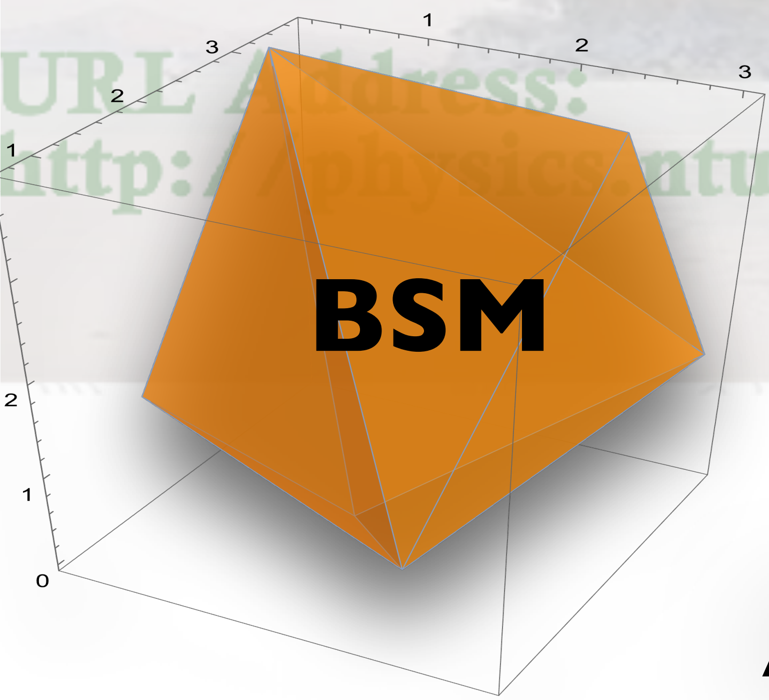
Corfu Summer Institute
on Elementary Particle Physics and Gravity
2023

URL Address:
http://physics.ntua.gr/corfu2023/

Cornering BSMs with Positivity



Alex Pomarol, IFAE & UAB (Barcelona)

based on 2211.12488 [hep-th] with C. Fernandez, F. Riva and F. Sciotti
2307.04729 [hep-th] with T. Ma and F. Sciotti

Motivation

E

Effective Field Theory (EFT)

good tool to describe exp. data!

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UV completion?

more ambitious goal!

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☞ **BSMs** { Dark Matter, Axions, Composite Higgs } → UV completion?

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more ambitious goal!

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tackle by demanding **Positivity bounds**

(⊕ Lorentz, analyticity, locality, crossing)

☞ It has been shown in many recent examples that they can provide very **powerful** constraints

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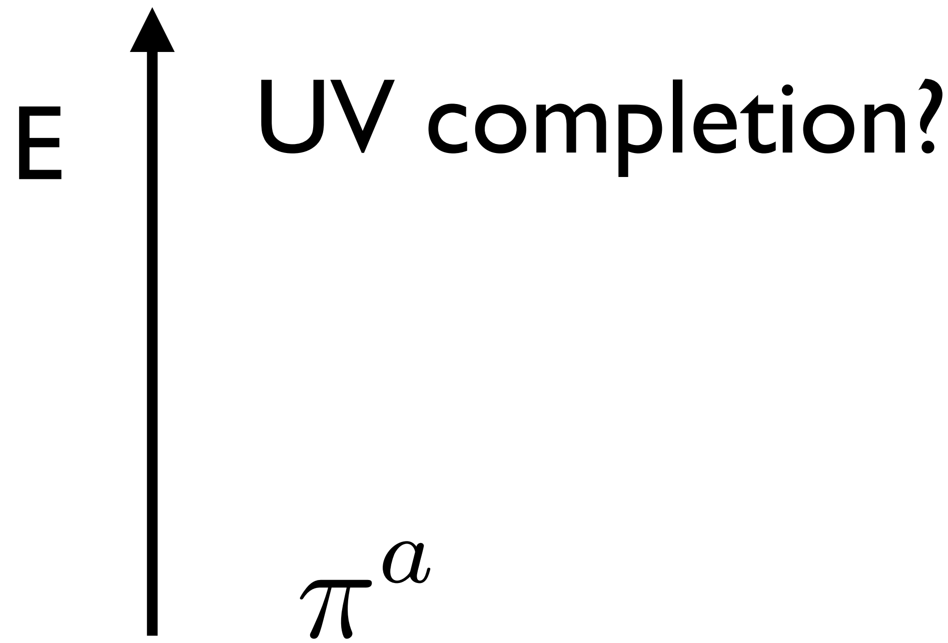
UV completion for a theory of pions



UV completion for a theory of pions

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UV completion?

π^a

simplifying assumption:

☞ weakly-coupled theories (tree-level)

UV completion for a theory of pions

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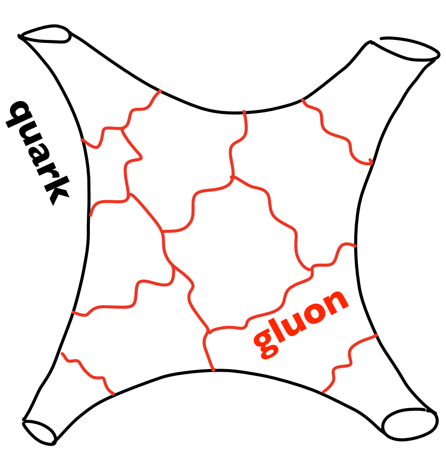
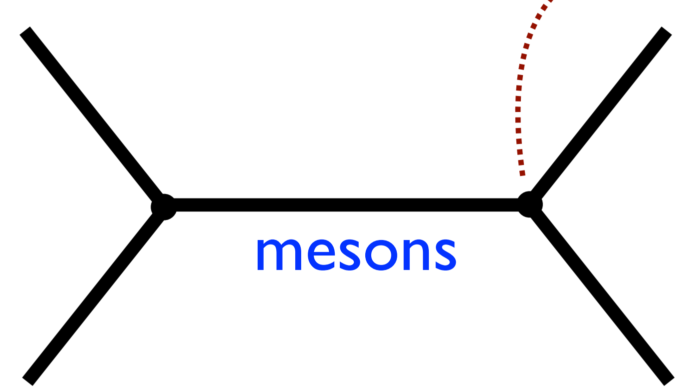
Also aiming **strongly-coupled gauge theories** (QCD) in the **large-N_c** limit:

quarks, gluons
$SU(N_c)$

$N_c \to \infty$

mesons ($q\bar{q}$ states), glueballs





$g \sim 1/\sqrt{N_c}$

+ other channels

Positivity bounds

N. Arkani-Hamed, T.-C. Huang, and Y.-T. Huang, arXiv: 2012. 15849

C. de Rham, S. Melville, A. J. Tolley, and S.-Y. Zhou, arXiv: 1702.06134

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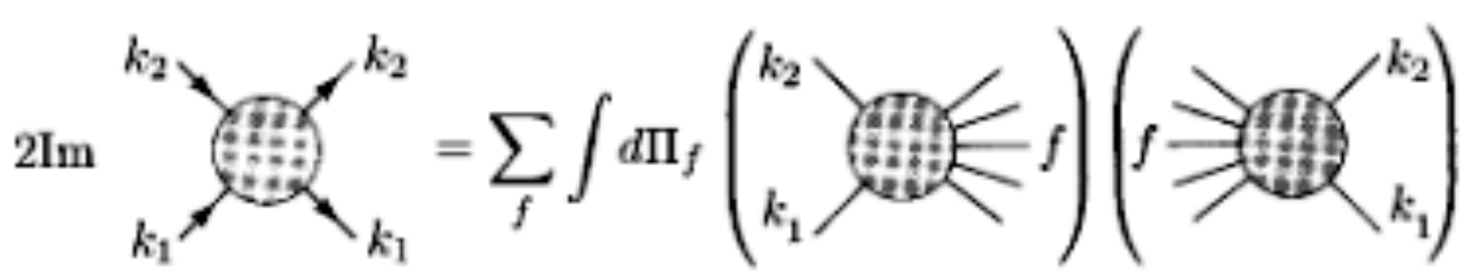
and much more…

- **Generalizations of the optical theorem**

forward limit:



≥ 0

Peskin & Schroeder

Positivity bounds

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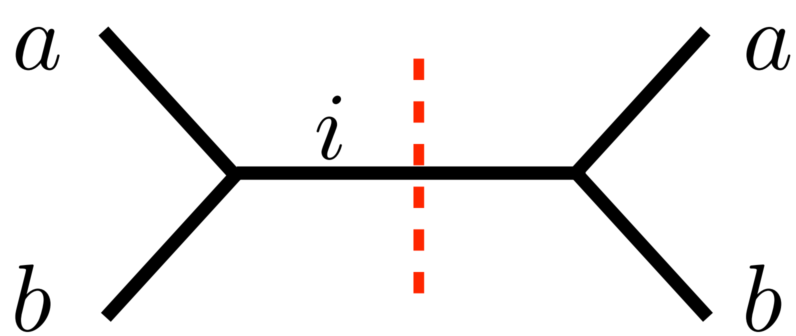
- **Generalizations of the optical theorem:**

forward limit (tree-level):



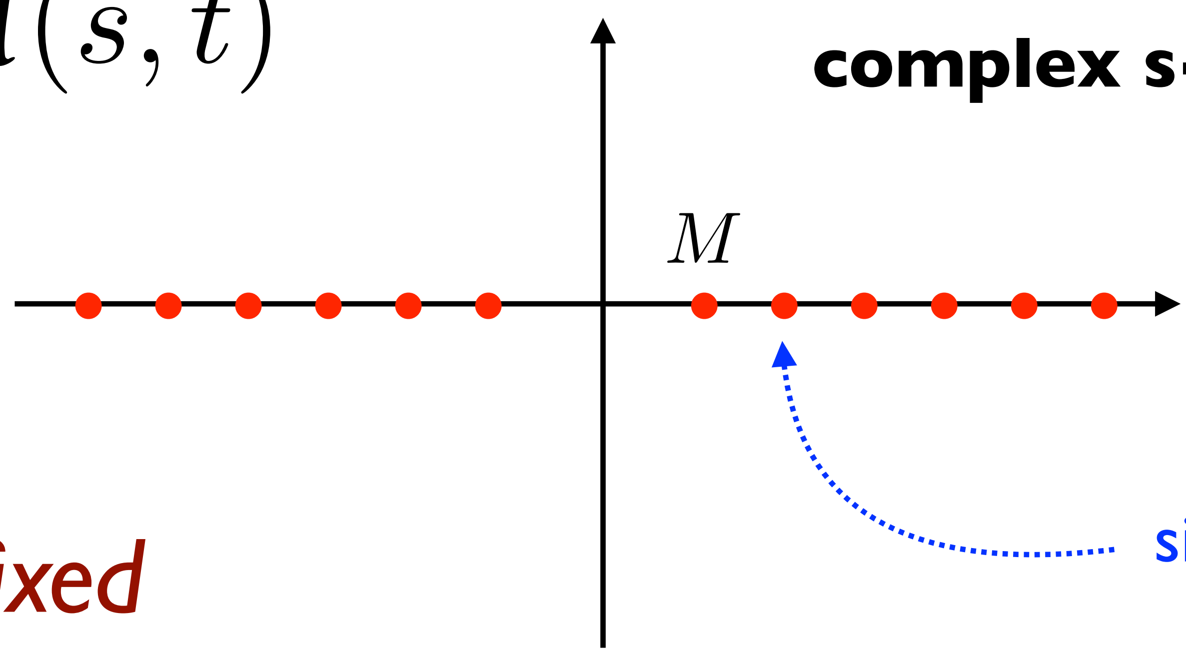
$$\propto \quad |g_{abi}|^2 \quad \geq 0$$

Positivity bounds on (tree-level mediated) amplitudes

Analytical structure of amplitudes:

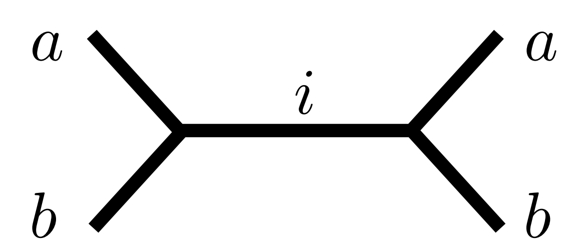
$$\mathcal{M}(s,t)$$

complex s-plane



u fixed

simple poles due to states in the s-channel:



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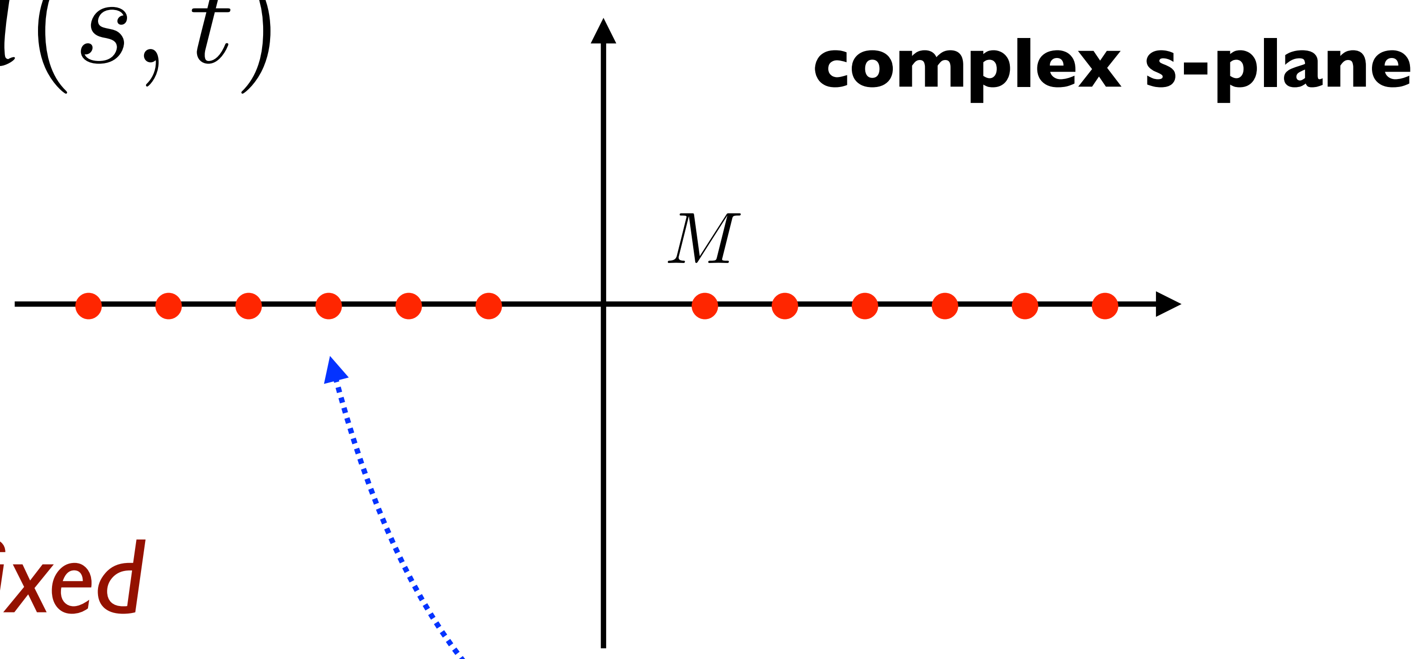
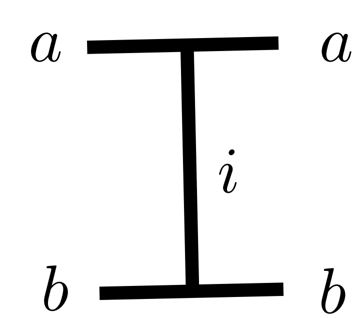


u fixed

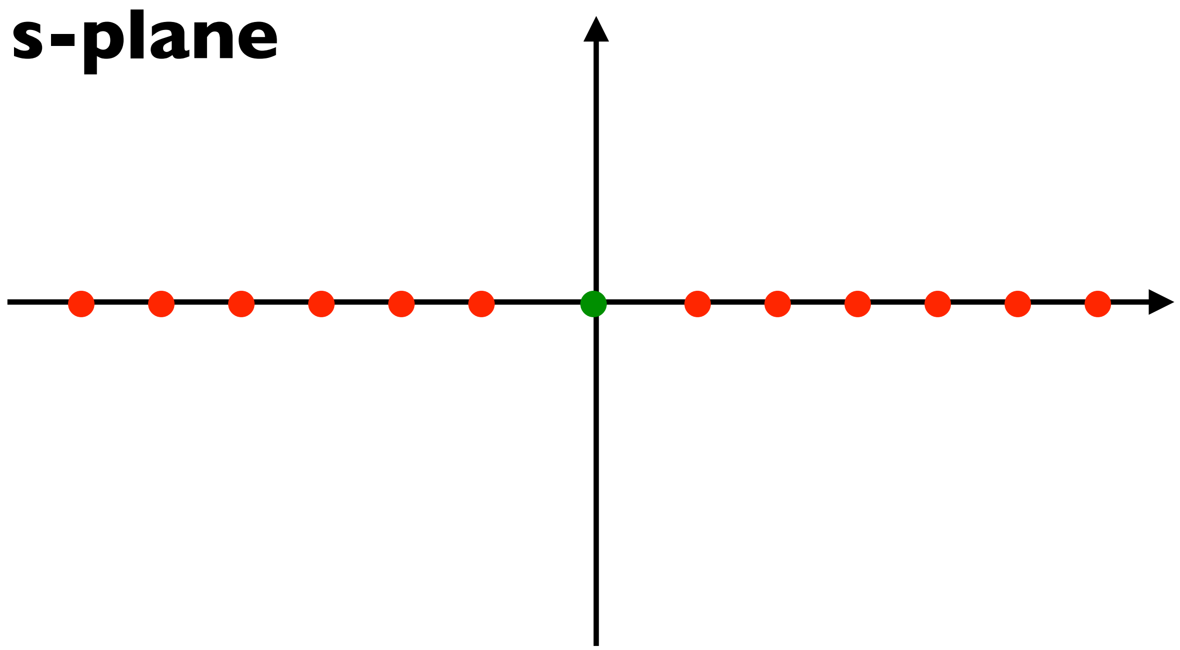
$s = -t-u-4m^2$

simple poles due to states in the t-channel:



