ATMOSPHERIC NEUTRINO OCTANT

FROM

FLAVOUR SYMMETRY

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REFERENCES

P.H. Frampton,
Second Octet Favored for Non-Maximal θ_{23}.
Mod.Phys.Lett. **A32,** 1750101 (2017).
`arXiv:1707.01383[hep-ph]`.

C.Coriano, P.H. Frampton and P. Santorelli,
Atmospheric Neutrino Octant from Flavour Symmetry.
`arXiv:2305.10463[hep-ph].`

Introduction

In this talk we shall discuss an applicatio of the finite non-abelian group $T^{'}$, of order $g = 24$, to the problem of neutrino mixing between the three neutrinos ν_e, ν_μ, ν_τ in particular to the important unanswered question of whether the atmospheric mixing angle θ_{23} is in the first octant $0 < \theta_{23} < 45^0$ or in the second octant $45^0 < \theta_{23} < 90^0$.

The group $T^{'}$ is often introduced as the double-cover of the group T which is simpler to visualise as the symmetry of a tetrahedron which is easily seen to coincide with the even elements of the symmetric group S_4. The relationship of $T^{'}$ to T as a double-cover is strictly analogous to that of $SU(2)$ to $SO(3)$. There is a subtlety, however, in that the relationship is not one of a subgroup because of the fact that $SO(3) \not\subset SU(2)$ and $T \not\subset T^{''}$.

This subtlety is a global property of these groups which does not interfere with local properties such as the table of Kronecker products as can be seen in multiplication tables for the irreducibl representations of $T(1, 1^{'}, 1^{''}; 3)$ and $T^{'}(1, 1^{'}, 1^{''}; 2, 2^{'}, 2^{''}; 3)$, respectively.

Table 1: Multiplication table for irreducible representations of T

	1	$1^{'}$	$1^{"}$	3
1	1	$1^{'}$	$1^{"}$	3
$1^{'}$	$1^{'}$	$1^{"}$	1	3
$1^{"}$	$1^{"}$	1	$1^{'}$	3
3	3	3	3	$1+1^{'}+1^{"}+3+3$

Table 2: Multiplication table for the irreducible representations of $T^{'}$

	1	$1^{'}$	$1^{"}$	2	$2^{'}$	$2^{"}$	3
1	1	$1^{'}$	$1^{"}$	2	$2^{'}$	$2^{"}$	3
$1^{'}$	$1^{'}$	$1^{"}$	1	$2^{'}$	$2^{"}$	2	3
$1^{"}$	$1^{"}$	1	$1^{'}$	$2^{"}$	2	$2^{'}$	3
2	2	$2^{'}$	$2^{"}$	$1+3$	$1^{'}+3$	$1^{"}+3$	$2+2^{'}+2^{"}$
$2^{'}$	$2^{'}$	$2^{"}$	2	$1^{'}+3$	$1^{"}+3$	$1+3$	$2+2^{'}+2^{"}$
$2^{"}$	$2^{"}$	2	$2^{'}$	$1^{"}+3$	$1+3$	$1^{'}+3$	$2+2^{'}+2^{"}$
3	3	3	3	$2+2^{'}+2^{"}$	$2+2^{'}+2^{"}$	$2+2^{'}+2^{"}$	$1+1^{'}+1^{"}+3+3$

We note that omitting the rows and columns corresponding to doublets from Table(2) leads precisely to Table(1) despite the fact that globally $T^{'} \not\supset T$.
Curiously, it was $T^{'}$ as in Table 2 which was the first of these two attractive flavour symmetries to be used in 1995 in particle theory, at a time when neutrino masses, and hence the PMNS mixing matrix, were not of interest. This makes sense because the doublets of $T^{'}$ are necessary to achieve the correct CKM matrix.

T and T-prime

When neutrino masses and oscillations were established, starting in 1998, the PMNS matrix became the centre of attention For the neutrinos, only triplets and singlets are necessary and so the smaller group T as in Table 1 achieved great success * especially in the hands of Ernest Ma and others.

To accommodate quarks and the CKM matrix it was most useful to extend T of Table 1 to $T^{'}$ of Table 2 because the successes of T could be retained and, by using the $T^{'}$ doublets and a $(2+1)$ family structure, one could the achieve excellent fits to the mixing angles of the CKM matrix by the Chapel Hill group.

