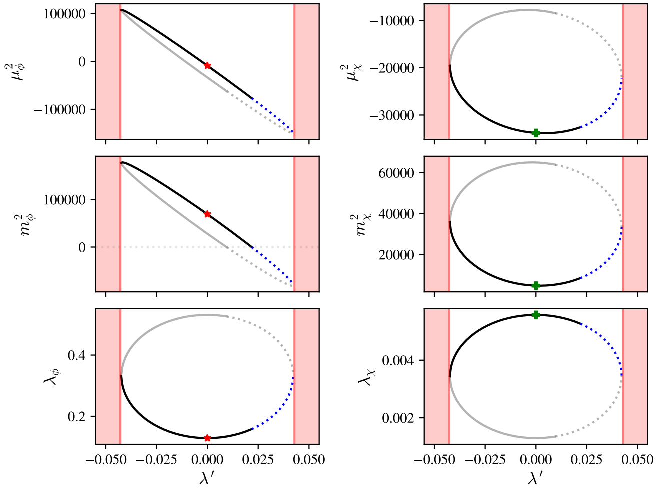
$$\text{Conditions 1,2:} \quad \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial v}\right|_{\text{min}} = \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial w}\right|_{\text{min}} = 0$$

$$\text{Conditions 3,4:} \quad \left.\begin{pmatrix} \partial_v^2 V^{[1]}_{\text{OPT}} & \partial_v \partial_w V^{[1]}_{\text{OPT}} \\ \partial_w \partial_v V^{[1]}_{\text{OPT}} & \partial_w^2 V^{[1]}_{\text{OPT}} \end{pmatrix}\right|_{\text{min}} \to \begin{pmatrix} M_h^2 & 0 \\ 0 & M_s^2 \end{pmatrix}$$

$$\text{Conditions 5,6:} \quad \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial m_h^2}\right|_{\text{min}} = \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial m_s^2}\right|_{\text{min}} = 0$$

- We investigate the parametrization in the free parameter λ'

Parametrization of the singlet model



Phase transitions

Finite temperature corrections

- One-loop effective potential ⟶ No new parameters at $T > 0$

$$V_{\text{eff}}^{[1]}(\{v_k\}, T) = V_{\text{cl}}(\{v_k\}) + \sum_{i \in \mathbf{P}} \left[V_i^{(1)}(\{v_k\}) + V_{\text{T},i}^{(1)}(\{v_k\}, T) \right]$$

Finite temperature corrections

- One-loop effective potential $\longrightarrow$ No new parameters at $T > 0$

$$V_{\mathrm{T}}^{(1)}(\{v_k\}, T) = \frac{T^4}{2\pi^2} \sum_{i \in \mathbf{P}} n_i J_{\pm}^{(i)}\left(m_i^2(\{v_k\}), T\right)$$

[arXiv:hep-ph/9212235]

$$J_{\pm}\left(m_i^2, T\right) = \begin{cases} \mathcal{I}_{-}\left(\frac{m_i^2}{T^2}\right) - \frac{\pi}{6}\left(\frac{\overline{m}_i^3}{T^3} - \frac{m_i^3}{T^3}\right), & \text{if } i = \text{ scalars, longitudinal modes}, \\ \mathcal{I}_{\pm}\left(\frac{m_i^2}{T^2}\right), & \text{if } i = \text{ fermions/transverse modes}. \end{cases}$$

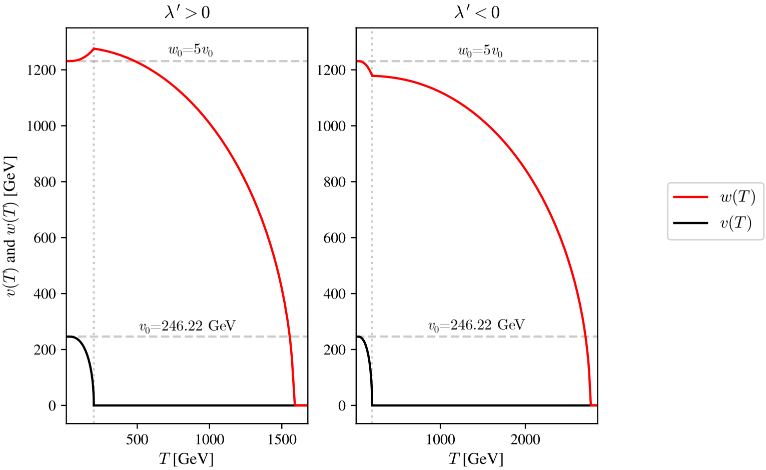
(1) $\mathcal{I}_{\pm}(m_i^2/T^2)$ and m_i^3 is real if $m_i^2 > 0$

$\rightarrow$ OPT gives $m_i^2 > 0$ ✓

(2) $\overline{m}_i$ is the thermal mass $\rightarrow \overline{m}_i^2 > 0$ is required at all T for $\overline{m}_i^3$ to be real

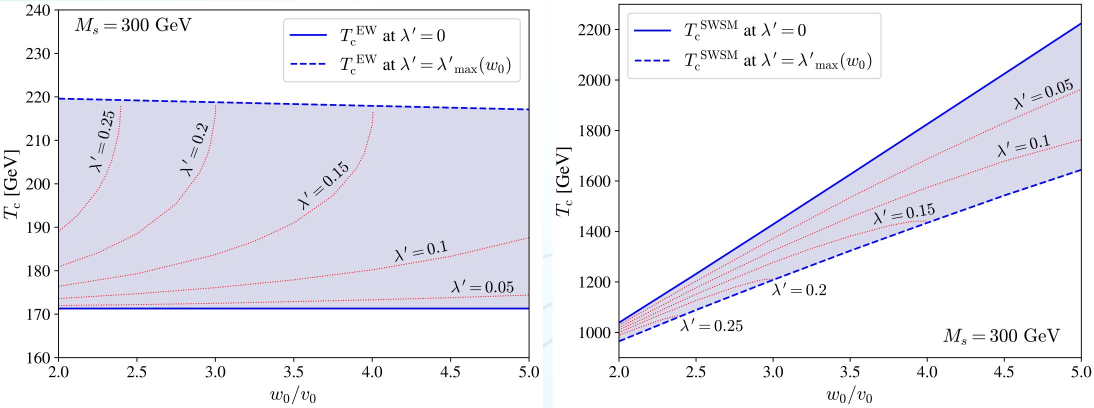
$\rightarrow$ $\overline{m}^2_{\text{scalar}} < 0$ is possible above a high T if $\lambda' < 0$ because $\overline{m}^2_{\text{scalar}} \supset c\lambda' T^2$

Example phase transition



Critical temperatures in a specific model

Superweak extension of SM: [arXiv:1812.11189] or talk by Z. Trócsányi on Tuesday 10:30-11:00



Conclusions

- We presented a simple method for obtaining a real effective potential
  1. Proof of concept: SM effective potential
  2. Example: Singlet scalar extension of SM & Superweak extension of SM

- At finite T there are 3 sources of imaginary parts in the effective potential:
  1. $T = 0$ part for $m_i^2 < 0 \quad \rightarrow \quad$ solved ✓
  2. $T > 0$ part for $m_i^2 < 0 \quad \rightarrow \quad$ solved ✓
  3. $T > 0$ part for $T > T'$ if $\lambda' < 0 \quad \rightarrow \quad$ Model issue, not of perturbation theory

• Thank You for Your Attention! •