2023 "Workshop on the Standard Model and Beyond
02/09/2023, Corfù

Anomalies and Dynamics in Strongly-coupled Gauge Theories, New Criteria for Different Phases. Lessons from susy gauge theories

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Plan

Part 1: Intro: Strongly-coupled (chiral) gauge theories

Part 2: Generalized symmetries, Anomalies and Dynamics

Part 3: New criteria for color confinement and other phases

Part 4: Lessons from supersymmetry

Bolognesi, KK, Shifman, PRD '18
Bolognesi, KK., PRD '19
Bolognesi, KK, Luzio
JHEP '20, JHEP '20, PRD '21
JHEP '21, JHEP '22, IJMPA '23
JHEP '23

Giacomelli, KK JHEP '12,'13,'16

Part 1: Introduction

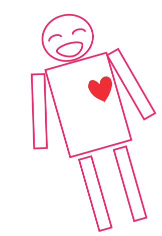
A challenge for theorists

Understand better the dynamics of
strongly-coupled chiral gauge theories

WHY

(i) We live in a chiral world $O(10^{-6}) \sim O(10^{0})\ cm$

e.g., DNA spirals



Left-right symmetry (spontaneously) broken

(ii) Standard models of the fundamental interactions

$$SU(3)_{QCD} \times (SU(2)_L \times U_Y(1))_{GWS} \qquad (*) \qquad O(10^{-16})\ cm$$

(iii) GUTs SU(5), SO(10) … ? (**) $O(10^{-29}\ cm)$

◆ (*) (**) are chiral gauge theories, but weakly coupled: well-understood in perturbation theory

Gauge-anomaly cancellation

◆ (*) to be regarded as a (very good) low-energy effective action

masses, neutrinos, families, … , Higgs, … ?????

◆ Surprisingly little is known today about strongly-coupled (asymptotically-free) chiral gauge theories

◆ Cfr. vectorlike theories, e.g.,

QCD (50 years of successful studies);
$\mathcal{N}=2$ susy gauge theories (Seiberg-Witten solution ~ 30 yrs)

Some examples: SU(N) gauge theories w fermions

("theoretical laboratories")

Bolognesi, KK, Shifman '18
Bolognesi, KK '19
Bolognesi, KK, Luzio '20-'23

(i) $\psi^{\{ij\}} , \quad \eta_i^B , \qquad (i,j = 1,2,\ldots,N \ , \quad B = 1,2,\ldots,N+4) \ ,$

$\square\!\square \oplus (N+4)\,\bar{\square}$ (+ p pairs $\square \oplus \bar{\square}$)

Bars, Yankielowicz (BY)
$\psi\eta$ model
(1,0) model

(ii) $\chi_{[ij]} \ , \qquad \tilde{\eta}^{B\,j} \ , \qquad B = 1,2,\ldots,(N-4) \ ,$ $\qquad \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + (N-4)\,\square$

(0,1) model
$\chi\eta$ model
Georgi-Glashow (GG)

(iii) $\psi^{\{ij\}} \ , \qquad \chi_{[ij]} \ , \qquad \eta_i^A \ , \qquad A = 1,2,\ldots 8 \ ,$ $\qquad \square\!\square \oplus \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \oplus 8 \times \bar{\square}$

$\psi\chi\eta$ model (1,1) model

(iv) $\frac{N-4}{k}\,\psi^{\{ij\}} \oplus \frac{N+4}{k}\,\chi_{[ij]} \qquad \frac{N-4}{k}\,\square\!\square \oplus \frac{N+4}{k}\,\bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

(v) $\psi' s \sim \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} \Big\} \frac{N}{2}$ (N= even)

(vi) $N_f \ (\eta \oplus \tilde{\eta})$ (QCD)

(vii) $N_f \ \lambda$ (adjoint QCD)

A well-known tool - ’t Hooft anomaly matching conditions - unfortunately, is not sufficiently stringent

e.g.,

- $\psi\eta$ model
- $\chi\eta$ model

Bars, Yankielovicz '81
Appelquist, Cohen, Schmaltz, Shrock '99
Appelquist, Duan, Sannino, '00

$\psi\eta$ model

$$\psi^{\{ij\}}\,, \quad \eta_i^B\,, \qquad B = 1,2,\ldots,N+4 \qquad\qquad G = \;SU(N)_{\rm c} \times SU(N+4)_{\rm f} \times U(1)\,,$$

$$\Box\!\Box \oplus (N+4)\,\bar{\Box}$$

"$\psi\eta$"

(A) Confining, SU(N+4)x U(1) symmetric phase (no condensates)

massless baryons ~ $B^{[AB]} = \psi^{ij}\eta_i^A\eta_j^B\,, \qquad A,B = 1,2,\ldots,N+4$

(&)

	fields	$SU(N)_{\rm c}$	$SU(N+4)$	$U(1)$
UV	ψ	$\Box\!\Box$	$\frac{N(N+1)}{2}\cdot(\cdot)$	$N+4$
	η^A	$(N+4)\cdot\bar{\Box}$	$N\;\cdot\;\Box$	$-(N+2)$
IR	$B^{[AB]}$	$\frac{(N+4)(N+3)}{2}\cdot(\cdot)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$-N$

(B) Color-flavor locked (Higgs) phase

$$\langle\psi^{\{ij}\eta_i^{B\}}\rangle = C\,\delta^{jB}\,, \qquad j,B = 1,2,\ldots N$$

$$G \to G' = SU(N)_{\rm cf} \times SU(4)_{\rm f} \times U'(1)$$

The anomaly matching **OK,** $\frac{N^2+7N}{2}$ massless baryons
$8N+1$ Nambu-Goldstone

♦ Massless baryons and (NG) bosons in L.E.

	fields	$SU(N)_{\rm cf}$	$SU(4)_{\rm f}$	$U'(1)$
UV	ψ	$\Box\!\Box$	$\frac{N(N+1)}{2}\cdot(\cdot)$	1
	η^{A_1}	$\overline{\Box\!\Box} \oplus \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$N^2\;\cdot\;(\cdot)$	-1
	η^{A_2}	$4\cdot\bar{\Box}$	$N\;\cdot\;\Box$	$-\frac{1}{2}$
IR	$B^{[A_1B_1]}$	$\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$\frac{N(N-1)}{2}\cdot(\cdot)$	-1
	$B^{[A_1B_2]}$	$4\cdot\bar{\Box}$	$N\;\cdot\;\Box$	$-\frac{1}{2}$

Back to p.7

Standard 't Hooft anomaly matching in the case (A)

fields	$SU(N)_c$	$SU(N+4)$	$U(1)$
ψ	$\square\!\square$	$\frac{N(N+1)}{2}\cdot(\cdot)$	$N+4$
η^A	$(N+4)\cdot\bar{\square}$	$N\cdot\square$	$-(N+2)$
$B^{[AB]}$	$\frac{(N+4)(N+3)}{2}\cdot(\cdot)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$-N$

Table 6: Chirally symmetric phase of the $(1,0)$ model

Anomaly	$A_{UV}(\psi,\eta)$	$A_{IR}(B)$
$SU(N+4)^3$	N	$N+4-4$
$U(1)SU(N+4)^2$	$-(N+2)\cdot N$	$-N\cdot(N+4-2)$
$U(1)^3$	$(N+4)^3\frac{N(N+1)}{2}-(N+2)^3N(N+4)$	$-N^3\frac{(N+4)(N+3)}{2}$
$U(1)$	$(N+4)\frac{N(N+1)}{2}-(N+2)N(N+4)$	$-N\frac{(N+4)(N+3)}{2}$
$\mathbb{Z}_{N+2}\,SU(N+4)^2$	0	$N+2$
$\mathbb{Z}_{N+4}\,SU(N+4)^2$	N	$2\cdot(N+4-2)$

Table 7: UV-IR Anomaly matching in Chirally symmetric phase

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Part 2: Anomalies and Dynamics: new, more powerful constraints ('18-'23)

0. Tools: generalized symmetries and anomalies
1. $(\mathbb{Z}_2)_F$ anomaly
2. (Dynamical) Higgs phase
3. Strong anomaly and phases
4. Dynamical Abelianization
5. More general DSB

Tool: Generalized symmetries

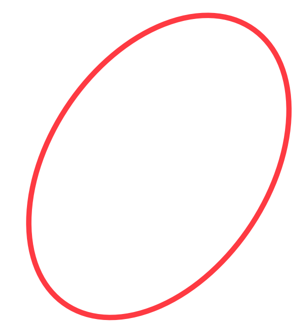
Gaiotto, Kapustin, Seiberg, Willet,
Gaiotto, Kapustin, Komargodski, Seiberg,
Aharony, Seiberg, Tachikawa,
Poppitz, Kikuchi, Tanizaki, Sakai, Shimizu, Yonekura,
'05 -

♦ From 0-form symm. (acting on local operators)
to k-form symmetries (acting on line, surface, etc operators)

- e.g. the center Z_N symmetry in SU(N) YM

Wilson loop
Polyakov loop

$$e^{i\oint_\gamma A} \to \Omega_N \, e^{i\oint_\gamma A}\,, \qquad \Omega_N = e^{2\pi i/N}\mathbb{1} \in \mathbb{Z}_N \qquad (*)$$



♦♦ "Gauging" the 1-form discrete Z_N symmetry $\qquad \widetilde{a} = a + \frac{1}{N}B_c^{(1)}$

$$N B_c^{(2)} = dB_c^{(1)}\,, \quad B_c^{(2)} \to B_c^{(2)} + \mathrm{d}\lambda_c\,, \qquad B_c^{(1)} \to B_c^{(1)} + N\lambda_c\,,$$

$$\widetilde{a} \to \widetilde{a} + \lambda_c\,. \quad \longrightarrow$$

$$\frac{1}{8\pi^2}\int_{\Sigma_4} \mathrm{tr}\, F^2 \longrightarrow \frac{1}{8\pi^2}\int_{\Sigma_4} \mathrm{tr}\,(\tilde{F}(\tilde{a}) - B_c^{(2)})^2 \quad \longrightarrow$$

fractional 1/N 't Hooft Flux

$\longrightarrow$ CP/T broken at $\theta = \pi$ (SU(N) YM) (even N)

Gaiotto, Kapustin,
Komargodski, Seiberg, '17

$$a \equiv t_R^b A_\mu^b dx^\mu$$

$$F^2 \equiv F \wedge F = \frac{1}{2} F^{\mu\nu} F^{\rho\sigma} dx_\mu dx_\nu dx_\rho dx_\sigma = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} d^4x$$

Differential form notation

♦♦♦ Gauging 1-form (color-flavor locked) Z_N center symmetry

- 1-form (color-flavor locked) Z_N

$$\mathcal{P}e^{i\oint_L a} \to e^{\frac{2\pi i}{N}}\mathcal{P}e^{i\oint_L a}\,; \quad \psi^k \to e^{\frac{2\pi i \mathcal{N}_k}{N}}\psi^k\,, \qquad \mathbb{Z}_N \subset SU(N)\,;$$

$$\Pi_i\, e^{i\oint_L A_i} \to \left(e^{2\pi i \sum_{i,k} q_k^{(i)}}\right)\Pi_i\, e^{i\oint_L A_i}\,; \qquad \psi^k \to e^{2\pi i \sum_i q_k^{(i)}}\psi^k\,, \qquad U_i(1)\,;$$

$$\sum_i q_k^{(i)} = -\frac{\mathcal{N}_k}{N}\,, \qquad \forall k$$

- Gauging it $\quad N B_c^{(2)} = dB_c^{(1)}\,, \qquad \widetilde{a} = a + \frac{1}{N}B_c^{(1)}$ $\qquad$ SU(N) -> U(N)

$$B_c^{(2)} \to B_c^{(2)} + \mathrm{d}\lambda_c\,, \qquad B_c^{(1)} \to B_c^{(1)} + N\lambda_c\,,$$

$$\widetilde{a} \to \widetilde{a} + \lambda_c\,.$$

$\psi\eta$ model

$$\tilde{A} \to \tilde{A} - \lambda_c\,, \qquad A_0 \to A_0 + \frac{N}{2}\lambda_c$$

$$U(1)_{\psi\eta} \qquad (\mathbb{Z}_2)_F$$

$$\oint \lambda_c = \frac{2\pi\ell}{N}$$

1-form gauge fn

Gauge inv. kin. terms

$$d + \mathcal{R}_S(\tilde{a}_c - \frac{1}{N}B_c^{(1)}) + \frac{N+4}{2}(\tilde{A} + \frac{1}{N}B_c^{(1)}) + A_0 - \frac{1}{2}B_c^{(1)}$$

$$d - (\tilde{a}_c - \frac{1}{N}B_c^{(1)}) - \frac{N+2}{2}(\tilde{A} + \frac{1}{N}B_c^{(1)}) - (A_0 - \frac{1}{2}B_c^{(1)})$$

→ (%)

1. $(\mathbb{Z}_2)_F$ anomaly

Bolognesi, KK, Luzio '19

- All BY and GG models have a non anomalous $(\mathbb{Z}_2)_F$ symmetry (fermion parity) $\subset L_+^\uparrow$

$\psi_i \to -\psi_i$: In type I models (*) (e.g., even N "$\psi\eta$ model"),

because $\Delta S = \sum_i c_i \times \frac{1}{8\pi^2}\int_{\Sigma_4} \mathrm{tr}_i\, F_{\mu\nu}\tilde{F}^{\mu\nu} \times (\pm\pi) = 2\pi\mathbf{Z}$

with $\boxed{\sum_i c_i = 2\mathbf{Z} \neq 0}$!   instanton #

Type I models (*) : all BY and GG models with N, p even
Type II: others

- In type I models, the symmetry group space is disconnected:

$$G = \frac{SU(N) \times SU(N+4) \times U_{\psi\eta}(1) \times Z_2}{Z_N}$$

(*) $\psi\eta$ model Even N

- In type II BY and GG models, no Z_2 and $\sum_i c_i = 0$

$\psi\eta$ model Odd N

No new results w.r.t. the conventional 't Hooft anomaly algorithm

- Type I models: gauging of the 1-form color-flavor locked Z_N symmetry

$$\Delta S^{(\text{Mixed anomaly})} = (\pm\pi)\cdot \sum_{\text{fermions}} \Big(\underbrace{d(R)\mathcal{N}(R)^2 - N\cdot D(R)}_{N^2}\Big)\, \underbrace{\frac{1}{8\pi^2}\int_{\Sigma_4} (B_c^{(2)})^2}_{1/N^2} = \pm\pi$$

Master formula for BY, GG
Bolognesi, KK, Luzio, '21, '22
(%)

In IR, the massless baryons do not support the Z_2 anomaly

The confining, symmetric vacuum (&) is inconsistent ($)

No problem in Higgs phase

Objection to (%), ($)

Smith, Karasik, Lohitsiri, Tong, '20

$$\oint B_c^{(1)} = 2\pi \; ; \quad \oint A_0 = \frac{2\pi}{2}$$ A_0 singular (Z_2 vortex) !!