*Note that T was often called $A(4)$.

One of the outstanding questions in neutrino physics is the correct octant of the PMNS angle θ_{23} which remains an ambiguity even from the latest experimental data. This can be, and has already been, addressed using the $T^{'}$ flavour symmetry.

Two other outstanding questions in neutrino physics are the hierarchy issue, and the CP violation phase. As far as we can see, both of these pressing issues cannot be resolved even by $T^{'}$ flavour symmetry without more input from experiment. For the latter CP issue, an important follow-up question will be whether the phase is related to the phase necessary for generating matter-antimatter asymmetry in leptogenesis.

The goal of the present talk is to revisit the octant ambiguity discussed in 2017 in the light of much new experimental data which we take from the November 2022 entry of the PDG, and in the light of all the interest in this ambiguity.

There remains the question therefore of an octant prediction from $T^{'}$ using the much more precise recent experimental data on neutrinos, as we could use it as a black-white litmus test. If this prediction is correct, it with bolster our confidence even further in this flavour symmetry. If it were incorrect, it would cast doubt on all the successes with the six PMNS and CKM mixing angles.

We recall that of the many free parameters in the Standard Model, these 6 are successf predicted by $T^{'}$ flavour symmetry while the other 22 free parameters are not yet predicted by anything. Having said that, this impressive flavour symmetry has not so far made any progress with fermion masses, although not from lack of trying.

Comparison to latest neutrino data

The most robust $T^{'}$ prediction, invariant under leptonic CP violation, is this one, relating θ_{13} with θ_{23}:

$$\theta_{13} = \sqrt{2}\left|\frac{\pi}{4} - \theta_{23}\right| \qquad (1)$$

We can evaluate the LHS and RHS of Eq. (1) using the November 2022 data from PDF, as follows. We use the 3σ data.
The LHS is 0.131 for the first octant.
It is 0.148 for the second octant.
The RHS of Eq. (1) is in the range from 0.143 to 0.156.
The unique $T^{'}$ prediction is therefore that θ_{23} is in the second octant. This agrees with the 2017 result which used less precise data.

We should point out that if we restrict attention to only the leptonic sector, Eq. (1) can actually be derived using T flavour symmetry. This is an academic remark, because successfully to include quarks, and the CKM matrix, we must use $T^{'}$ flavour symmetry.

Discussion

Therefore we stick our necks out about the $T^{'}$ flavour symmetry, buoyed by its success with all six of the PMNS and CKM mixing angles. We predict confidently that θ_{23} is in the second octant. If this turns out experimentally to be the correct resolution of the octant ambiguity, we shall gain even more confidence in $T^{'}$. If this prediction is refuted experimentally, we shall admit that $T^{'}$ is fatally flawed, and never discuss it again.

As for the hierarchy problem which depends on the sign of $\triangle m_{23}^2$, all we can say is that most model building leads to a normal hierarchy which leads us to suspect that normal is more likely than the inverted hierarchy. But this is admittedly prejudice and only experiment can resolve this ambiguity by establishing the sign for $\triangle m_{23}^2$.

Regarding the CP-violating phase δ_{CP} in the PMNS matrix, the present prejudice is $\delta_{CP} \simeq -90^0$ although we do not really know this.

Perhaps the deepest question in neutrino physics is whether δ_{CP} is related to δ_{LG}, the CP-violating phase occurring within leptogenesis in the decay of right-handed neutrinos. In a general model, this is not the case. The phase δ_{LG} is all important for establishing the correct mechanism involved in matter-antimatter asymmetry. In one special case (FGY) there is a direct connection between δ_{LG} and δ_{CP}. If this is the case in Nature, the long-baseline neutrino experiments which will measure δ_{CP} are simultaneously shedding light on one of the greatest mysteries in the early universe, matter-antimatter asymmetry.

Establishing such a relationship requires knowledge of the right-handed neutrinos which are possibly super-heavy and this renders establishing such a linkage extremely challenging at least in the foreseeable future. A similar remark is valid for the see-saw model of neutrino masses.

Thank you for your attention