A smoking gun signature of 3HDM

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Science Foundation Ireland — For what's next

IRISH RESEARCH COUNCIL — An Chomhairle um Thaighde in Éirinn

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Based on

work in progress and arXiv: 2309.XXXX

Corfu summer institute: **Workshop on the Standard Model and Beyond**

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The Standard Model and its shortcomings

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- No significant deviation from the SM
- No signs of new physics

But no explanation for

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- Fermion mass hierarchy
- Vacuum stability
- Dark Matter & ...

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  The recently discovered 125-GeV scalar can be a portal to the dark sector.
  problem: Current direct and indirect detection as well as relic density bound strongly constrain the simplistic possibilities.

BSMs to the rescue

Scalar extensions with a Z_2 symmetry: 3HDM: SM + 2 scalar doublets

CP-conserving I(2+1)HDM

$$\phi_1, \phi_2, \phi_3$$

$$g_{Z_2} = diag(-1, -1, +1)$$

$$VEV = (0, 0, v)$$

[JHEP 1401 (2014) 052], [Phys. Rev. D 90, 075015 (2014)], [arXiV: 1907.12522]

The scalar potential with explicit CPC

$$
\begin{aligned}
V_{3HDM} &= V_0 + V_{Z_2} \\
V_0 &= \sum_{i}^{3}\left[-\mu_i^2(\phi_i^\dagger\phi_i) + \lambda_{ii}(\phi_i^\dagger\phi_i)^2\right] \\
&\quad + \sum_{i,j}^{3}\left[\lambda_{ij}(\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j) + \lambda'_{ij}(\phi_i^\dagger\phi_j)(\phi_j^\dagger\phi_i)\right] \\
V_{Z_2} &= -\mu_{12}^2(\phi_1^\dagger\phi_2) + \lambda_1(\phi_1^\dagger\phi_2)^2 + \lambda_2(\phi_2^\dagger\phi_3)^2 + \lambda_3(\phi_3^\dagger\phi_1)^2 + h.c.
\end{aligned}
$$

The Z_2 symmetry

$$\phi_1 \to -\phi_1, \quad \phi_2 \to -\phi_2, \quad \phi_3 \to \phi_3, \quad \text{SM fields} \to \text{SM fields}$$

[Phys.Lett.B 695 (2011) 459-462]

Parameters of the model

- All parameters of the potential to be real
- "dark" parameters $\lambda_1, \lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda'_{12}$ (values have been fixed in agreement with the theoretical constraints.)
- $\mu_1^2 = n\mu_2^2, \quad \lambda_3 = n\lambda_2, \quad \lambda_{31} = n\lambda_{23}, \quad \lambda'_{31} = n\lambda'_{23}$
- fixed by the Higgs mass $\mu_3^2 = v^2\lambda_{33} = m_h^2/2$

6 important parameters

- Mass splittings μ_{12}^2 , λ_2
- Higgs-DM coupling $\lambda_2, \lambda_{23}, \lambda'_{23}$
- Mass scale of inert particles μ_2^2

[$Eur.Phys.J.C$80(2020)2, 135]

The mass eigenstates

The doublet compositions

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{H_1^0 + iA_1^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{H_2^0 + iA_2^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} G^+ \\ \frac{v + h + iG^0}{\sqrt{2}} \end{pmatrix}$$

The mass eigenstates

$$\begin{aligned} H_1 &= \cos\theta_h H_1^0 + \sin\theta_h H_2^0, \quad & A_1 &= \cos\theta_a A_1^0 + \sin\theta_a A_2^0 \\ H_2 &= \cos\theta_h H_2^0 - \sin\theta_h H_1^0, \quad & A_2 &= \cos\theta_a A_2^0 - \sin\theta_a A_1^0 \\ H_1^\pm &= \cos\theta_c \phi_1^\pm + \sin\theta_c \phi_2^\pm, \quad & H_2^\pm &= \cos\theta_c \phi_2^\pm - \sin\theta_c \phi_1^\pm \end{aligned}$$

H_1 is assumed to be the DM candidate

- **Input parameters:**

DM mass m_{H_1}, Mass of second CP-even scalar m_{H_2}, Higgs-DM coupling $g_{H_1 H_1 h}$, angles θ_c, θ_a and n.