Positivity bounds on (tree-level mediated) amplitudes

This simple structure allows to get dispersion relations:



$$\frac{\mathcal{M}(s,t)}{s^{k+1}}$$

$k = 1, 2, 3, \ldots$

u fixed

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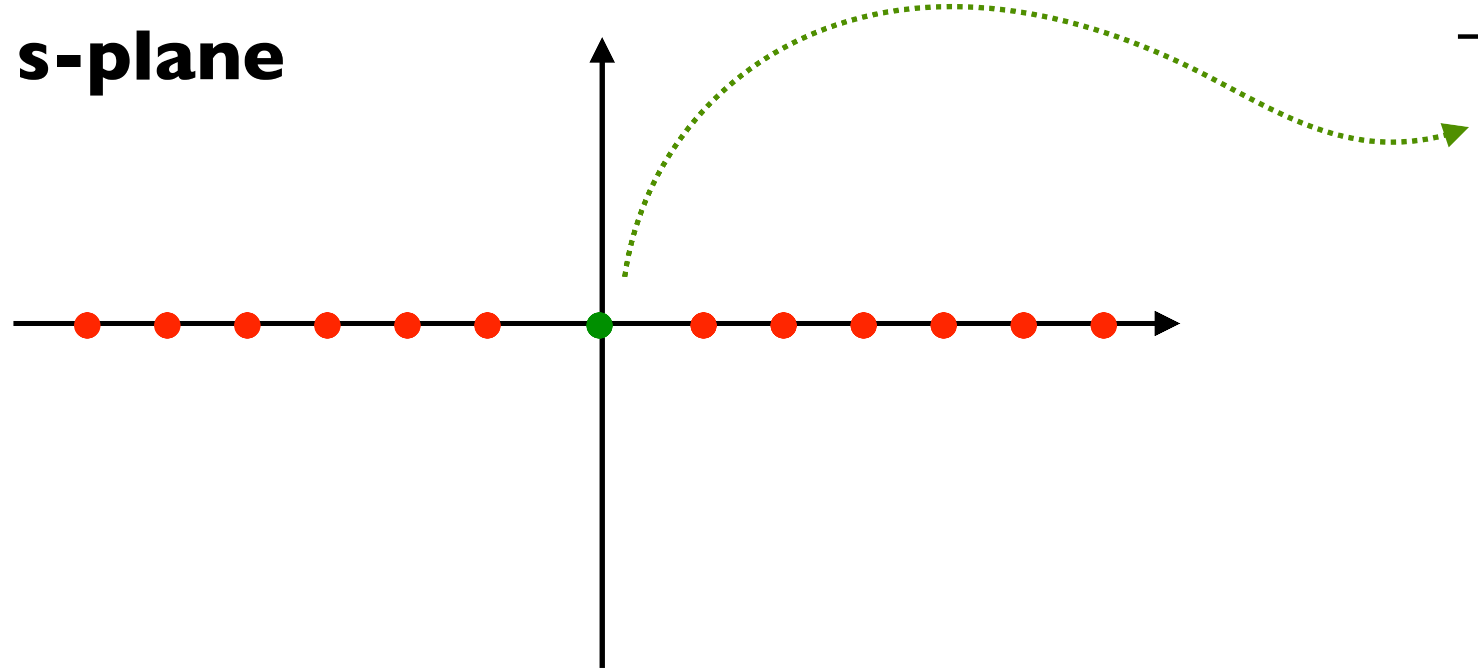

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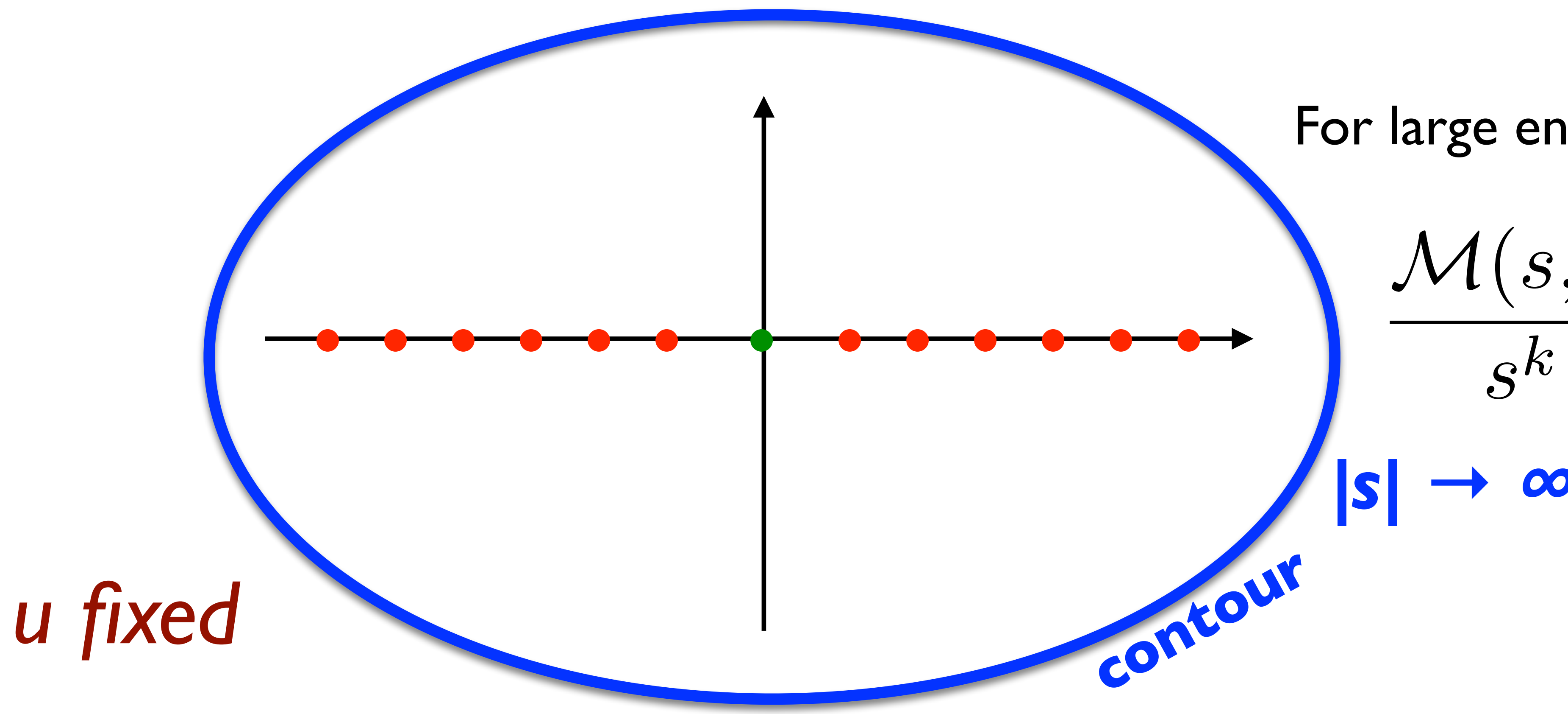


For large enough k, we have:

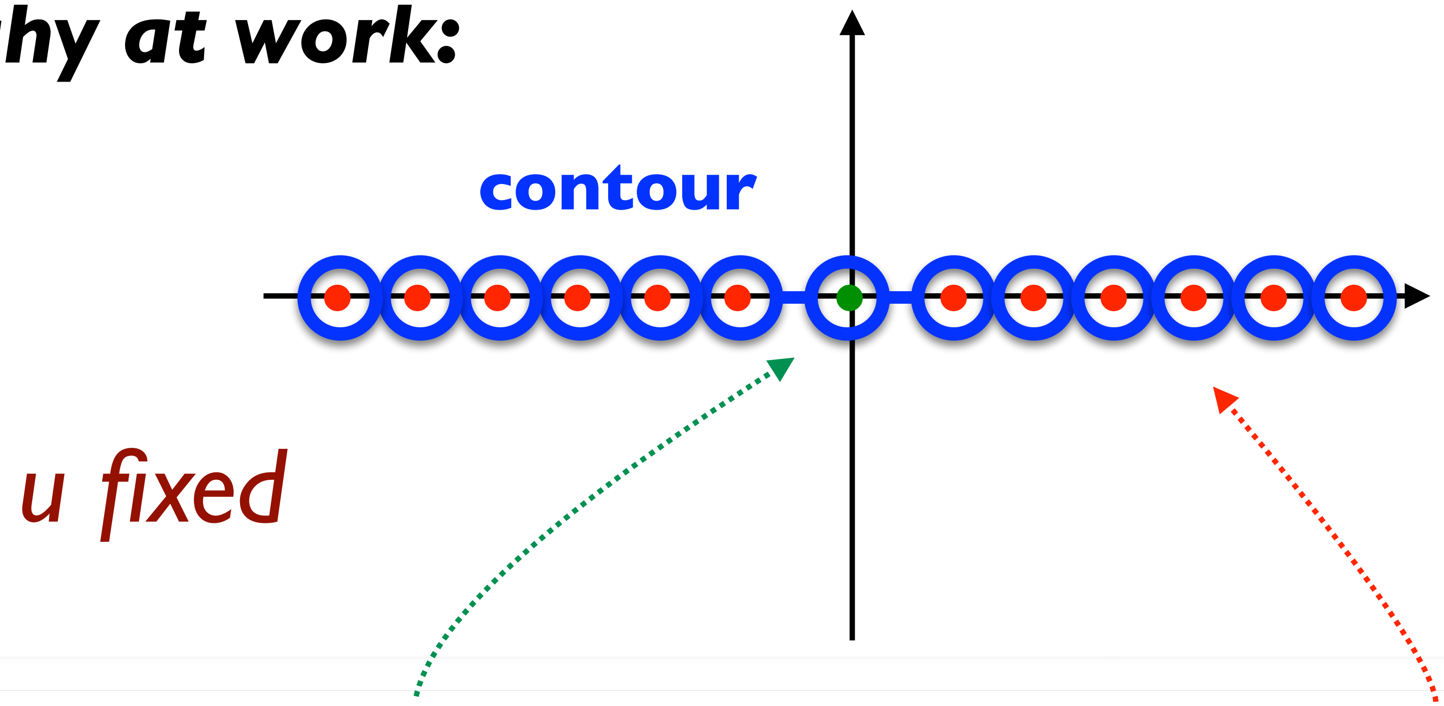
$$\frac{\mathcal{M}(s,t)}{s^k} \underset{|s|\to\infty}{\longrightarrow} 0$$

☞
$$\oint \frac{\mathcal{M}(s,t)}{s^{k+1}} = 0$$

Positivity bounds on (tree-level mediated) amplitudes

This simple structure allows to get dispersion relations:

Cauchy at work:



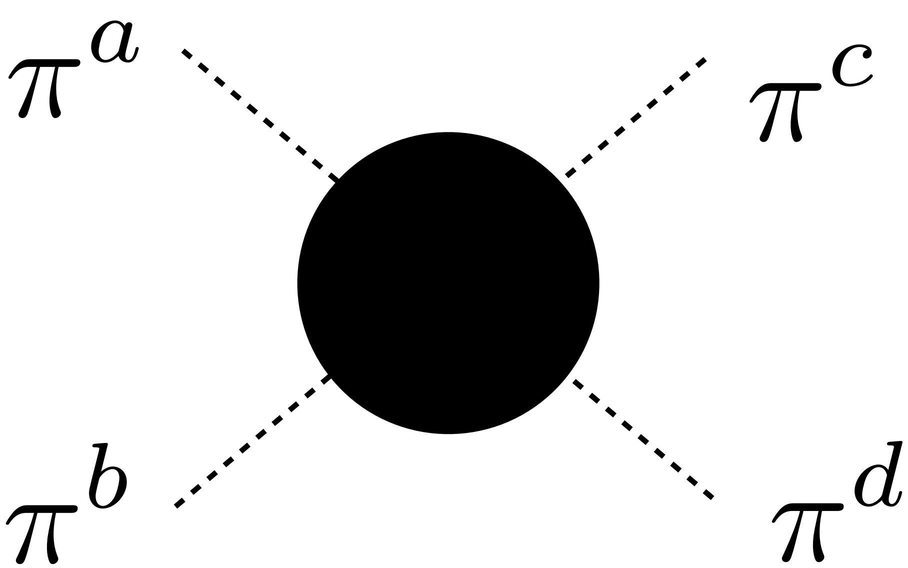
residue at the origin + sum of residues at the mass poles = 0

(low-energy EFT parameters **related to** *masses and couplings of mesons)*

pion-pion scattering

J. Albert and L. Rastelli, arXiv: 2203.11950

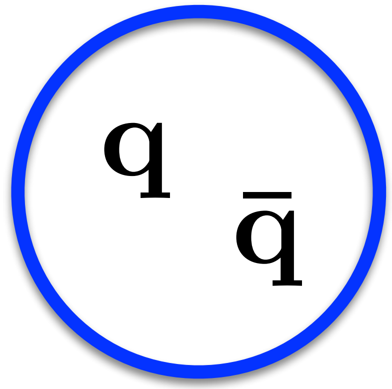
Lets assume an **SU(2)** (isospin) global symmetry



$\pi^a \in \mathbf{3}$ *massless*

Goldstones from
SU(2)⊗SU(2)→SU(2)

Extra condition from large-N_c:

Mesons = 

Isospin = $I = 1/2 \otimes 1/2 = 0, 1$

☞ **no I =2 states**

Extra condition from large-N$_c$:

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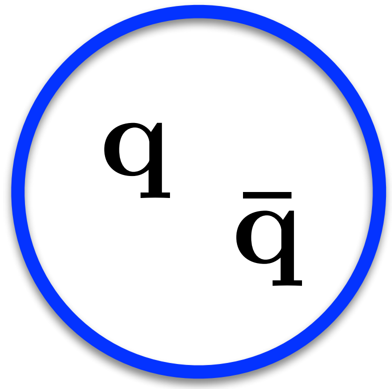

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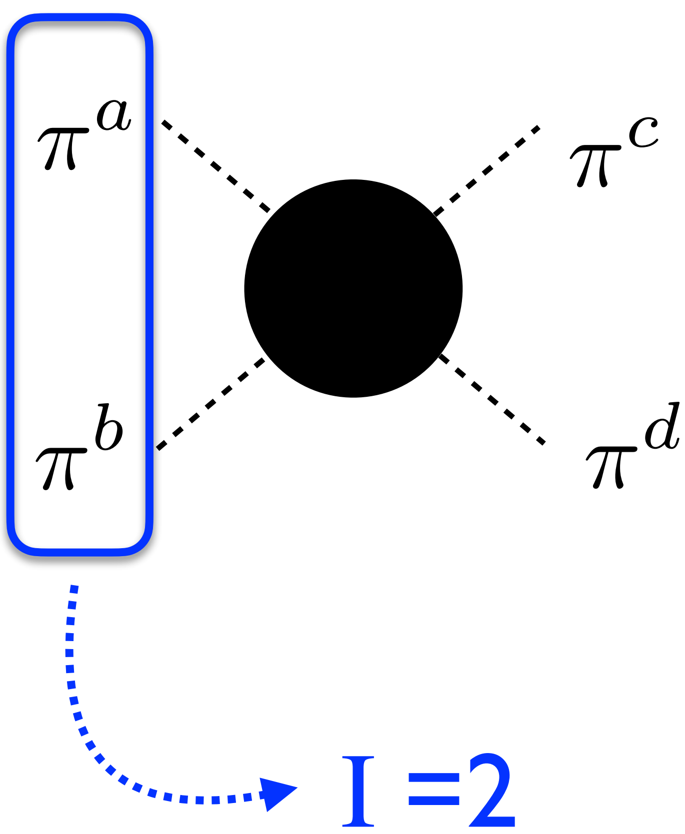
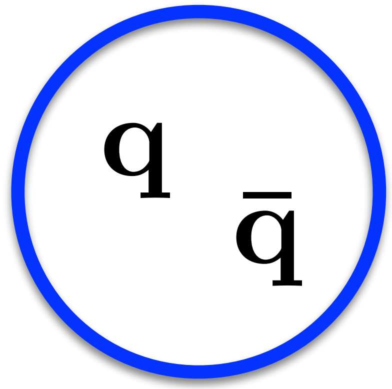


$\mathcal{M}_s^{I=2}$ cannot have poles in s

Extra condition from large-N_c:

