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Running Vacuum approach to the Quantum Vacuum
Theoretical & Phenomenological Implications

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Workshop on Standard Model and Beyond (Corfu August 27-September 7, 2023)

Guidelines of the Talk

- Vacuum energy and the CC Problem
- Dynamical **DE** and Running Vacuum Models
- **Running Vacuum in QFT and beyond**
- **RVM** and ΛCDM troubles (H_0- and σ_8 tensions)
- Conclusions

Interpretation of Einstein's eqs.

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda\, g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}}$$

1915

1917

Geometry $\Longleftrightarrow$ **Energy**

$$\nabla^{\mu} G_{\mu\nu} = 0, \text{ where } G_{\mu\nu} = R_{\mu\nu} - (1/2) g_{\mu\nu} R$$

$$\nabla_{\mu}\Lambda = \partial_{\mu}\Lambda = 0 \quad \Rightarrow \quad \Lambda = \text{const}\,. \quad !!$$

if $\nabla^{\mu}\left(G_N T_{\mu\nu}\right) = 0 \ldots$!!!

$$\rho_{\Lambda} = \frac{\Lambda}{8\,\pi\, G_N}$$

Cosmological Constant

Dark Energy

➤ The old CC problem as a fine tuning problem

The CC problem stems from realizing that the effective or physical vacuum energy is the sum of two terms:

$$\boxed{\rho_{\Lambda\rm phys} = \rho_{\Lambda\rm vac} + \rho_{\Lambda\rm ind}}$$

$$S_{EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{|g|}\,(R - 2\Lambda_{\rm vac}) = \int d^4x\sqrt{|g|}\left(\frac{1}{16\pi G_N}R - \rho_{\Lambda\rm vac}\right)$$

$$\boxed{\rho_{\Lambda\rm vac} = \frac{\Lambda}{8\pi G_N}}$$

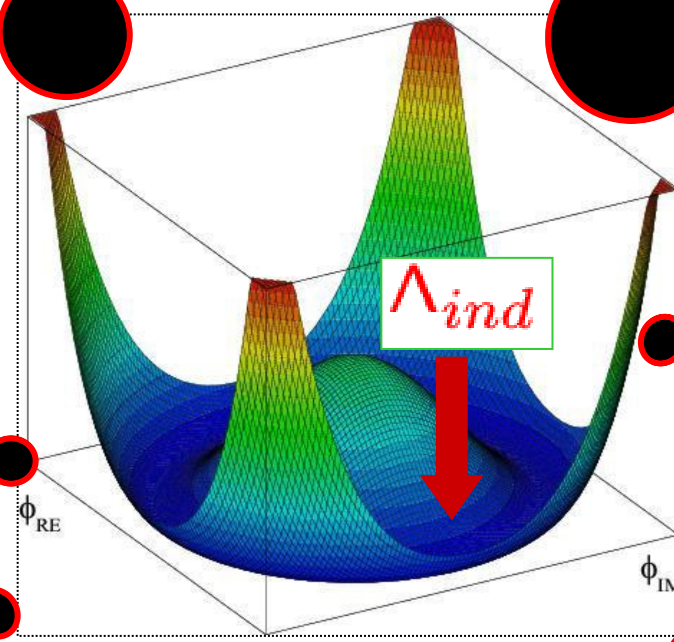
Vacuum bare term in Einstein eqs.

$$R_{ab} - \frac{1}{2}g_{ab}R = -8\pi G_N\left(\langle\tilde{T}^{\varphi}_{ab}\rangle + T_{ab}\right) = -8\pi G_N\, g_{ab}\,(\rho_{\Lambda\rm vac} + \rho_{\Lambda\rm ind} + T_{ab})$$

Quantum effects $\Rightarrow$ $\rho_{\Lambda\rm ind} = \langle V(\varphi)\rangle + \text{ZPE}$

Vacuum energy = bubbles+SSB

$$\frac{1}{2}\,\hbar\,\omega_k$$



Λ in the SM and beyond

Source	Effect (GeV^4)	Λ/Λ_{exp}
electron 0-point	10^{-16}	10^{31}
QCD chiral	10^{-4}	10^{43}
QCD gluon	10^{-2}	10^{45}
Electroweak SM	10^{+9}	10^{56}
typical GUT	10^{+64}	10^{111}
Quantum Gravity	10^{+76}	10^{123} !!

!!

$$\rho_\Lambda^0 = \Omega_\Lambda^0\,\rho_c^0 \simeq 6\,h^2 \times 10^{-47}\,\mathrm{GeV}^4 \simeq 3 \times 10^{-47}\,\mathrm{GeV}^4$$

$$\boxed{m_\Lambda \equiv \sqrt[4]{\rho_\Lambda^0} \simeq 2-3\ \mathrm{meV}}$$

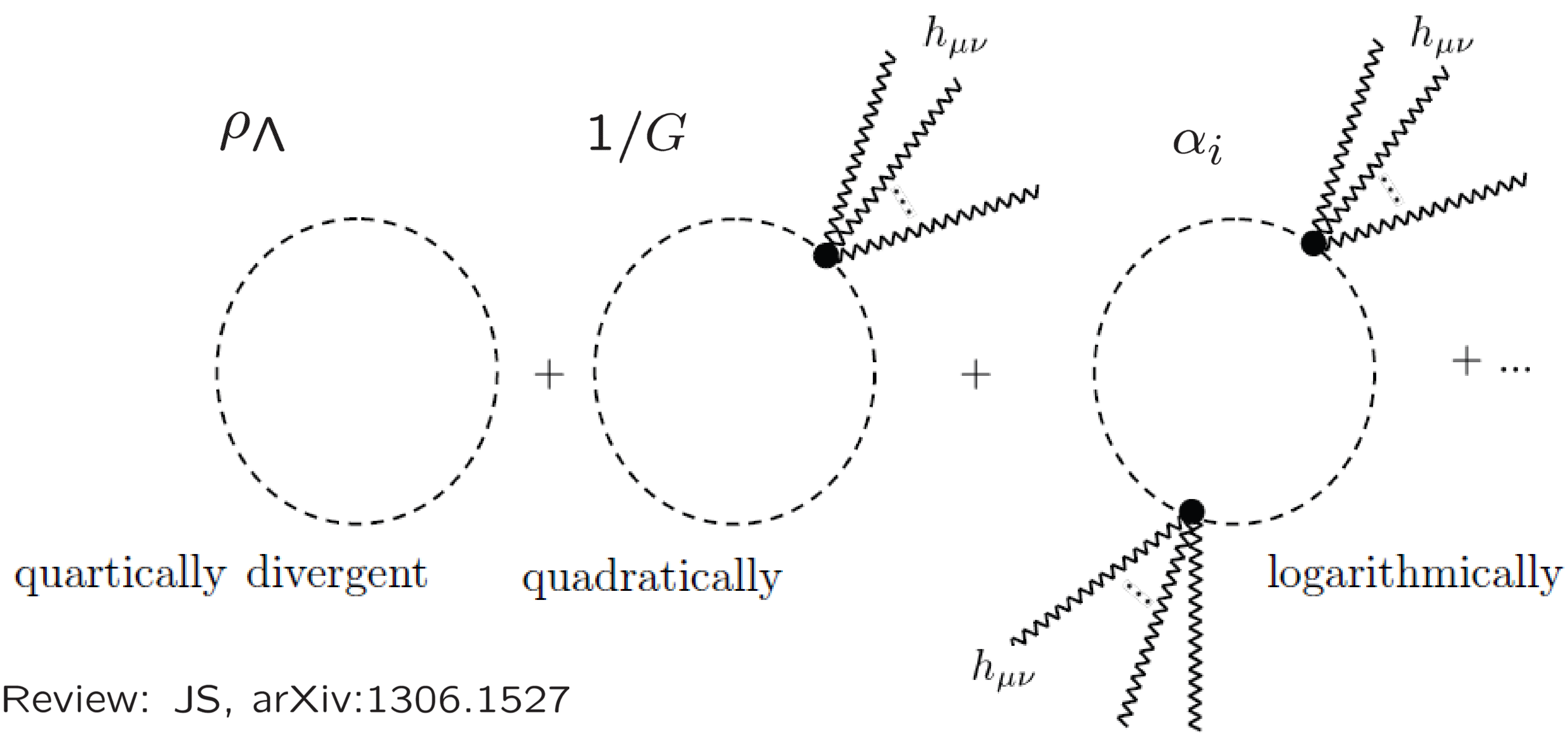
➢ Introducing an external gravitational field: QFT in curved spacetime!

