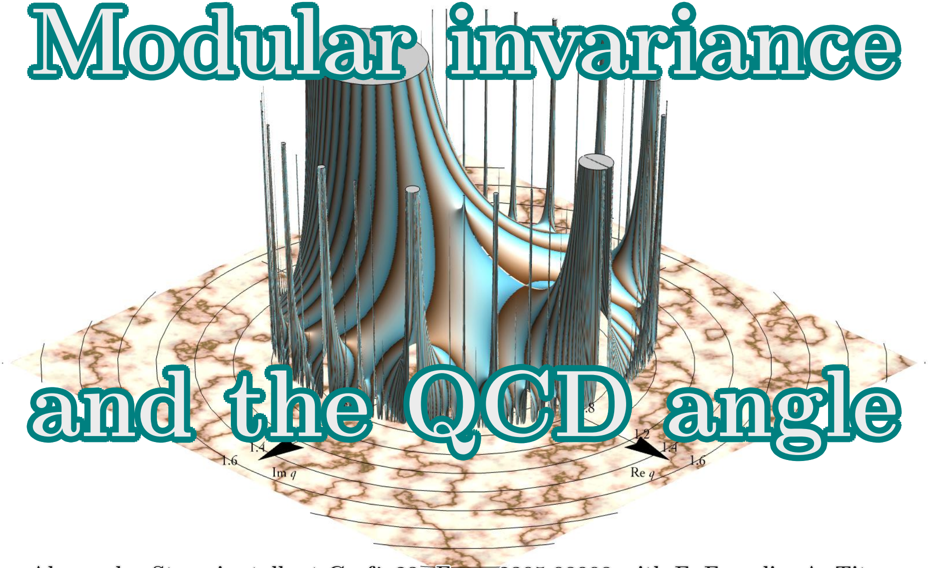
Modular invariance and the QCD angle



Alessandro Strumia, talk at Corfù 23. From 2305.08908 with F. Feruglio, A. Titov.

Introduction

The QCD θ puzzle

Data show:

- large CP violation in quark mixing $\delta_{\rm CKM} \sim 1$;
- no neutron electric dipole implying $\bar\theta = \theta_{\rm QCD} + \arg\det M_q \lesssim 10^{-10}$

$$\mathscr{L}_{\rm QCD} = \sum_q \bar q (i \not{D} - m_q) q - \frac{1}{4} {\rm Tr}\ G^2 + \theta_{\rm QCD} \frac{g_3^2}{32\pi^2} {\rm Tr}\ G\tilde G$$

but both originate from quark Yukawa couplings.

Solutions

- Axion. But: not observed so far; quality problem.
- Special models (Nelson-Barr, 0...). But: ad hoc?

Modular invariance

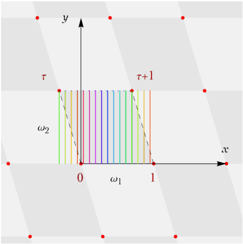
Allows a theory of CP and flavour that provides a neat solution.

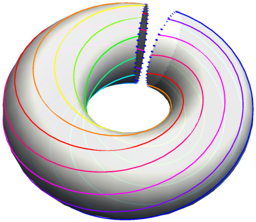
The string motivation

Modular invariance can be done as math independently from its string motivation.

Super-strings in 4+6 dimensions are real. Chiral families of fermions can arise from compactifications on spaces with a complex structure. So CP can be a geometric symmetry spontaneously broken by the compactification.

Literature focused on $N = 1$ supersymmetry, that needs a Ricci-flat compactification with complex structure. Simplest geometry: compactification on orbi-folded 6d flat tori T^3. We only need a 2d flat torus T, obtained writing a 2d space as $z = x + iy$ and imposing a PacMan lattice identification $z = z + \omega_1$ and $z = z + \omega_2$:





$\tau = \omega_2/\omega_1$ tells the geometry: $\mathrm{Im}\,\tau$ is the relative radius, $\mathrm{Re}\,\tau$ is the twisting.

Modular invariance

Modular invariance is a sub-group of global reparametrizations:

$$\omega_2 \to a\omega_2 + b\omega_1, \qquad \omega_1 \to c\omega_2 + d\omega_1$$

gives an equivalent lattice if a, b, c, d are integers with $ad - bc = 1$.