Mesons = 

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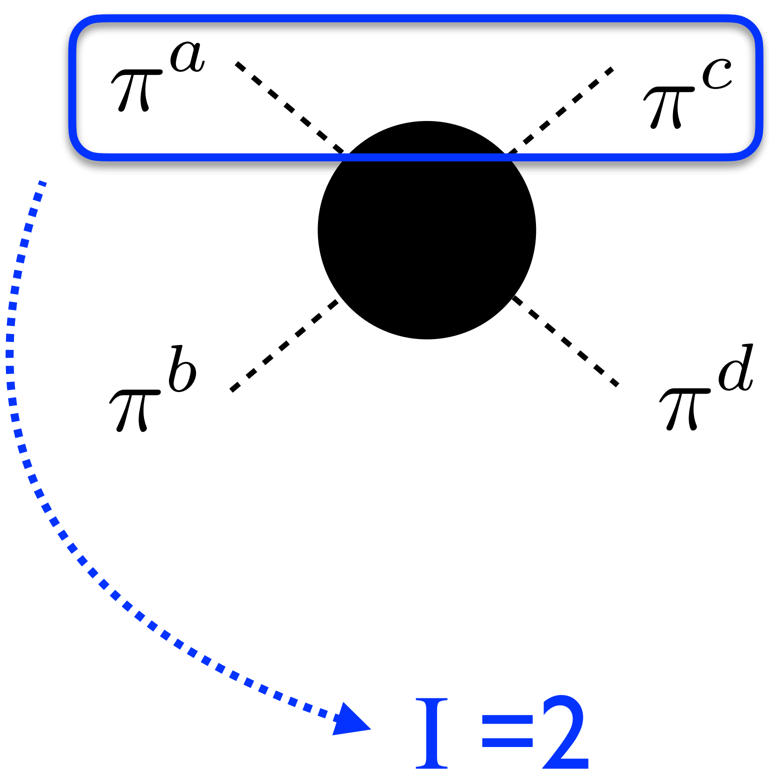


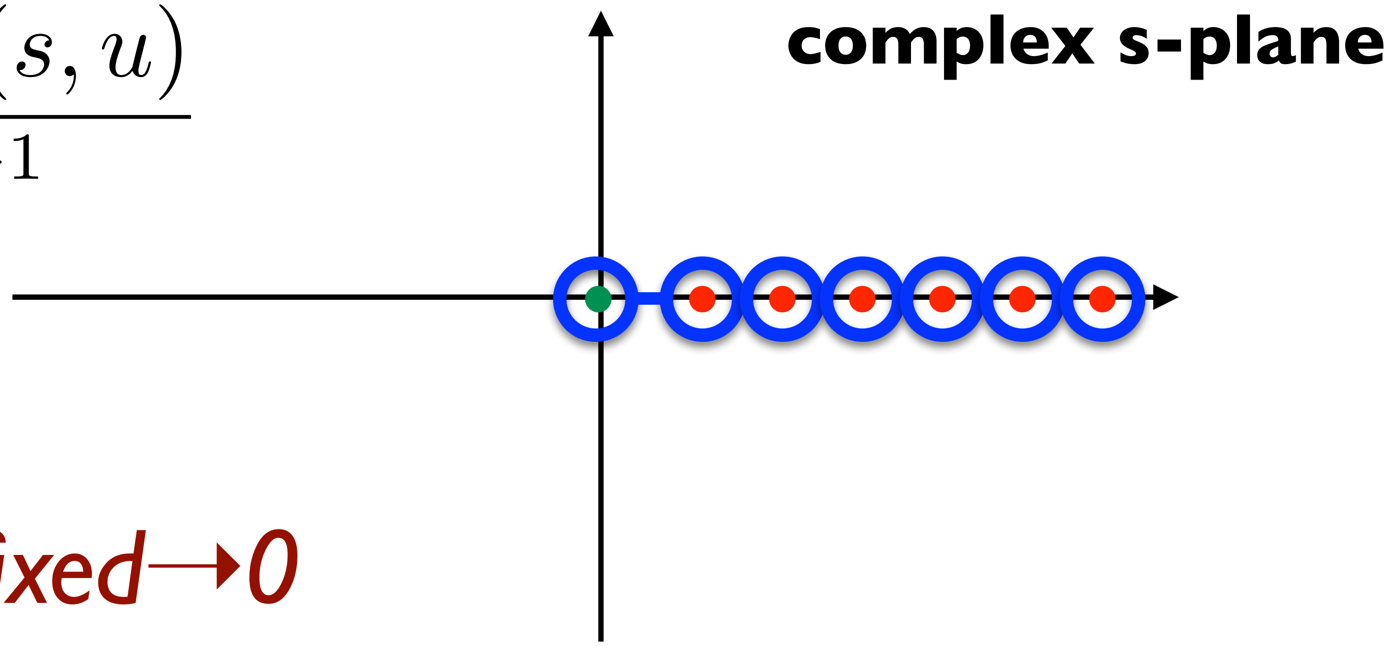
$\mathcal{M}_t^{I=2}$ cannot have poles in t

Working with $\mathcal{M}_t^{I=2}(s,u)$

(that cannot have poles in the t-channel)

crossing s↔u invariant

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}}$$



u fixed→0

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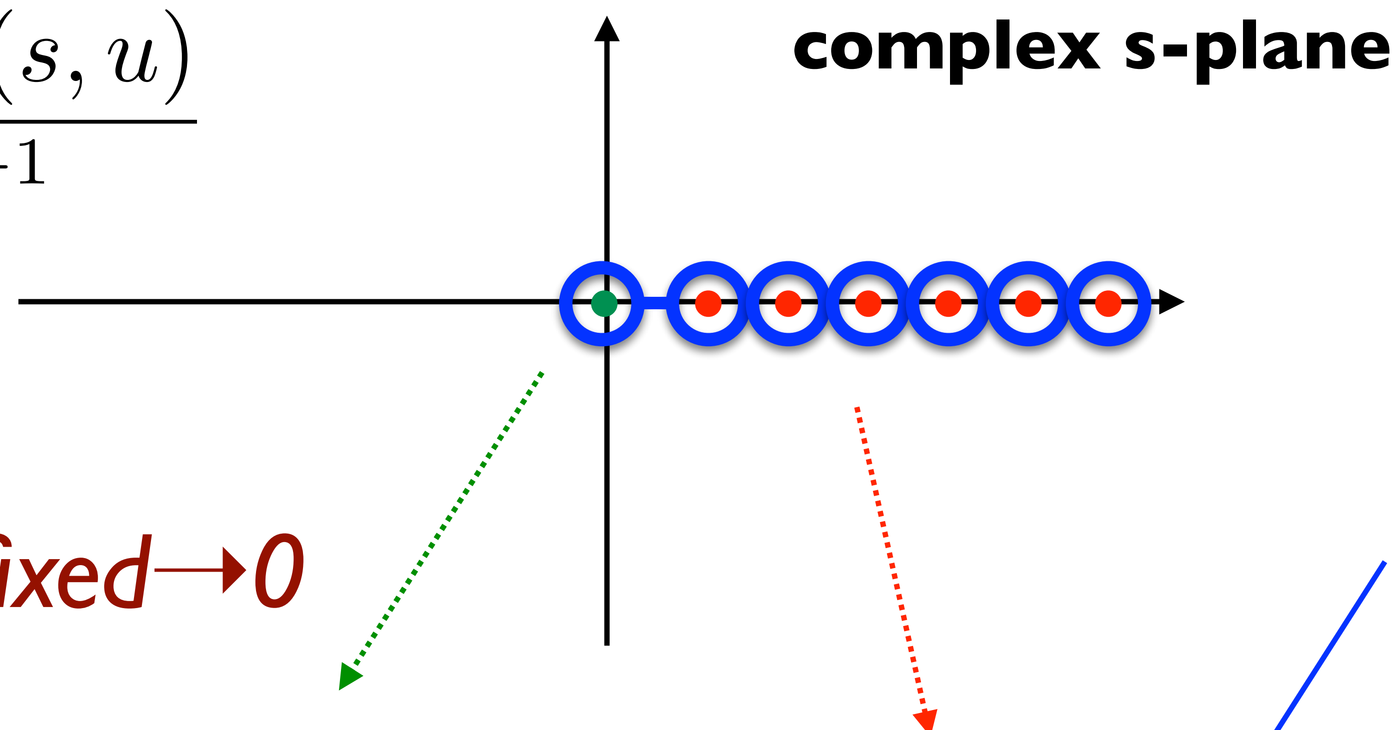

u fixed→0

partial-wave expansion

$$\mathrm{Res}\frac{\mathcal{M}_t^{I=2}(s,u)}{s^{k+1}} = \sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^{2k}} P_{J_i}\left(1+\frac{2u}{m_i^2}\right)$$

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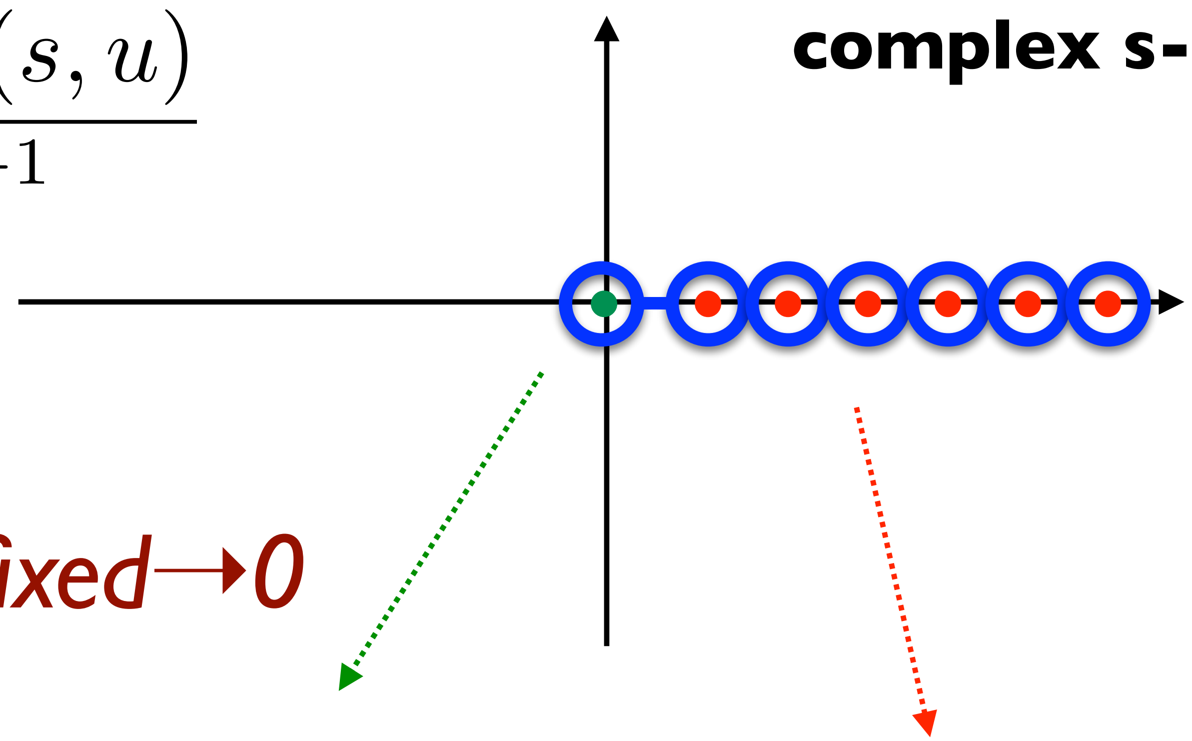

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Wilson coefficients

$$\mathcal{M}_t^{I=2}(s,u) \xrightarrow{s,u\to 0} g_{1,0}(s+u) + g_{2,0}(s^2+u^2) + g_{2,1}su + \cdots$$

Legendre pol. and derivatives (all positive!)

small u expansion:

$$k=1: \qquad g_{1,0}+g_{2,1}u+g_{3,1}u^2+\ldots = \sum_i |g_{\pi\pi\,i}|^2\left(\frac{P_{J_i}(1)}{m_i^2}+2\frac{P'_{J_i}(1)}{m_i^4}u+2\frac{P''_{J_i}(1)}{m_i^6}u^2+\ldots\right),$$

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$$\begin{cases} g_{n,0} = \sum_i \frac{g_{i\pi\pi}^2}{m_i^{2n}} \\ g_{n+1,1} = \sum_i \frac{g_{i\pi\pi}^2\, J_i(J_i+1)}{m_i^{2(n+1)}} \end{cases}$$

all states contribute positively!

small u expansion:

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due to crossing, overconstrained system!

☞ **infinite constraints in the spectrum and couplings**

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e.g. $$\sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3)=0$$

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☞ **also from dispersion relations at fixed t**

Implications of Positivity bounds

Lets assume at $|s| \to \infty$ & either t or u fixed:

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s} \to 0$$

k=1

Infinite set of Sum Rules:

$$\sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{10}} J_i(J_i-1)(J_i+1)(J_i+2)(J_i^2+J_i-15) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{14}} J_i(J_i-2)(J_i-1)(J_i+1)(J_i+2)(J_i+3)(J_i^2+J_i-28) = 0$$

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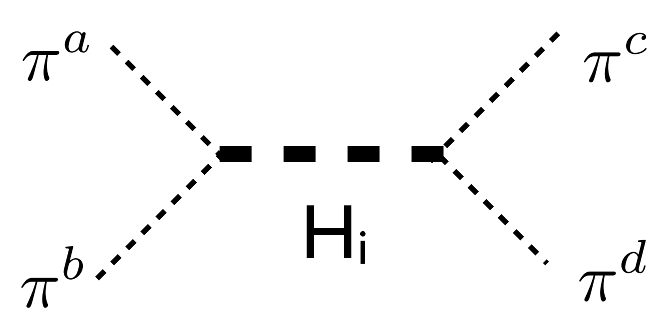
⋮

No constraints for $J_i = 0$ states

➥ **possible UV completion:**

Theory of Scalars (*Higgs mechanism*)



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spin-1 must be in the spectrum with the largest coupling

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☞ **Vector Meson Dominance** (VMD), assumed in the past to explain QCD experimental data

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$$\vdots$$

spin-2 must be in the spectrum

Infinite set of Sum Rules:

$$\sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{10}} J_i(J_i-1)(J_i+1)(J_i+2)(J_i^2+J_i-15) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{14}} J_i(J_i-2)(J_i-1)(J_i+1)(J_i+2)(J_i+3)(J_i^2+J_i-28) = 0$$

$$\vdots$$

spin-3 must be in the spectrum

Infinite set of Sum Rules:

$$\sum_i \frac{|g_{\pi\pi\,i}|^2}{m_i^6} J_i(J_i+1)(J_i-2)(J_i+3) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{10}} J_i(J_i-1)(J_i+1)(J_i+2)(J_i^2+J_i-15) = 0$$

$$\sum_i \frac{|g_{\pi\pi i}|^2}{m_i^{14}} J_i(J_i-2)(J_i-1)(J_i+1)(J_i+2)(J_i+3)(J_i^2+J_i-28) = 0$$

⋮

↓

non-scalar UV completions require **all spin states**
with couplings to pions decreasing with J

From the constraints, we find numerically (~50 constraint, J_{max}~1000):

Upper bound on couplings

(normalized to m_i^2/F_π^2)

J	$\|g_{\pi\pi i}\|^2$
1	0.78
2	0.18
3	0.03

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(normalized to m_i^2/F_π^2)

J	$\lvert g_{\pi\pi i}\rvert^2$	Exp. QCD
1	0.78	→ 0.5
2	0.18	→ 0.18
3	0.03	

Constraints on Wilson coefficients

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1 \mathrm{Tr}^2\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_2 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right) + L_3 \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U \partial_\nu U^\dagger \partial^\nu U\right)$$

$$\mathbf{e}^{\mathbf{i}\,\sigma^{\mathbf{a}}\pi^{\mathbf{a}}/\mathbf{F}_\pi}$$

Constraints on Wilson coefficients

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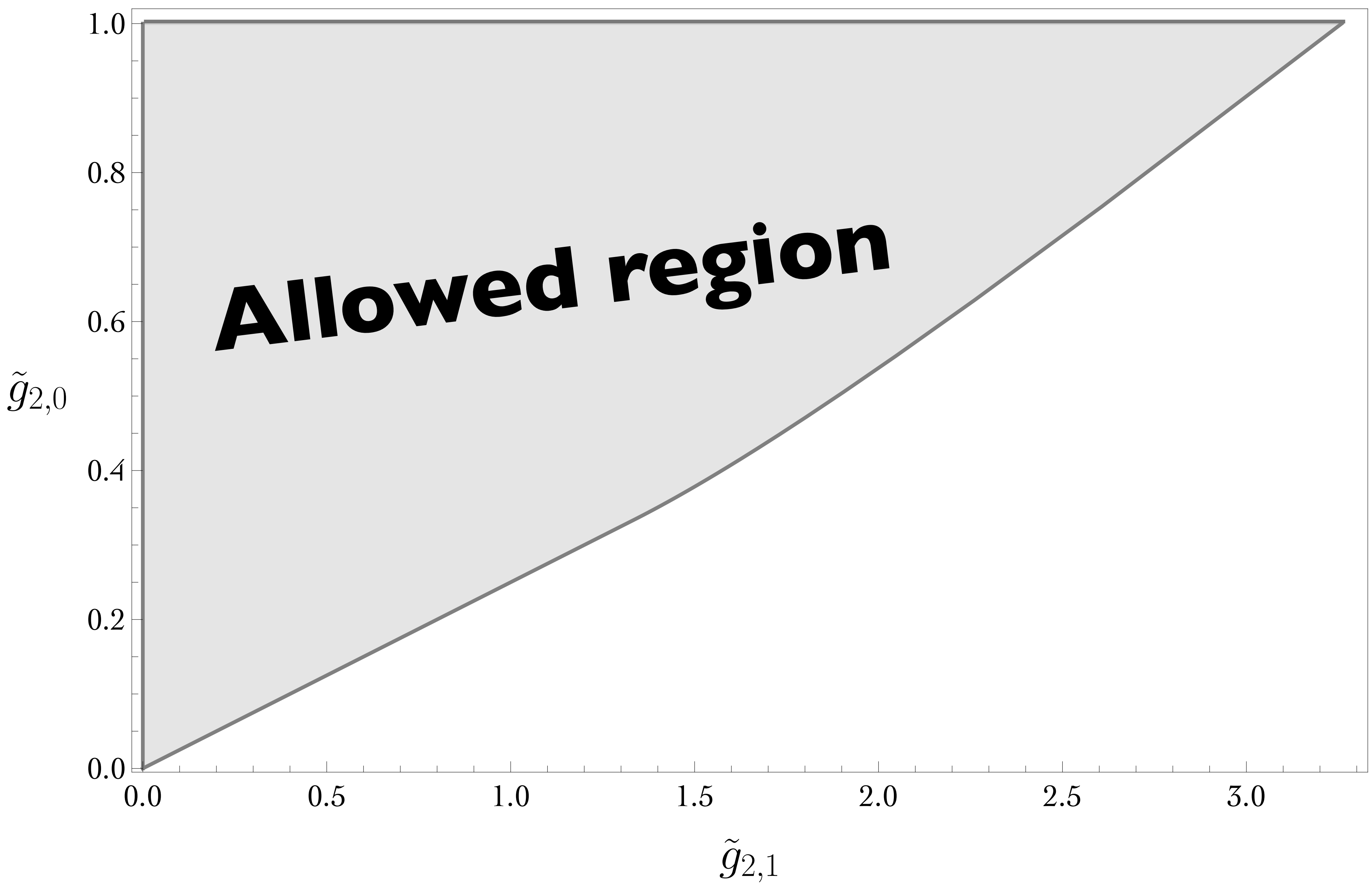
O(s²): $\tilde{g}_{2,0} = 4(2L_1 + 3L_2 + L_3)\frac{M^2}{F_\pi^2}$, $\tilde{g}_{2,1} = 16 L_2 \frac{M^2}{F_\pi^2}$

mass of the 1st meson



"Polyhedronal" bounds

EFTs are "**EFT-hedron**"