BKL '19

Propose: use $\oint A_0 = 2\pi \longrightarrow \oint B_c^{(1)} = 4\pi$: No Z_2 anomaly !!!?

(*)

…. But (*) is a trivial Z_2 holonomy $\psi \to \psi \, , \quad \eta \to \eta \, ,$!!

- Cure: consider $\psi\eta$ model with a regulator Dirac pair $q, \tilde{q}$ and a singlet scalar ϕ w/ Yukawa ("X - ray model")

Bolognesi, KK, Luzio JHEP '23 in press

$$\Delta L = g \, \phi q \tilde{q} + h.c. \qquad \langle \phi \rangle = v \gg \Lambda_{\psi\eta}$$

$$\frac{SU(N)_c \times \tilde{U}(1) \times U_0(1)}{\mathbb{Z}_N}$$

	$SU(N)_c$	$SU(N+4)$	$U(1)_{\psi\eta}$	$U(1)_V$	$U_0(1)$	$\tilde{U}(1)$
ψ	$\square\square$	$(\cdot)$	$\frac{N+4}{2}$	0	1	$\frac{N+4}{2}$
η	$\bar{\square}$	$\square$	$-\frac{N+2}{2}$	0	-1	$-\frac{N+2}{2}$
q	$\square$	$(\cdot)$	0	1	1	$\frac{N+2}{2}$
$\tilde{q}$	$\bar{\square}$	$(\cdot)$	0	-1	1	$-\frac{N+2}{2}$
ϕ	$(\cdot)$	$(\cdot)$	0	0	-2	0

→ Mixed anomalies

- $\tilde{A} - (B_c^{(2)})^2$

$$\delta S_{\delta\alpha} = \frac{\tilde{C}}{8\pi^2} \int_{\Sigma_4} (B_c^{(2)})^2 \, \delta\alpha$$

- $A_0 - (B_c^{(2)})^2$

$$\delta S_{\delta\alpha_0} = \frac{C_0}{8\pi^2} \int_{\Sigma_4} (B_c^{(2)})^2 \, \delta\alpha_0 \, , \qquad C_0 = N^2(N+3)$$

$\xrightarrow{v}$ $(\mathbb{Z}_2)_F$ anomaly (%) → ($)

2. (Dynamical) Higgs phase

Appelquist, Duan, Sannino, '00
Bolognesi, KK, Luzio, JHEP '20

E.g., "$\psi\eta$" model

$$\square\!\square \oplus (N+4)\,\bar{\square}$$

<u>Color-flavor locked VEV</u>

$$\langle \psi^{\{ij}\eta_i^{B\}} \rangle = C\,\delta^{jB}\,, \qquad j, B = 1, 2, \ldots N$$

$$G \to G' = SU(N)_{\rm cf} \times SU(4)_{\rm f} \times U'(1)$$

Massless baryons and (NG) bosons in L.E.

	fields	$SU(N)_{\rm cf}$	$SU(4)_{\rm f}$	$U'(1)$
UV	ψ	$\square\!\square$	$\frac{N(N+1)}{2}\cdot(\cdot)$	1
	η^{A_1}	$\overline{\square\!\square} \oplus \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$N^2\cdot(\cdot)$	-1
	η^{A_2}	$4\cdot\bar{\square}$	$N\cdot\square$	$-\frac{1}{2}$
IR	$B^{[A_1B_1]}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{N(N-1)}{2}\cdot(\cdot)$	-1
	$B^{[A_1B_2]}$	$4\cdot\bar{\square}$	$N\cdot\square$	$-\frac{1}{2}$

- The conventional anomaly matching manifest
- OK with the Z_2 anomaly (%) = impossibility of gauging 1-form Z_N

$$\frac{SU(N)\times U_{\psi\eta}\times \mathbf{Z}_2}{\mathbf{Z}_N}$$

$\frac{N^2+7N}{2}$ massless baryons

$8N+1$ Nambu-Goldstone

It is "matched" in the IR: $U_{\psi\eta}(1)$ hence the 1-form Z_N symmetry itself is spontaneously broken.

- N.B. For confining, symmetric vacuum (&), (%) means a matching failure (inconsistency)

3. Strong anomaly and phases

♦ QCD (N_f =2) and the $U_A(1)$ problem

$$\langle U \rangle = \langle \bar\psi_R \psi_L \rangle \neq 0 \quad \xrightarrow{?} \quad m_\eta \gg m_\pi$$

$U(2)_L \times U(2)_R \to SU(2)_V \times U(1)_V$
($U_A(1)$ vs $SU_A(2)$ NG bosons)

't Hooft '74
Witten '79,'80
Veneziano '79
Rosenzweig, Schechter, Trahern '80
Di Vecchia, Veneziano '80
Kawarabayashi, Ohta '80
Nath, Arnowitt '81

- Ans. $\partial_\mu J^\mu_A = N_f \dfrac{g^2}{32\pi^2} F_{\mu\nu}\tilde F^{\mu\nu}$; $\displaystyle\int d^4x \frac{g^2}{32\pi^2} F_{\mu\nu}\tilde F^{\mu\nu} = \mathbf{Z}$
- $L_{\rm eff}$: $L = L_0 + \hat L$,

$$\hat L = \frac{i}{2} q(x) \log\det U/U^\dagger + \frac{N}{a_0 F_\pi^2} q^2(x) - \theta\, q(x)$$

$$q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde F^{a,\mu\nu}$$

Anomalous $U_A(1)$ variation $\quad \Delta S = 2N_f\alpha$

reproduced by the log det term $\quad \rightarrow \quad m_\eta \quad$!

Q: Does (a multi-valued) $\log\det U/U^\dagger$ make sense?

Ans: Yes, as $\quad U = \langle U \rangle\, e^{i\frac{t^a\pi^a + t^0\eta}{F_\pi}} = \text{const.}\,[\mathbf{1} + \frac{i}{F_\pi}(t^a\pi^a + t^0\eta) + \ldots]$

♦ Invert the logic: $L_{\rm eff}$ with the strong-anomaly log term **implies**

$$\langle U \rangle = \langle \bar\psi_R \psi_L \rangle \neq 0$$ **i.e., XSB** with massless pions

- Apply the same logic in chiral gauge theories

Bolognesi, KK, Luzio, JHEP '20

  - Demand that the low-energy effective degrees of freedom (i.e. the phase) be such that $L_{\rm eff}$ with the strong-anomaly term can be written in terms of them

- e.g. the "$\chi\eta$" model : an SU(N) theory with fermions

$$\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array} \oplus (N-4)\,\bar{\square}$$

Georgi-Glashow

Strong-anomaly effective action

$$\hat{L} = \frac{i}{2} q(x) \log (\chi\eta)^{N-4} \chi\chi + \text{h.c.} \qquad q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a,\mu\nu}$$

$$(\chi\eta)^{N-4}\chi\chi \equiv \epsilon_{i_1 i_2 \ldots i_N} \epsilon_{m_1 m_2 \ldots m_{N-4}} (\chi\eta)^{i_1 m_1} (\chi\eta)^{i_2 m_2} \ldots (\chi\eta)^{i_{N-4} m_{N-4}} \chi^{i_{N-3} i_{N-2}} \chi^{i_{N-1} i_N}$$

$\langle \chi\eta \rangle \neq 0$, $\langle \chi\chi \rangle \neq 0$: dynamical Higgs phase!

N=5 Veneziano '81

Cfr. confining chirally symmetric vacuum with massless baryons ($B \sim \chi\eta\eta$) only, fails ($)

Fermion zero-mode counting

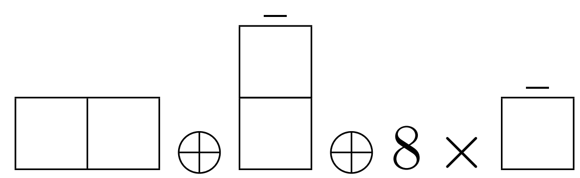
- Dynamical Higgs phase favored also in the "$\psi\eta$" model as well as in **all** generalized BY and GG models

N even or odd

4. Dynamical Abelianization

♦ "$\psi\chi\eta$" <u>model:</u> SU(N) theory with Weyl fermions

Goity, Peccei, Zeppenfeld, '85
Eichten, Peccei, Preskill, Zeppenfeld, '86



$$G = SU(N) \times \frac{U(1)_{\psi\chi} \times \tilde{U}(1) \times SU(8)}{\mathbb{Z}_N \times \mathbb{Z}_{8/N^*}}$$

with nonanomalous U(1) symmetries

$$\tilde{U}(1): \qquad \psi \to e^{2i\alpha}\psi\,, \quad \chi \to e^{-2i\alpha}\chi\,, \quad \eta \to e^{-i\alpha}\eta\,,$$

$$U(1)_{\psi\chi}: \qquad \psi \to e^{i\frac{N-2}{N^*}\beta}\psi\,, \qquad \chi \to e^{-i\frac{N+2}{N^*}\beta}\chi\,, \qquad \eta \to \eta$$

• **Assume :**

$$\langle \psi^{ik}\chi_{kj}\rangle = \Lambda^3 \begin{pmatrix} c_1 & & \\ & \ddots & \\ & & c_N \end{pmatrix}^{i}_{j} \qquad c_n \in \mathbb{C}\,, \qquad \sum_n c_n = 0$$

Bolognesi, KK, Shifman '18

→ Fate of the symmetries and the structure of L_{eff}

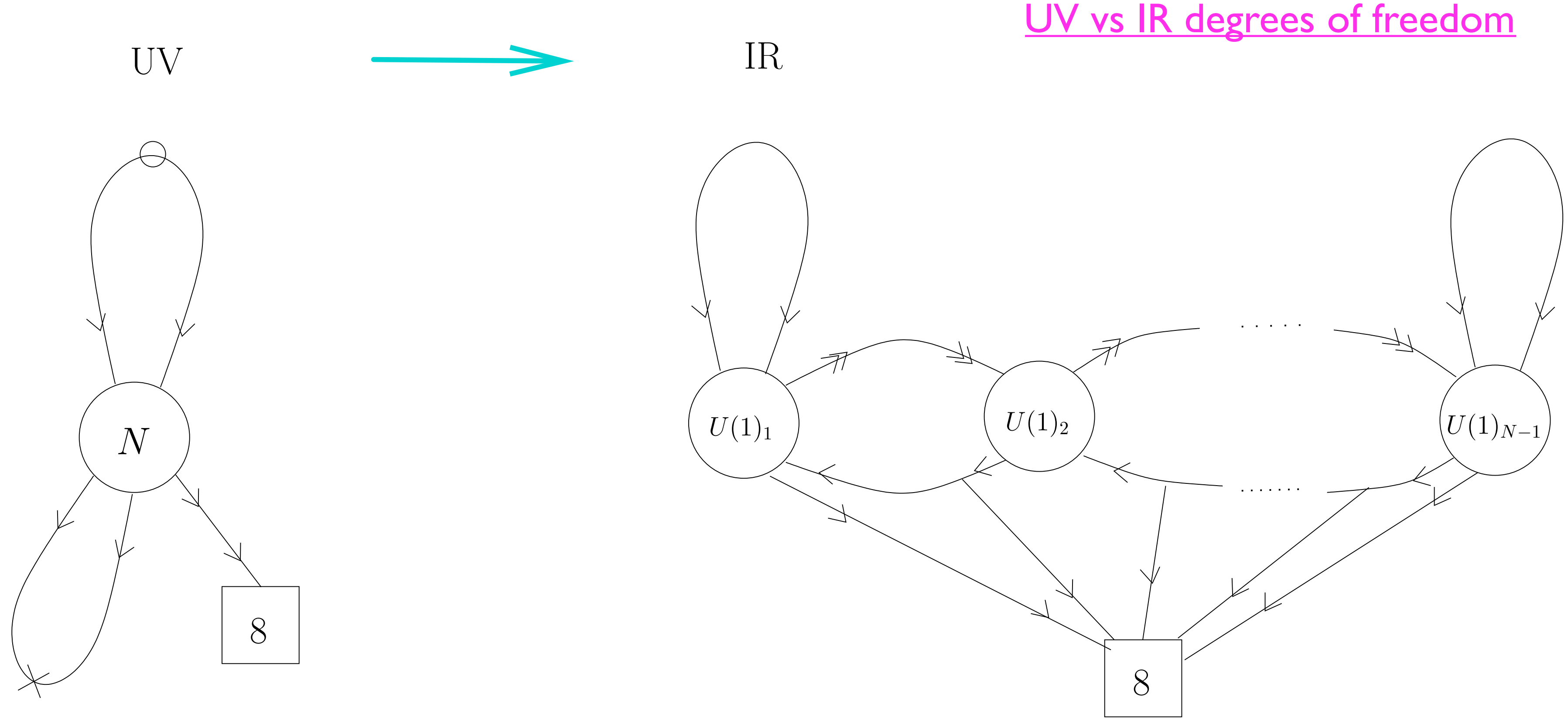
Cfr. <u>Seiberg-Witten solutions</u> in $\mathcal{N}=2$
susy models (elementary adj scalar)

$\langle \psi\chi \rangle \neq 0$ →

$$SU(N) \times \frac{SU(8)_{\rm f} \times \tilde{U}(1) \times U(1)_{\psi\chi}}{\mathbb{Z}_N \times \mathbb{Z}_{8/N^*}} \longrightarrow \frac{\prod_{\ell=1}^{N-1} U(1)_\ell \times SU(8)_{\rm f} \times \tilde{U}(1)}{\prod_{\ell=1}^{N-1} \mathbb{Z}_\ell \times \mathbb{Z}_N \times \mathbb{Z}_2}$$

- $\tilde{U}(1)$: non anom.; unbroken → Manifest symmetry in IR
- $U(1)_{\psi\chi}$ non anom.; <u>broken</u> → 1 NG boson (π) ; massless fermions
- $U(1)_{an}$ anom.; unbroken → strong anomaly effective action
- unbroken <u>"discrete"</u> symmetries

<u>UV vs IR degrees of freedom</u>



Let us now check the D.A. against the mixed-anomalies

Bolognesi, KK, Luzio '22

$$G = SU(N) \times \frac{U(1)_{\psi\chi} \times \tilde{U}(1) \times SU(8)}{\mathbb{Z}_N \times \mathbb{Z}_{8/N^*}}$$

- Gauge the color-flavor locked 1-form $\mathbb{Z}_N = SU(N) \cap \tilde{U}(1)$ symmetry.