In diagrammatic form, $\Rightarrow$ expansion $\sqrt{-g}$ around Minkowski space,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h = \eta^{\mu\nu} h_{\mu\nu}$$

$$\boxed{\sqrt{-g} = 1 + \frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h_{\mu\nu}h^{\mu\nu} + \mathcal{O}\left(h^3\right)}$$



Review: JS, arXiv:1306.1527

RVM: inflation and cosmological expansion

Consider the class of time evolving vacuum models following a power series of the Hubble rate: (phenomenological approach)

I. Shapiro and J. Solà (2000,2003,2009)

J. Solà and H. Stefancic (2005,2006)

J. Solà (2007) ...

$$\Lambda(H) = c_0 + c_1 H + c_2 H^2 + c_3 H^3 + c_4 H^4 + \ldots$$

Reviews:

J. Solà (2011,2013,2014,,2016)

N. Mavromatos, J. Solà (2020)

(**"stringy-RVM" ...**)

Better fit than the ΛCDM and alleviates H_0 and σ_8-tensions

Vacuum energy density: $\rho_\Lambda(H) = \Lambda(H)/(8\pi G)$

At low energy:

$$\Lambda(H) = c_0 + c_2 H^2 = \Lambda_0 + 3\,\nu\,(H^2 - H_0^2)$$

proposed (RG) equation for the vacuum energy density of the expanding Universe

(Ansatz)

$$\frac{d\rho_\Lambda(\mu)}{d\ln\mu^2} = \frac{1}{(4\pi)^2}\left[\sum_i B_i M_i^2\,\mu^2 + \sum_i C_i\,\mu^4 + \sum_i \frac{D_i}{M_i^2}\,\mu^6 + \ldots\right]$$

$$\Big\Downarrow \quad \mu^2 = aH^2 + b\dot{H}$$

$$\rho_\Lambda(H,\dot{H}) = \boxed{a_0} + a_1\,\dot{H} + \boxed{a_2\,H^2} + a_3\,\dot{H}^2 + \boxed{a_4\,H^4} + a_5\,\dot{H}\,H^2$$

(Generalized Ansatz:)

$$\Big\Downarrow \quad \mu^2 = H^2$$

$$\rho_\Lambda(H) = \frac{3}{8\pi G_N}\left(c_0 + \nu H^2 + \frac{H^4}{H_I^2}\right)$$

Distinctive from Starobinsky's inflation !!

Can this be substantiated in QFT or string theory?

Adiabatic renormalization of the **VED** in QFT in a FLRW background: absence of quartic mass terms

C. Moreno-Pulido and JSP arXiv:2005.03164 (EPJ-C) arXiv:2201.05827

- The gravitational field equations read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{matter},$$

where Λ is the Cosmological constant, with energy density $\rho_\Lambda \equiv \Lambda/(8\pi G_N)$. (this is not yet the physical VED)

Consider a toy-model (but non-trivial) calculation of the VED.

- We will suppose that there is only one matter field contribution to the EMT in $T_{\mu\nu}^{matter}$ in the form of a real scalar field, ϕ.

(**nonminimal coupling** ξ)

$$S[\phi] = -\int d^4x\sqrt{-g}\left(\frac{1}{2}g_{\mu\nu}\partial_\nu\phi\partial_\mu\phi + \frac{1}{2}(m^2+\xi R)\phi^2\right)$$

(**no SSB contribution!**)

- The Energy-Momentum tensor (EMT) associated to the scalar field is

$$T_{\mu\nu}(\phi) = (1-2\xi)\,\partial_\mu\phi\partial_\nu\phi + \left(2\xi - \frac{1}{2}\right) g_{\mu\nu}\partial^\sigma\phi\partial_\sigma\phi$$
$$-2\xi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi + \xi G_{\mu\nu}\phi^2 - \frac{1}{2}m^2 g_{\mu\nu}\phi^2.$$

- We can take into account the quantum fluctuations of the field ϕ by considering the expansion of the field around its background (or classical mean field) value ϕ,

$$\phi(\tau, \mathbf{x}) = \phi_b(\tau) + \delta\phi(\tau, \mathbf{x}),$$

$$\boxed{\langle T_{\mu\nu}^{vac}\rangle \equiv -\rho_\Lambda g_{\mu\nu} + \langle T_{\mu\nu}^{\delta\phi}\rangle.}$$

Total
vacuum contribution

(needs renormalization!!)

$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu = a^2(\tau)\eta_{\mu\nu}\,dx^\mu dx^\nu$$

$\text{sign}(g_{\mu\nu}) = (-,+,+,+)$

Fluctuations split in Fourier modes:

$$\delta\phi(\tau,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}a}\int d^3k\left[A_{\mathbf{k}}e^{i\mathbf{kx}}h_k(\tau) + A^{\dagger}_{\mathbf{k}}e^{-i\mathbf{kx}}h^*_k(\tau)\right]$$

$$(\Box - m^2 - \xi R)\delta\phi(\tau,x) = 0 \quad \Rightarrow \quad \boxed{h''_k + \Omega^2_k h_k = 0,} \quad \textbf{(mode equation)}$$

$$h'_k h^*_k - h_k h^{*\prime}_k = i$$

$$\Omega^2_k \equiv k^2 + a^2m^2 + \boxed{a^2(\xi - 1/6)R} \quad \textbf{(non-trivial!)}$$

The solution is $\quad h_k(\tau) \sim \dfrac{e^{i\int^{\tau} W_k(\tau_1)d\tau_1}}{\sqrt{W_k(\tau)}},$

$$W^2_k = \Omega^2_k - \frac{1}{2}\frac{W''_k}{W_k} + \frac{3}{4}\left(\frac{W'_k}{W_k}\right)^2$$

In order to solve this equation we should use the **WKB approximation** or **adiabatic regularization**. (slowly varying) Ω_k **!!**

$$W_k = \omega_k^{(0)} + \omega_k^{(2)} + \omega_k^{(4)} \ldots ,$$ (**Adiabatic expansion**)$^{(*)}$