Constraints on Wilson coefficients

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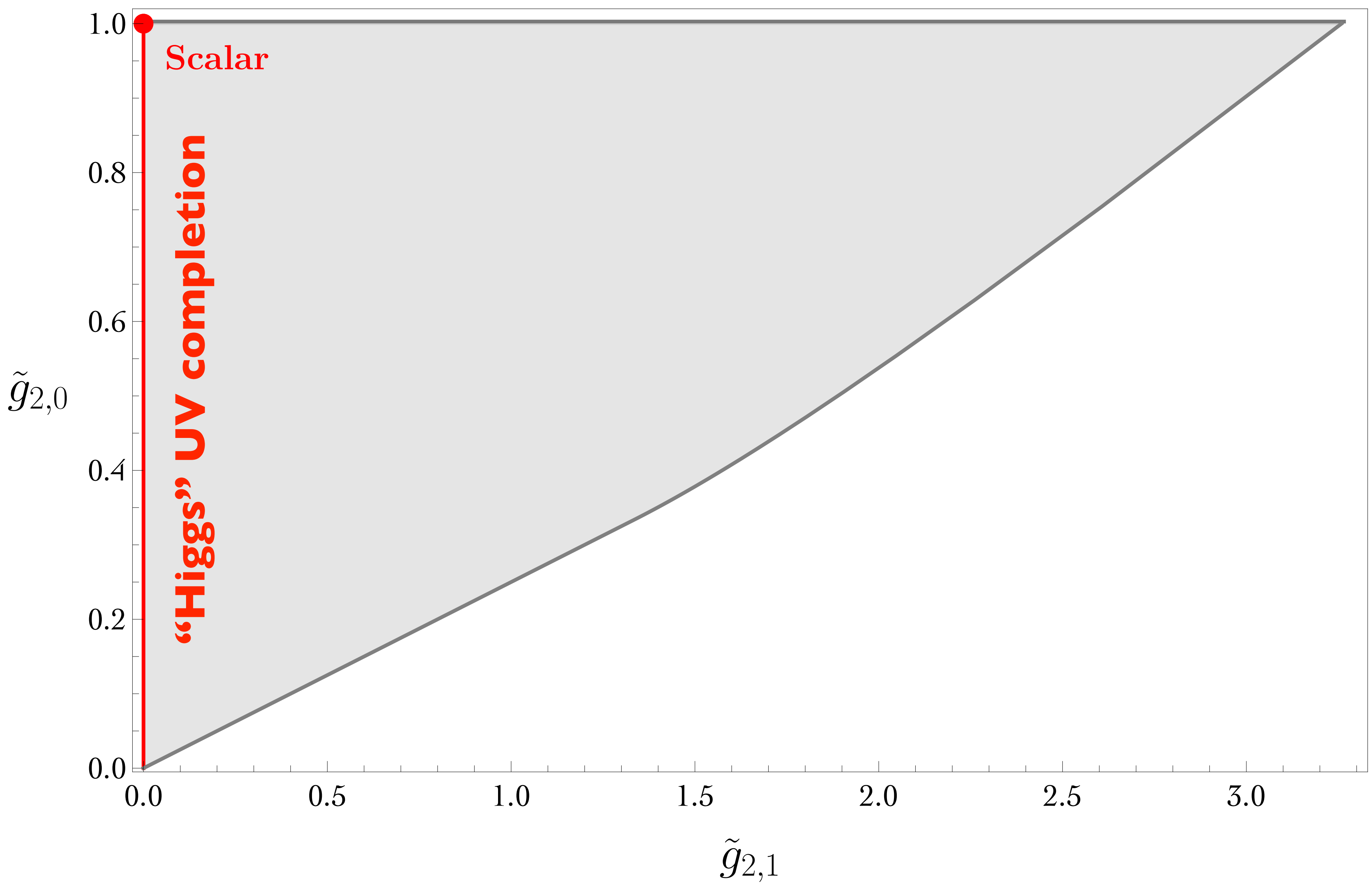

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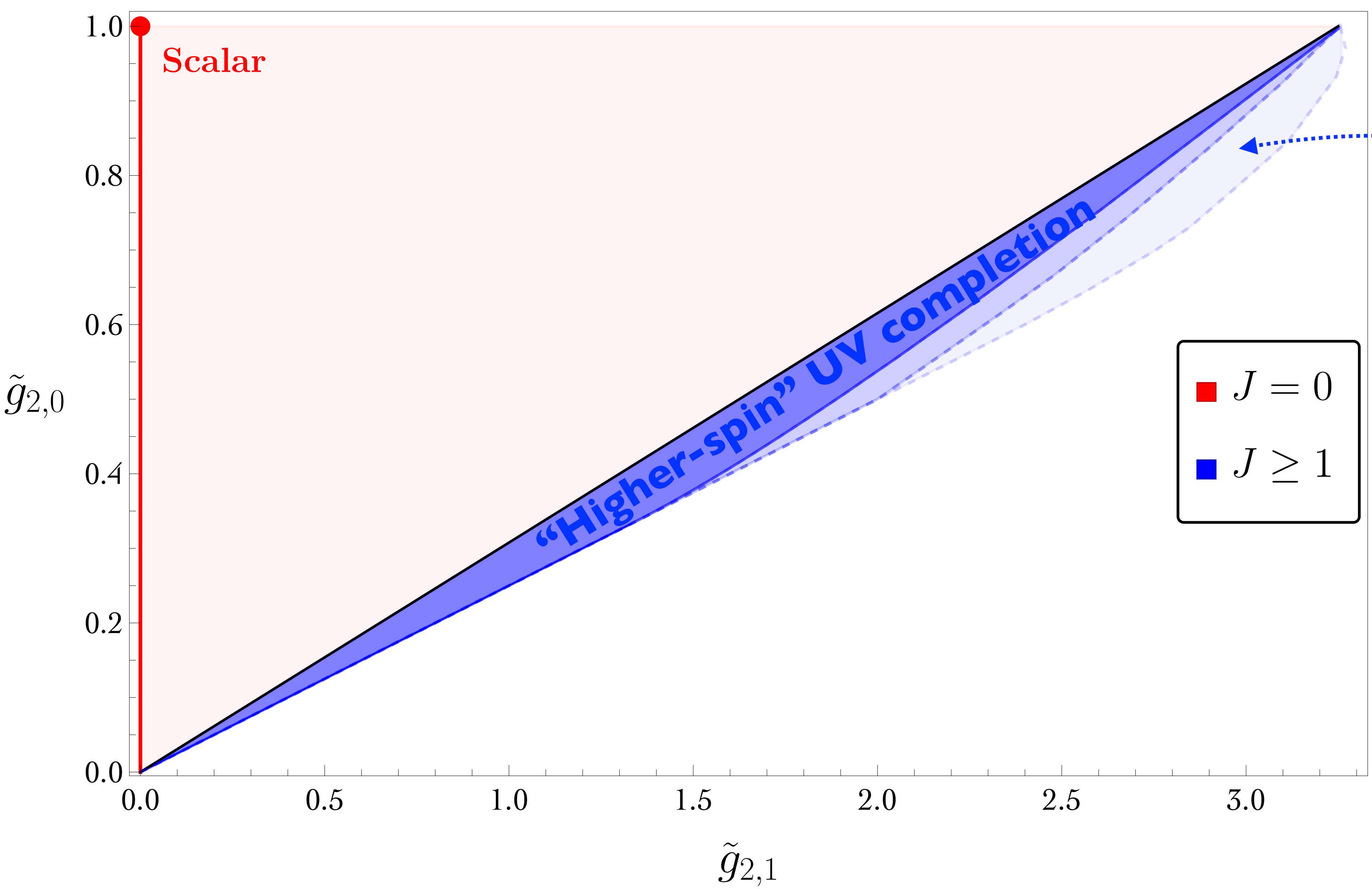

obtained numerically

Constraints on Wilson coefficients

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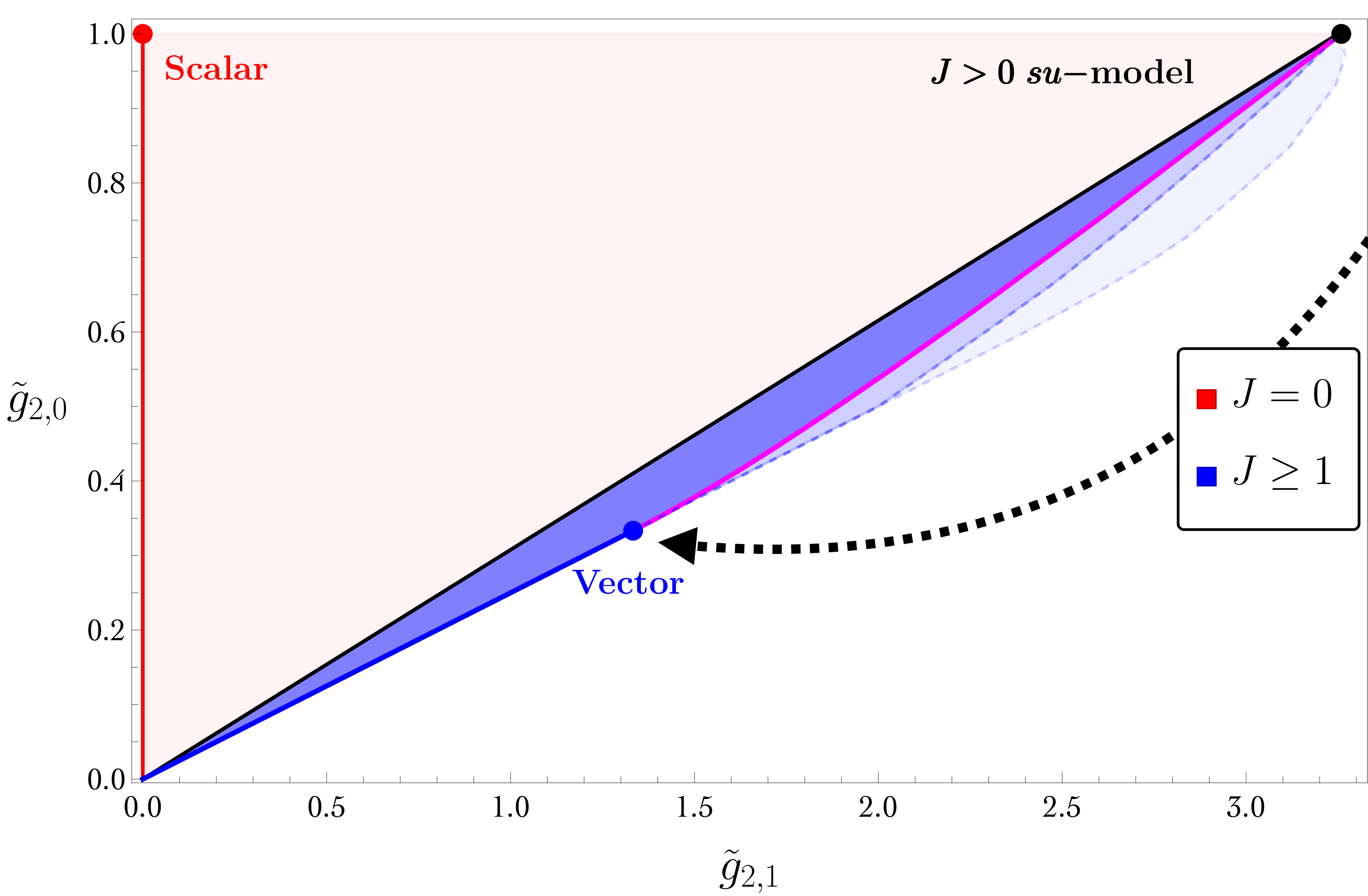

spin-1 model

Constraints on Wilson coefficients

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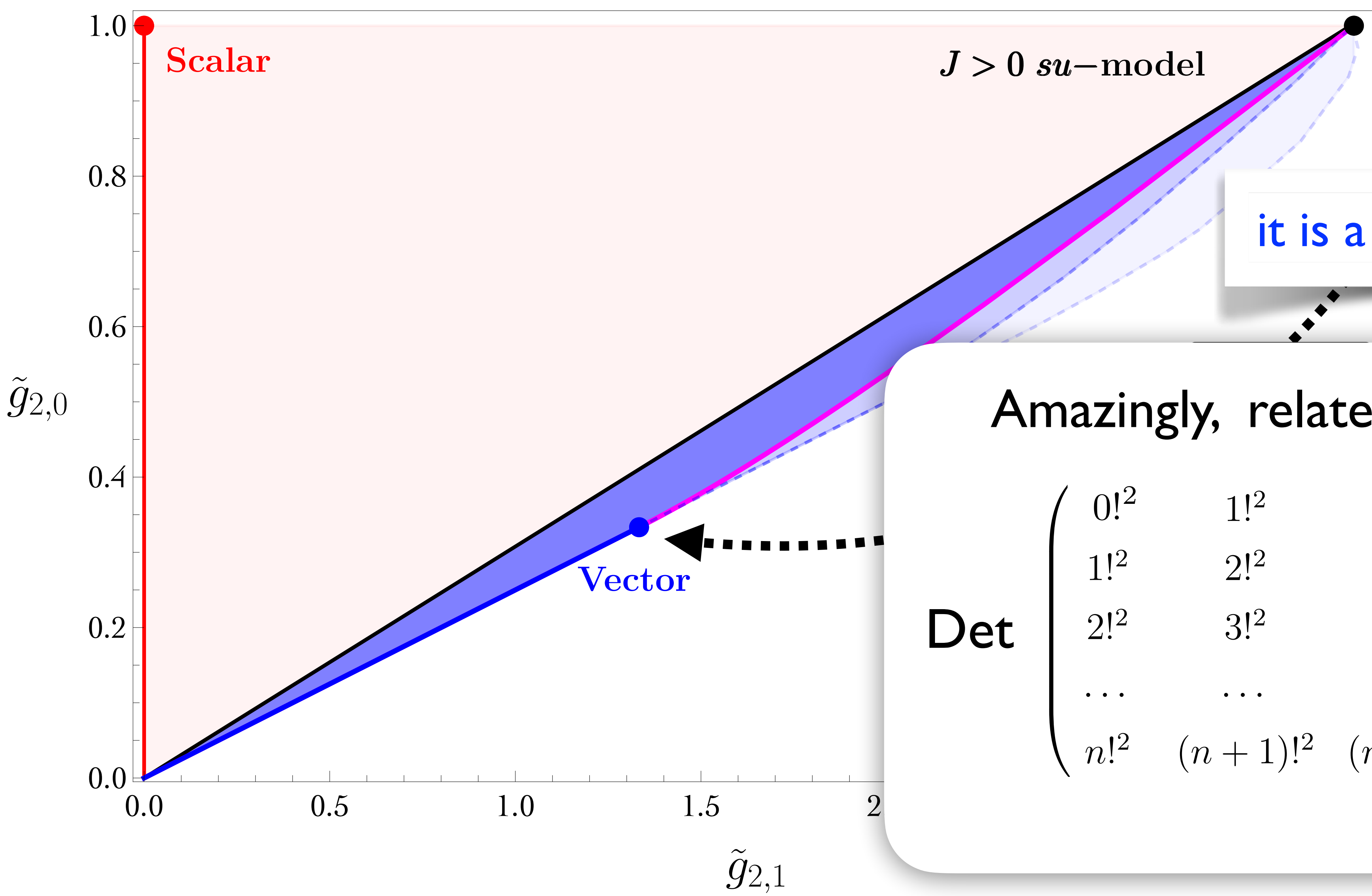

spin-1 model

it is a kink of the boundary

Amazingly, related to the fact that

$$\mathrm{Det}\begin{pmatrix} 0!^2 & 1!^2 & 2!^2 & \cdots & n!^2 \\ 1!^2 & 2!^2 & 3!^2 & \cdots & (n+1)!^2 \\ 2!^2 & 3!^2 & 4!^2 & \cdots & (n+2)!^2 \\ \cdots & \cdots & \cdots & \ddots & \vdots \\ n!^2 & (n+1)!^2 & (n+2)!^2 & \cdots & (2n)!^2 \end{pmatrix} = 0$$