Various mixed anomalies: (p.63)

	$\tilde{U}(1)$	$U(1)_{\psi\chi}$	$(\mathbb{Z}_{N+2})_\psi$	$(\mathbb{Z}_{N-2})_\chi$	$SU(8)_\eta$	$\mathbb{Z}_{N^*}$	$\mathbb{Z}_{4/N^*}$
Mixed Anomalies	$\checkmark$	X	X	X	$\checkmark$	$\checkmark$	$\checkmark$
Dyn. Abel.	$\checkmark$	X	X	X	$\checkmark$	$\checkmark$	$\checkmark$

Assumption of the dynamical Abelianization is consistent

cfr. Sheu, Shifman '22

5. More general dynamical symmetry breaking DSB

Bolognesi, KK, Luzio '23

$\langle \psi \chi \rangle \neq 0$ (adj. repr) but

$$\langle \psi \chi \rangle = \mathrm{diag.}\,(c_1 \mathbf{1}_{n_1}, c_2 \mathbf{1}_{n_2}, \ldots)\,, \qquad \sum_i c_i n_i = 0$$

Non Abelian IR-free subgroup(s) surviving in the IR?

$SU(N) \to SU(n) \times \cdots$

- SU(N) models with $\frac{N-4}{k}\ \Box\!\Box \oplus \frac{N+4}{k}\ \bar{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$

  - $N = 5,\ k = 1$: $\Box\!\Box \oplus 9\ \bar{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$

    $G_C = SU(5) \to SU(3) \times SU(2) \times U(1)$

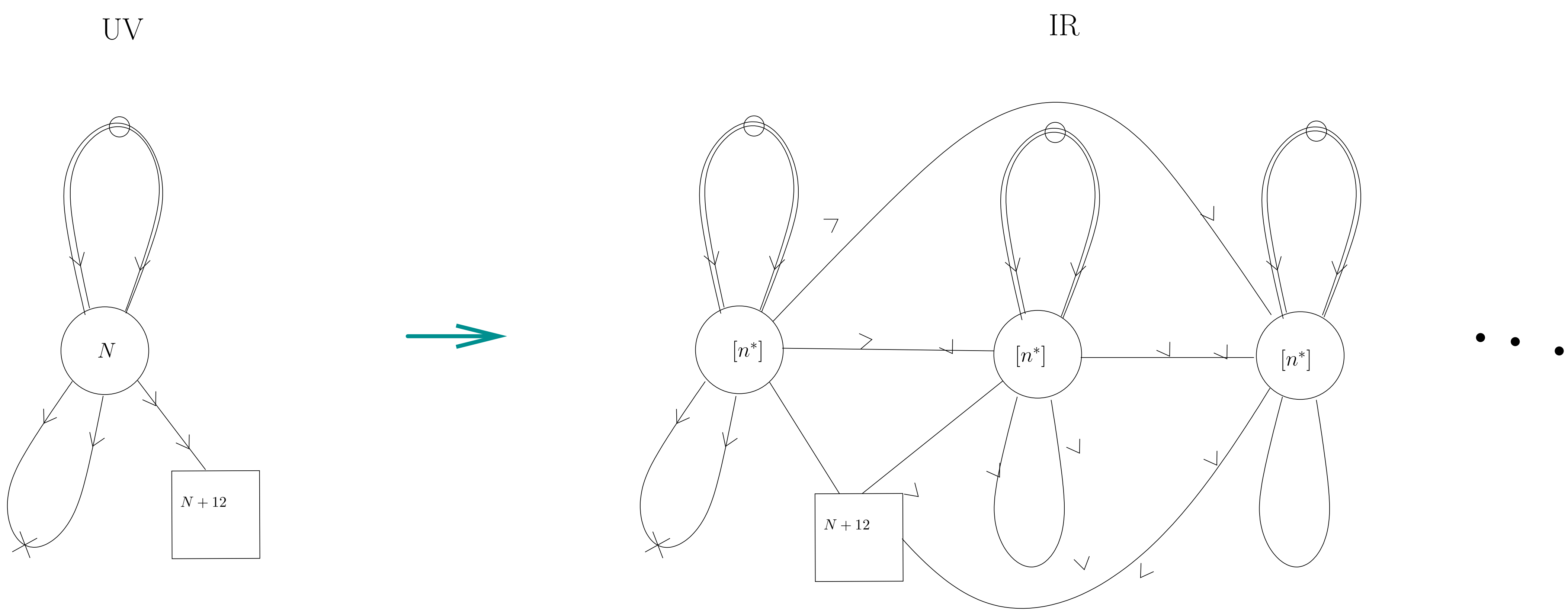
    $G_F = SU(9) \times U_0(1) \to SU(8) \times U_0(1)'$

    $\beta(SU(3)) > 0$
    $\beta(SU(2)) < 0$

  - $N = 6, k = 2$: No breaking w IR-free NonAbelian groups

- SU(N) theory with $2\ \Box\!\Box \oplus \bar{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} \oplus (N+12)\ \bar{\Box}$

N	$[n^*]$	$SU(N) \to \cdots$
3	2	$SU(3) \to SU(2) \times U(1)$
4	2	$SU(4) \to SU(2) \times SU(2) \times U(1)$
5	2	$SU(5) \to SU(2) \times SU(2) \times U(1)^2$
6	2	$SU(6) \to SU(2) \times SU(2) \times SU(2) \times U(1)^2$
$\vdots$	$\vdots$	$\vdots$
$N \to \infty$	$[n^*]$	$SU(N) \to \prod_1^7 SU([n^*]) \times SU(N - 7[n^*]) \times U(1)^7$



UV
IR
N
$N+12$
$[n^*]$
$[n^*]$
$[n^*]$
$N+12$

SU(5) model with

$$G_C = SU(5) \to SU(3) \times SU(2) \times U(1)$$

$$\square\!\square \oplus 9\, \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$$

	fields	$SU(3)$	$SU(2)$	$U(1)$	$SU(9)$	$U_0(1)$
UV	ψ^{ij}	$\square\!\square$	$(\cdot)$	4	$(\cdot)$	$\frac{9}{N+2}$
	ψ^{iJ}	$\square$	$\square$	-1	$(\cdot)$	$\frac{9}{N+2}$
	ψ^{JK}	$(\cdot)$	$\square\!\square$	-6	$(\cdot)$	$\frac{9}{N+2}$
	χ^A_{ij}	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}} = \square$	$(\cdot)$	-4	$\square$	$-\frac{1}{N-2}$
	χ^A_{iJ}	$\bar{\square}$	$\square$	1	$\square$	$-\frac{1}{N-2}$
	χ^A_{JK}	$(\cdot)$	$(\cdot)$	6	$\square$	$-\frac{1}{N-2}$

Table 1: $\psi\chi$ model, $N=5$, $k=1$. $A=1,2,\ldots,9$, $i,j=1,2,3$; $J,K=4,5$.

$$\langle \psi\chi^{(9)} \rangle \propto \mathrm{diag}(2v, 2v, 2v, -3v, -3v)$$

	fields	$SU(3)$	$SU(2)$	$U(1)$	$SU(8)$	$U_0(1)'$
IR	ψ^{ij}	$\square\!\square$	$(\cdot)$	4	$(\cdot)$	
	ψ^{JK}	$(\cdot)$	$\square\!\square$	-6	$(\cdot)$	
	χ^9_{ij}	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}} = \square$	$(\cdot)$	-4	$(\cdot)$	
	χ^B_{ij}	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}} = \square$	$(\cdot)$	-4	$\square$	
	χ^B_{iJ}	$\bar{\square}$	$\square$	1	$\square$	
	χ^9_{JK}	$(\cdot)$	$(\cdot)$	6	$(\cdot)$	
	χ^B_{JK}	$(\cdot)$	$(\cdot)$	6	$\square$	
	$\phi^B \sim \Re(\psi^{iJ}\chi^B_{Ji})$	$(\cdot)$	$(\cdot)$	$(\cdot)$	$\square$	
	$\pi^B \sim \Im(\psi^{iJ}\chi^B_{Ji})$	$(\cdot)$	$(\cdot)$	$(\cdot)$	$\square$	
	$\pi^9 \sim \Im(\psi^{iJ}\chi^9_{Ji})$	$(\cdot)$	$(\cdot)$	$(\cdot)$	$(\cdot)$	

Table 2: $N=5$, $k=1$, $\psi\chi$ model: massless fermions. $B=1,2,\ldots,8$.

???

?

	fields	$SU(3)$	$SU(2)$	$U_Y(1)$	$SU(3)$
IR	u_R^c	$\square$	$(\cdot)$	$-\frac{4}{3}$	$\square$
	q_L	$\square$	$\square$	$\frac{1}{3}$	$\square$
	e_R^c	$(\cdot)$	$(\cdot)$	2	$\square$
	d_R^c	$\square$	$(\cdot)$	$\frac{2}{3}$	$\square$
	ψ_L	$(\cdot)$	$\square$	-1	$\square$
	ν_R^c	$(\cdot)$	$(\cdot)$	0	$\square$
	ϕ	$(\cdot)$	$\square$	1	$(\cdot)$

Part 3: Criteria for confinement and other phases

Reflections

- In chiral BY and GG models, a putative confinement phase with fully unbroken global symmetries (no condensates forming) is inconsistent.

- BY and GG models are (likely) in color-flavor locked dynamical Higgs phase;
- In $"\psi\chi\eta"$ and other models with $\langle\psi\chi\rangle \neq 0$ in adj repr.

  → Dynamical Abelianization (Coulomb phase)

  → More general DSB (nonAbelian IR gauge group)

- In QCD (vector-like) the SU(3) color is confined.

What is confinement ?

Color confinement ?

Def. A Particles with color (e.g., quarks) cannot be freely propagating, i.e., "confined" inside color-singlet hadrons (mesons and baryons).

Def. B Wilson-loop, Polyakov-loop criteria

- $W(\gamma) = Tr\{\mathcal{P}\, e^{i \oint_\gamma A_\mu dx^\mu}\}$ $\quad A_\mu \equiv A_\mu^a T^a$

  $\lim_{\gamma \to \infty} \langle W(\gamma) \rangle = \begin{cases} e^{-A} & \text{confinement} \\ e^{-L} & \text{Higgs} \end{cases}$

  area law

  perimeter law

  $\lim_{T \gg R} |\langle W(R,T) \rangle| \sim e^{-TV(R)}$

- $P(\mathbf{r}) = \frac{1}{N} Tr\{\mathcal{T} e^{i \int_0^\beta d\tau A_0(\mathbf{r},\tau)}\}$

  Center symmetry $\quad P(\mathbf{r}) \to \mathbb{Z}_N P(\mathbf{r})$

  $\lim_{\beta \to \infty} |\langle P(\mathbf{r}) \rangle| = 0$

  Center symmetry unbroken (Confinement)

  $\langle P(\mathbf{r}) \rangle = Z_Q / Z = e^{-\beta F}$

  $A_\mu = t^a A_\mu^a dx^\mu$

- Lattice simulation $\rightarrow$ SU(N) YM is in confinement phase!
- But there is nothing to confine in YM theory !!
  - massless quarks $\rightarrow$ no center symmetry; the string splits, area law lost
  - what distinguish Confinement and Higgs phase (both perimeter law) ?

- Def. A is also problematic. Gauge non-invariant (colored) operators/states as gauge invariant ones, in a given gauge.

- e.g., Weinberg-Salam SU(2)xU(1) theory

Brout-Englert-Higgs '64
CMS, Atlas '12

Higgs VEV $\langle\phi\rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$, $v \neq 0$ really means $\langle \sum_{i=1}^{2} \phi^{i\,*}\phi^i \rangle \neq 0$

Also, the neutrino and electron in $\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$:

't Hooft

really mean $\nu_L \sim \phi^\dagger \cdot \psi_L \, , \quad e_L \sim \epsilon_{\alpha\beta}\, \psi_L^\alpha \phi^\beta$

- Does it mean that there are no distinctions (Higgs and confinement)?