Constraints

- **Vacuum stability**: scalar potential V bounded from below
- **Perturbative unitarity**: eigenvalues Λ_i of the high-energy scattering matrix fulfill the condition $|\Lambda_i| < 8\pi$
- **Collider**: bounds on masses of the scalars
  - Limits from gauge bosons width:
  $m_{H_i} + m_{H_j^\pm} \geq m_W, \; m_{H_i} + m_{H_j} \geq m_Z, \; 2\, m_{H_{1,2}^\pm} \geq m_Z$
  - Limits on charged scalar mass and lifetime:
  $m_{H_i^\pm} \geq 70 \; \mathrm{GeV}, \quad \tau \leq 10^{-7}\,\mathrm{s} \to \; \Gamma_{\mathrm{tot}} \geq 10^{-18} \; \mathrm{GeV}$
  - Allowed by Higgs invisible branching ratio, $Br(h \to inv.) < 19\%$
  - Allowed by Higgs total decay width, $\mu^{tot}(h)$ as well as Higgs signal strength data.
- **DM constraints**: Relic density, Direct and indirect detection bounds.

Relevant DM scenario

In the low mass region ($m_{H_1} < m_Z$)

We can have multiple scenarios but we are interested in

- **Scenario I: coannihilation** with $H_2, A_{1,2}$:

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} < M_{H_1^{\pm},H_2^{\pm}}$$

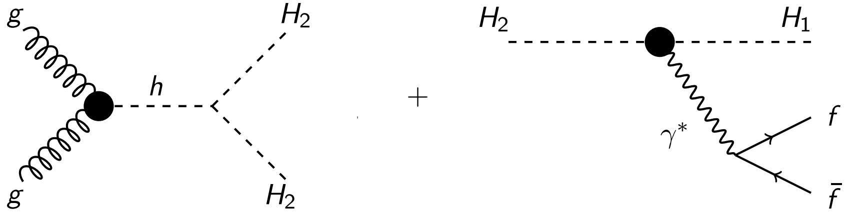
[JHEP 09 (2018) 059]

CPC DM at the LHC

Looking for a **smoking-gun** signal of the 3HDM which is not allowed in the 2HDM with one inert doublet.

Smoking gun Signal

- We focused on,



In the CPC I(2+1)HDM, a process contributing to the $\not{E}_T l^+ l^+ l^- l^-$ signature is

$$gg \to h \to H_2 H_2 \to H_1 H_1 \gamma^* \to H_1 H_1 l^+ l^- l^+ l^-,$$

where the off-shell γ^* splits into $l^+ l^-$ and the H_1 states escape detection and will give $\not{E}_T$.

Smoking gun Signal

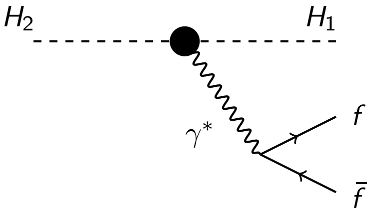


Figure: Radiative decay of the heavy neutral particle $H_2 \to H_1\gamma^* \to H_1 l^+ l^-$.

Smoking gun Signal

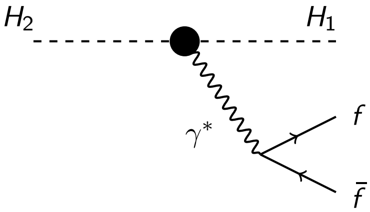
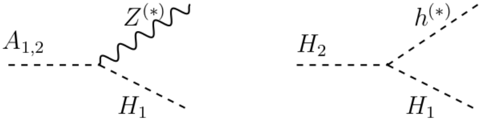


Figure: Radiative decay of the heavy neutral particle $H_2 \to H_1\gamma^* \to H_1 l^+ l^-$.

- $m_{H_2} - m_{H_1}$ is very small
- H_2, into the lightest inert state, H_1, and a virtual photon, which then would split into a light $l\bar{l}$ pair.