Constraints on Wilson coefficients

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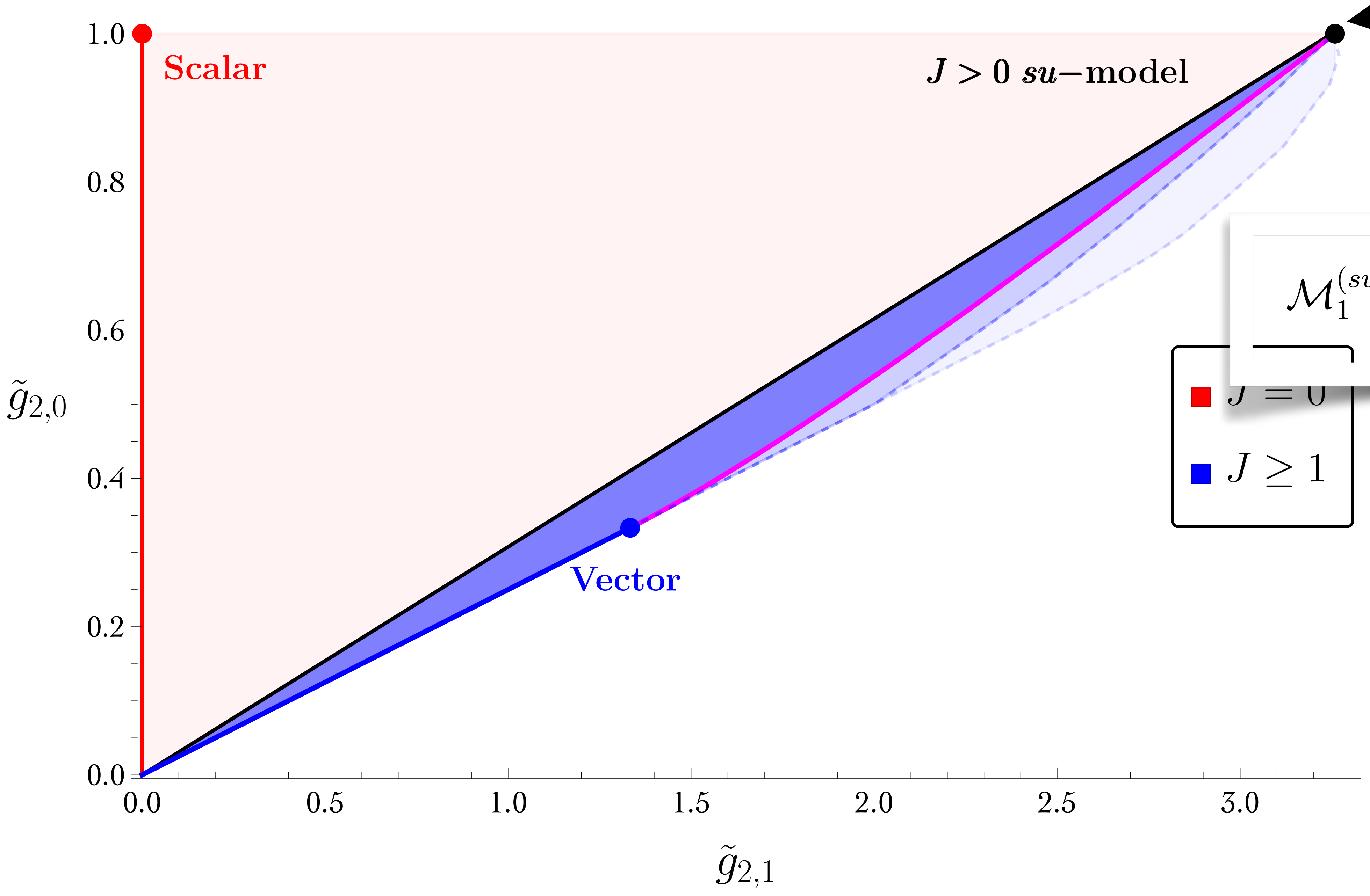

the other kink:

$$\mathcal{M}_1^{(su)}(s,u) = \frac{m^2(s+u) + \lambda s u}{(s-m^2)(u-m^2)}$$

J=1,2,3,... states of equal mass

Constraints on Wilson coefficients

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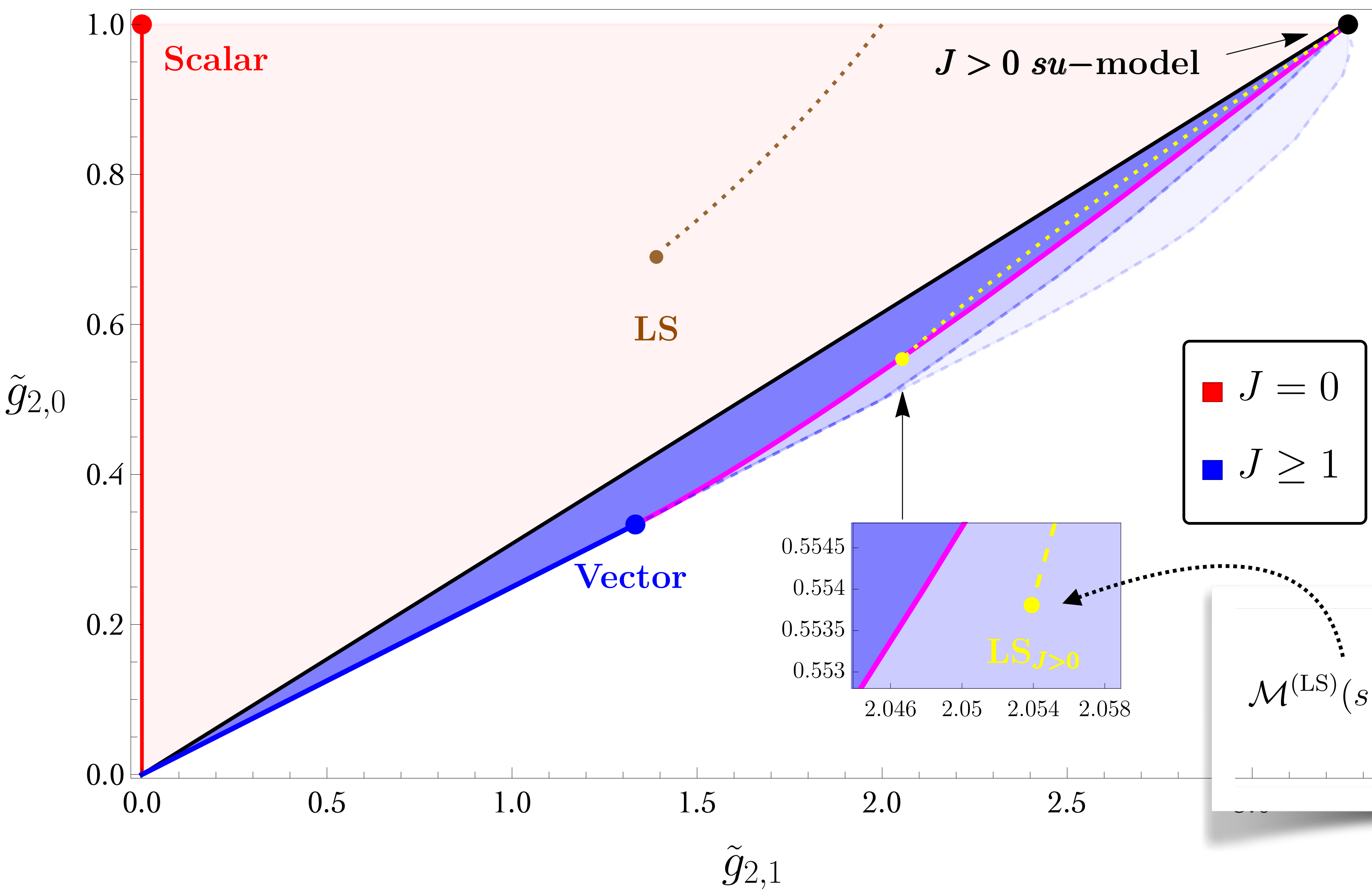
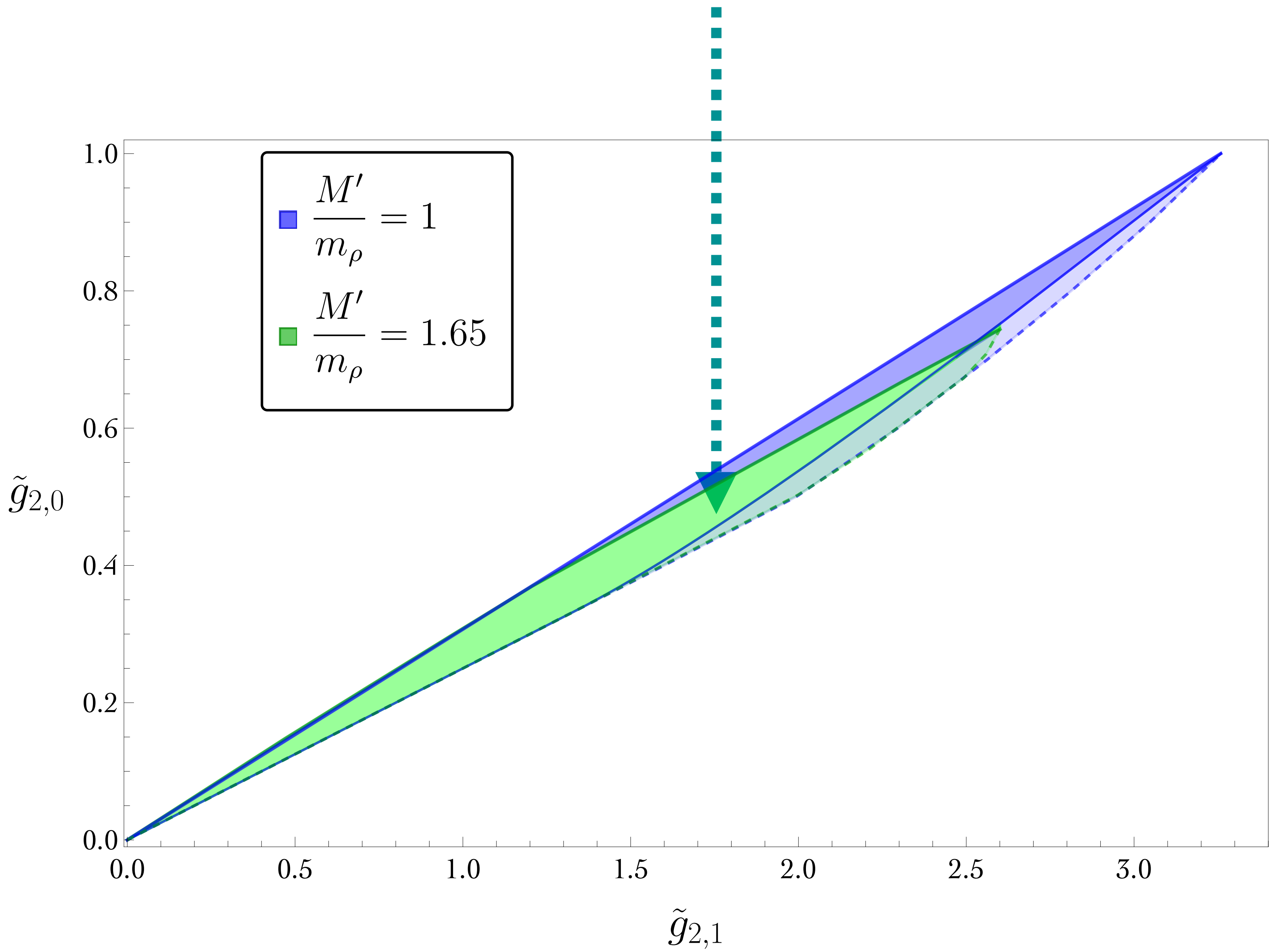


Lovelace-Shapiro:

$$\mathcal{M}^{(\mathrm{LS})}(s,u) = \frac{\Gamma\left(\frac{1}{2} - \frac{s}{2m_\rho^2}\right)\Gamma\left(\frac{1}{2} - \frac{u}{2m_\rho^2}\right)}{\Gamma\left(\frac{t}{2m_\rho^2}\right)}$$

$m_n^2 = m_\rho^2(2n+1)\,, \quad n = 0, 1, 2, \ldots$

stronger bounds if we assume that, as in QCD, J>1 mesons are heavier



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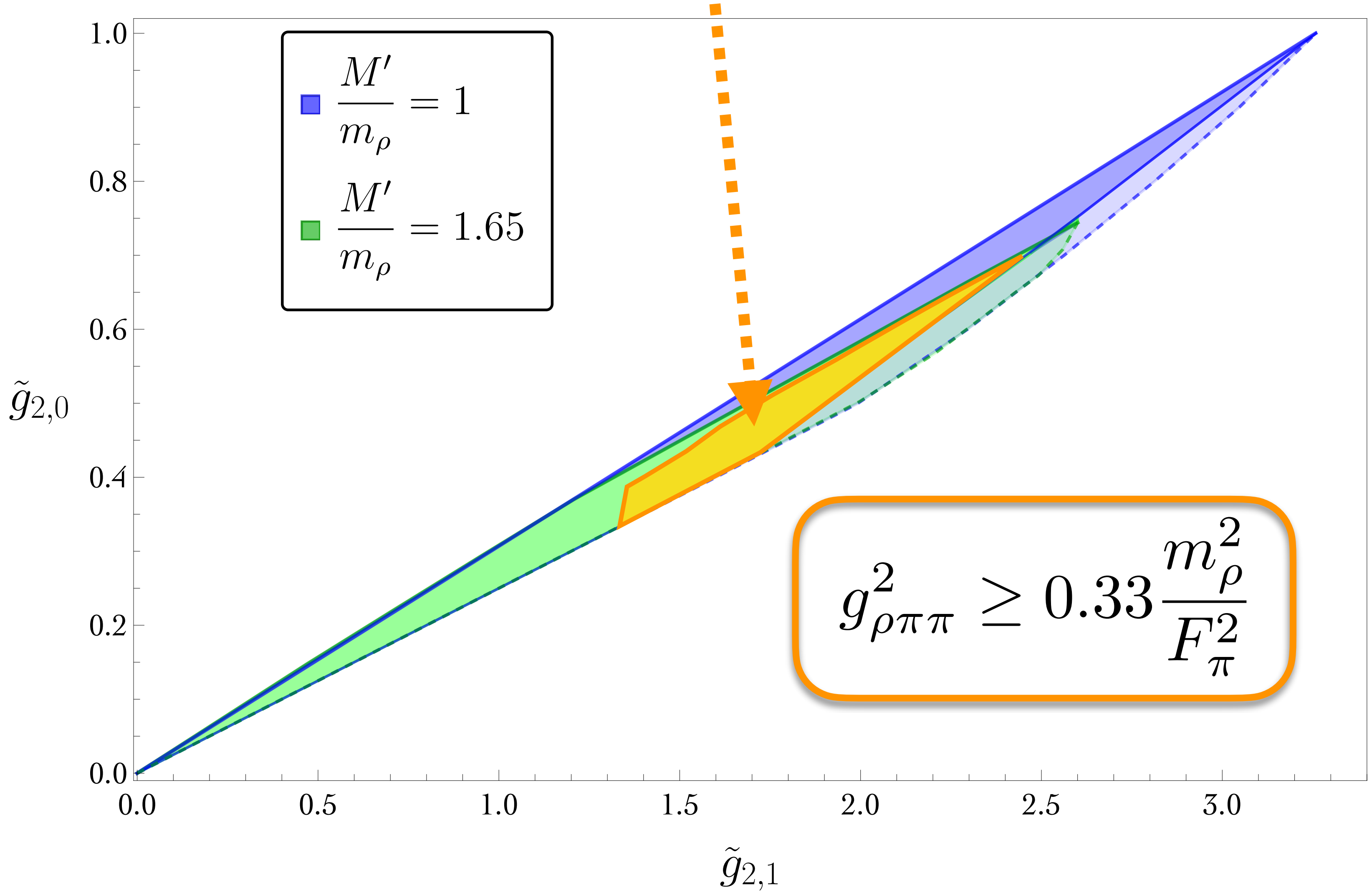
+ the spin-1 meson, the ρ, has a non-zero coupling to ππ