$\langle \sum_1^2 \phi^{i\,*}\phi^i \rangle \neq 0 \, , \quad \langle\phi\rangle = 0$

No, There are differences in the spectrum

Abbott, Farhi '81

- In $\psi\eta$ model, the NG boson $\sum_{n,j}^{N} \psi^{nj}\eta_j^n = \mathrm{Tr}(\psi\eta) \propto \mathbf{1} + \frac{i}{F_\pi^{(0)}}\phi_0 + \ldots$

~ gauge-invariant $\det U, \quad U^{k\ell} \equiv \psi^{kj}\eta_j^\ell$

Dynamical Higgs phase

but the "confining" system, $\langle \det U \rangle \neq 0, \ \ \langle \psi\eta \rangle = 0,$ has a different symmetry.

- Dynamical Higgs phase and Elitzur's theorem

Def. C   <u>Confinement</u> = dual superconductivity (dual Meissner effect)   't Hooft '81 Seiberg, Witten '94

$U(1)^2 \subset SU(3)_{color}$ A particle has el. and mag. q. numbers

$(n_1, n_2; m_1, m_2)$,

U(1)$_I$ e m charges = $n_1 e_1, m_1 g_1,$ $e_1 g_1 = n/2$ (Dirac)

Def. Dirac unit between two particles

$$\mathcal{D} \equiv \sum_i (n_i^{(1)} m_i^{(2)} - n_i^{(2)} m_i^{(1)})$$

Criterion $\langle M^{(1)} \rangle \neq 0$ → Particles (2) with $\mathcal{D} \neq 0 \quad \mathrm{Mod}\ 3$ are confined

e.g. $\langle M_{(0,0;1;0)} \rangle \neq 0,$ → $\mathrm{Quark}_{(1,0;0;0)}$ is confined!

(magnetic monopole condensation)

- Def. C is also problematic:
  - The idea involves Abelian monopoles
  - $\langle M_b^a \rangle = \delta_b^a \Lambda \neq 0 \quad \to \quad \mathrm{Confinement} \quad XSB \to SU(N_f)_V$

    But too many NG bosons; also doubling of the Regge trajectories   Yung

☞ Confinement = NonAblelian dual Meissner effect ?
<u>Condensation of (strongly-coupled) NonAbelian monoples ?</u>

(Tentative) New criteria

- The phase of an SU(N) gauge theory NOT determined by the underlying pure SU(N) YM theory (in the "confinement phase"),

- BUT by **elementary (scalar), composite (bifermion), or solitonic** condensates and by the type of NG bosons they produce:

  - No colored NG bosons → Confinement (e.g., YM, QCD, Susy QCD/YM)
  - $N^2 - 1$ colored NG bosons → Higgs phase (e.g., BY and GG models, GWS)
  - N (N -1) colored NG bosons → Dynamical Abelianization ($\psi\chi\eta$, $\mathcal{N} = 2$ Susy)
    (Coulomb phase, or dual Higgs)
  - Other groups of colored NG bosons → Partial Higgs/confinement/Coulomb
    (being explored..)

Part 4: Supersymmetry and strongly-coupled gauge theories

"SUSY 40"
'80 - '22

Montonen-Olive duality
Susy instanton calculus,
Veneziano-Yankielovicz,
Seiberg's duality in SQCD,
Seiberg-Witten,
Witten-Olive, Witten
Generalized KK anomaly,
Argyres-Douglas, SCFT, EHIY
Argyres-Plesser-Seiberg-Witten
GST duality,
SCFT and confinement
(Susy-inspired) results on
NonAbelian vortices and monopoles

→ Some reflections

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→ Some reflections

1. SYM, SQCD

Taylor-Veneziano-Yankielowicz '83
KK '84
Affleck-Dine-Seiberg '84
Amati, KK, Meurice, Rossi, Veneziano, '88

- $\mathcal{N}=1$ SQCD

  - $\det m \neq 0 \quad \langle\lambda\lambda\rangle =$

$$m_i\langle Q_i\tilde{Q}_i\rangle = \text{indep. of } i = \Lambda_1^{\frac{3n_c-n_f}{n_c}}\left(\prod_{i=1}^{n_f} m_j^{1/n_c}\right)\cdot e^{2\pi i k/n_c}, \quad k=1,2,\ldots,n_c :$$

(Exact)

$$m \to 0 \; : \quad \begin{array}{ll} \langle Q\tilde{Q}\rangle \to \infty\,, & N_f < N_c \\ \langle\lambda\lambda\rangle\,,\langle Q\tilde{Q}\rangle \to 0\,, & N_f > N_c \end{array}$$

(run-away vacua)

  - $m=0$: QMS (flat directions)

Seiberg's EM duality, phases, SCFT

Seiberg '94

N_f	$< N_c$	N_c	$N_c+1 \leq N_f < \frac{3N_c}{2}$	$\frac{3N_c}{2} < N_f \leq 3N_c$	$> 3N_c$
Phases	No vacua	finite vacua	Free magnetic phase	SCFT	Infrared free
IR Deg. freedom	-	$M, B, \bar{B}$	$M, B, \bar{B}$	$Q, \tilde{Q}$ or $q, \tilde{q}, M$	$Q, \tilde{Q}$

2. SCFT (superconformal th) $\mathcal{N}=2$ SYM, SQCD

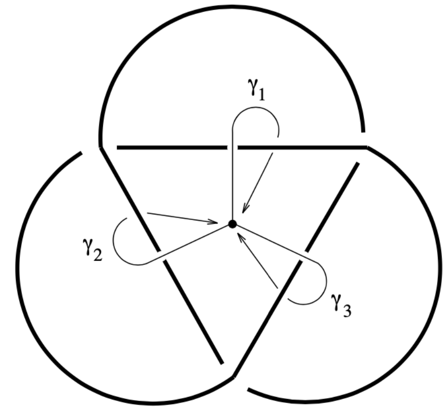
Seiberg-Witten '94

$u \equiv Tr\langle\Phi^2\rangle, \quad v \equiv Tr\langle\Phi^3\rangle, \quad etc.$

- SU(3) SYM

$$4u^3 = 27\tilde{v}^2 \ , \quad \tilde{v} = v \pm 2\Lambda^3$$

→ Nonlocal U(1) SCFT



Argyres-Douglas '95

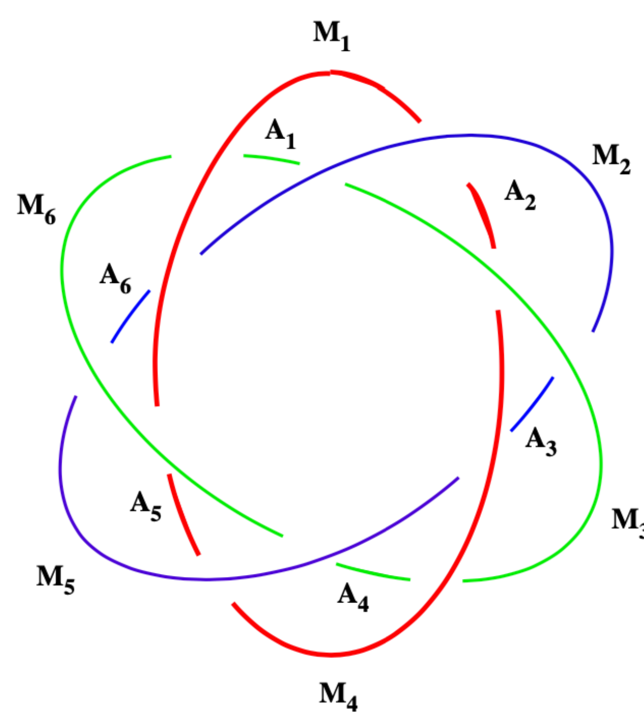
$$\mathbf{n}^{(1)} = (1,0;0,0)$$
$$\mathbf{n}^{(2)} = (0,0;-1,0)$$
$$\mathbf{n}^{(3)} = (1,0;-1,0)$$

- SU(3) N_F =4 SQCD

Auzzi, Grena, KK '02

$$u = 3m^2 \ , \quad v = 2m^3$$

→ Nonlocal SU(2) x U(1) SCFT

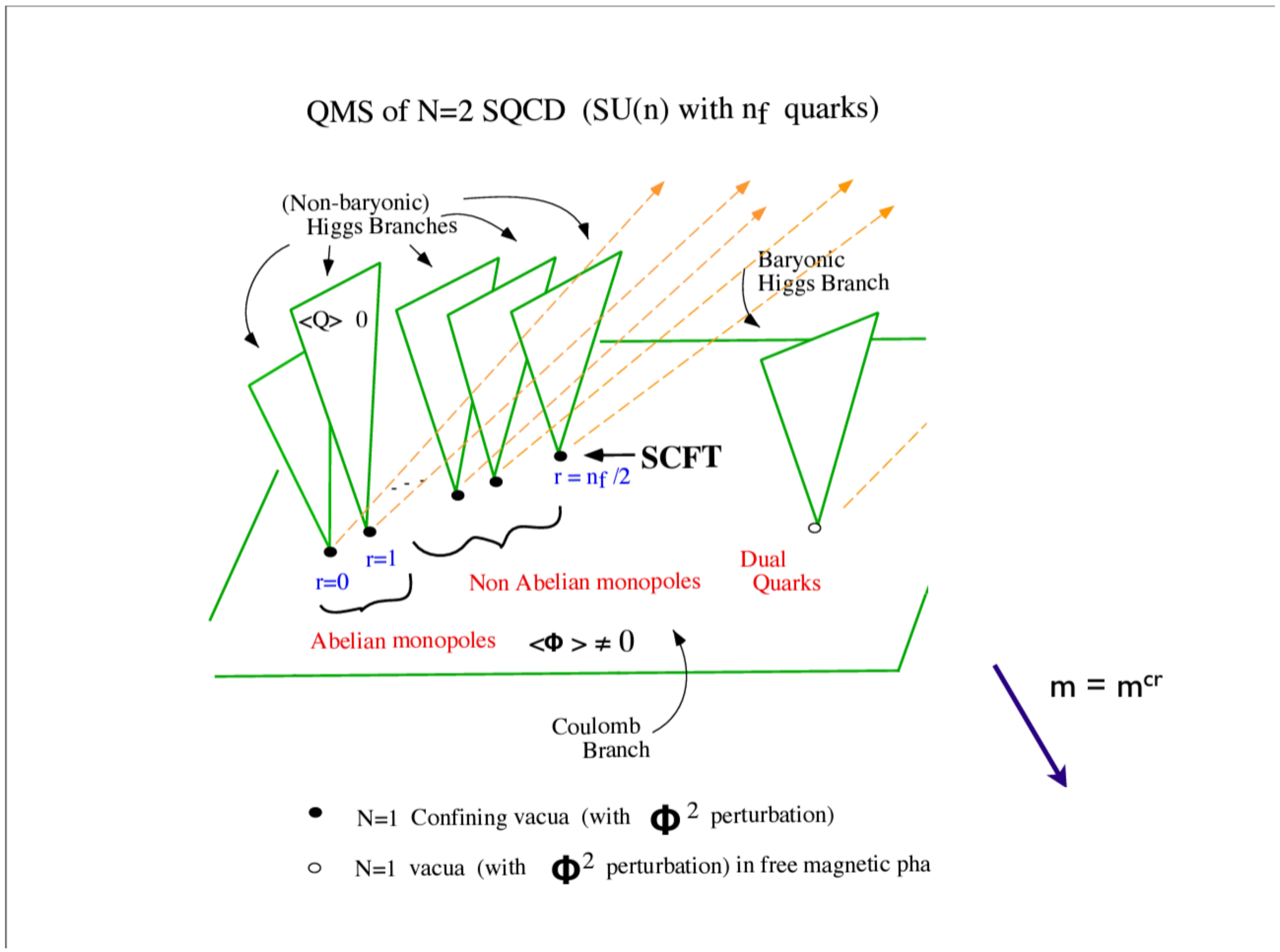


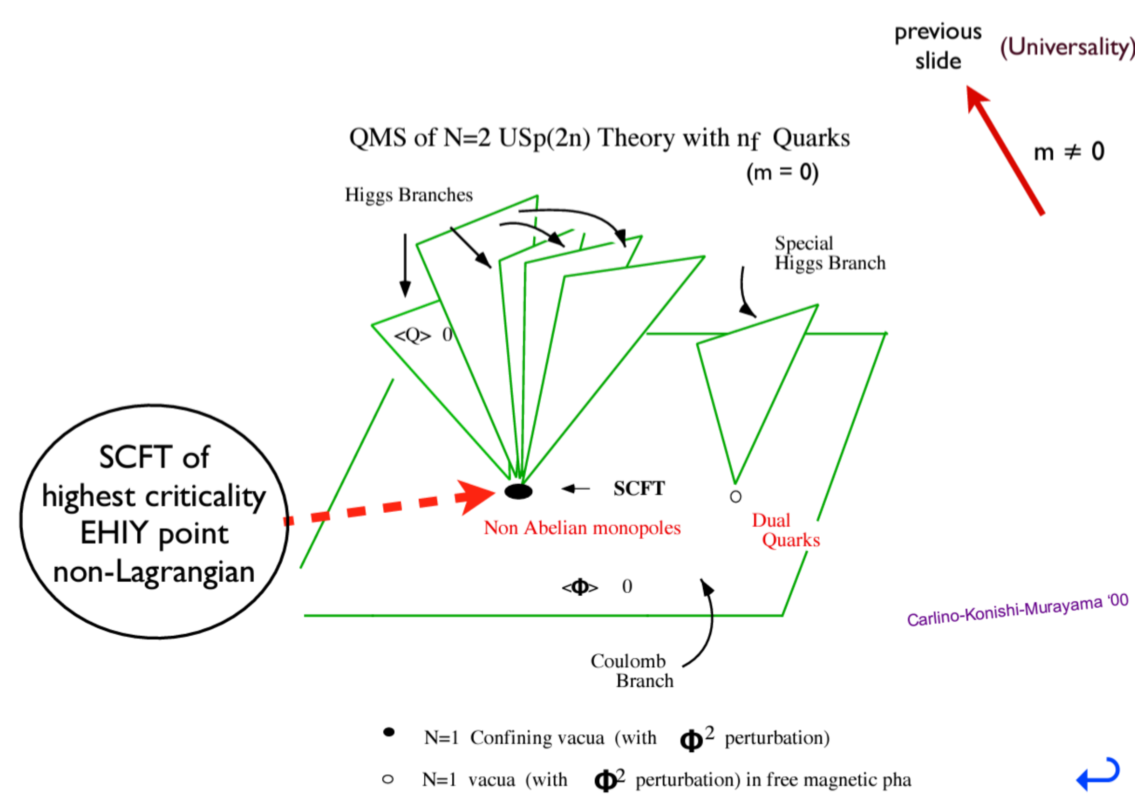
Matrix	Charge
M_1, M_4	$(\pm 1, 1, 0, 0)^4$
A_2, A_5	$(\pm 1, -1, \mp 1, 0)^4$
M_2, M_5	$(\pm 2, 2, \mp 1, 0)$
A_3, A_6	$(\pm 2, -2, \pm 1, 0)$
M_3, M_6	$(0, 2, \pm 1, 0)$
A_1, A_4	$(\pm 4, -2, \mp 1, 0)$