Inert cascade decays at the LHC

When there is a large mass splitting between DM and other inert particles:

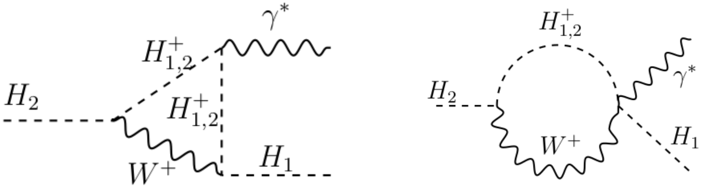


It can give the tree level process $E^T_{miss} + l^+l^-l^+l^-$:

$pp \to H_2H_2/A_{1,2}A_{1,2} \to H_1H_1Z^*Z^* \to H_1H_1l^+l^-l^+l^-$

Inert cascade decays at the LHC

When there is a small mass splitting between DM and other inert particles (winning scenarios):



It can give the loop level process $E^T_{miss} + l^+l^-l^+l^-$:

$pp \to H_2H_2/A_{1,2}A_{1,2} \to H_1H_1\gamma^*\gamma^* \to H_1H_1l^+l^-l^+l^-$

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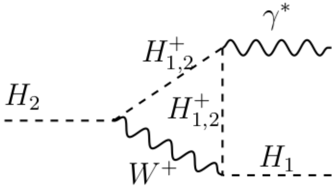

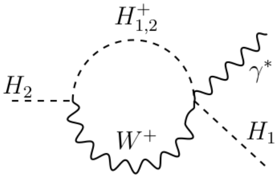

It can give the loop level process $E^T_{miss} + l^+l^-l^+l^-$:

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The **smoking gun** channel

- We are looking for Benchmarks with small mass gap (Δm) between H_2 and H_1

BPs	m_{H_1}	m_{H_2}	Δm	n	$g_{H_1 H_1 h}$	θ_h	$\sigma(pp \to H_1 H_1 2\mu^+ 2\mu^-)$
$BP1 : I_5^{50}$	50	55	5	0.83	0.01	0.105	6.923 fb
$BP2 : I_{10}^{50}$	50	60	10	0.70	0.01	0.103	4.0 fb

Table: Parameter choices of our Benchmark points (BPs)

Signal and backgrounds

- **Signal:** At least 3−lepton with at least one pair of Opposite sign $\mu + \not{E}_T$.

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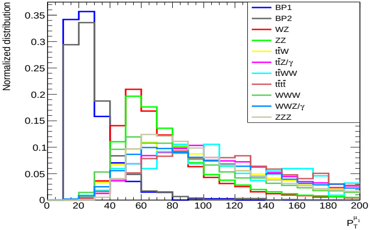
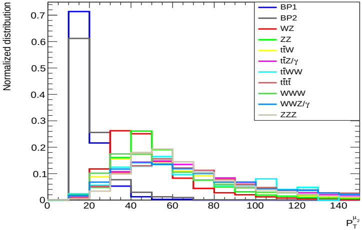
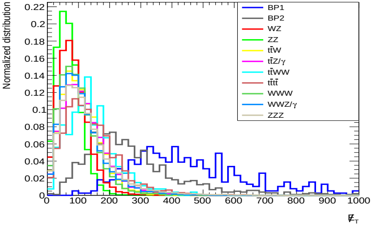
- **Signal:** At least 3−lepton with at least one pair of Opposite sign $\mu + \not{E}_T$.
- **Backgrounds:** 1) **Di-boson**, $VV(V: W, Z, \gamma)$: Mainly WZ/γ and ZZ have large contribution where both V can decay leptonically.
  2) **Tri-boson**, $VVV(V: W, Z, \gamma)$: Mainly consider WWZ/γ, WWW and ZZZ. All vector bosons are supposed to decay leptonically.
  3) $t\bar{t}X$,(X: W, Z, γ, WW, $t\bar{t}$: The fully leptonic decay mode of $t\bar{t}X$ can give us atlast three lepton with at least one pair of μ with opposite charge.