So the low-energy EFT contains a modulus superfield τ invariant under SL(2, $\mathbb{Z}$)

$$\tau \to \frac{a\tau + b}{c\tau + d}.$$

Unusual: appears integrating out infinite states, because of how strings experience the geometry (e.g. $R = 1/R$).

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So the low-energy EFT contains a modulus superfield τ invariant under $\mathrm{SL}(2, \mathbb{Z})$

$$\tau \to \frac{a\tau + b}{c\tau + d}.$$

Unusual: appears integrating out infinite states, because of how strings experience the geometry (e.g. $R = 1/R$). Matter fields Φ transform with weight k_Φ

$$\Phi \to (c\tau + d)^{-k_\Phi}\Phi$$

such that that the minimal global SUSY action with $h \ll \bar{M}_{\rm Pl}$

$$\begin{aligned} K &= -h^2 \ln(-i\tau + i\tau^\dagger) + \sum_\Phi \frac{\Phi^\dagger e^{2V}\Phi}{(-i\tau + i\tau^\dagger)^{k_\Phi}}, \\ W &= Y^u_{ij}(\tau) H_u u_{Ri} Q_j + Y^d_{ij}(\tau) H_d d_{Ri} Q_j \end{aligned}$$

is modular invariant if Yukawa couplings transform with definite weights

$$Y^q_{ij}(\tau) \to (c\tau + d)^{k^q_{ij}} Y^q_{ij}(\tau) \qquad k^q_{ij} = k_{q_{Ri}} + k_{q_{Lj}} + k_{H_q}.$$

Modular invariance, flavour and CP

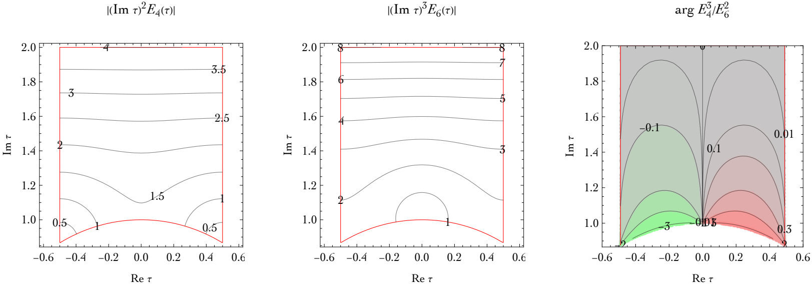
Modular invariance contains symmetries used in flavour models ($S_3, A_4 \ldots$) spontaneously broken in a specific way by τ. It gave predictive flavour models assuming that Yukawas are modular functions with no singularity i.e. '*forms*'. Few exist:

Weight k	0	1, 2, 3	4	6	8	10	12	14	$\cdots$
Forms	1	–	E_4	E_6	$E_8 = E_4^2$	$E_{10} = E_4 E_6$	E_4^3, E_6^2	$E_{14} = E_4^2 E_6$	$\cdots$

where the Eisenstein series transform nicely thanks to lattice summation

$$E_k(\tau) = \frac{1}{2\zeta(k)} \sum_{(m,n)\neq(0,0)} \frac{1}{(m+n\tau)^k}.$$

Key: $E_4^3 \neq E_6^2$ have different phases.



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Key: $E_4^3 \neq E_6^2$ have different phases. This is CP, that acts as $\tau \to -\tau^\dagger$, $\Phi \to \Phi^\dagger$.

1) We assume that $\mathrm{Re}\,\tau$ is the source of CP violation and that

$Y_{ij}^q(\tau) = c_{ij}^q \, F_{k_{ij}^q}(\tau)$ where c is real and F_k is a modular form with weight k.

2) We assume that the Higgs don't break modular invariance, $k_{H_u} + k_{H_d} = 0$.

Motivated but not new, so far. End of the review. Now the talk begins.

Solving the $\theta_{\rm QCD}$ puzzle

We assume that modular invariance has no QCD anomaly i.e.

$$A = \sum_{i=1}^{3} (2k_{Q_i} + k_{u_{R_i}} + k_{d_{R_i}}) = 0.$$

As simple as that. End of the talk?