- Argyres-Plesser-Seiberg-Witten, Eguchi-Hori-Ito-Yang, Gaiotto, … …

SCFT points of $\mathcal{N} = 2$ SQCD with SU(N), USp(2N)

Argyres, Plesser,Seiberg '96
Carlino, KK, Murayama '00
Di Pietro, Giacomelli '11





3. IR CFT (conformally inv fixed points) ~ confinement

Nielsen-Froggart

- ♦ Naively, diametrically opposite concepts

Banks-Zacks,

SQCD Seiberg

- In systems with parameters (N_F , g, QMS), however, they may be close to each other, as the parameters are varied

- Small relevant deformation (perturbation, or produced by the system itself)

  → deviation of the RG flow: CFT → Confinement

Seiberg-Witten '94

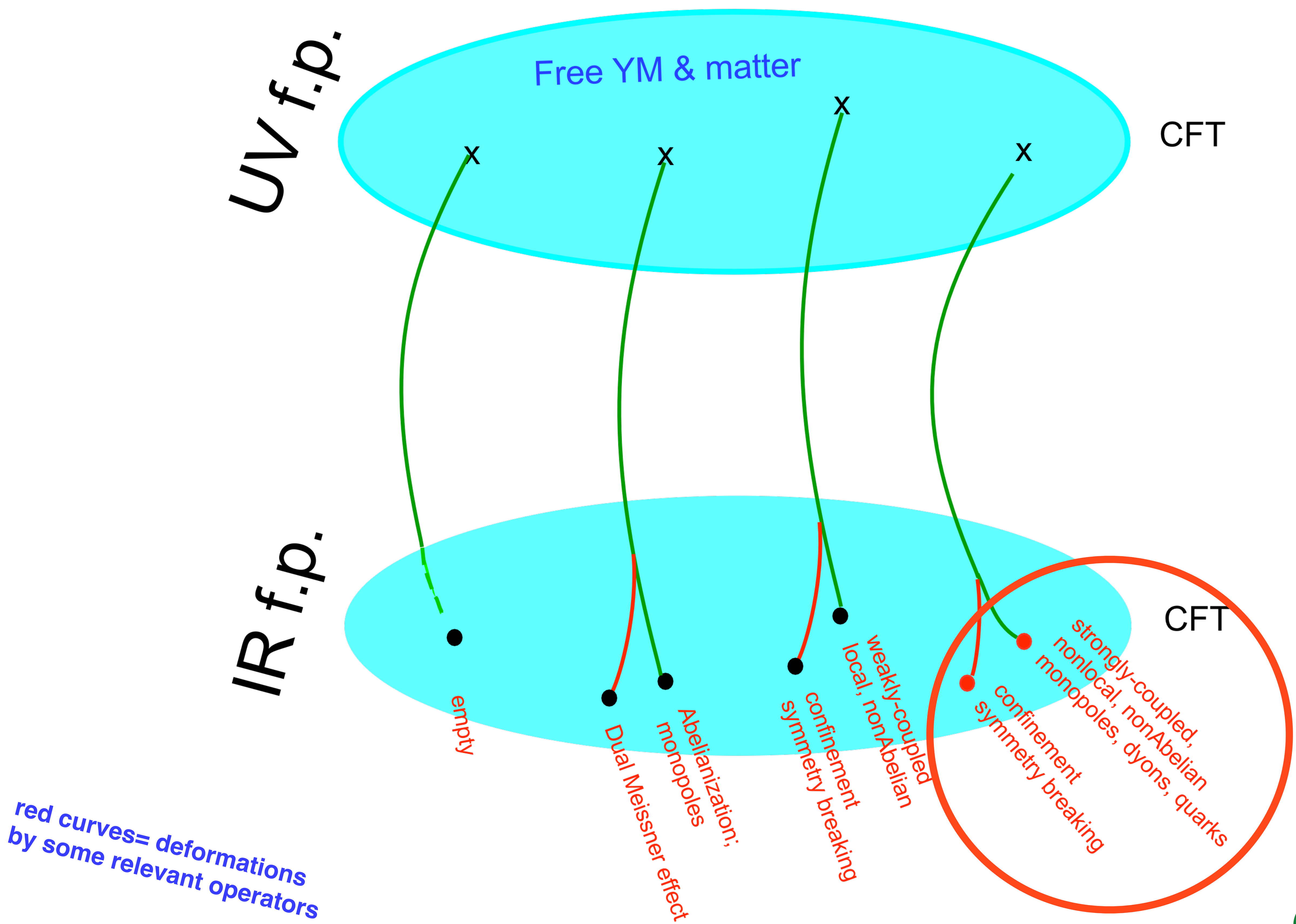
$\Delta L = \mu \Phi^2|_F + h.c.$

- ♦ The same degrees of freedom describing the CFT f.p. describe confinement vacuum nearby

- ♦ Interesting nonAbelian CFT's are strongly coupled (cfr. Abelian dual superconductor)

☞ A difficulty!

Confinement and RG flow



4. How to study strongly-coupled conformal IR fixed points (and show near-by confinement)

- S duality in exact conformal theories (w arbitrary g)

$$g = \infty \;\leftrightarrow\; g_D = 1/g \sim 0$$

$\mathcal{N} = 4$

$\mathcal{N} = 2, \; N_F = 4$

Argyres-Seiberg '07

- GST duality: apply Argyres-Seiberg to the SCFT IR fixed points

Gaiotto-Seiberg-Tachikawa '11
Giacomelli '12

- GST allows us to study a singular SCFT, to deform it to get confinement and XSB

Giacomelli, KK, '12,'13
Bolognesi, Giacomelli, KK '16

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Giacomelli, KK, '12,'13
Bolognesi, Giacomelli, KK '16

RG flows

Bolognesi, Giacomelli, KK '16

Real-world QCD
N=0 SCFT

$$a_{UV} = \frac{11 N_f N_c}{360} + \frac{31}{180}(N_c^2 - 1)$$

$$c_{UV} = \frac{1}{20} N_f N_c + \frac{N_c^2 - 1}{10}$$

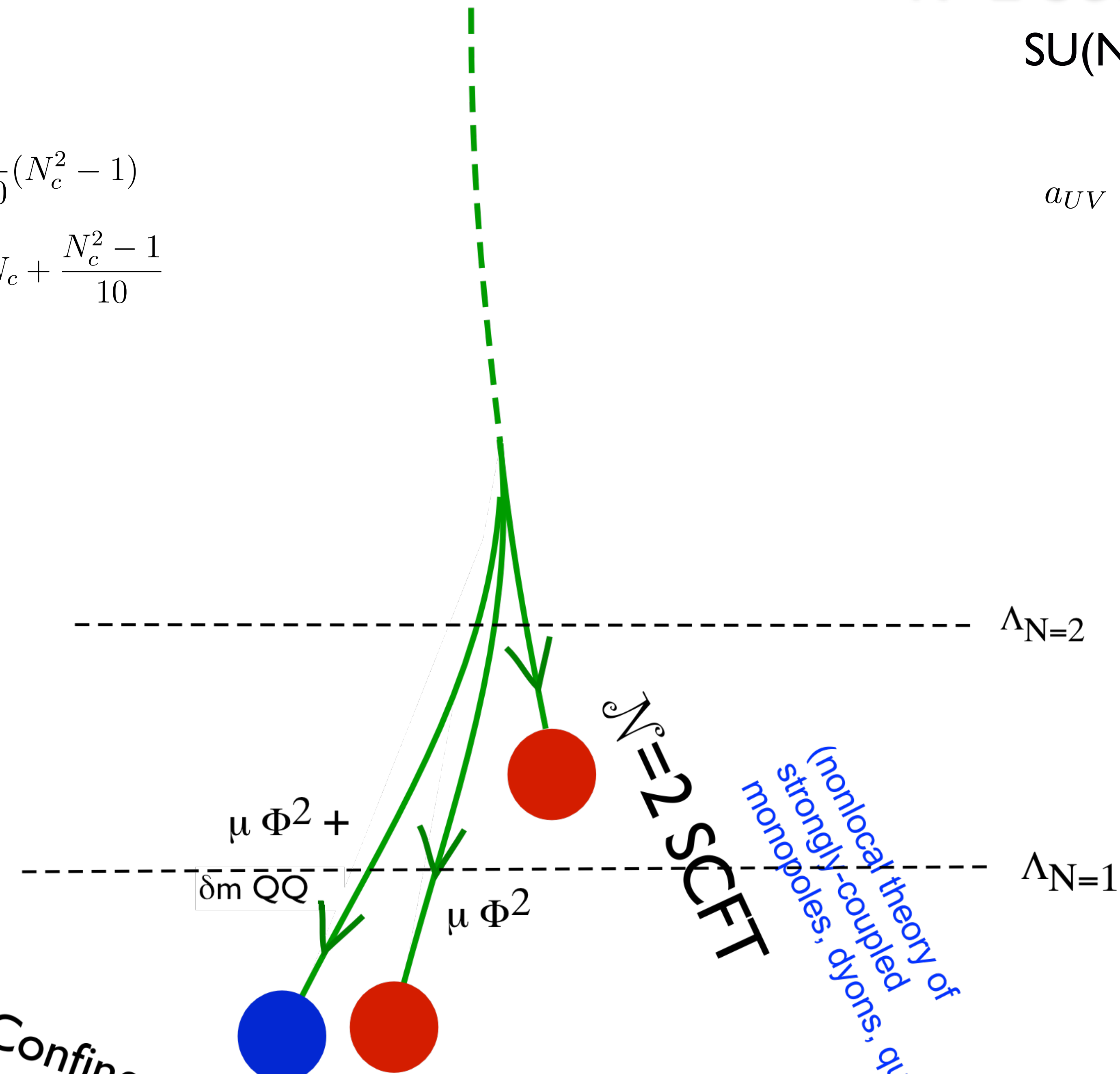
$N_f < \frac{11}{2} N_c$

$\vee$

$$a_{IR} = \frac{N_f^2 - 1}{360}$$

$$c_{IR} = \frac{N_f^2 - 1}{120} \; ;$$

UV



Confinement
Flavor Symmetry Breaking

$\mathcal{N}$=1 SCFT
(local theory of massless mesons)
in adjoint repr of G_F

N=2 SCFT

$u_i = 0 \quad m_i = \Lambda/N$

SU(N), N_F =2N-1

$$a_{UV} = \frac{7N^2 - N - 5}{24}$$

$$c_{UV} = \frac{4N^2 - N - 2}{12}$$

$\vee$

$$a_{N=2SCFT} = \frac{7N(N-1)}{24}$$

$$c_{N=2SCFT} = \frac{N(N-1)}{3}$$

$\vee$

$$a_{IR} = \frac{(2N-1)^2 - 1}{48}$$

$$c_{IR} = \frac{(2N-1)^2 - 1}{24}$$

THE END

Thank you for your attention !

1. $\langle\lambda\lambda\rangle$

- $\mathcal{N}=1$ SYM

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle = \frac{\mu}{T_G} \left\langle \mathrm{Tr}\phi^2 \right\rangle ,$$

es of μ; by matching the dynamical scales as $\Lambda^3_{\mathcal{N}=1} = \mu\,\Lambda^2_{\mathcal{N}=2}$
onsidered here) upon decoupling the adjoint matter, one finds

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SU(r+1)} = \Lambda^3_{\mathcal{N}=1} ,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SO(2r+1)} = 2^{\frac{4}{2r-1}-1}\Lambda^3_{\mathcal{N}=1} ,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{USp(2r)} = 2^{1-\frac{2}{r+1}}\Lambda^3_{\mathcal{N}=1} ,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SO(2r)} = 2^{\frac{2}{r-1}-1}\Lambda^3_{\mathcal{N}=1} ,$$

KK-Ricco '03

(Exact) ← Seiberg-Witten

adj mass $\mathcal{N}=1$ perturbation, Konishi-anomaly, decoupling

- OK w/ results from SQCD (weak instanton calc.)