Distributions







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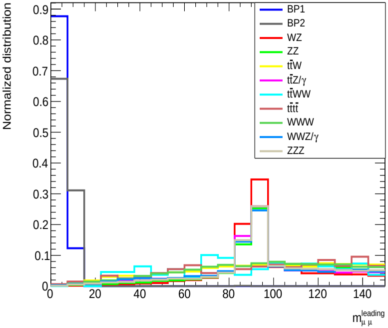

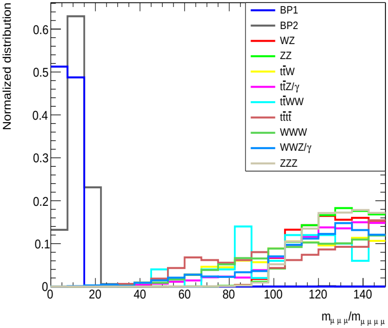

Figure: Normalized distribution of invariant mass of two leading muons and invariant mass of all muons fir signal BPs and backgrounds.

Distributions

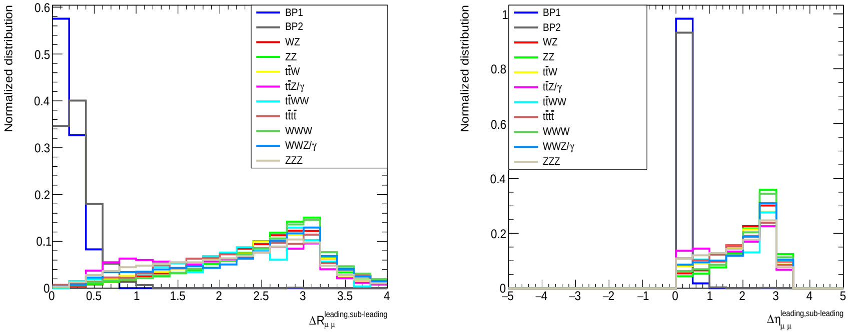

Figure: Normalized Distribution of ΔR and $\Delta\eta$ of leading and sub-leading muon for signal BPs and backgrounds.

Results

Cuts:

1) **Pre-selection Cut**: We are looking for events where we can have at least three or four muons in final state with no $b-jet$.

2) **Cut-A**:
- $m_{\mu\mu}^{leading}$ and $m_{\mu\mu}^{\Delta R_{min}}$ has to be less than 50 GeV.
- $m_{\mu\mu\mu}/m_{\mu\mu\mu\mu}$ has to be less than 70 GeV.

3) **Cut-B**:
- $m_{\mu\mu}^{leading}$ and $m_{\mu\mu}^{\Delta R_{min}}$ has to be less than 20 GeV.
- $m_{\mu\mu\mu}/m_{\mu\mu\mu\mu}$ has to be less than 30 GeV.
- $\Delta R_{\mu\mu}^{leading,sub-leading} < 1.0$ and $\Delta R_{\mu\mu}^{leading,sub-sub-leading} < 1.2$.
- $\Delta \eta_{\mu\mu}^{leading,sub-leading} < 1.0$ and $\Delta \eta_{\mu\mu}^{leading,sub-sub-leading} < 1.0$.

Results

Datasets	Cross-section (fb)	**Pre-selection Cut**	**Cut − A**	**Cut − B**
$BP1$	6.961	17	16	16
$BP2$	3.733	59	58	58
WZ	163.4068	97691	9	0
ZZ	16.554	22614	2	0
WWW	0.248862	185	3	0
WWZ/γ	0.04978	96	1	0
ZZZ	9.3516×10^{-3}	16	0	0
$t\bar{t}W$	0.606	114	2	0
$t\bar{t}Z/\gamma$	0.3045	136	1	0
$t\bar{t}WW$	1.279×10^{-3}	0	0	0
$t\bar{t}t\bar{t}$	1.51359×10^{-3}	0	0	0

Table: Signal and background events cross-section at and Number of Events after cuts at $\sqrt{s} = 14$ TeV and $\mathcal{L} = 3000 fb^{-1}$ for ≥ 3-μ + $\not{E}_T$ final state.