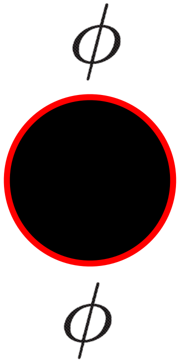
$$\begin{cases} \omega_k^{(2)} = \dfrac{a^2\Delta^2}{2\omega_k} + \dfrac{a^2R}{2\omega_k}(\xi - 1/6) - \dfrac{\omega_k''}{4\omega_k^2} + \dfrac{3\omega_k'^2}{8\omega_k^3}\,, \\ \omega_k^{(4)} = -\dfrac{1}{2\omega_k}\left(\omega_k^{(2)}\right)^2 + \dfrac{\omega_k^{(2)}\omega_k''}{4\omega_k^3} - \dfrac{\omega_k^{(2)''}}{4\omega_k^2} - \dfrac{3\omega_k^{(2)}\omega_k'^2}{4\omega_k^4} + \dfrac{3\omega_k'\omega_k^{(2)'}}{4\omega_k^3}\,. \end{cases}$$

$$\begin{cases} \omega_k^{(0)} \equiv \omega_k = \sqrt{k^2 + a^2M^2}, \\ \omega_k' = a^2\mathcal{H}\dfrac{M^2}{\omega_k}, \quad \omega_k'' = 2a^2\mathcal{H}^2\dfrac{M^2}{\omega_k} + a^2\mathcal{H}'\dfrac{M^2}{\omega_k} - a^4\mathcal{H}^2\dfrac{M^4}{\omega_k^3}\,. \end{cases}$$

The non-appearance of the odd adiabatic orders is justified by means of general covariance.

Explains why only even powers of H:

$$\boxed{\Lambda(H) = c_0 + \cancel{c_1 H} + c_2 H^2 + \cancel{c_3 H^3} + c_4 H^4 + \ldots}$$



one-loop

$T_{00}^{\delta\phi}$ up to 4th adiabatic order:

$$\left\langle T_{00}^{\delta\phi} \right\rangle = \int dk k^2 \left[|h_k'|^2 + \left(\omega_k^2 + a^2\Delta^2\right)|h_k|^2 \right.$$
$$\left. \left(\xi - \frac{1}{6}\right)\left(-6\mathcal{H}^2|h_k|^2 + 6\mathcal{H}(h_k' h_k^* + h_k^{*\prime} h_k)\right) \right]$$

unrenormalized

ZPE

UV-divergent !!

$$\left\langle T_{00}^{\delta\phi} \right\rangle = \frac{1}{8\pi^2 a^2}\int dk k^2 \left[2\omega_k + \frac{a^4 M^4 \mathcal{H}^2}{4\omega_k^5} - \frac{a^4 M^4}{16\omega_k^7}(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 + 8\mathcal{H}'\mathcal{H}^2 + 4\mathcal{H}^4) \right.$$
$$+ \frac{7a^6 M^6}{8\omega_k^9}(\mathcal{H}'\mathcal{H}^2 + 2\mathcal{H}^4) - \frac{105 a^8 M^8 \mathcal{H}^4}{64\omega_k^{11}}$$
$$+ \left(\xi - \frac{1}{6}\right)\left(-\frac{6\mathcal{H}^2}{\omega_k} - \frac{6a^2 M^2 \mathcal{H}^2}{\omega_k^3} + \frac{a^2 M^2}{2\omega_k^5}(6\mathcal{H}''\mathcal{H} - 3\mathcal{H}'^2 + 12\mathcal{H}'\mathcal{H}^2) \right.$$
$$\left. -\frac{a^4 M^4}{8\omega_k^7}(120\mathcal{H}'\mathcal{H}^2 + 210\mathcal{H}^4) + \frac{105 a^6 M^6 \mathcal{H}^4}{4\omega_k^9} \right)$$
$$\left. + \left(\xi - \frac{1}{6}\right)^2 \left(-\frac{1}{4\omega_k^3}(72\mathcal{H}''\mathcal{H} - 36\mathcal{H}'^2 - 108\mathcal{H}^4) + \frac{54 a^2 M^2}{\omega_k^5}(\mathcal{H}'\mathcal{H}^2 + \mathcal{H}^4) \right) \right]$$
$$+ \frac{1}{8\pi^2 a^2}\int dk k^2 \left[\frac{a^2\Delta^2}{\omega_k} - \frac{a^4\Delta^4}{4\omega_k^3} + \frac{a^4 \mathcal{H}^2 M^2 \Delta^2}{2\omega_k^5} - \frac{5}{8}\frac{a^6 \mathcal{H}^2 M^4 \Delta^2}{\omega_k^7} \right.$$
$$\left. + \left(\xi - \frac{1}{6}\right)\left(-\frac{3a^2\Delta^2\mathcal{H}^2}{\omega_k^3} + \frac{9a^4 M^2 \Delta^2 \mathcal{H}^2}{\omega_k^5} \right) \right] + \ldots,$$

- We compute terms up to 4th order because the divergences are only present up to this adiabatic order.
- We define the renormalized ZPE in curved space-time at the scale M as follows:

$$\boxed{\langle T_{00}^{\delta\phi}\rangle_{Ren}(M) \equiv \langle T_{00}^{\delta\phi}\rangle(m) - \langle T_{00}^{\delta\phi}\rangle^{(0-4)}(M)}$$

$$\langle T_{00}^{\delta\phi}\rangle_{\rm Ren}(M) = \frac{a^2}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4\ln\frac{m^2}{M^2}\right)$$
$$-\left(\xi - \frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2}\left(m^2 - M^2 - m^2\ln\frac{m^2}{M^2}\right) + \left(\xi - \frac{1}{6}\right)^2\frac{9\left(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 - 3\mathcal{H}^4\right)}{16\pi^2 a^2}\ln\frac{m^2}{M^2} + \dots$$

$$\Downarrow$$

$$\boxed{\mathcal{M}_{\rm Pl}^2(M)G_{\mu\nu} + \rho_\Lambda(M)g_{\mu\nu} + \alpha(M)\ {}^{(1)}H_{\mu\nu} = \langle T_{\mu\nu}^{\delta\phi}\rangle_{\rm ren}(M)\,.}$$

$$\mathcal{M}_{\rm Pl}^2(M) = \frac{G^{-1}(M)}{8\pi}$$

Off-shell subtraction: $\Longrightarrow$ **Exploring different scales**

$$\blacktriangleright\ \textbf{VED} = \quad \rho_\Lambda + \mathrm{ZPE}$$

$$\langle T_{\mu\nu}^{\rm vac}\rangle \equiv -\rho_\Lambda g_{\mu\nu} + \langle T_{\mu\nu}^{\delta\phi}\rangle \quad \Longrightarrow \quad \rho_{\rm vac}(M) = \rho_\Lambda(M) + \frac{\left\langle T_{00}^{\delta\phi}\right\rangle_{Ren}(M)}{a^2}$$

arXiv:2207.07111

$$\rho_{\rm vac}(M) = \rho_\Lambda(M) + \frac{1}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4\ln\frac{m^2}{M^2}\right)$$
$$+\left(\xi-\frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2a^2}\left(M^2 - m^2 + m^2\ln\frac{m^2}{M^2}\right) + \left(\xi-\frac{1}{6}\right)^2\frac{9\left(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 - 3\mathcal{H}^4\right)}{16\pi^2a^4}\ln\frac{m^2}{M^2} + \cdots$$

$$\left.\begin{array}{l}\text{in Minkowski space } (H=0)\\ \rho_{\rm vac}(M) \quad \text{must be RG invariant}\end{array}\right\} \quad \beta_{\rho_\Lambda}(M) = M\frac{\partial\rho_\Lambda(M)}{\partial M} = \frac{1}{2(4\pi)^2}(M^2-m^2)^2$$

Joan Solà (Corfu 2023)

➢ Beta Function of the VED

$$\beta_{\rho_{\rm vac}} = M\frac{\partial\rho_{\rm vac}(M)}{\partial M}$$

$$\boxed{= \left(\xi - \frac{1}{6}\right)\frac{3H^2}{8\pi^2}\left(M^2 - m^2\right)}$$

$$+ \left(\xi - \frac{1}{6}\right)^2 \frac{9\left(\dot{H}^2 - 2H\ddot{H} - 6H^2\dot{H}\right)}{8\pi^2}$$

(Higher order, negligible for current universe)

arXiv:2207.07111

$$\beta_{\rho_{\rm vac}} \propto \cancel{m^4}$$

!!