Solving the $\theta_{\rm QCD}$ puzzle

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$$A = \sum_{i=1}^{3} (2k_{Q_i} + k_{u_{R_i}} + k_{d_{R_i}}) = 0.$$

- $\det M_q$ is a modular form with weight $A = 0$, which is a real constant so

$$\arg \det M_u M_d = 0 \qquad \theta_{\rm QCD} = 0.$$

- $\delta_{\rm CKM} \propto {\rm Im}\ \det[Y_u^\dagger Y_u, Y_d^\dagger Y_d]$ has no special modular properties and $E_4^3 \neq E_6^2$ introduces physical CP violation, $\delta_{\rm CKM} \sim 1$.
- Quark kinetic matrices Z_q can be made canonical via a quark linear transformation that affects q masses and mixing but not $\bar{\theta}$. Minimal Kähler:

$$Y_{ij}^q|_{\rm can} = c_{ij}^q (2{\rm Im}\,\tau)^{k_{ij}^q/2} F_{k_{ij}^q}(\tau).$$

- Supersymmetry must be broken by modular-invariant dynamics such that the gluino mass M_3 is real. SUSY not needed at the weak scale.

The Minimal MSSM Model

Simplest model: modular weights $k_Q = k_{u_R} = k_{d_R} = (-6, 0, +6)$ so $\det Y_q$ is real:

$$Y_q|_{\mathrm{can}} = \begin{array}{c} \\ q_{R1} \\ q_{R2} \\ q_{R3} \end{array} \begin{array}{c} \begin{array}{ccc} q_{L1} & q_{L2} & q_{L3} \end{array} \\ \begin{pmatrix} 0 & 0 & c^q_{13} \\ 0 & c^q_{22} & c^q_{23}(2\mathrm{Im}\,\tau)^3 E_6(\tau) \\ c^q_{31} & c^q_{32}(2\mathrm{Im}\,\tau)^3 E_6(\tau) & (2\mathrm{Im}\,\tau)^6 \left[c^q_{33} E_4^3(\tau) + c'^q_{33} E_6^2(\tau)\right] \end{pmatrix} \end{array}.$$

A numerical or approximate diagonalisation

$$y_3 \simeq y_{33}, \quad y_2 \simeq y_{22}, \quad y_1 \simeq -\frac{y_{13}y_{31}}{y_{33}}, \qquad \theta_{23} \simeq \frac{y_{32}}{y_{33}}, \quad \theta_{13} \simeq \frac{y_{31}}{y_{33}}, \quad \theta_{12} \simeq \frac{y_{31}y_{23}}{y_{22}y_{33}}$$

shows that all quark masses and mixings can be reproduced with comparable c

$$c^u_{ij} \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.56 \\ 0 & -1.86 & 0.87 \\ 1.29 & 4.14 & 3.51, 1.40 \end{pmatrix} \qquad c^d_{ij} \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.55 \\ 0 & -2.59 & 4.59 \\ 0.378 & 0.710 & 0.734, 1.76 \end{pmatrix}$$

for $\tan\beta = 10$ and $\tau = 1/8 + i$. No predictions.

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for $\tan\beta = 10$ and $\tau = 1/8 + i$. No predictions. Leptons with $k_L = k_{e_R} = k_Q$:

$$
c^e_{ij} = 10^{-3} \begin{pmatrix} 0 & 0 & 1.29 \\ 0 & 5.95 & 0.35 \\ -2.56 & 1.47 & 1.01, 1.32 \end{pmatrix} \qquad c^\nu_{ij} = \frac{1}{10^{16}\,{\rm GeV}} \begin{pmatrix} 0 & 0 & 3.4 \\ 0 & 7.1 & 1.2 \\ 3.4 & 1.2 & 0.19, 0.95 \end{pmatrix}.
$$

MSSM models

Less minimal modular weights such as $(-6, \pm 2, 6)$ give asymmetric matrices.