Significance

- we calculated the projected significance ($\mathcal{S}$) in the $3\mu + \not{E}_T$ channel for each benchmark point, for **14 TeV LHC** with **3000 fb**$^{-1}$. The significance $\mathcal{S}$ is defined as follows:

$$\boxed{\mathcal{S} = \sqrt{2[(S+B)\mathrm{Log}(1+\frac{S}{B}) - S]}} \tag{1}$$

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$$\boxed{\mathcal{S} = \sqrt{2[(S+B)\text{Log}(1+\frac{S}{B}) - S]}} \tag{1}$$

BP	$\mathcal{S}(Pre - selection)$	$\mathcal{S}(Cut - A)$
$BP1$	0.05 σ	3.77 σ
$BP2$	0.17 σ	13.67 σ

- with **Cut − B** we will end up with only signal events
- L = 300 fb^{-1} also can give us fully background elimaned signals after **Cut − A** itself.

Summary

- Inert Doublet Model
  - a good DM model with rich phenomenology, however, very constrained.
- CP-Conserving in I(2+1)HDM
  - SM-like active sector: $H_3 \equiv h^{SM}$
  - The inert sector: $H_{1,2}, A_{1,2}, H^{\pm}_{1,2}$, $H_1 \rightarrow$ DM
  - less constrained DM sector with low mass DM particle
  - New **Smoking-gun** signature at the LHC: m_{H_2} and m_{H_1} are close
  - Good signal significance in $3l + \not{E}_T$ channel over backgrounds at HL-LHC.

Back-up slides

BSMs to the rescue

Solution: Scalar extensions with a Z_2 symmetry:

- 2HDM: SM + scalar doublet
  - Type-I, Type-II, ...: $\phi_1,\ \phi_2 \Rightarrow$ CPV, ~~DM~~
  - IDM - I(1+1)HDM: $\phi_1,\ \phi_2 \Rightarrow$ DM, ~~CPV~~
- 3HDM: SM + 2 scalar doublets
  - Weinberg model: $\phi_1,\ \phi_2,\ \phi_3 \Rightarrow$ CPV, ~~DM~~
  - I(1+2)HDM: $\phi_1,\ \phi_2,\ \phi_3 \Rightarrow$ DM, CPV
  - I(2+1)HDM: $\phi_1,\ \phi_2,\ \phi_3 \Rightarrow$ CPV, DM

Dark Matter (DM)

around 25 % of the Universe is:

- cold
- non-baryonic
- neutral
- very weakly interacting

$\Rightarrow$ **W**eakly **I**nteracting **M**assive **P**article

- stable due to the discrete symmetry

$$\underbrace{\mathrm{DM}\ \mathrm{DM} \to \mathrm{SM}\ \mathrm{SM}}_{\text{pair annihilation}}, \quad \underbrace{\mathrm{DM} \not\to \mathrm{SM}, \ldots}_{\text{stable}}$$

Higgs-portal DM

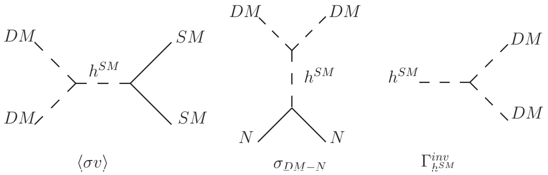
Simplest realisation: the SM with Φ_{SM} + Z_2-odd scalar S:

$$S \to -S, \quad \text{SM fields} \to \text{SM fields}$$

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial S)^2 - \frac{1}{2} m_{DM}^2 S^2 - \lambda_{DM} S^4 - \lambda_{hDM} \Phi_{SM}^2 S^2$$

Higgs-portal interaction:

$$\text{SM sector} \overset{\text{Higgs}}{\longleftrightarrow} \text{DM sector}$$



given by the same coupling

2HDM with CP-violation (~~DM~~)

The general scalar potential

$$
\begin{aligned}
V &= \mu_1^2(\phi_1^\dagger\phi_1) + \mu_2^2(\phi_2^\dagger\phi_2) - \left[\mu_3^2(\phi_1^\dagger\phi_2) + h.c.\right] \\
&+ \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&+ \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + h.c.\right].
\end{aligned}
$$

$$Z_2 \text{ symmetry} \Rightarrow \lambda_6 = \lambda_7 = 0$$

The doublets composition with $\tan\beta = v_2/v_1$

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + h_1^0 + ia_1^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + h_2^0 + ia_2^0}{\sqrt{2}} \end{pmatrix}$$