$$g^2_{\rho\pi\pi} \geq 0.33 \frac{m^2_\rho}{F^2_\pi}$$

stronger bounds if we assume that, as in QCD, J>1 mesons are heavier

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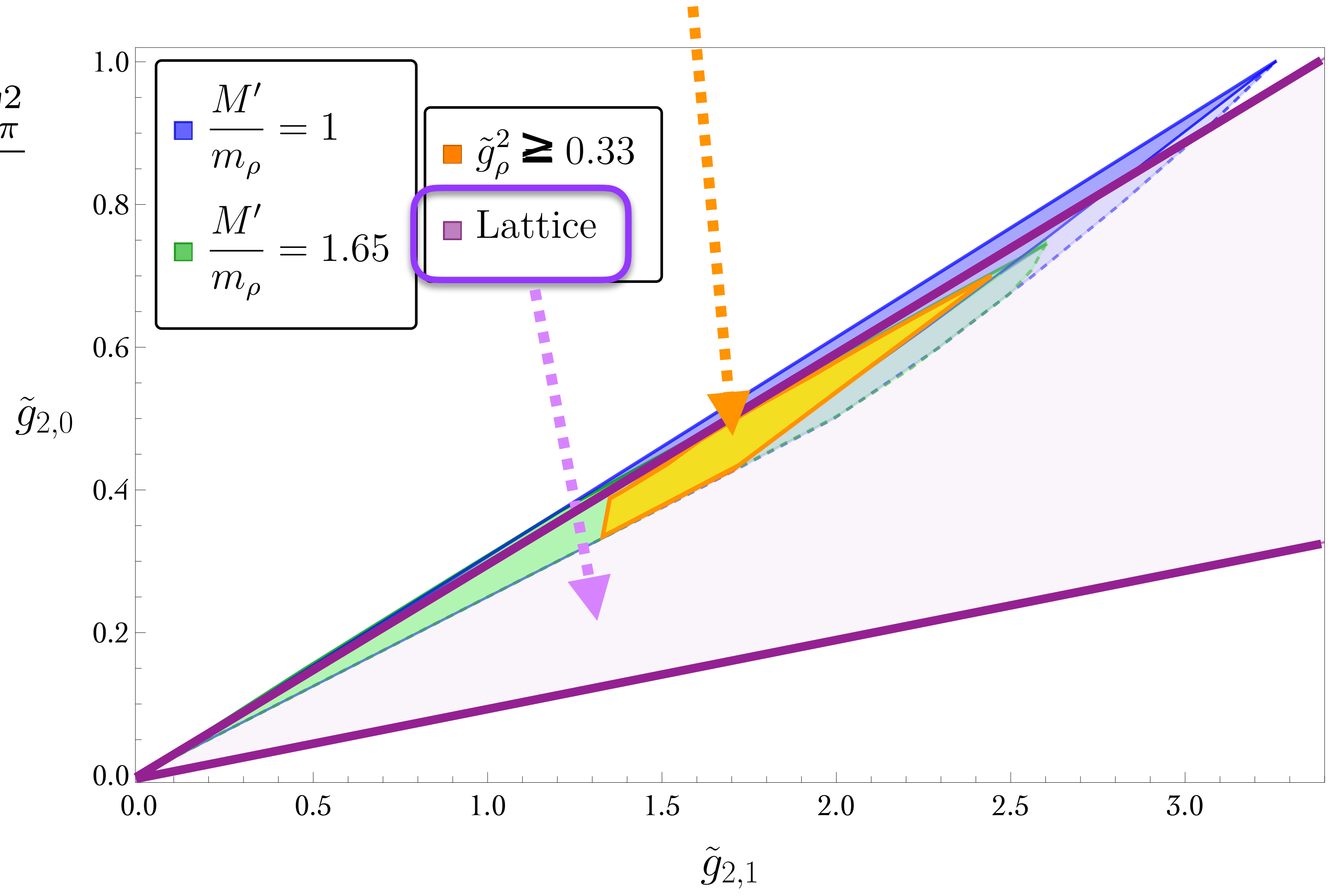
$$\tilde{g}_\rho^2 \equiv \frac{g_{\rho\pi\pi}^2 F_\pi^2}{m_\rho^2}$$



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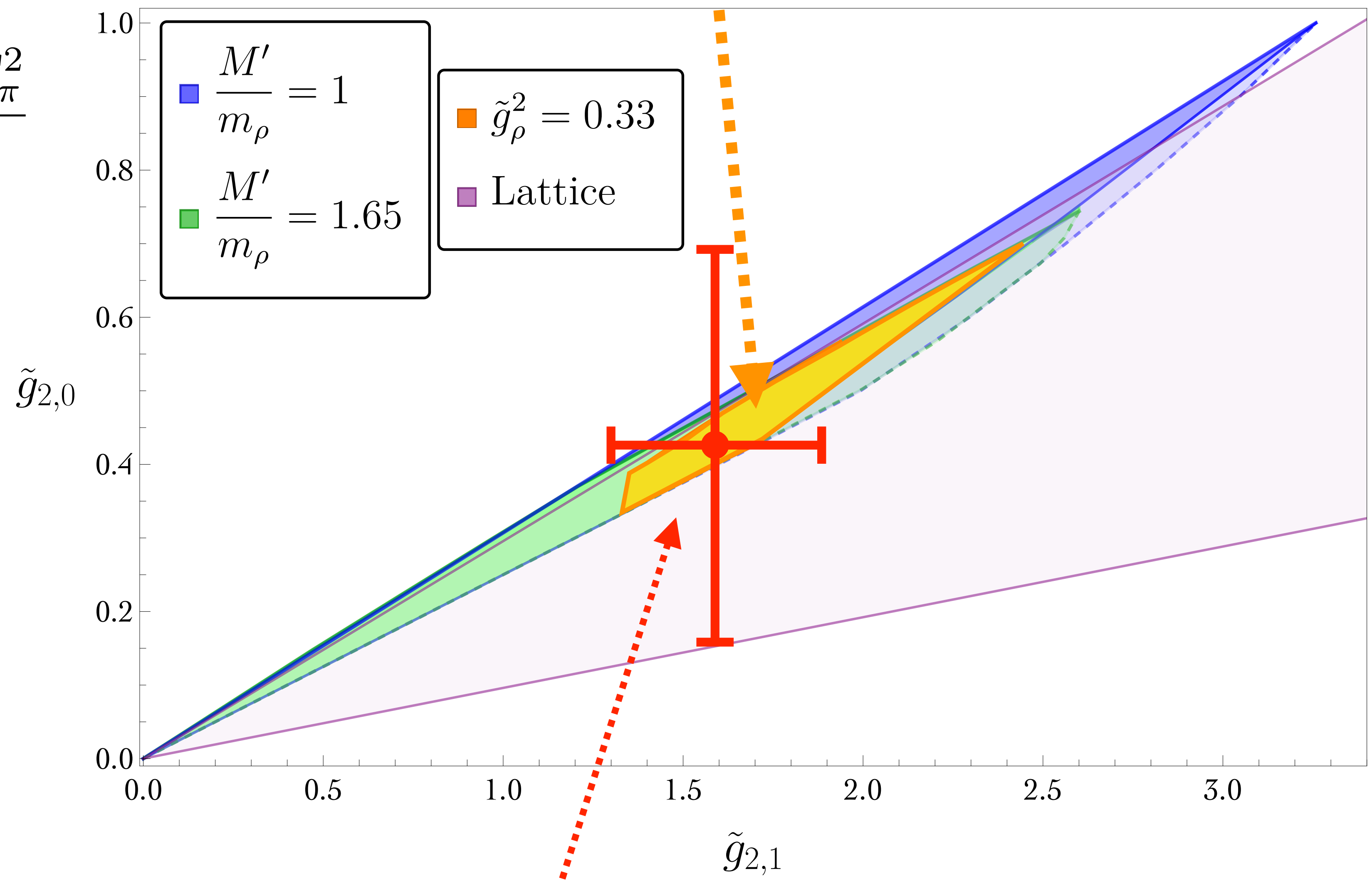

Experimental QCD data

Explaining the success of holography

AdS/QCD:

5D model for QCD mesons (spin=0,1):

$SU(2)_L \times SU(2)_R$ model:

Erlich+Katz+Son+Stephanov 05
Da Rold+Pomarol 05

$$\mathcal{L}_5 = \frac{M_5}{2} Tr\left[-L_{MN}L^{MN} - R_{MN}R^{MN} + |D_M\Phi|^2 + 3|\Phi|^2\right]$$

	Experiment	AdS$_5$	Deviation
m_ρ	775	824	+6%
m_{a_1}	1230	1347	+10%
m_ω	782	824	+5%
F_ρ	153	169	+11%
F_ω/F_ρ	0.88	0.94	+7%
F_π	87	88	+1%
$g_{\rho\pi\pi}$	6.0	5.4	−10%
L_9	$6.9 \cdot 10^{-3}$	$6.2 \cdot 10^{-3}$	−10%
L_{10}	$-5.2 \cdot 10^{-3}$	$-6.2 \cdot 10^{-3}$	−12%
$\Gamma(\omega \to \pi\gamma)$	0.75	0.81	+8%
$\Gamma(\omega \to 3\pi)$	7.5	6.7	−11%
$\Gamma(\rho \to \pi\gamma)$	0.068	0.077	+13%
$\Gamma(\omega \to \pi\mu\mu)$	$8.2 \cdot 10^{-4}$	$7.3 \cdot 10^{-4}$	−10%
$\Gamma(\omega \to \pi ee)$	$6.5 \cdot 10^{-3}$	$7.3 \cdot 10^{-3}$	+12%

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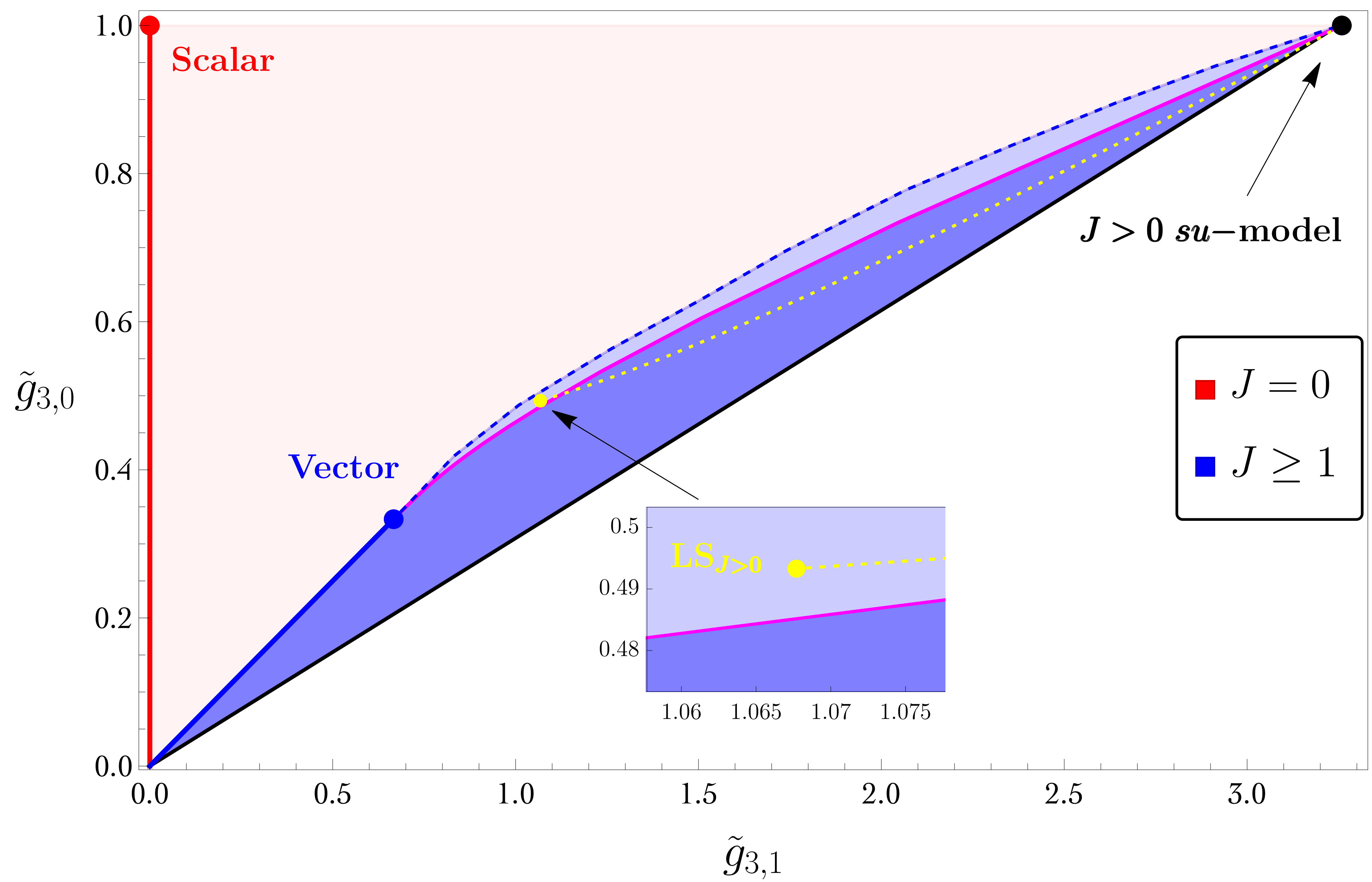
Success can be understood from positivity bounds that restrict J>1 mesons to contribute little to low-energy observables

Similar structure for higher-order Wilson coeff.

$O(s^3)$:



$U(1)_A$ axial anomaly

Introducing the η' (Goldstone of an anomalous symmetry):

$$U(2)\otimes U(2) \to U(2)$$

$$\hookrightarrow SU(2)\otimes SU(2)\otimes U(1)_A \otimes U(1)$$

- WZW term: *5-goldstone int.*
- Adding external gauge-bosons:

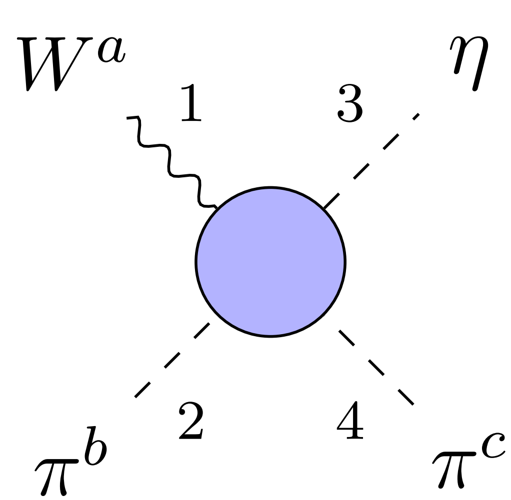
$$\pi \to \gamma\gamma$$

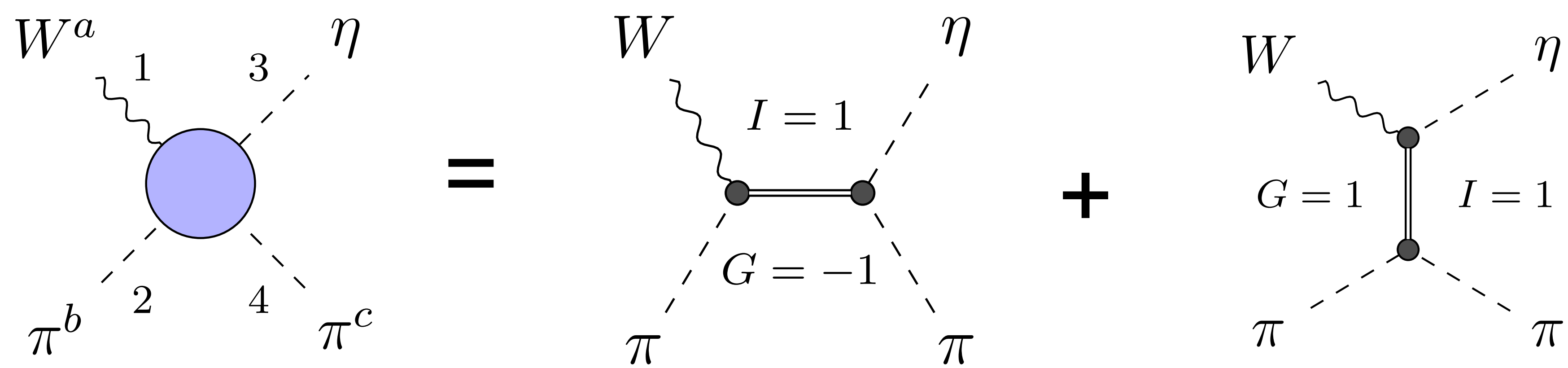
$\Big\} \propto \kappa$

$$\kappa = \frac{N_c}{12\pi^2 F_\pi^3}$$

two q_L, q_R model

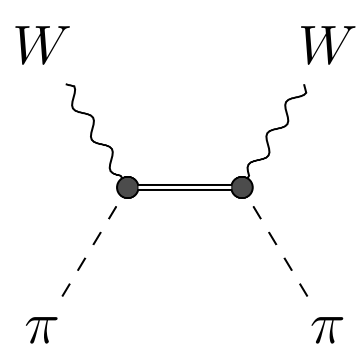
but also





a) It cannot be mediated by scalars

☞ axial anomaly **discards** theories with **only** scalar resonances

b) Bounded by 

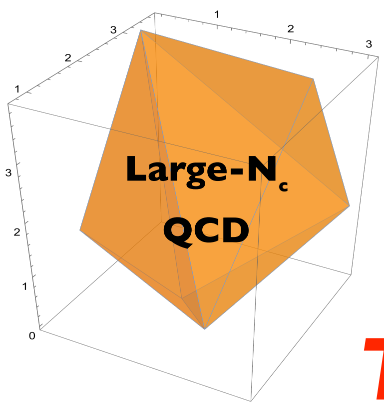
☞ bound on the anomaly:

$$\frac{\kappa}{\sqrt{\mathcal{P}/F_\pi^2}} \leq \frac{1}{\sqrt{2}}$$

pion polarizabilities

Conclusions

- Positivity bounds from ***Crossing + Analyticity + Unitarity*** shows the **"EFT-hedron"** structure of the Chiral Lagrangian at large-N_c



☞ ***Allows to get information on possible UV completions for a theory of Goldstones!***

Two possibilities {
- scalars (Higgs mechanism)
- Higher-spin (states with all J needed)

- Higher-spin (J>1) states are strongly constrained, giving a possible explanation for VMD & the success of holographic QCD

- **Axial anomaly** can distinguish between the two possibilities

Bounded from above:

$$\frac{\kappa}{\sqrt{\mathcal{P}/F_\pi^2}} \leq \frac{1}{\sqrt{2}}$$

what theory saturates it?