Yukawa matrices $Y_{u,d}$	Modular weights $(u_L, d_L)_{1,2,3}$	Modular weights $u_{R1,2,3}$	Modular weights $d_{R1,2,3}$	Alternative bigger weights $(u_L, d_L)_{1,2,3}$	Alternative bigger weights $u_{R1,2,3}$	Alternative bigger weights $d_{R1,2,3}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_6 \\ 1 & E_6 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ -2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4^2 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4 E_6 \\ 1 & E_6 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^3 + E_6^2 \\ 1 & E_4 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$

Deconstruction to $\mathrm{U}(1)_{\mathrm{FN}}$?

Quark hierarchies are reproduced somehow like in $\mathrm{U}(1)_{\mathrm{FN}}$ Froggatt-Nielsen.

- The hierarchy comes from the modular '6' e.g. $(2\mathrm{Im}\,\tau)^6 = 64$ for $\tau = i + \cdots$.

Can $\mathrm{U}(1)_{\mathrm{FN}}$ also give $\bar{\theta} \ll \delta_{\mathrm{CKM}}$? Weights become charges, one scalar η gives

$$\text{powers} \qquad F_k(\tau) \to \eta^k \qquad \text{instead of modular forms.}$$

If positive powers of η^* are avoided by SUSY, one gets $\bar{\theta} = 0$ but $\delta_{\mathrm{CKM}} = 0$.

FN needs two scalars η, η' with different charges and CP phases.

- Modular automatically provides $E_4^3 \neq E_6^2$.

Negative powers of η' would screw up, model building of heavy states needed.

- Modular forms forbid E_4^3/E_6^2.

Conclusion: maybe not impossible, not nice. Back to modular forms

Non-minimal models with heavy quarks

Not needed. But let's play the same game adding optional extra vector-like quarks, with modular weights such that the full theory is non-anomalous

$$A = 0 \qquad \Rightarrow \qquad \bar{\theta} = \arg\det M_q + \theta_{\rm QCD} = 0 \qquad \text{as} \qquad \left\{ \begin{array}{l} \det M_q \text{ is real} \\ \theta_{\rm QCD} = 0. \end{array} \right.$$

A possible choice is $\pm 6, \pm 2, 0$. Exercise: integrate out the heavy quarks. Interesting because modular invariance seems to become anomalous in the low-energy EFT (e.g. $0, 2, 6$). The anomaly cancels with the gauge kinetic function.

- Yukawas become modular *functions* with poles at the values of τ where the heavy quarks get massless; this screws predictivity;
- $\det M_q^{\rm light}$ is complex, because of functions and of modular weight $A_{\rm light} \neq 0$;
- modular invariance has a QCD anomaly $A_{\rm light} \neq 0$ generating $\theta_{\rm QCD}^{\rm light} \neq 0$;
- the gauge kinetic-function is complex and $\bar{\theta} = \arg\det M_q^{\rm light} + \theta_{\rm QCD}^{\rm light} = 0$.

This optional complication is needed in supergravity and superstrings...

Supergravity

The worst $\bar{\theta} \sim h^2/\bar{M}_{\rm Pl}^2$ would be acceptable, as $h \lesssim 10^{-5}\bar{M}_{\rm Pl}$ is allowed.
But strings etc motivate $h = n\bar{M}_{\rm Pl}$. Supergravity gives new complicated effects:

- the Kähler potential K and the super-potential W unify in

$$G = K/\bar{M}_{\rm Pl}^2 + \ln|W/\bar{M}_{\rm Pl}^3|^2;$$

- a modular transformation of τ implies a Kähler transformation:
  W acquires modular weight $k_W = h^2/\bar{M}_{\rm Pl}^2 > 0$;
- a Kähler transformation implies an extra phase rotation of fermions (quarks and the gluino), so the modular anomaly becomes

$$A = A_{\rm quark} + A_{\rm gluino} = \sum_{i=1}^{3} (2k_{Qi} + k_{u_{Ri}} + k_{d_{Ri}} - 2k_W) + 3k_W.$$

- $A = 0$ again implies $\bar{\theta} \propto \arg M_3^3 \det M_q = 0$.

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- $A = 0$ again implies $\bar{\theta} \propto \arg M_3^3 \det M_q = 0$. But...

the gluino gets involved, does not mix with quarks, has $k_W > 0$. Some quark remains massless within the MSSM. Non-minimal models are needed.