CP-mixed mass eigenstates

- 2×2 charged mass-squared matrix

$$\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} \Rightarrow \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

- 4×4 neutral mass-squared matrix

$$\begin{pmatrix} a_1^0 \\ h_1^0 \\ a_2^0 \\ h_2^0 \end{pmatrix} \Rightarrow \begin{pmatrix} G^0 \\ H_1 \\ H_2 \\ H_3 \end{pmatrix}$$

CPV severely constrained from SM data

The Inert Doublet Model (~~CPV~~)

Scalar potential V invariant under a Z_2-transformation:

$$Z_2: \quad \phi_1 \to \phi_1, \quad \phi_2 \to -\phi_2, \quad \text{SM fields} \to \text{SM fields}$$

$$\begin{aligned} V &= -\frac{1}{2}\left[m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2\right] + \frac{1}{2}\left[\lambda_1 \left(\phi_1^\dagger \phi_1\right)^2 + \lambda_2 \left(\phi_2^\dagger \phi_2\right)^2\right] \\ &+ \lambda_3 \left(\phi_1^\dagger \phi_1\right)\left(\phi_2^\dagger \phi_2\right) + \lambda_4 \left(\phi_1^\dagger \phi_2\right)\left(\phi_2^\dagger \phi_1\right) + \frac{1}{2}\lambda_5 \left[\left(\phi_1^\dagger \phi_2\right)^2 + \left(\phi_2^\dagger \phi_1\right)^2\right] \end{aligned}$$

- All parameters are real $\to$ no CP violation
- Only ϕ_1 couples to fermions
- The whole Lagrangian is explicitly Z_2-symmetric

DM in the IDM

The Inert minimum

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

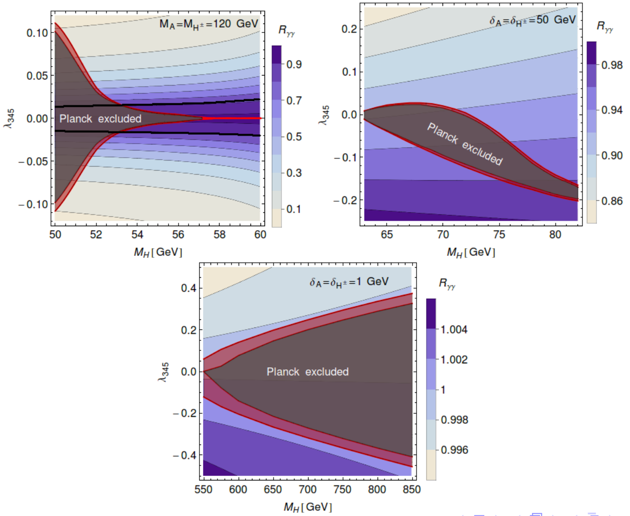
- Z_2-symmetry survives the EWSB

$$g_{Z_2} = diag(+1, -1)$$
$$VEV = (v, 0)$$

- ϕ_1 is active (plays the role of the SM-Higgs)
- ϕ_2 is “dark” or inert (with 4 dark scalars $H, A, H^{\pm}$)

$\rightarrow$ the lightest scalar is **a candidate for the DM**

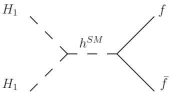
$h \to \gamma\gamma$ signal strength (JHEP 09 (2013) 055)



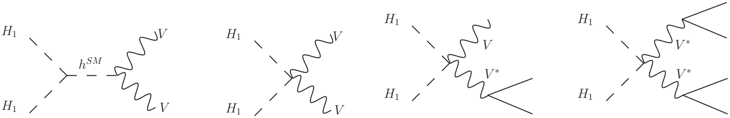
CP-conserving I(2+1)HDM

Dark Matter Annihilation

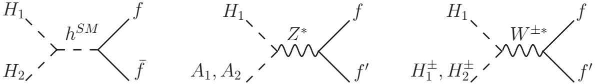
- annihilation through Higgs into fermions; dominant channel for $M_{DM} < M_h/2$