Squark, gaugino condensates, K anomaly, decoupling of quarks

Shifman, Vainshtein, Zakharov, '83, '85
Davies,Hollowood,Khoze,Mattis '99

cfr. 4/5 puzzle

- $\langle\lambda\lambda\rangle \neq 0\,, \quad Z_{2N} \to Z_2$ → XSB & N vacua (SU(N)) Witten

- $\langle\lambda\lambda\rangle \neq 0$ Toron?

- $\Delta L_{\mathcal{N}=0} = m_\lambda \lambda\lambda + h.c.$ → SU(2) YM

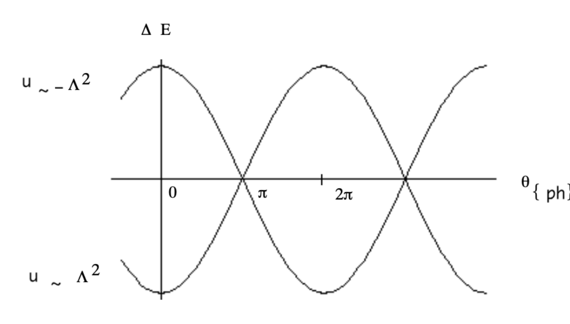


Figure 2: Energy density in the two minima

KK '93
Evans, Hsu, Schwetz '93
Di Vecchia, Veneziano '80
Mixed anomaly '17

(ii) $(N_\psi, N_\chi) = (0,1)$ model

“$\chi\eta$”

Georgi-Glashow SU(5) GUT
Appelquist, Duan, Sannino, ‘00
Poppitz
BKS ‘18

$$\chi_{[ij]}\,, \qquad \tilde\eta^{B\,j}\,, \qquad B = 1,2,\ldots,(N-4)$$

$$SU(N)_{\rm c} \times SU(N-4)_{\rm f} \times U(1) \qquad \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + (N-4)\,\square$$

(A) Confinement with no XSB (&)

massless baryons: $B^{\{CD\}} = \chi_{[ij]}\,\tilde\eta^{i\,C}\tilde\eta^{j\,D}$

looks consistent

fields	$SU(N)_c$	$SU(N-4)$	$U(1)$
χ	$\bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{N(N-1)}{2}\cdot(\cdot)$	$N-4$
$\tilde\eta^A$	$(N-4)\cdot\square$	$N\cdot\square$	$-(N-2)$
$B^{\{AB\}}$	$\frac{(N-4)(N-3)}{2}\cdot(\cdot)$	$\square\square$	$-N$

(B) Color-flavor locked dyn. Higgs

$$\langle\chi_{[ij]}\tilde\eta^{B\,j}\rangle = \text{const.}\,\Lambda^3\delta_i^B \qquad \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \otimes \square \to \bar{\square} \oplus \ldots\,.$$

$$\to SU(N-4)_{\rm cf} \times U(1)' \times SU(4)_{\rm c}$$

The same massless baryons B^{CD} do the job

No NG bosons; complementarity (?)

fields	$SU(4)_c$	$SU(N-4)_{cf}$	$U'(1)$
$\chi_{i_1 j_1}$	$\frac{(N-4)(N-5)}{2}\cdot(\cdot)$	$\bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	N
$\chi_{i_1 j_2}$	$(N-4)\cdot\bar{\square}$	$4\cdot\bar{\square}$	$\frac{N}{2}$
$\chi_{i_2 j_2}$	$\bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{4\cdot 3}{2}\cdot(\cdot)$	0
$\tilde\eta^{A,i_1}$	$(N-4)^2\cdot(\cdot)$	$\square\square \oplus \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$-N$
$\tilde\eta^{A,i_2}$	$(N-4)\cdot\square$	$4\cdot\square$	$-\frac{N}{2}$
$B^{\{AB\}}$	$\frac{(N-4)(N-3)}{2}\cdot(\cdot)$	$\square\square$	$-N$

- But $\chi_{[i_2,j_2]}$ still around and strongly coupled

$\to \langle\chi\chi\rangle \neq 0$ ········▸ Dark Matter?

$$\bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \otimes \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \to \bar{\begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}} \oplus \ldots$$

Tumbling (?): SU(4) confined

to p.7

Theoretical laboratories : chiral SU(N) gauge theories with Weyl fermions

(battle fields?)

$$N_\psi \, \yng(2) \oplus N_{\tilde\psi} \, \overline{\yng(2)} \oplus N_{\tilde\chi} \, \yng(1,1) \oplus N_\chi \, \overline{\yng(1,1)} \oplus N_{\tilde\eta} \, \yng(1) \oplus N_\eta \, \overline{\yng(1)} \oplus N_{adj} \, \text{adj} \; :$$

$\psi \quad \tilde\psi \quad \tilde\chi \quad \chi \quad \tilde\eta \quad \eta \quad \lambda$

N's constrained by gauge-anomaly -free condition

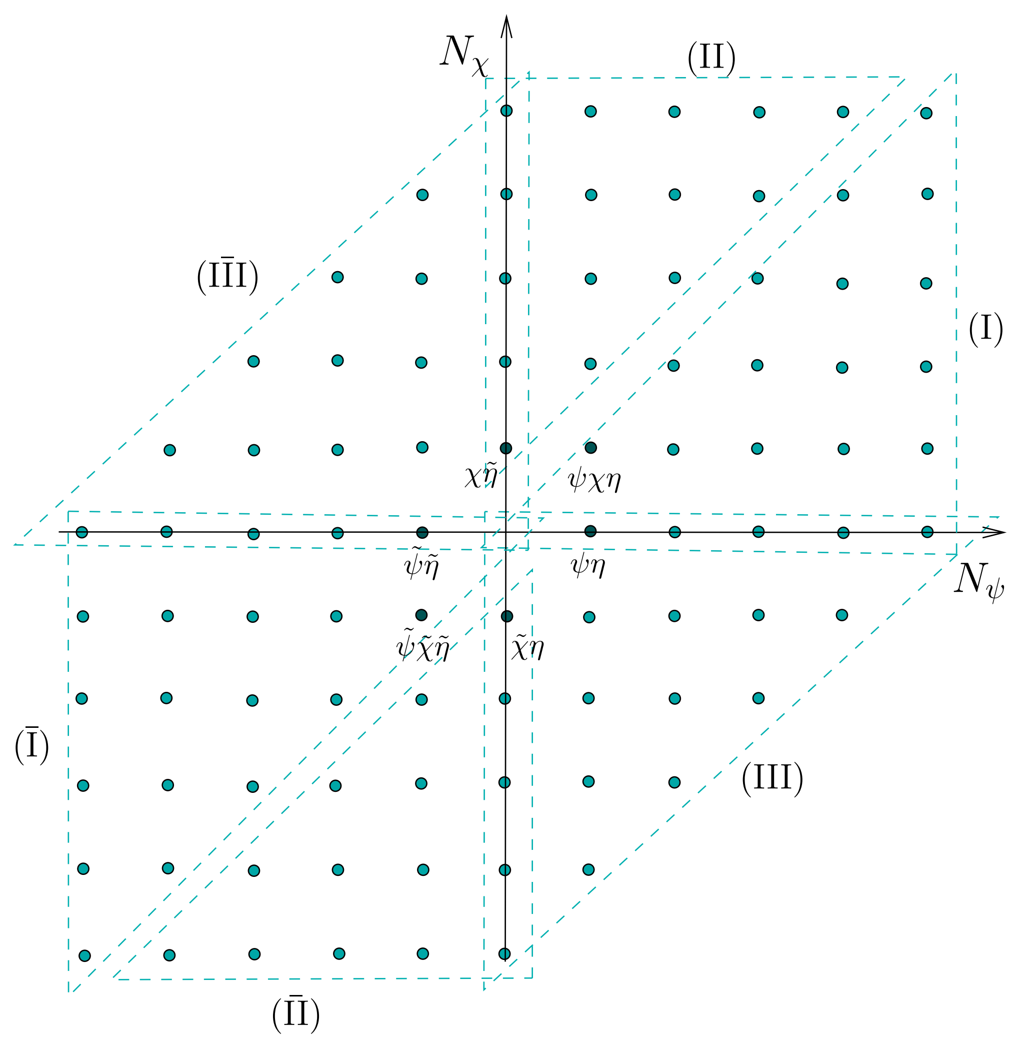


Figure 1: All the possible (N_ψ, N_χ) models that are AF[***] at large N.

*** AF or CFT?

Part 4: Supersymmetry and strongly-coupled gauge theories

"SUSY 40"
'80 - '22

Montonen-Olive duality
Susy instanton calculus,
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NonAbelian vortices and monopoles

→ Some reflections

1. $\langle \lambda\lambda \rangle$

- $\mathcal{N}=1$ SYM

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle = \frac{\mu}{T_G} \langle \mathrm{Tr}\phi^2 \rangle \ ,$$

es of μ; by matching the dynamical scales as $\Lambda^3_{\mathcal{N}=1} = \mu\, \Lambda^2_{\mathcal{N}=2}$
onsidered here) upon decoupling the adjoint matter, one finds

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SU(r+1)} = \Lambda^3_{\mathcal{N}=1} \ ,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SO(2r+1)} = 2^{\frac{4}{2r-1}-1} \Lambda^3_{\mathcal{N}=1} \ ,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{USp(2r)} = 2^{1-\frac{2}{r+1}} \Lambda^3_{\mathcal{N}=1} \ ,$$

$$\left\langle \frac{\mathrm{Tr}\lambda^2}{16\pi^2} \right\rangle_{SO(2r)} = 2^{\frac{2}{r-1}-1} \Lambda^3_{\mathcal{N}=1} \ ,$$

KK-Ricco '03

(Exact)

Seiberg-Witten

adj mass $\mathcal{N}=1$
perturbation,
Konishi-anomaly,
decoupling

- OK w/ results from SQCD (weak instanton calc.)

Squark, gaugino condensates, K anomaly, decoupling of quarks

Shifman, Vainshtein, Zakharov, '83, '85
Davies,Hollowood,Khoze,Mattis '99

cfr. 4/5 puzzle

- $\langle \lambda\lambda \rangle \neq 0 \ , \quad Z_{2N} \to Z_2$ → XSB & N vacua (SU(N)) Witten

- $\langle \lambda\lambda \rangle \neq 0$ Toron?

- $\Delta L_{\mathcal{N}=0} = m_\lambda \lambda\lambda + h.c.$ → SU(2) YM

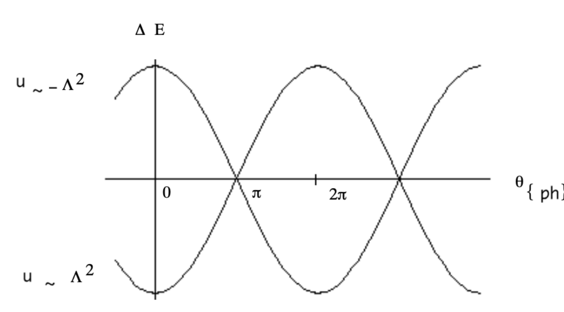


Figure 2: Energy density in the two minima

KK '93
Evans, Hsu, Schwetz '93
Di Vecchia, Veneziano '80
Mixed anomaly '17

- $\mathcal{N}=1$ SQCD

Taylor-Veneziano-Yankielowicz '83
KK '84
Affleck-Dine-Seiberg '84
Amati, KK, Meurice, Rossi, Veneziano, '88

- $\det m \neq 0 \quad \langle\lambda\lambda\rangle =$

$$m_i\langle Q_i\tilde{Q}_i\rangle = \text{indep. of } i = \Lambda_1^{\frac{3n_c-n_f}{n_c}}\left(\prod_{i=1}^{n_f} m_j^{1/n_c}\right)\cdot e^{2\pi i k/n_c}, \quad k=1,2,\ldots,n_c:$$

(Exact)

$m \to 0$:

$\langle Q\tilde{Q}\rangle \to \infty\,, \qquad N_f < N_c$ (run-away vacua)

$\langle\lambda\lambda\rangle\,, \langle Q\tilde{Q}\rangle \to 0\,, \qquad N_f > N_c$

- $m = 0$: QMS (flat directions)

Seiberg's EM duality, phases, SCFT

Seiberg '94

N_f	$< N_c$	N_c	$N_c+1 \leq N_f < \frac{3N_c}{2}$	$\frac{3N_c}{2} < N_f \leq 3N_c$	$> 3N_c$
Phases	No vacua	finite vacua	Free magnetic phase	SCFT	Infrared free
IR Deg. freedom	-	$M, B, \bar{B}$	$M, B, \bar{B}$	$Q, \tilde{Q}$ or $q, \tilde{q}, M$	$Q, \tilde{Q}$

2. SCFT (superconformal th) $\mathcal{N}=2$ SYM, SQCD

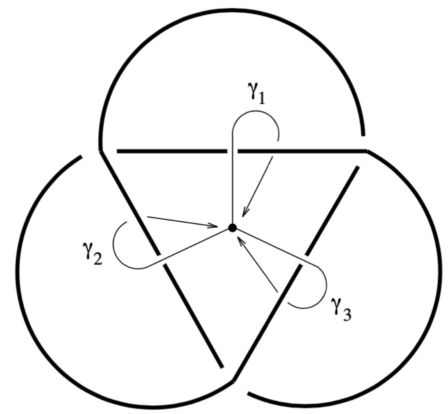
Seiberg-Witten '94

$u \equiv Tr\langle\Phi^2\rangle, \quad v \equiv Tr\langle\Phi^3\rangle, \quad etc.$