E.g. an extra $\lambda' \in 8$ of SU(3) with modular charge opposite to the gluino resurrects the previously discussed models. Integrating λ' out gives EFT with anomalies and modular functions, that solve $\bar{\theta} = 0$ as discussed in the previous slide.

Superstrings

This suggests more general sugra models: assume

1. Full theory with non-anomalous modular invariance, $A = 0$;
2. Integrating out heavy states only gives poles at $\tau = i\infty$.
3. No modular anomaly from quarks, $\sum_{i=1}^{3}(2k_{Qi} + k_{u_{Ri}} + k_{d_{Ri}} - 2k_W) = 0$.

The resulting effective sugra field theory:

- Allows negative powers of Dedekind $\eta(\tau) = [(E_4^3(\tau) - E_6^2(\tau))/12^3]^{1/24}$ only.
- Extra phases e.g. $\eta \to e^{i\theta}(c\tau + d)^{1/2}\eta$ mess math without affecting physics.
- Modular anomaly cancelled by gauge kinetic function, $f \ni 3k_W \ln \eta/(4\pi^2)$.
- $\bar{\theta} = \theta_{\rm QCD} + \arg M_3^3 \det M_q = 0$ as both depend on η.

This remembers you something stringy? Maybe the proposed understanding of the $\theta_{\rm QCD}$ puzzle could be realized in toroidal string compactifications:

1.✓ Modular invariances of superstrings are non-anomalous; anomalies appears in the effective QFT of massless states.

2.✓ Integrating out ∞ towers of states with mass $\propto n + m\tau$ gives Dedekind η with de-compactification poles.

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3.~~✓~~ Anomaly-free EFT? We don't know if strings realise the MSSM... $\langle\tau\rangle$?

Deviations from $\bar{\theta} = 0$

- **Non-renormalizable operators** controlled by SL(2,$\mathbb{Z}$).
- **Extra scalars** must get CP-conserving vev.
- **Extra moduli** must get CP-conserving vev or have good modular charges.
- **SM** gives $\bar{\theta} \sim 10^{-18}$ at 4 loops.
- **SUSY breaking** corrects M_q at 1 loop unsuppressed by $v/m_{\rm SUSY}$:

$$\delta\bar{\theta} \approx \frac{\alpha_3}{4\pi} \sum_{q=u,d} \frac{\operatorname{Im}\operatorname{Tr}\left[(Y_q)^{-1} m^2_{\tilde{q}_R} A_q m^2_{\tilde{Q}}\right]}{m^5_{\rm SUSY}}$$

A problem if squarks violate flavour/CP differently from quarks.
Avoided assuming gauge or anomaly mediation below the τ modulus mass:

$$m_{\rm SUSY} < \Lambda_{\rm SUSY} < \Lambda_{\rm flavour} \sim M_\tau$$

Then loops respect the SM $\mathrm{U}(3)_Q \otimes \mathrm{U}(3)_{u_R} \otimes \mathrm{U}(3)_{d_R}$ flavour structure:

$$\bar{\theta} \lesssim \operatorname{Im}\left[(Y_u^\dagger Y_u)^2 (Y_d^\dagger Y_d)^2 (Y_u^\dagger Y_u)(Y_d^\dagger Y_d)\right] \approx \frac{M_t^4 M_b^4 M_c^2 M_s^2}{v^{12}} J_{\rm CP} \tan^6\beta \sim 10^{-28} \tan^6\beta.$$

Conclusions

QCD θ puzzle understood

A non-anomalous modular invariance allows theory of CP and flavour where:

- ⋆ $\bar{\theta} \ll \delta_{\rm CKM} \sim 1$;
- ⋆ q and ℓ masses and mixings reproduced up to order one coefficients.

The simple general idea works in supersymmetry or supergravity, with modular forms or functions, with or without heavy colored states. Maybe strings?

Phenomenology: how can this be ~~tested~~ confirmed?

- $V(\tau) = V(-\tau^*)$, CP walls can be inflated away.
- SUSY and τ could be heavy, $\Gamma(\tau \to \nu_R \nu_R) \sim M_\tau^3/h^2$ (leptogenesis?).
- The fermionic τ could be stable LSP DM.

Light new particles not needed, all can be Planck-heavy... no signals.