➢ VED evolution

$$\rho_{\rm vac}(M,H)-\rho_{\rm vac}(M_0,H_0)=\frac{3\left(\xi-\frac{1}{6}\right)}{16\pi^2}\left[H^2\left(M^2-m^2+m^2\ln\frac{m^2}{M^2}\right)\right.$$
$$\left.-H_0^2\left(M_0^2-m^2+m^2\ln\frac{m^2}{M_0^2}\right)\right]+\cdots,$$

$M = H$ and $M_0 = H_0$

for the current universe

C.Moreno-Pulido and JSP (2020,2022) recent !

$$\boxed{\rho_{\rm vac}(H)=\rho^0_{\rm vac}+\frac{3\nu_{\rm eff}(H)}{8\pi}\,m^2_{\rm Pl}\,(H^2-H_0^2)+\mathcal{O}(H^4)}$$

$$\boxed{\nu_{\rm eff}(H)\equiv\frac{1}{2\pi}\left(\xi-\frac{1}{6}\right)\frac{m^2}{m^2_{\rm Pl}}\left(-1+\ln\frac{m^2}{H^2}-\frac{H_0^2}{H^2-H_0^2}\ln\frac{H^2}{H_0^2}\right)}$$

naturally small parameter

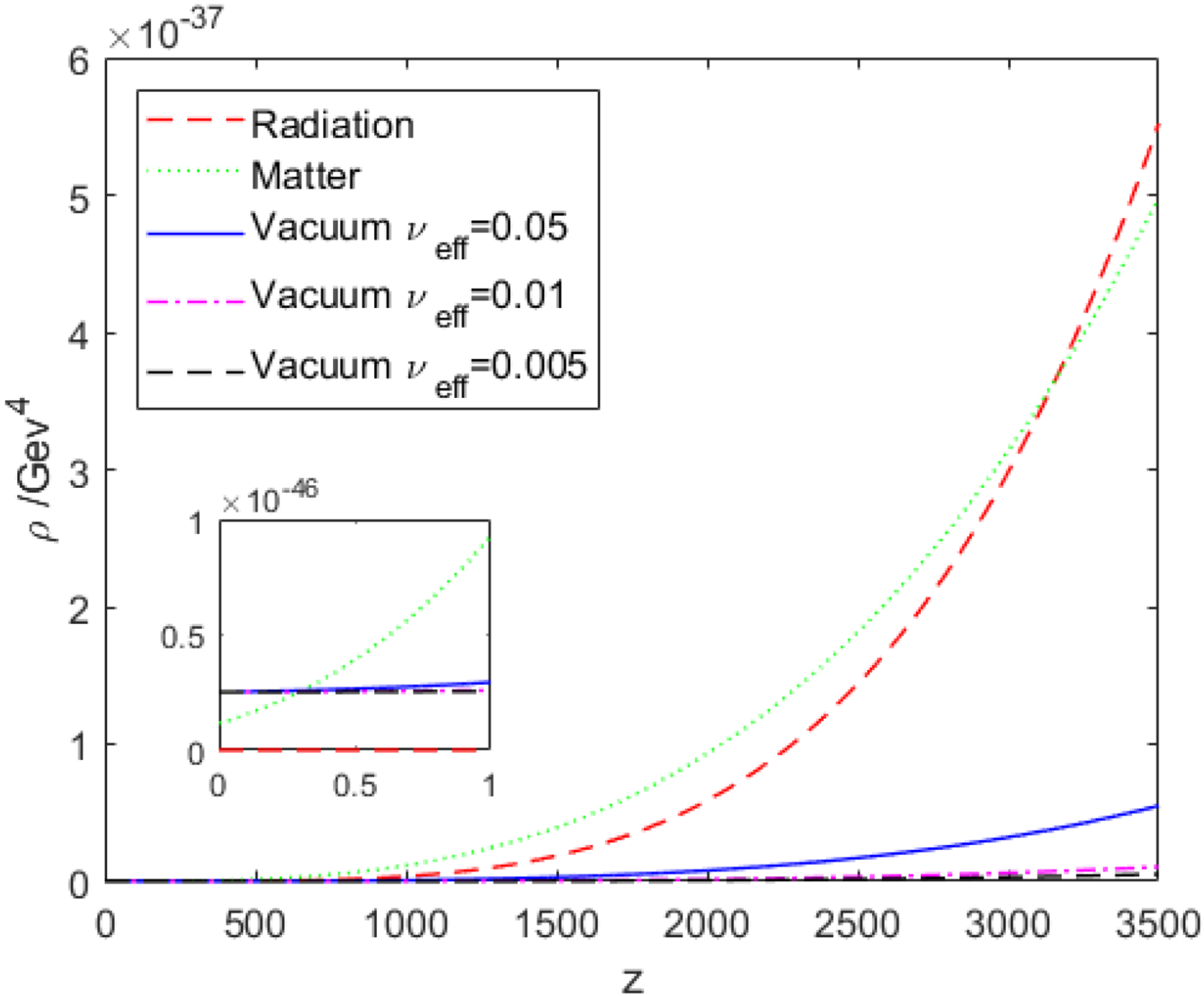
RVM structure !!

J. Solà (2011,2013,2014,,2016)

from action: 0710.4151

(J.Phys.A **41** (2008) 164066)

$$\nu_{\rm eff}\simeq\epsilon\ln\frac{m^2}{H_0^2}\qquad\epsilon\equiv\frac{1}{2\pi}\left(\xi-\frac{1}{6}\right)\frac{m^2}{m^2_{\rm Pl}}$$


$\times 10^{-37}$
6
5
4
3
2
1
0
ρ /Gev^4
Radiation
Matter
Vacuum ν_{eff}=0.05
Vacuum ν_{eff}=0.01
Vacuum ν_{eff}=0.005
$\times 10^{-46}$
1
0.5
0
0
0.5
1
0
500
1000
1500
2000
2500
3000
3500
z

Recall where we come from:

$$\boxed{\begin{aligned}\rho_{\rm vac}(M) &= \rho_\Lambda(M) + \frac{1}{128\pi^2}\left(-M^4 + 4m^2M^2 - 3m^4 + 2m^4\ln\frac{m^2}{M^2}\right)\\ &+\left(\xi-\frac{1}{6}\right)\frac{3\mathcal{H}^2}{16\pi^2a^2}\left(M^2 - m^2 + m^2\ln\frac{m^2}{M^2}\right) + \left(\xi-\frac{1}{6}\right)^2\frac{9\left(2\mathcal{H}''\mathcal{H} - \mathcal{H}'^2 - 3\mathcal{H}^4\right)}{16\pi^2a^4}\ln\frac{m^2}{M^2} + \cdots\end{aligned}}$$

The dots $\cdots$ are 6th order adiabatic terms: $\boxed{\mathcal{O}\left(H^6\right)}$ These are finite !!

but difficult to compute!