- SU(3) SYM

$4u^3 = 27\tilde{v}^2\ , \quad \tilde{v} = v \pm 2\Lambda^3$

→ Nonlocal U(1) SCFT



Argyres-Douglas '95

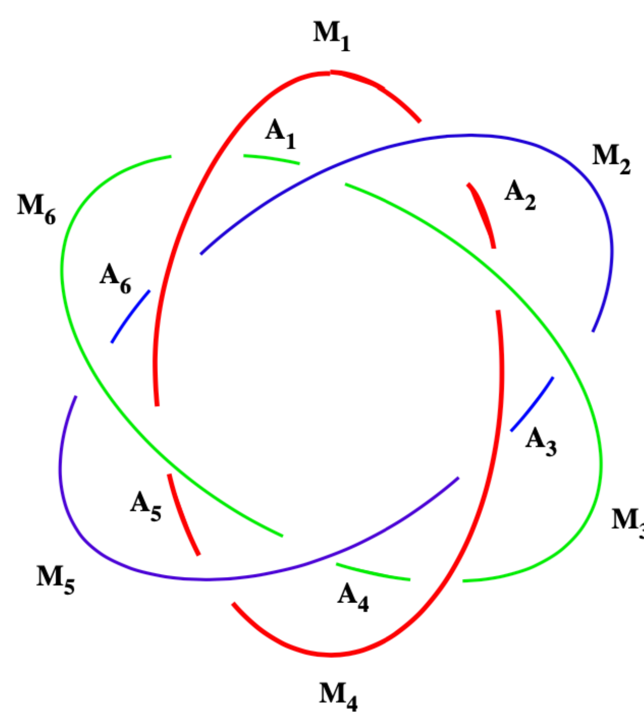
$$\mathbf{n}^{(1)} = (1,0;0,0)$$
$$\mathbf{n}^{(2)} = (0,0;-1,0)$$
$$\mathbf{n}^{(3)} = (1,0;-1,0)$$

- SU(3) N_F =4 SQCD

Auzzi, Grena, KK '02

$u = 3m^2\ , \quad v = 2m^3$

→ Nonlocal
SU(2) x U(1) SCFT

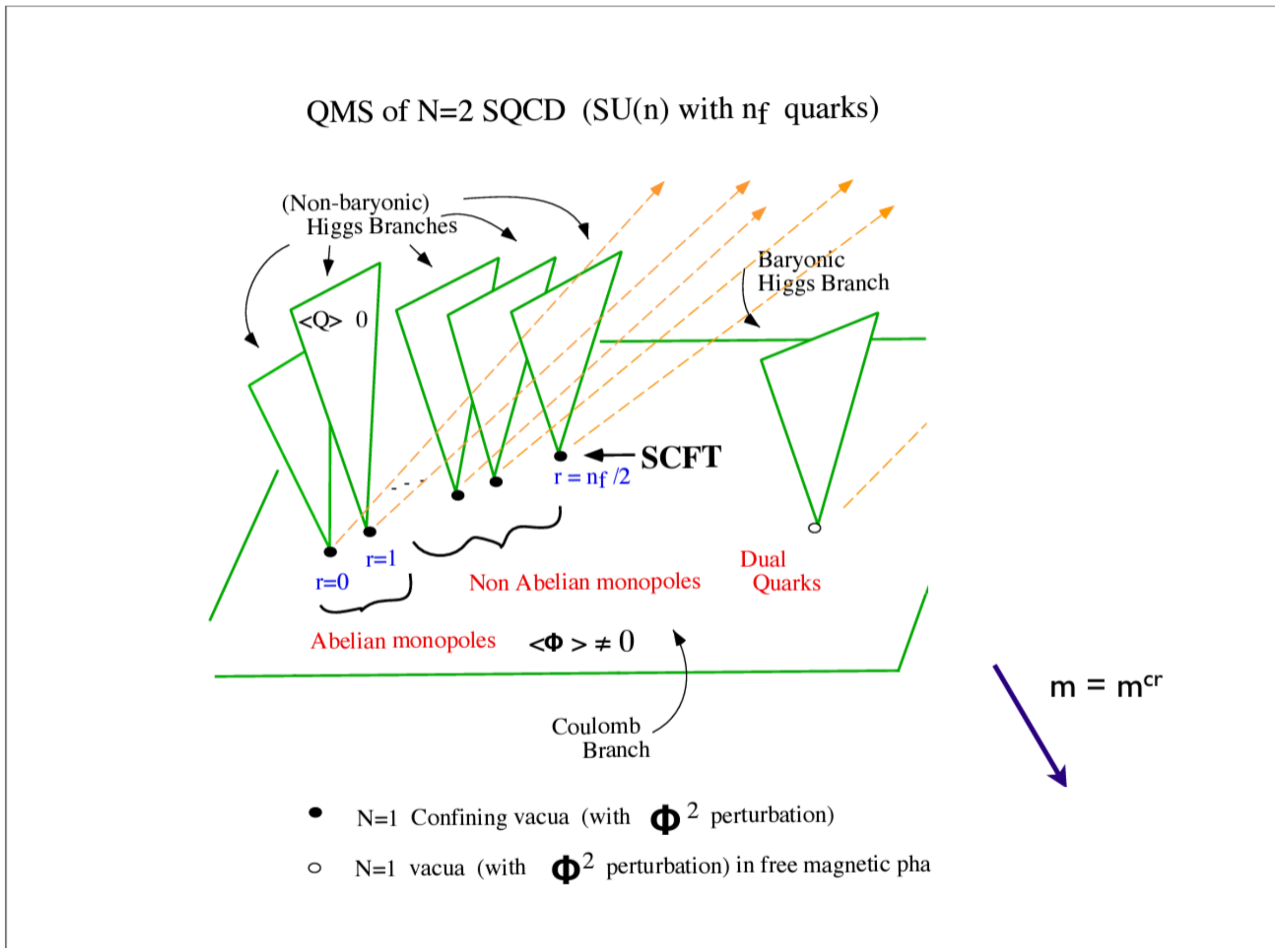


Matrix	Charge
M_1, M_4	$(\pm1,1,0,0)^4$
A_2, A_5	$(\pm1,-1,\mp1,0)^4$
M_2, M_5	$(\pm2,2,\mp1,0)$
A_3, A_6	$(\pm2,-2,\pm1,0)$
M_3, M_6	$(0,2,\pm1,0)$
A_1, A_4	$(\pm4,-2,\mp1,0)$

- Argyres-Plesser-Seiberg-Witten, Eguchi-Hori-Ito-Yang, Gaiotto, … …

SCFT points of $\mathcal{N}=2$ SQCD with SU(N), USp(2N)

Argyres, Plesser,Seiberg '96
Carlino, KK, Murayama '00
Di Pietro, Giacomelli '11

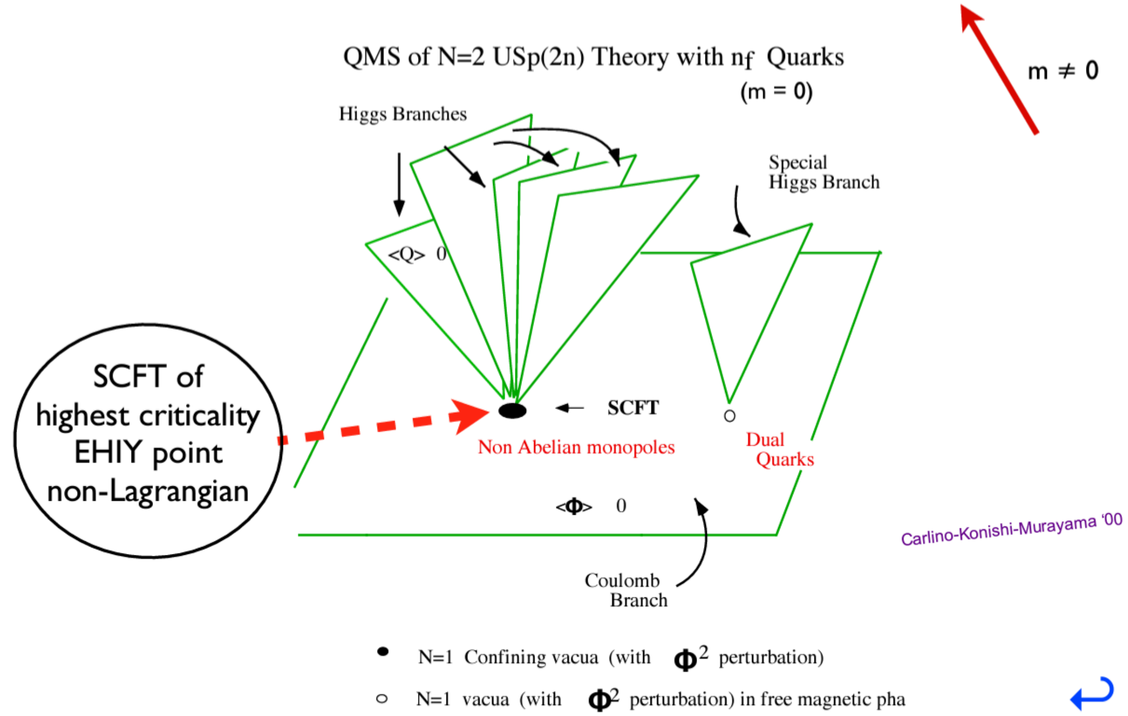


$m = m^{cr}$

previous slide

(Universality)

$m \neq 0$



IR CFT (conformally inv fixed points) ~ confinement

Nielsen-Froggart

- Naively, diametrically opposite concepts

Banks-Zacks,

SQCD Seiberg

- In systems with parameters (N_F , g, QMS), however,
  they may be close to each other, as the parameters are varied

- Small relevant deformation (perturbation, or produced by the system itself)

  → deviation of the RG flow: CFT → Confinement

Seiberg-Witten '94

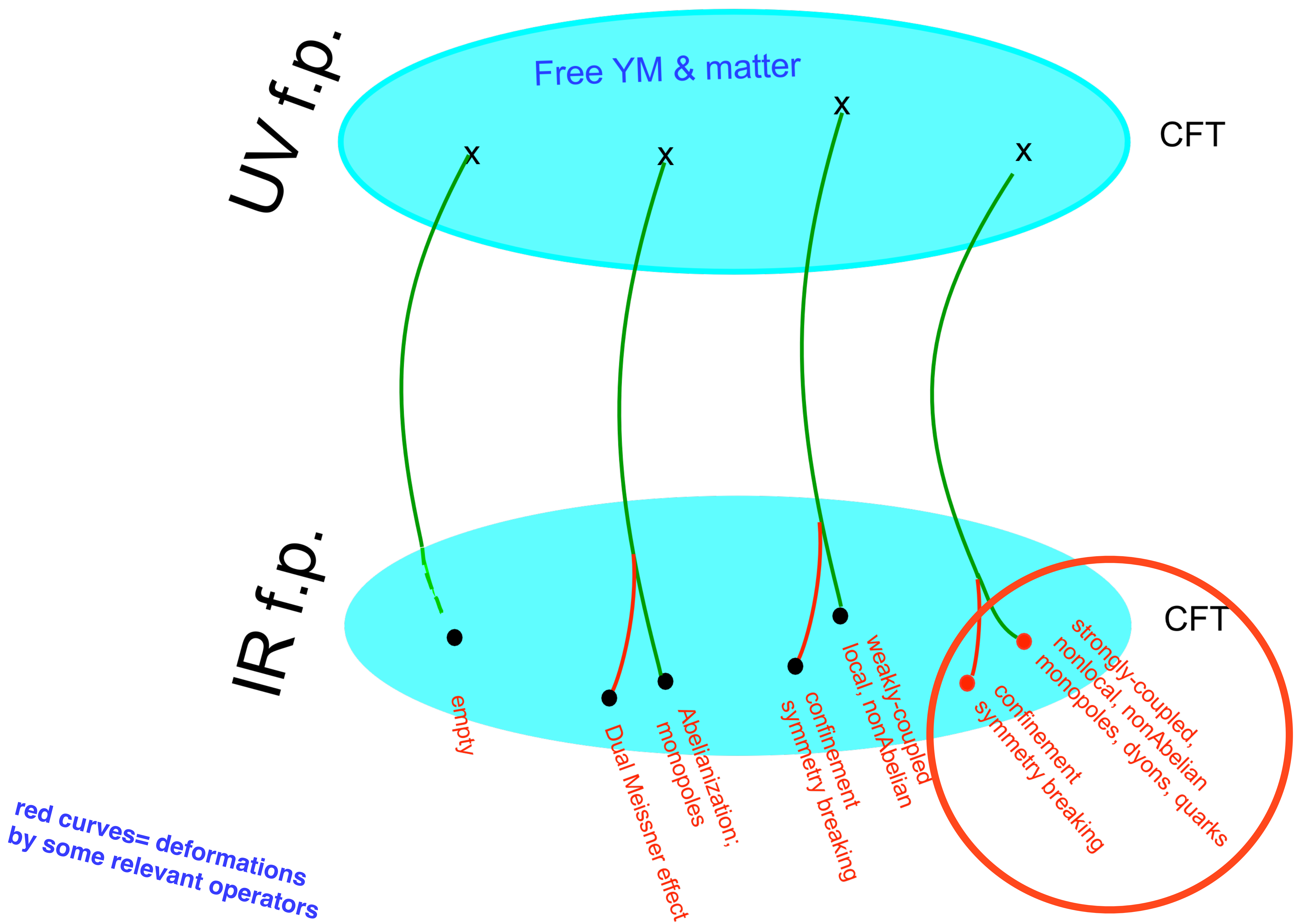
$\Delta L = \mu \Phi^2|_F + h.c.$

- The same degrees of freedom describing the CFT f.p. describe confinement vacuum nearby

- Interesting nonAbelian CFT's are strongly coupled (cfr. Abelian dual superconductor)

☞ A difficulty!