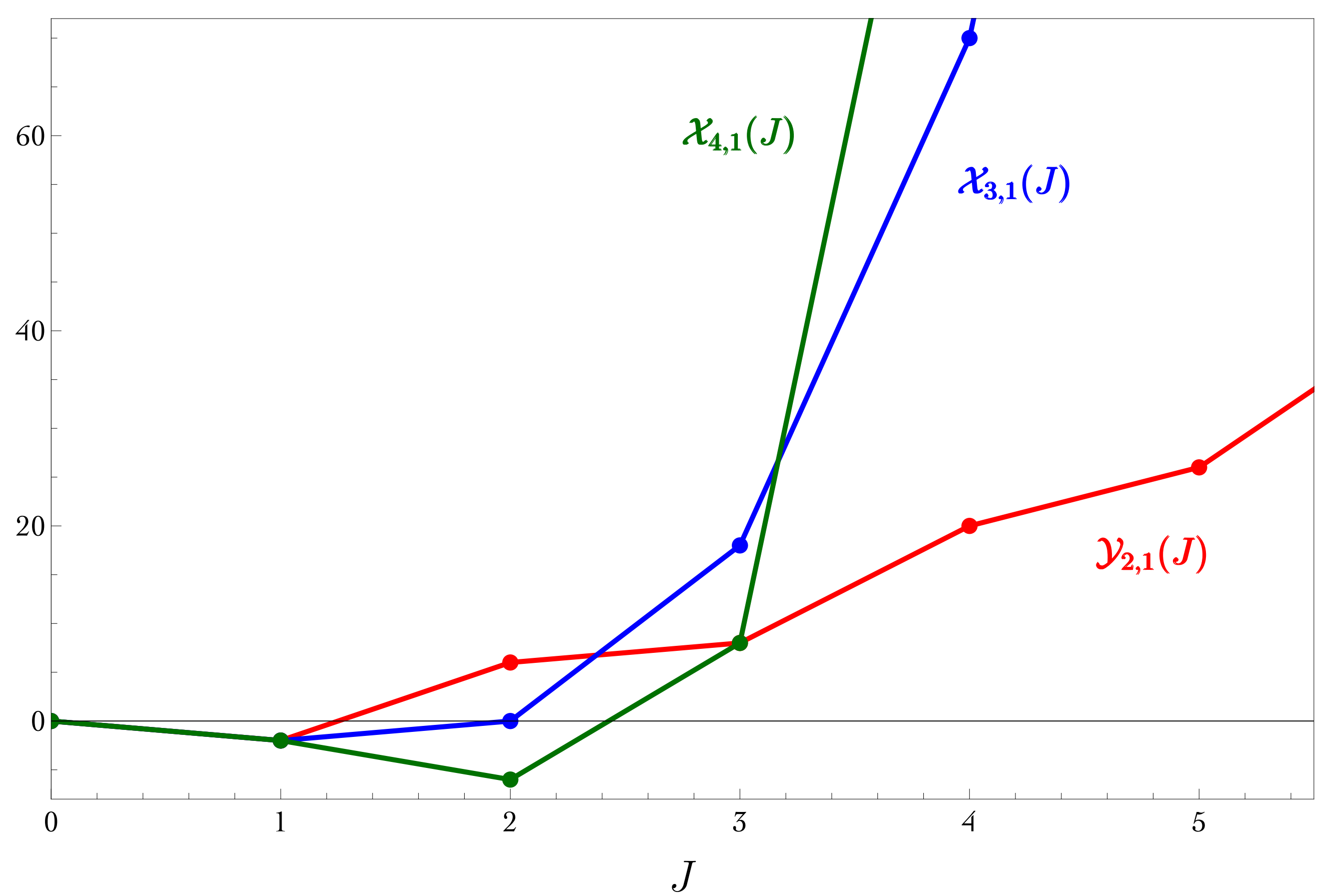
➥ potential interest to constrain DM scenarios (e.g. SIMPs)

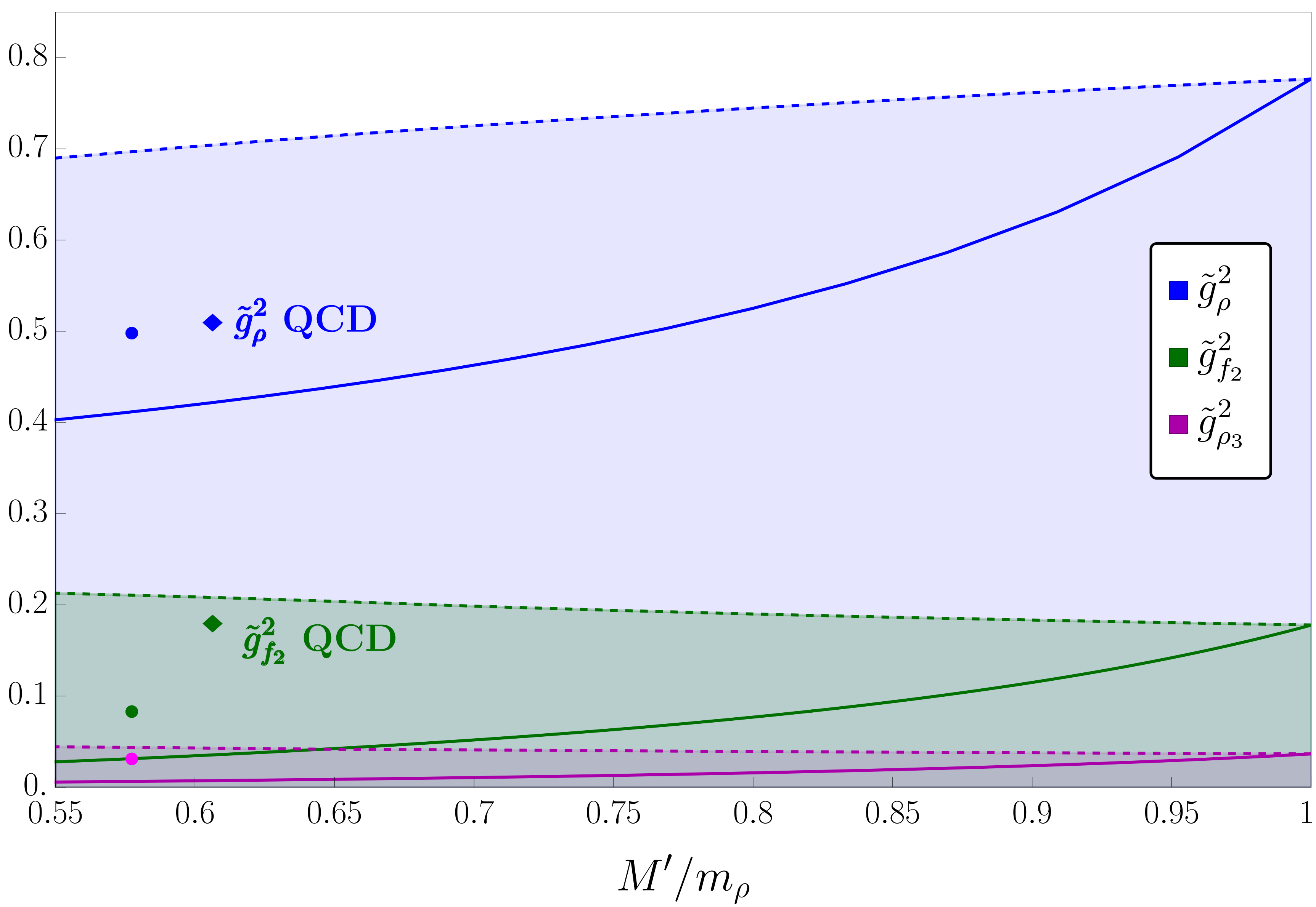

RESTRICTED AREA
MONITORED
BY VIDEO
CAMERA

$$0 = \sum_i \frac{|g_{\pi\pi\, i}|^2}{m_i^{2n}} \left(\underbrace{\frac{2^{n-1}}{(n-1)!^3} P_{J_i}^{(n-1)}(1) - \mathcal{J}_i^2}_{\mathcal{X}_{n,1}} \right)$$

n=2,3,4,...

$\mathcal{J}^2 \equiv J(J+1)$

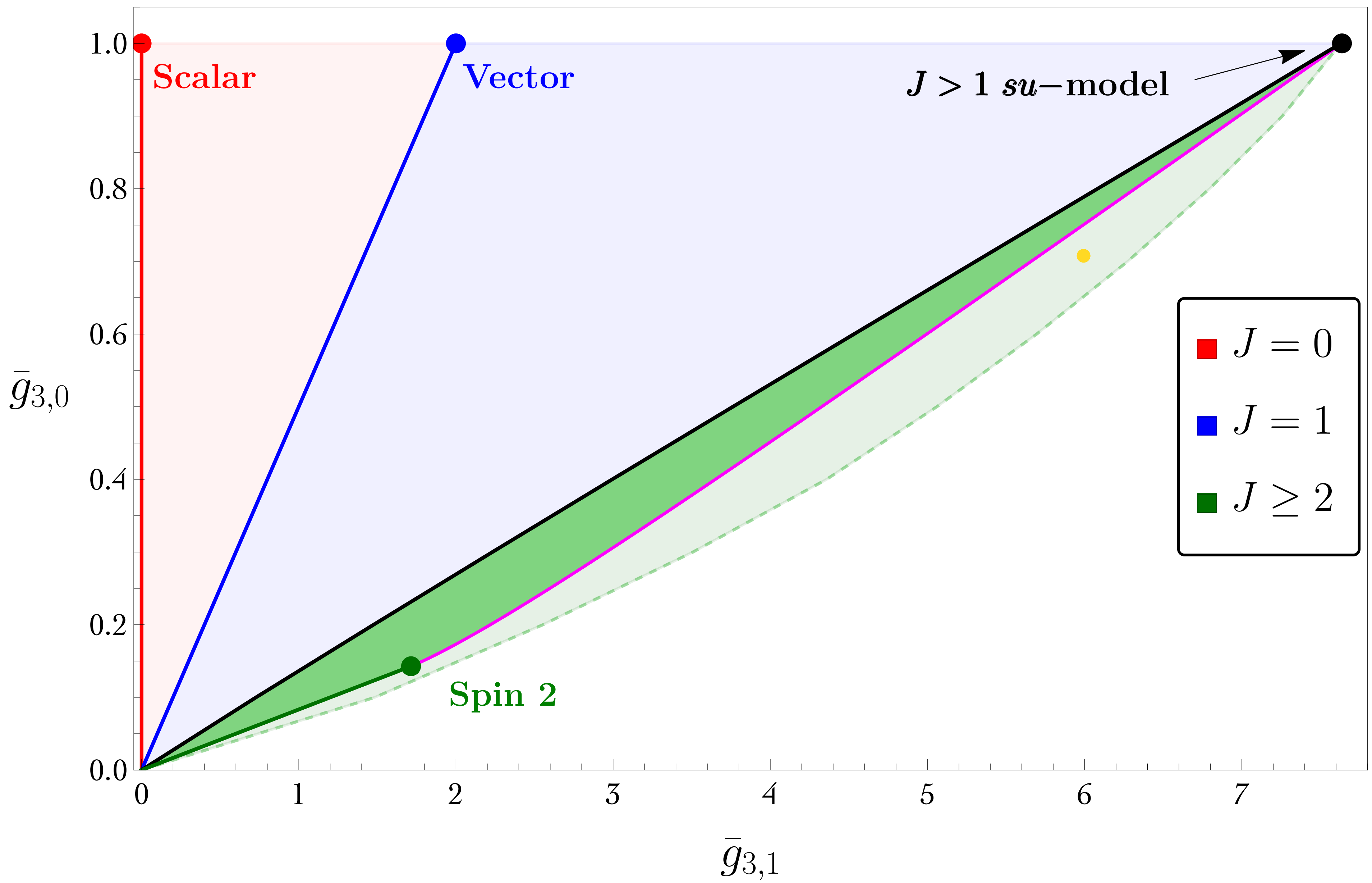




0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1.
M'/m_ρ
$\tilde{g}^2_\rho$ QCD
$\tilde{g}^2_{f_2}$ QCD
$\tilde{g}^2_\rho$
$\tilde{g}^2_{f_2}$
$\tilde{g}^2_{\rho_3}$

Lets assume at s→∞ and either t or u fixed:

$$\frac{\mathcal{M}_t^{I=2}(s,u)}{s^2} \to 0$$



1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
$\bar{g}_{3,0}$
$\bar{g}_{3,1}$
Scalar
Vector
$J > 1$ su−model
Spin 2
$J = 0$
$J = 1$
$J \geq 2$

C The su-models

Let us consider the most general theory of a degenerate spectrum that contributes to the four-pion amplitude $\mathcal{M}(s,u)$ [7, 8]. This means that all states have equal mass m, and therefore the denominator of this amplitude is fixed to be $\mathcal{M}(s,u) \propto 1/((s-m^2)(u-m^2))$. If we further demand that Eq. (6a) and Eq. (6b) are satisfied for $k_{\min} = 1$, we are led to

$$\mathcal{M}(s,u) = \frac{a_1 m^4 + a_2 m^2 (s+u) + a_3 su}{(s-m^2)(u-m^2)}\,, \tag{91}$$

where a_i are constants. The Adler's zero condition fixes $a_1 = 0$. Then, aside from a global multiplicative factor, the amplitude has only one free parameter. We can write it as

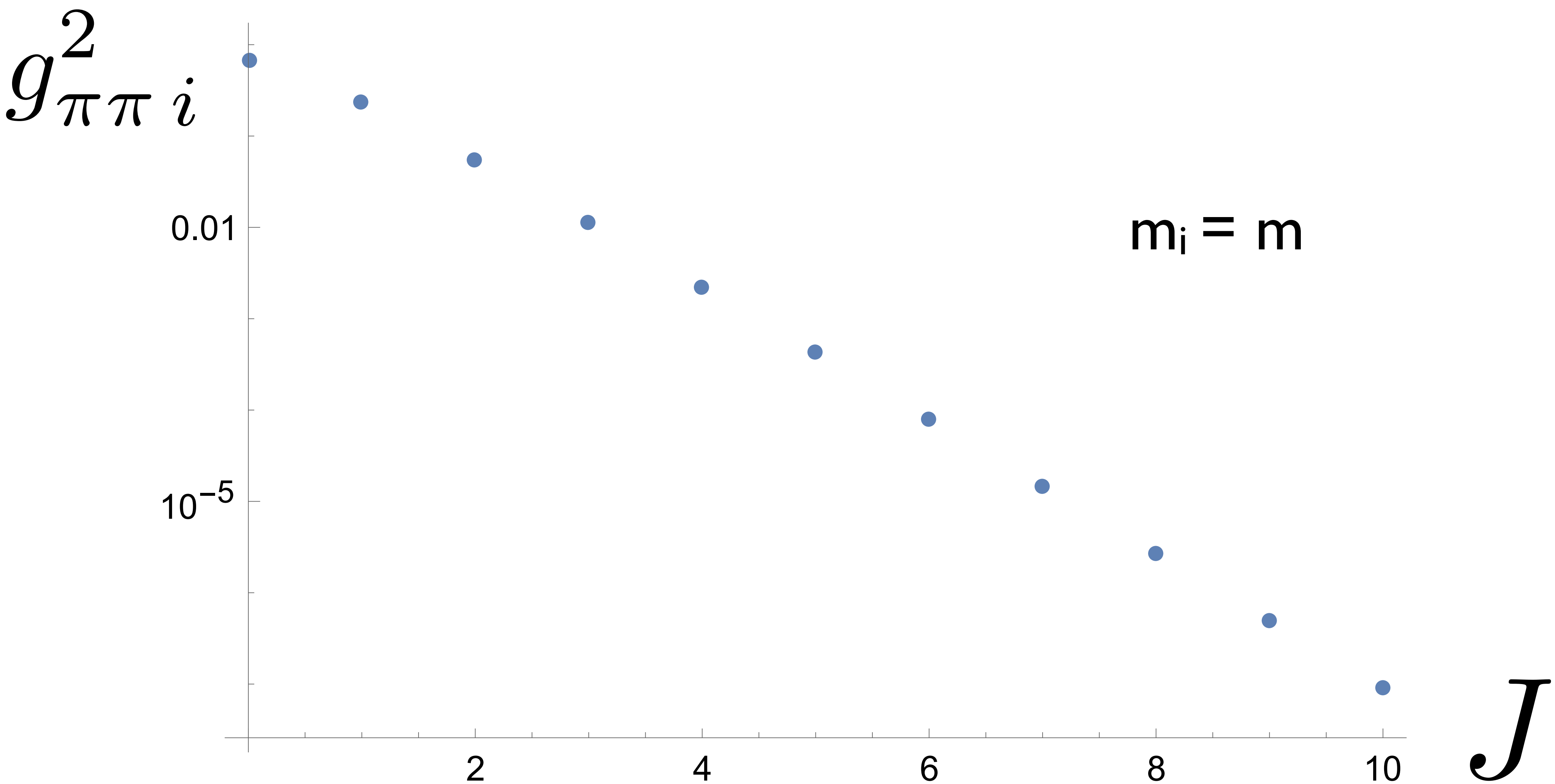
$$\mathcal{M}_1^{(su)}(s,u) = \frac{m^2(s+u) + \lambda su}{(s-m^2)(u-m^2)}\,, \tag{92}$$

where the possible values of λ are determined by unitarity. Indeed, imposing the positivity of the residues of Eq. (92), we obtain

$$-2 \leq \lambda \leq \frac{2\ln 2 - 1}{1 - \ln 2}\,. \tag{93}$$

In the limiting case $\lambda = -2$, the residues of all $J > 0$ states are zero, and we are left with the scalar amplitude Eq. (22). In the other limit,

$$\lambda = \frac{2\ln 2 - 1}{1 - \ln 2} \simeq 1.26\,, \tag{94}$$


$g^2_{\pi\pi\, i}$
0.01
10^{-5}
$m_i = m$
2
4
6
8
10
J

D The Lovelace-Shapiro amplitude

The Lovelace-Shapiro (LS) amplitude for the scattering of four pions is defined as [26, 27]

$$\mathcal{M}^{(\mathrm{LS})}(s,u) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(u))}{\Gamma(1-\alpha(s)-\alpha(u))} \ , \tag{105}$$

where $\alpha(s) = \alpha_0 + \alpha' s$ is referred as the Regge trajectory. We will fix the values of α_0 and α' by requiring that Eq. (106) satisfies the Adler zero condition, $\mathcal{M}^{(\mathrm{LS})}(s,u) \to 0$ for $s,u \to 0$, and that the first pole of Eq. (106) occurs for $s = m_\rho^2$. These two conditions lead to $\alpha_0 = 1/2$ and $\alpha' = 1/(2m_\rho^2)$ [66] and then we can write

$$\mathcal{M}^{(\mathrm{LS})}(s,u) = \frac{\Gamma\left(\frac{1}{2} - \frac{s}{2m_\rho^2}\right)\Gamma\left(\frac{1}{2} - \frac{u}{2m_\rho^2}\right)}{\Gamma\left(\frac{t}{2m_\rho^2}\right)} \ . \tag{106}$$

By looking at the poles of Eq. (106), one can see that the LS amplitude corresponds to a theory of higher-spin states with masses

$$m_n^2 = m_\rho^2(2n+1)\,, \quad n = 0,1,2,\dots \,. \tag{107}$$

For a given n, there are at most $n+1$ states with spin $J = 0, 1, ..., n+1$. Furthermore, Eq. (106) satisfies the condition Eq. (6a) and Eq. (6b) with $k_{\min} = 1$.