➢ QFT-driven INFLATION

C.Moreno-Pulido and JSP (2022) arXiv:2201.05827

$$\boxed{\rho_{\rm vac}^{\rm inf} = \frac{\langle T_{00}^{\delta\phi}\rangle_{\rm Ren}^{\rm 6th}(m)}{a^2} = \frac{\tilde{\xi}}{80\pi^2 m^2} H^6 + f(\dot{H}, \ddot{H}, \dddot{H}\ldots)}$$

$$\boxed{\tilde{\xi} = \left(\xi - \frac{1}{6}\right) - \frac{2}{63} - 360\left(\xi - \frac{1}{6}\right)^3}$$

$$\begin{aligned}
\langle T_{00}^{\delta\phi}\rangle_{\rm ren}^{(6)}(m) = & \frac{a^2}{20160\pi^2 m^2}\left(-8H^6 - 36H^4\dot{H} - 20\dot{H}^3 + 42H^3\ddot{H} + 3\ddot{H}^2 - 6\dot{H}\dddot{H}\right.\\
& \left.+84H^2\dot{H}^2 + 36H^2\dddot{H} + 60H\dot{H}\ddot{H} + 6H\ddddot{H}\right)\\
& + \left(\xi - \frac{1}{6}\right)\frac{a^2}{160\pi^2 m^2}\left(2H^6 + 12H^4\dot{H} + 8\dot{H}^3 - 14H^3\ddot{H} - \ddot{H}^2 + 2\dot{H}\dddot{H} - 34H^2\dot{H}^2\right.\\
& \left.-12H^2\dddot{H} - 24H\dot{H}\ddot{H} - 2H\ddddot{H}\right)\\
& + \left(\xi - \frac{1}{6}\right)^2\frac{3a^2}{32\pi^2 m^2}\left(-24H^4\dot{H} - 8\dot{H}^3 + 10H^3\ddot{H} + \ddot{H}^2 - 2\dot{H}\dddot{H} + 32H^2\dot{H}^2\right.\\
& \left.+12H^2\dddot{H} + 24H\dot{H}\ddot{H} + 2H\ddddot{H}\right)\\
& - \left(\xi - \frac{1}{6}\right)^3\frac{9a^2}{8\pi^2 m^2}\left(2H^2 + \dot{H}\right)\left(2H^4 - 19H^2\dot{H} + 2\dot{H}^2 - 6H\ddot{H}\right).
\end{aligned}$$

Generalization

$$\boxed{\Lambda(H) = c_0 + 3\nu H^2 + 3\alpha \frac{H^{n+2}}{H_I^n}}$$

$$\begin{cases} \rho_r(\hat{a}) = \tilde{\rho}_I(1-\nu)\dfrac{\hat{a}^{2n(1-\nu)}}{\left[1+\hat{a}^{2n(1-\nu)}\right]^{\frac{n+2}{n}}} \\ \\ \rho_\Lambda(\hat{a}) = \tilde{\rho}_I \dfrac{1+\nu\hat{a}^{2n(1-\nu)}}{\left[1+\hat{a}^{2n(1-\nu)}\right]^{\frac{n+2}{n}}} . \end{cases}$$

Minimal unified model at high energy (early universe):

- S. Basilakos,J.A.S Lima, and JS arXiv:1509.00163, arXiv:1307.6251
- JS and A. Gómez-Valent arXiv:1501.03832
- JS arXiv:1505.05863
- JS and H. Yu arXiv:1910.01638

+ **"Stringy RVM"**

See talk by Nick Mavromatos

N. Mavromatos and JSP arXiv:2012.07971

$$\boxed{\Lambda(t) = c_0 + 3\nu H^2 + 3\alpha \frac{H^4}{H_I^2}}$$

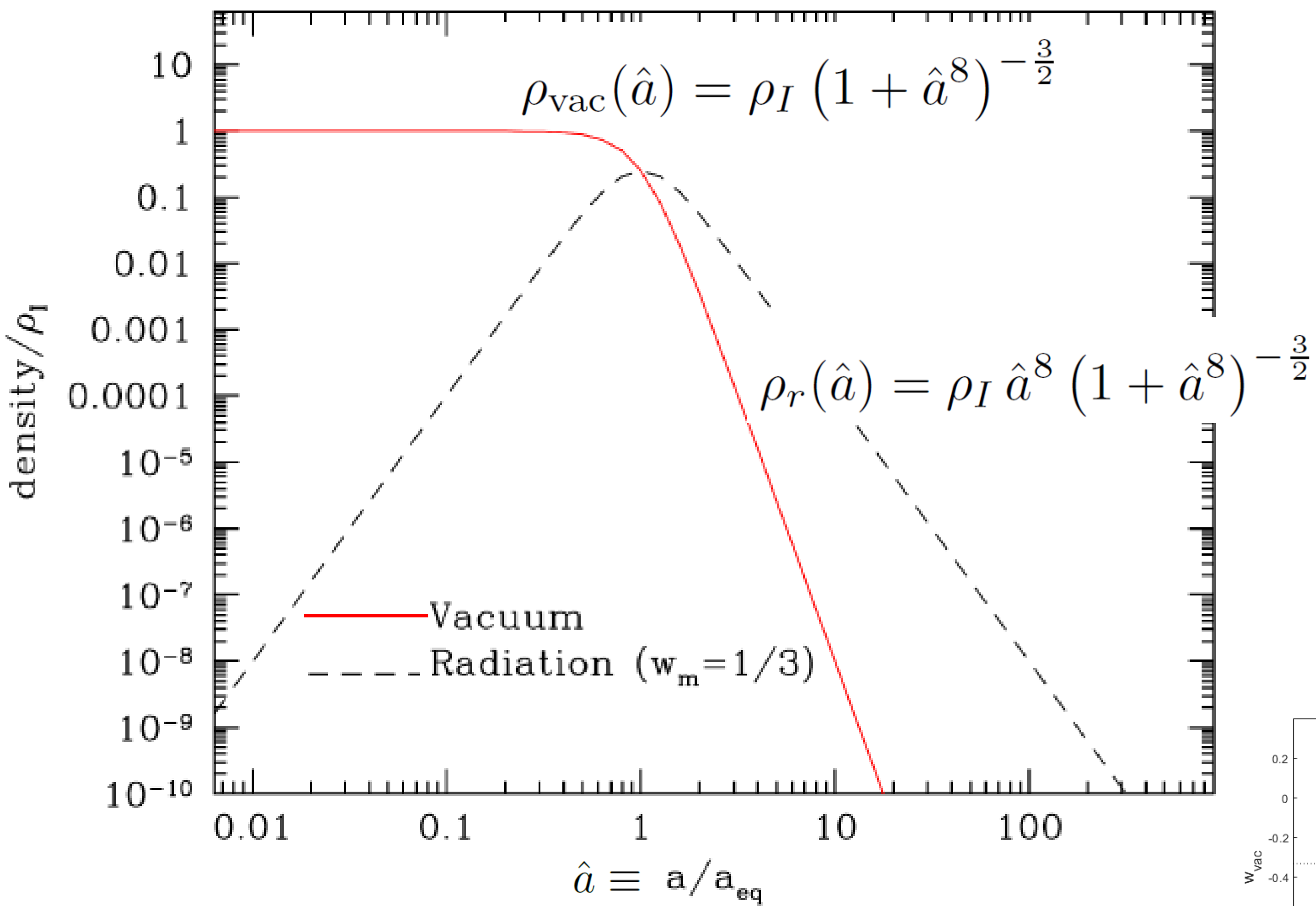
$$\dot{H} + \frac{3}{2}(1+\omega_m)H^2 \left[1 - \nu - \frac{c_0}{3H^2} - \alpha\frac{H^2}{H_I^2}\right] = 0$$