Confinement and RG flow



How to study strongly-coupled conformal IR fixed points
(and show near-by confinement)

- S duality in exact conformal theories (w arbitrary g)

$$g = \infty \;\; \leftrightarrow \;\; g_D = 1/g \sim 0$$

$\mathcal{N} = 4$

$\mathcal{N} = 2, \;\; N_F = 4$

Argyres-Seiberg '07

- GST duality: apply Argyres-Seiberg to the SCFT IR fixed points

Gaiotto-Seiberg-Tachikawa '11
Giacomelli '12

- GST allows us to study a singular SCFT, to deform it to get confinement and XSB

Giacomelli, KK, '12,'13
Bolognesi, Giacomelli, KK '16

RG flows

Bolognesi, Giacomelli, KK '16

UV

Real-world QCD
N=0 SCFT

$$a_{UV} = \frac{11 N_f N_c}{360} + \frac{31}{180}(N_c^2 - 1)$$

$$c_{UV} = \frac{1}{20} N_f N_c + \frac{N_c^2 - 1}{10}$$

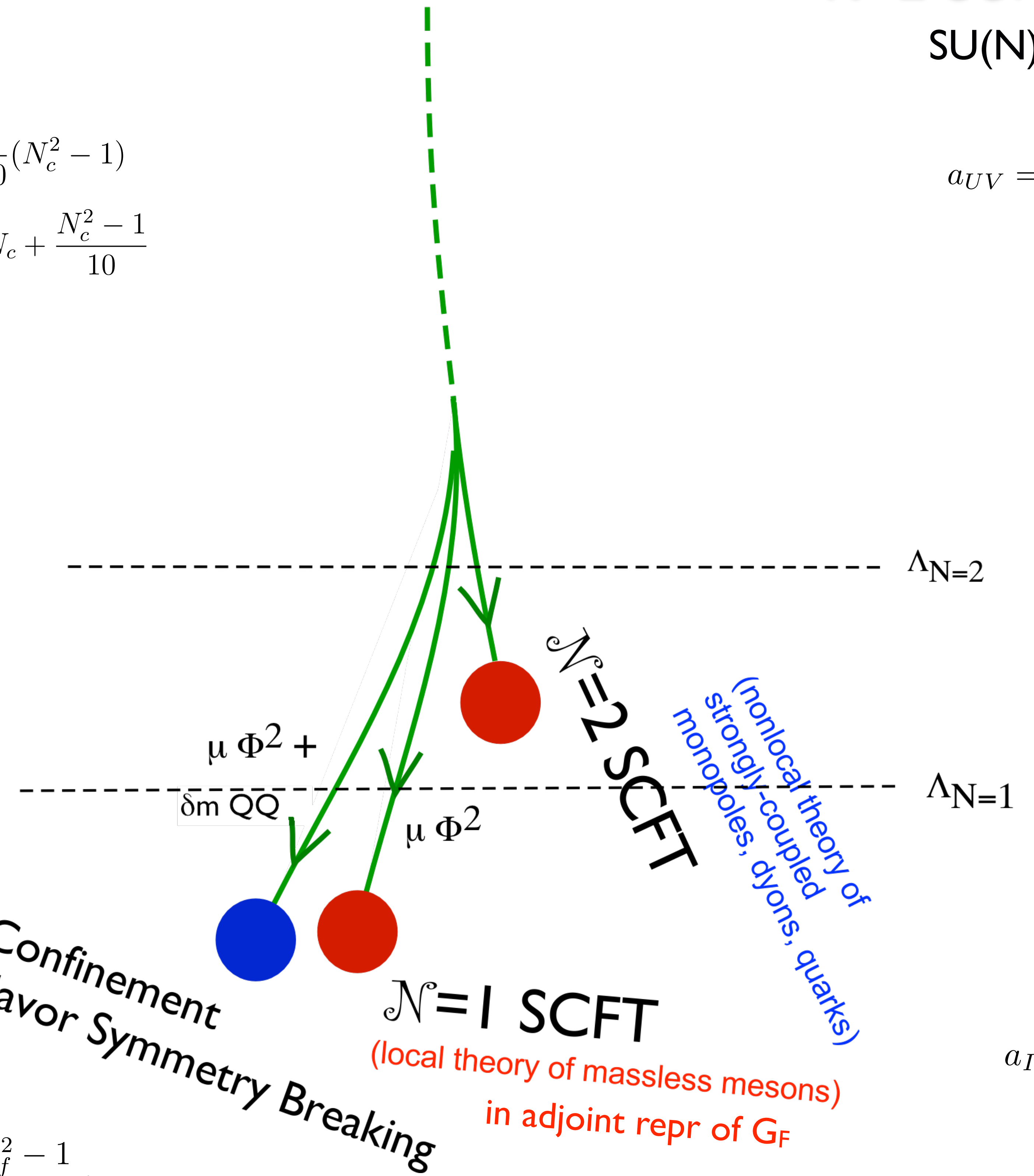
$N_f < \frac{11}{2} N_c$

N=2 SCFT $\quad u_i = 0 \quad m_i = \Lambda/N$

SU(N) SQCD, N_F =2N-1

$$a_{UV} = \frac{7N^2 - N - 5}{24}$$

$$c_{UV} = \frac{4N^2 - N - 2}{12}$$



$$a_{N=2SCFT} = \frac{7N(N-1)}{24}$$

$$c_{N=2SCFT} = \frac{N(N-1)}{3}$$

$$a_{IR} = \frac{N_f^2 - 1}{360}$$

$$c_{IR} = \frac{N_f^2 - 1}{120} \, ;$$

$$a_{IR} = \frac{(2N-1)^2 - 1}{48}$$

$$c_{IR} = \frac{(2N-1)^2 - 1}{24}$$

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To sum up: Susy gauge theories

☞

- Deep insights and understanding on:
  - Quantum monopoles and dyons; Dualities
  - (S)CFT, IR fixed-points
  - Hint: Confinement in QCD ~ close to nontrivial CFT

But

- Deform $\Delta L_{(\mathcal{N}=0)}$ to learn dynamics of non-Susy theories ? No (t easy…)
  - QMS (flat directions) in susy systems: where to start ?
  - Bifermion condensates

$$\langle \psi \eta \rangle \,, \quad \langle \chi \eta \rangle \,, \quad \langle \psi \chi \rangle \,, \qquad \langle \bar{q}_R q_L \rangle \quad \overset{m_q = 0}{\text{(QCD)}}$$

chiral superfield

$\Phi = A + \sqrt{2}\theta\psi + \dots$

all vanish by supersymmetry: Susy must be sp.ly broken

  - Interesting possible phases (dynamical Higgs, Abelianization, etc.) in chiral gauge theories : all out of reach of Susy cousins

AF or CFT

Up to 2 loops:

$$\alpha_{N_\psi,N_\chi} = \begin{pmatrix} -\frac{22\pi}{17N} & -\frac{24\pi}{13N} & -\frac{28\pi}{5N} & \frac{40\pi}{19N} & \frac{\pi}{2N} & \frac{8\pi}{77N} \\ -\frac{24\pi}{13N} & -\frac{2\pi}{N} & -\frac{8\pi}{N} & \frac{20\pi}{11N} & \frac{8\pi}{17N} & \frac{\pi}{10N} \\ -\frac{28\pi}{5N} & -\frac{8\pi}{N} & -\frac{14\pi}{N} & \frac{8\pi}{5N} & \frac{4\pi}{9N} & \frac{8\pi}{83N} \\ \frac{40\pi}{19N} & \frac{20\pi}{11N} & \frac{8\pi}{5N} & \frac{10\pi}{7N} & \frac{8\pi}{19N} & \frac{4\pi}{43N} \\ \frac{\pi}{2N} & \frac{8\pi}{17N} & \frac{4\pi}{9N} & \frac{8\pi}{19N} & \frac{2\pi}{5N} & \frac{8\pi}{89N} \\ \frac{8\pi}{77N} & \frac{\pi}{10N} & \frac{8\pi}{83N} & \frac{4\pi}{43N} & \frac{8\pi}{89N} & \frac{2\pi}{23N} \end{pmatrix} + O(1/N^2)\ .$$

$$\alpha < 0 : AF\ ; \qquad \alpha > 0 : CFT$$

$$N_\psi, N_\chi = 1, 2, \ldots 6$$

$$\beta(g) = -\frac{g}{4\pi}\left(\beta_0 \frac{\alpha}{(4\pi)} + \beta_1 \left(\frac{\alpha}{4\pi}\right)^2 + \ldots\right)$$

Actually all terms are of the same order in 1/N… Need 't Hooft 's 1/N expansion

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MAC like thinking

Raby-Dimopoulos-Susskind '80

$$A: \quad \psi\left(\square\square\right)\ \psi\left(\square\square\right) \quad \text{forming} \quad \begin{smallmatrix}\square\square\\ \square\square\end{smallmatrix}\ ;$$

$$B: \quad \chi\left(\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}\right)\ \chi\left(\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}\right) \quad \text{forming} \quad \overline{\begin{smallmatrix}\square\\ \square\\ \square\\ \square\end{smallmatrix}}\ ;$$

$$C: \quad \eta\left(\bar{\square}\right)\ \eta\left(\bar{\square}\right) \quad \text{forming} \quad \overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}\ ;$$

$$D: \quad \chi\left(\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}\right)\ \eta\left(\bar{\square}\right) \quad \text{forming} \quad \overline{\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}}\ ;$$

◆ $$E: \quad \psi\left(\square\square\right)\ \chi\left(\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}\right) \quad \text{forming an adjoint representation } (\tilde{\phi})\ ;$$

◆ $$F: \quad \psi\left(\square\square\right)\ \eta\left(\bar{\square}\right) \quad \text{forming} \quad \square \quad (\phi);$$

◆ $$G: \quad \tilde{\eta}\left(\square\right)\ \eta\left(\bar{\square}\right) \quad \text{forming} \quad (\cdot) \quad (\text{singlet})\ .$$

(QCD)

$$A: \quad \frac{2(N^2-4)}{N} - \frac{(N+2)(N-1)}{N} - \frac{(N+2)(N-1)}{N} = -\frac{2(N+2)}{N}\ ;$$

$$B: \quad \frac{2(N+1)(N-4))}{N} - \frac{(N+1)(N-2)}{N} - \frac{(N+1)(N-2)}{N} = -\frac{4(N+1)}{N}\ ;$$

$$C: \quad \frac{(N+1)(N-2))}{N} - \frac{N^2-1}{2N} - \frac{N^2-1}{2N} = -\frac{N+1}{N}\ ;$$

$$D: \quad \frac{3(N+1)(N-3))}{2N} - \frac{N^2-1}{2N} - \frac{(N+1)(N-2)}{N} = -\frac{2N+2}{N}\ ;$$

◆ $$E: \quad N - \frac{(N+2)(N-1)}{N} - \frac{(N+1)(N-2)}{N} = -\frac{N^2-4}{N}\ ;$$

◆ $$F: \quad \frac{N^2-1}{2N} - \frac{N^2-1}{2N} - \frac{(N+2)(N-1)}{N} = -\frac{(N+2)(N-1)}{N}\ ;$$

◆ $$G: \quad 0 - \frac{N^2-1}{2N} - \frac{N^2-1}{2N} = -\frac{N^2-1}{N}\ ,$$

(QCD)

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Bars, Yankielovicz ‘81
Appelquist, Cohen, Schmaltz, Shrock ‘99
Appelquist, Duan, Sannino, ‘00

(i) $(N_\psi, N_\chi) = (1, 0)$ model: a review

$$\psi^{\{ij\}}\,, \qquad \eta_i^B\,, \qquad B = 1, 2, \ldots, N+4$$

$$G = SU(N)_{\rm c} \times SU(N+4)_{\rm f} \times U(1)\,,$$

$$\square\!\square \oplus (N+4)\,\bar{\square}$$

“$\psi\eta$”

(A) Confining, SU(N+4)x U(1) symmetric phase with no condensates (&)

massless baryons ~ $B^{[AB]} = \psi^{ij}\eta_i^A\eta_j^B\,, \qquad A, B = 1, 2, \ldots, N+4$

(&)

	fields	$SU(N)_{\rm c}$	$SU(N+4)$	$U(1)$
UV	ψ	$\square\!\square$	$\frac{N(N+1)}{2}\cdot(\cdot)$	$N+4$
	η^A	$(N+4)\cdot\bar{\square}$	$N\cdot\square$	$-(N+2)$
IR	$B^{[AB]}$	$\frac{(N+4)(N+3)}{2}\cdot(\cdot)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$-N$

(B) CF locked (Higgs) phase

$$\langle\psi^{\{ij}\eta_i^{B\}}\rangle = C\,\delta^{jB}\,, \qquad j, B = 1, 2, \ldots N$$

$$G \to G' = SU(N)_{\rm cf} \times SU(4)_{\rm f} \times U'(1)$$

The anomaly matching **OK,** $\frac{N^2+7N}{2}$ massless baryons

$8N+1$ Nambu-Goldstone

	fields	$SU(N)_{\rm cf}$	$SU(4)_{\rm f}$	$U'(1)$
UV	ψ	$\square\!\square$	$\frac{N(N+1)}{2}\cdot(\cdot)$	1
	η^{A_1}	$\overline{\square\!\square} \oplus \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$N^2\cdot(\cdot)$	-1
	η^{A_2}	$4\cdot\bar{\square}$	$N\cdot\square$	$-\frac{1}{2}$
IR	$B^{[A_1B_1]}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{N(N-1)}{2}\cdot(\cdot)$	-1
	$B^{[A_1B_2]}$	$4\cdot\bar{\square}$	$N\cdot\square$	$-\frac{1}{2}$

- Massless baryons and (NG) bosons in L.E.

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