E The Coon amplitude

The Lovelace-Shapiro amplitude presented in Appendix D can be generalized to a larger class of amplitudes depending on an additional parameter q. This is the so-called Coon amplitude, which was first proposed in [28][11]:

$$\mathcal{M}_q(s,u) = C(\sigma,\tau,q)\prod_{n=0}^{\infty}\frac{(1-q^{n+1})\,(\sigma\tau - q^{n+1})}{(\sigma - q^{n+1})\,(\tau - q^{n+1})}\ , \tag{118}$$

where $\sigma = 1 + (q-1)(\alpha_0 + \alpha' s)$ and $\tau = 1 + (q-1)(\alpha_0 + \alpha' u)$. As explained in Appendix D, we take $\alpha_0 = 1/2$ and $\alpha' = 1/(2m_\rho^2)$. The parameter q takes values between 0 and 1, and in the limit $q \to 1$ we recover the LS amplitude Eq. (106). There is some freedom in the choice of the prefactor C, as long as it satisfies $\lim_{q\to 1} C(\sigma,\tau,q) = 1$.

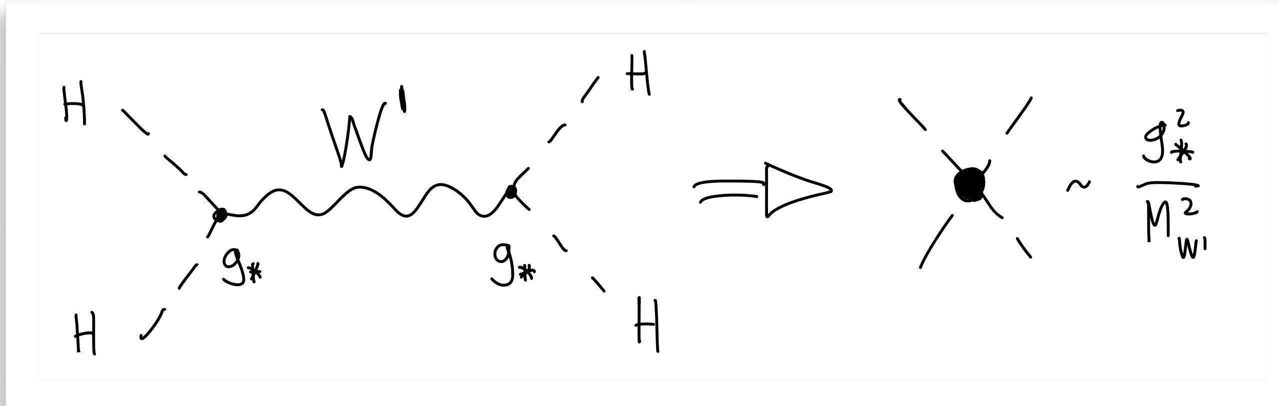
The Coon amplitude has an infinite number of simple poles at

$$s_n = m_\rho^2\frac{1+q-2q^{n+1}}{1-q}\ , \qquad n = 0,1,2,\ldots\ . \tag{119}$$

Impact on BSM searches at the LHC

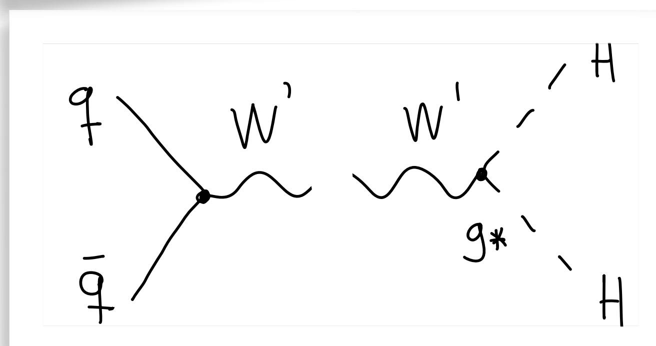
Higgs as a Pseudo-Goldtone boson:

Indirect probes:



deviations in Higgs coupling

Direct probes:



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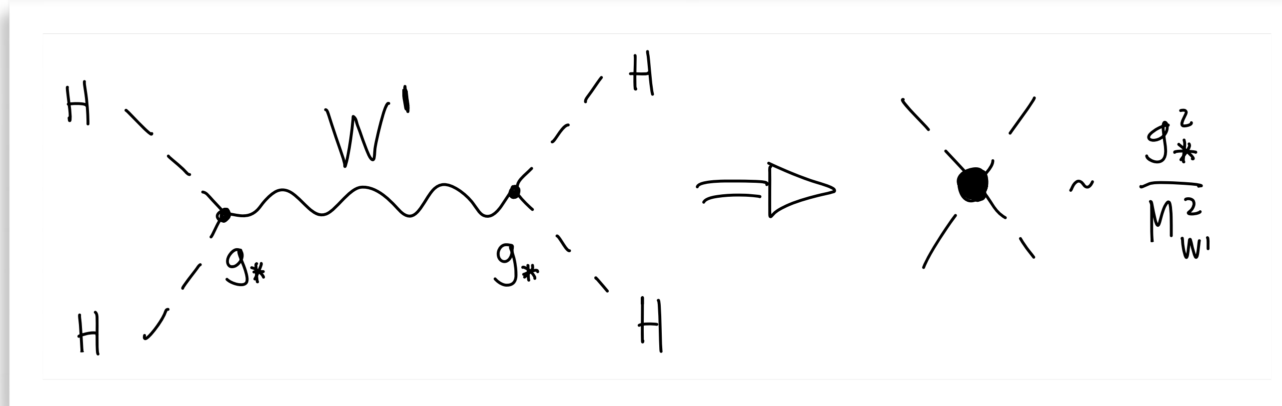
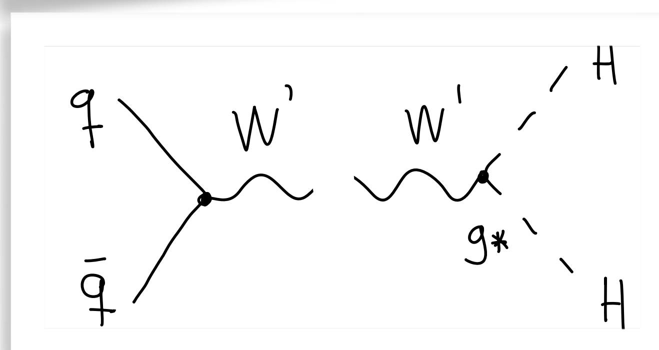
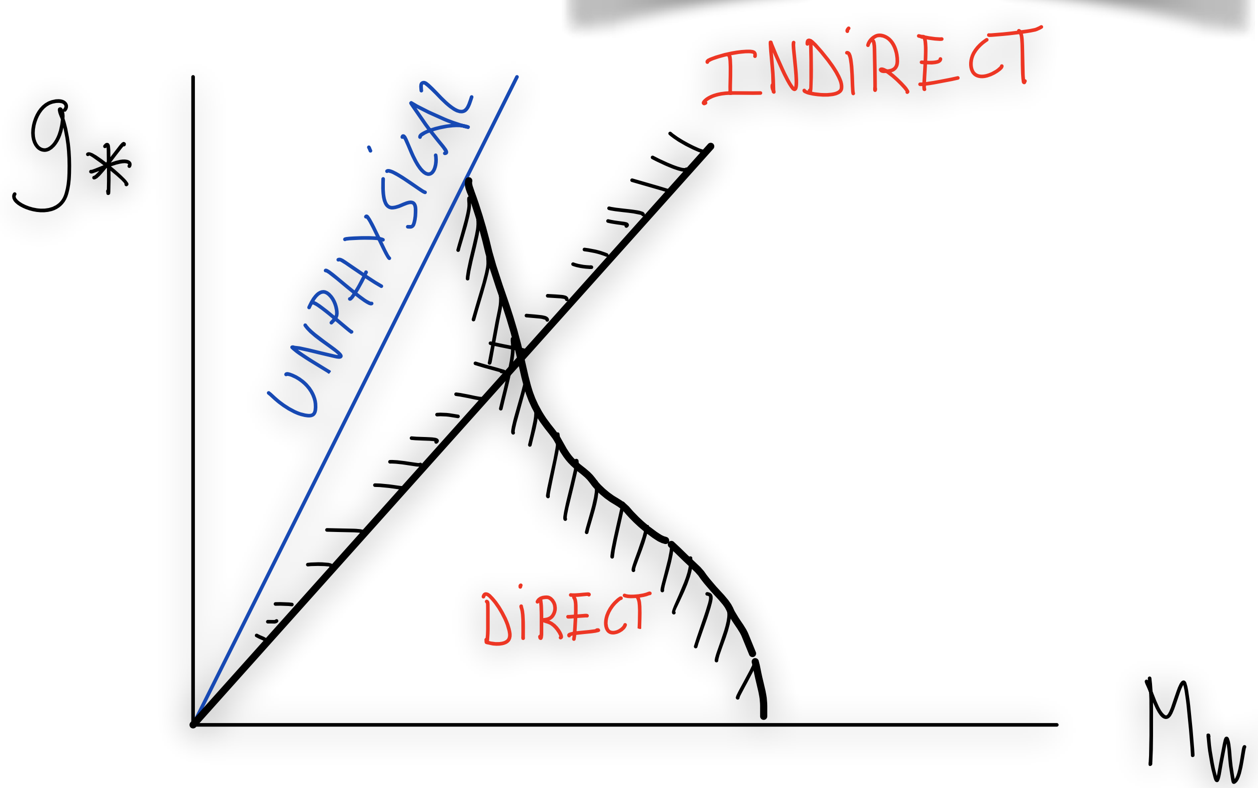


deviations in Higgs coupling

Direct probes:



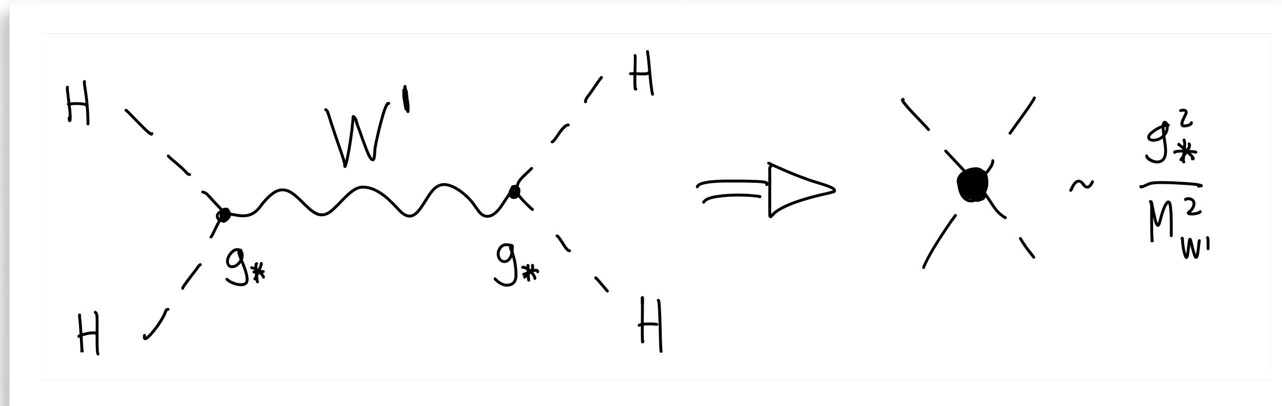


e.g. 1502.01701 [hep-th]

Impact on BSM searches at the LHC

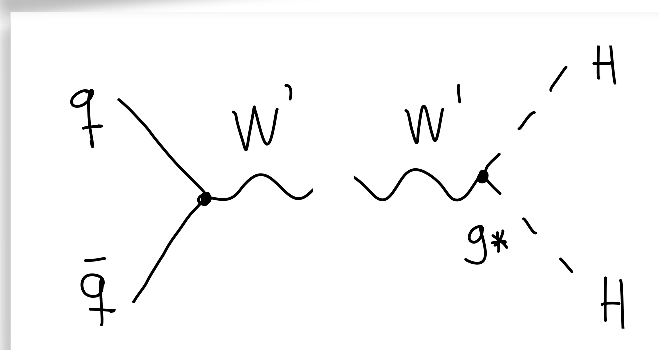
Higgs as a Pseudo-Goldtone boson:

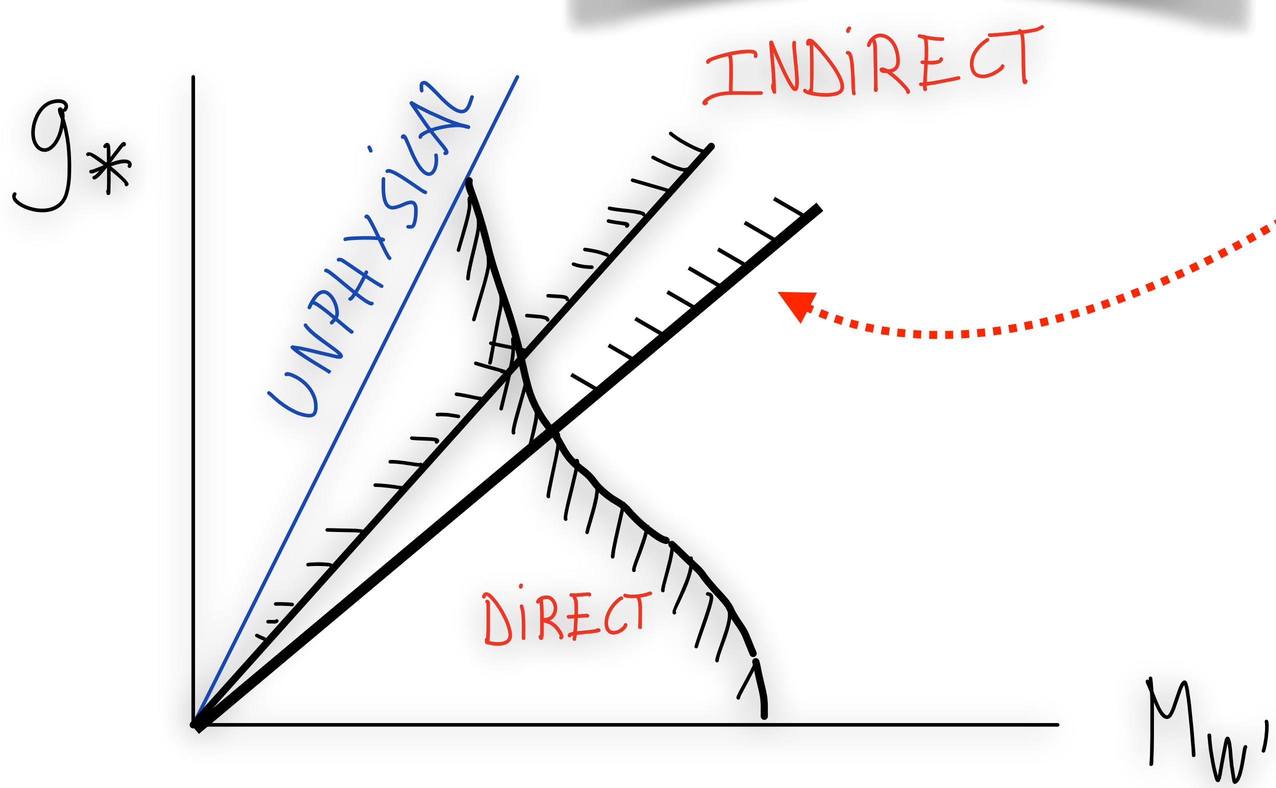
Indirect probes:



deviations in Higgs coupling

Direct probes:





J > 1 must **at least** contribute a **23%** to the Wilson coeff.

e.g. 1502.01701 [hep-th]