Inflationary solution:

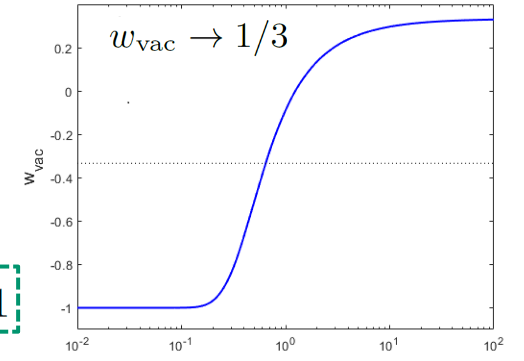
$$H^2 = (1-\nu)H_I^2/\alpha \quad !!$$

$$a(t) \propto e^{H_I t}.$$

RVM-inflation



(EoS)



$\boxed{w_{\rm vac} \simeq -1}$

QUANTUM VACUUM Pressure

Calculations up to 6th adiabatic order yield: arXiv:2201.05827

$$\boxed{P_{\rm vac}(M) = -\rho_{\rm vac}(M)} + \text{corrections} \;\; !!$$

$$+ f_2(M,\dot{H}) + f_4(M,H,\dot{H},...,\dddot{H}) + f_6(\dot{H},...,\overset{\cdot\cdot\cdot\cdot\cdot}{H})$$

e.g. $$f_2(M,\dot{H}) = \frac{\left(\xi-\frac{1}{6}\right)}{8\pi^2}\dot{H}\left(m^2 - M^2 - m^2\ln\frac{m^2}{M^2}\right)$$

Equation of State of the QUANTUM VACUUM (EoS)

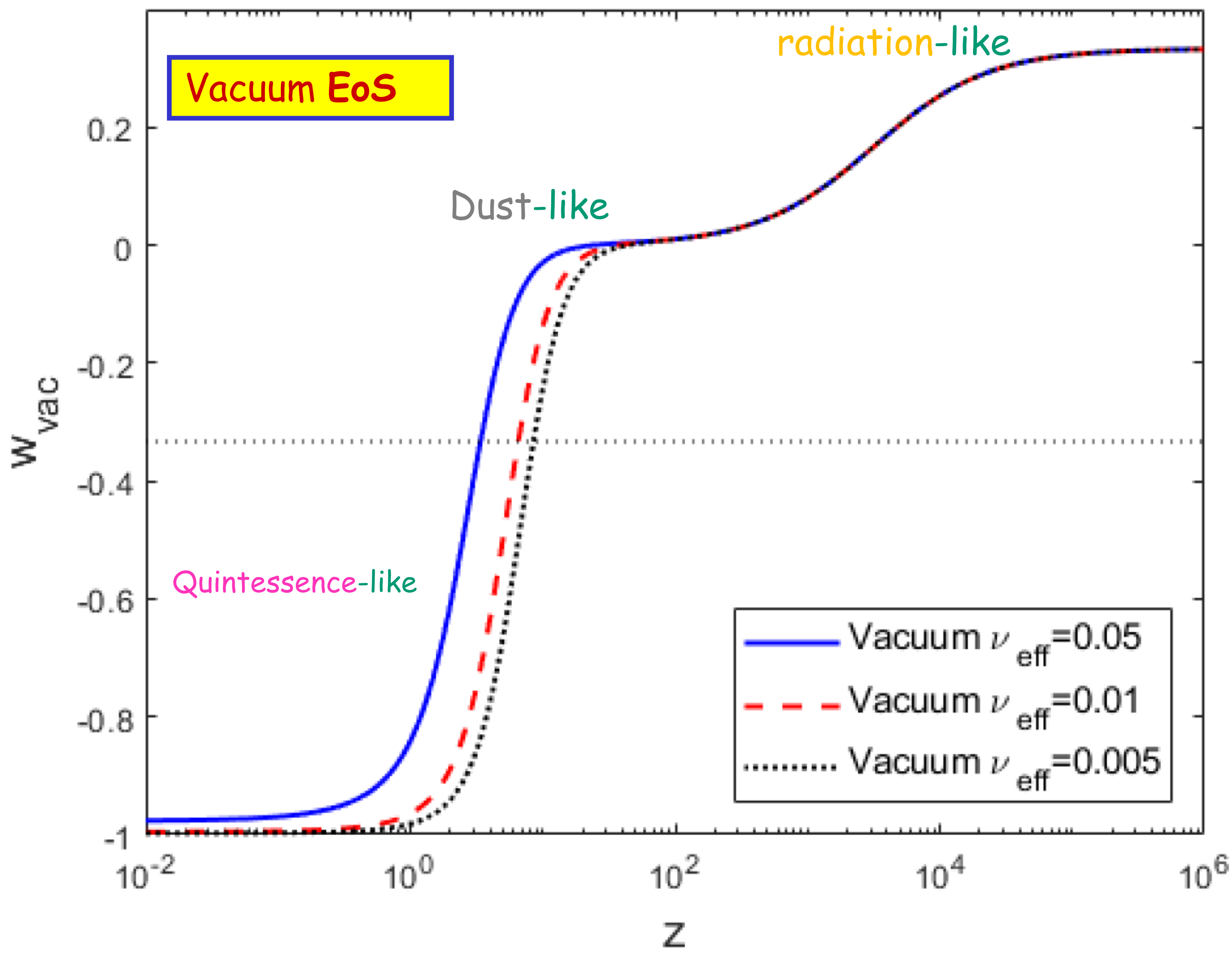
arXiv:2207.07111

$$w_{\rm vac}(H) \equiv \frac{P_{\rm vac}(H)}{\rho_{\rm vac}(H)} \simeq -1 + \frac{f_2(\dot{H})}{\rho_{\rm vac}(H)}$$

$$\simeq -1 + \left(\xi - \frac{1}{6}\right) \frac{\dot{H} m^2}{8\pi^2 \rho_{\rm vac}(H)} \left(1 - \ln \frac{m^2}{H^2}\right)$$

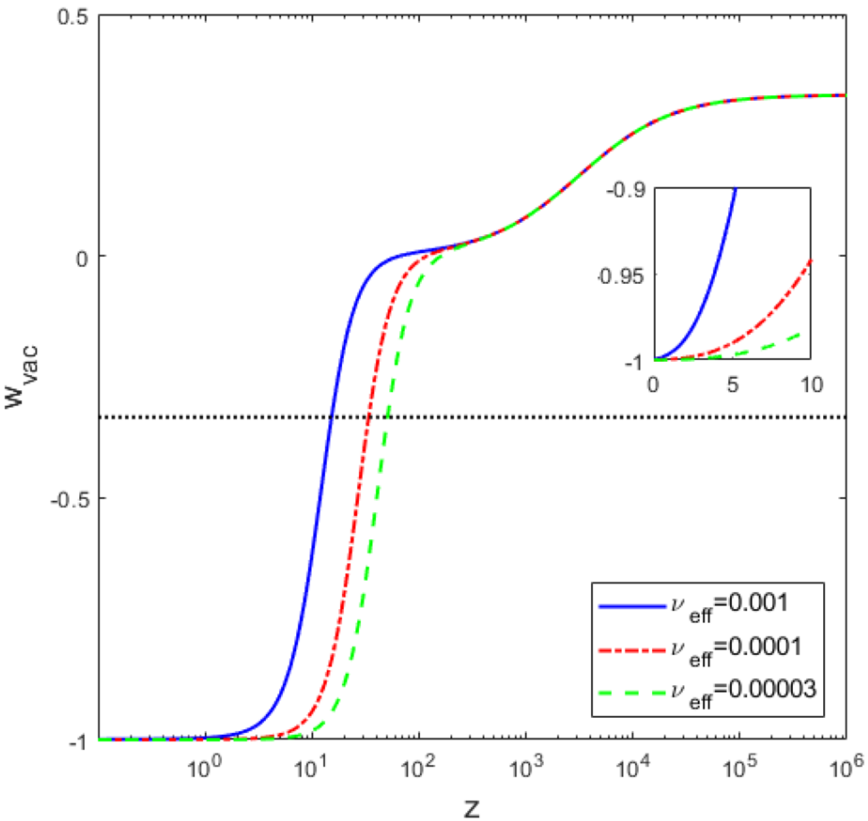
$$= -1 + \frac{\nu_{\rm eff}\left(\Omega_m^0 (1+z)^3 + \frac{4}{3}\Omega_r^0 (1+z)^4\right)}{\Omega_{\rm vac}^0 + \nu_{\rm eff}\left[-1 + \Omega_m^0 (1+z)^3 + \Omega_r^0 (1+z)^4 + \Omega_v^0\right]}$$

$$= \begin{cases} \frac{1}{3} & \text{for } \; z \gg z_{\rm eq} \; \text{ with } \; \Omega_r^0 (1+z) \gg \Omega_m^0, & \text{radiation behavior } (\nu_{\rm eff} \neq 0), \\ 0 & \text{for } \; \mathcal{O}(1) < z \ll z_{\rm eq} \; \text{ with } \; \Omega_m^0 \gg \Omega_r^0 (1+z), & \text{dust behavior } (\nu_{\rm eff} \neq 0), \\ \boxed{-1 + \nu_{\rm eff} \frac{\Omega_m^0}{\Omega_{\rm vac}^0} (1+z)^3} & \text{for } \; -1 < z < \mathcal{O}(1)\,, & \text{quintessence behavior } (\nu_{\rm eff} > 0) \end{cases}$$

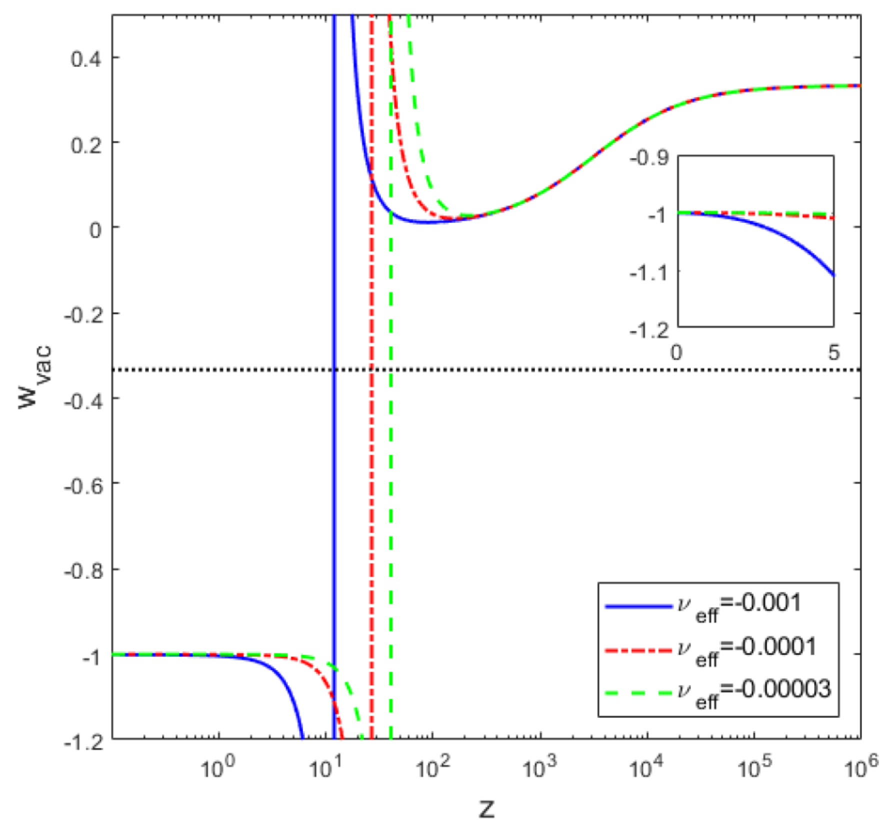


C. Moreno-Pulido and JSP, arXiv:2207.07111

quintessence-like $\nu_{\rm eff} > 0$



phantom-like $\nu_{\rm eff} < 0$



arXiv:2207.07111 [gr-qc] and 2301.05205 [gr-qc]

➢ Combined contribution Bosons+Fermions

$$S_\phi = -\int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}\left(m_\phi^2 + \xi R\right)\phi^2\right)$$

$$S_\psi(x) = -\int d^4x\sqrt{-g}\left[\frac{1}{2}i\left(\bar{\psi}\underline{\gamma}^\mu\nabla_\mu\psi - \left(\nabla_\mu\bar{\psi}\right)\underline{\gamma}^\mu\psi\right) + m_\psi\bar{\psi}\psi\right]$$

C. Moreno-Pulido, JSP & S.Cheraghchi
arXiv:2301.05205[gr-qc] (EPJC)

$$\boxed{\rho_{\rm vac}(H) = \rho_{\rm vac}(H_0) + \frac{3\nu_{\rm eff}}{8\pi}m_{\rm Pl}^2(H^2 - H_0^2)}$$

$$\boxed{\nu_{\rm eff} = \frac{1}{2\pi}\left[\sum_{j=1}^{N_{\rm s}}\left(\xi_j - \frac{1}{6}\right)\frac{m_{\phi_j}^2}{m_{\rm Pl}^2}\ln\frac{m_{\phi_j}^2}{H_0^2} - \frac{1}{3}\sum_{\ell=1}^{N_{\rm f}}\frac{m_{\psi_\ell}^2}{m_{\rm Pl}^2}\ln\frac{m_{\psi_\ell}^2}{H_0^2}\right]}$$

Running vacuum energy at the expense of matter non-conservation

$$\boxed{\rho_\Lambda = C_1 + C_2\, H^2\,.}$$

$$\rho_\Lambda(H) = \frac{3}{8\pi G}\left(c_0 + \nu H^2\right)$$

Bianchi identity

$$\dot\rho_\Lambda + \dot\rho_m + 3\,H(\rho_m + p_m) = 0$$

(matter non-conservation!!)

$$C_2 \propto \nu = \frac{M^2}{12\pi\, M_P^2}$$

$$\rho_m(z) = \rho_m^0\,(1+z)^{3(1-\nu)}$$

and "running" vacuum energy: (**RVM**)

$$\rho_\Lambda(z) = \rho_\Lambda^0 + \frac{\nu\,\rho_m^0}{1-\nu}\left[(1+z)^{3(1-\nu)} - 1\right]$$

Generalized Running vacuum energy emerging from QFT

$$\rho_{\rm vac}(H) = \frac{3}{8\pi G_N}\left(c_0 + \nu_{\rm eff} H^2 + \tilde{\nu}_{\rm eff}\dot{H}\right) + \mathcal{O}(H^4)$$

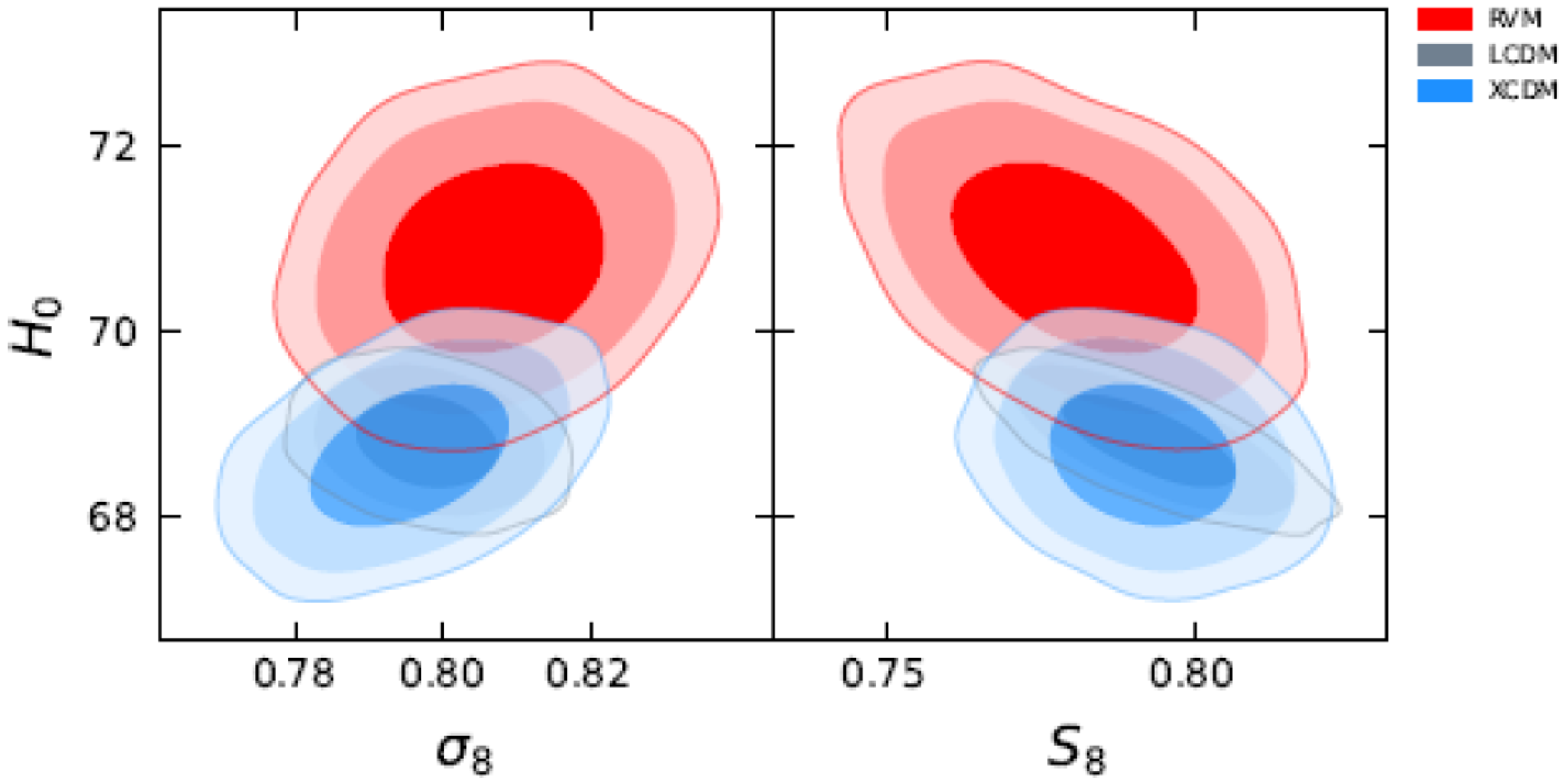
$$\tilde{\nu}_{\rm eff} = \nu_{\rm eff}/2$$

$$R = 12H^2 + 6\dot{H}$$

$$\rho_{\rm vac}(H) = \frac{3}{8\pi G_N}\left(c_0 + \frac{\nu}{12}R\right) \equiv \rho_{\rm vac}(R)$$

JSP, A.Gómez-Valent, J. de Cruz Pérez and C. Moreno-Pulido

arXiv:2304.11157



JSP, A. Gómez-Valent, J. de Cruz Pérez and C. Moreno-Pulido

arXiv:2304.11157

Baseline (No pol.) +SH0ES

Parameter	ΛCDM	type-I RRVM	type-I RRVM$_{\rm thr.}$	type-II RRVM	XCDM
H_0(km/s/Mpc)	68.94 ± 0.37	69.10 ± 0.44	68.48 ± 0.39	71.69 ± 0.80	68.61 ± 0.51
$\omega_{\rm b}$	0.02247 ± 0.00018	0.02240 ± 0.00022	0.02251 ± 0.00018	0.02280 ± 0.00024	0.02252 ± 0.00019
$\omega_{\rm dm}$	0.11630 ± 0.00083	0.11632 ± 0.00083	0.1220 ± 0.0019	0.1160 ± 0.0015	0.1157 ± 0.0010
$\Omega^0_{\rm m}$	0.2933 ± 0.0045	0.2919 ± 0.0062	0.3095 ± 0.0067	0.2702 ± 0.0068	0.2950 ± 0.0048
w_0	-1	-1	-1	-1	-0.981 ± 0.021
$\nu_{\rm eff}$	-	-0.00022 ± 0.00036	0.0193 ± 0.0055	0.00048 ± 0.00040	-
$\varphi_{\rm ini}$	-	-	-	$0.919^{+0.019}_{-0.022}$	-
$\varphi(0)$	-	-	-	$0.908^{+0.025}_{-0.028}$	-
$\tau_{\rm reio}$	0.0512 ± 0.0074	0.0494 ± 0.0084	$0.0595^{+0.0082}_{-0.0092}$	0.0528 ± 0.0085	0.0533 ± 0.0079
$\ln(10^{10}{\rm A_s})$	3.029 ± 0.016	3.027 ± 0.017	$3.047^{+0.017}_{-0.019}$	3.041 ± 0.017	3.032 ± 0.016
$n_{\rm s}$	0.9728 ± 0.0036	0.9715 ± 0.0044	0.9739 ± 0.0037	0.9915 ± 0.0070	0.9744 ± 0.0041
M	-19.396 ± 0.011	-19.392 ± 0013	-19.406 ± 0.011	-19.311 ± 0.024	-19.403 ± 0.013
σ_8	0.7939 ± 0.0068	0.801 ± 0.014	0.7719 ± 0.0094	0.794 ± 0.012	0.7876 ± 0.0096
S_8	0.785 ± 0.010	0.790 ± 0.014	0.784 ± 0.010	0.754 ± 0.017	0.781 ± 0.011
$r_{\rm d}$ (Mpc)	147.97 ± 0.30	147.85 ± 0.85	147.92 ± 0.30	141.3 ± 1.6	148.08 ± 0.32
ΔDIC	-	-0.10	$+10.06$	$+13.78$	-0.96

Summarized conclusions

- **Dynamical DE**: natural proposal for an **expanding Universe**
- The **RVM** based on a **running Λ** term in interaction with matter or **G** is theoretically **well motivated**
- **Running vacuum models** seem to describe **better** the observations SNIa+BAO+$H(z)$+LSS+CMB **than the ΛCDM**
- Provide a **consistent solution** to the main **tensions**
- These ideas may signal a **connection** between the the **LSS** of the Universe and the **quantum phenomena** in the **microcosmos**