- annihilation to gauge bosons; crucial for heavy masses



- coannihilation; when particles have similar masses



DM Annihilation Scenarios

(A) **no coannihilation effects**:

$$M_{H_1} < M_{H_2,A_1,A_2,H_1^\pm,H_2^\pm}$$

(I) **coannihilation** with $H_2, A_{1,2}$:

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} < M_{H_1^\pm,H_2^\pm}$$

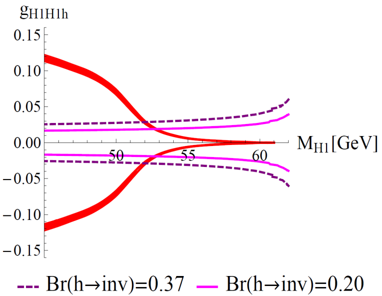
(G) **coannihilation** with $H_2, A_{1,2}, H_{1,2}^\pm$:

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} \approx M_{H_1^\pm,H_2^\pm}$$

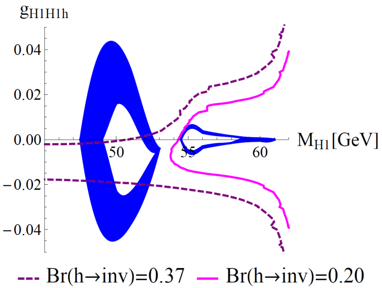
(H) **coannihilation** with $A_1, H_1^\pm$:

$$M_{H_1} \approx M_{A_1} \approx, H_1^\pm < M_{H_2,A_2,H_2^\pm}$$

LHC vs Planck $M_{DM} < M_h/2$



case A



case I

- $Br(h \to inv) < 37\%$ & $\Omega_{DM} h^2 \Rightarrow$
  - Case A: $M_{DM} \gtrsim 53\,\text{GeV}$
  - Case I: most masses are OK

Masses and mixing angles

- **The CP-even neutral inert fields**
The pair of inert neutral scalar gauge eigenstates, H_1^0, H_2^0, are rotated by

$$R_{\theta_h} = \begin{pmatrix} \cos\theta_h & \sin\theta_h \\ -\sin\theta_h & \cos\theta_h \end{pmatrix}, \text{with} \quad \tan 2\theta_h = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda_{\phi_1} - \mu_2^2 + \Lambda_{\phi_2}}$$

into the mass eigenstates, H_1, H_2, with squared masses

$$\begin{aligned} m_{H_1}^2 &= (-\mu_1^2 + \Lambda_{\phi_1})\cos^2\theta_h + (-\mu_2^2 + \Lambda_{\phi_2})\sin^2\theta_h - 2\mu_{12}^2\sin\theta_h\cos\theta_h, \\ m_{H_2}^2 &= (-\mu_1^2 + \Lambda_{\phi_1})\sin^2\theta_h + (-\mu_2^2 + \Lambda_{\phi_2})\cos^2\theta_h + 2\mu_{12}^2\sin\theta_h\cos\theta_h, \\ \text{where} \quad \Lambda_{\phi_1} &= \frac{1}{2}(\lambda_{31} + \lambda'_{31} + 2\lambda_3)v^2, \quad \Lambda_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} + 2\lambda_2)v^2. \end{aligned}$$

Masses and mixing angles

- **The charged inert fields**
The pair of inert charged gauge eigenstates, $\phi_1^\pm, \phi_2^\pm$, are rotated by

$$R_{\theta_c} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}, \text{with} \quad \tan 2\theta_c = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda'_{\phi_1} - \mu_2^2 + \Lambda'_{\phi_2}}$$

into the mass eigenstates, $H_1^\pm, H_2^\pm$, with squared masses

$$m^2_{H_1^\pm} = (-\mu_1^2 + \Lambda'_{\phi_1})\cos^2\theta_c + (-\mu_2^2 + \Lambda'_{\phi_2})\sin^2\theta_c - 2\mu_{12}^2 \sin\theta_c\cos\theta_c,$$

$$m^2_{H_2^\pm} = (-\mu_1^2 + \Lambda'_{\phi_1})\sin^2\theta_c + (-\mu_2^2 + \Lambda'_{\phi_2})\cos^2\theta_c + 2\mu_{12}^2 \sin\theta_c\cos\theta_c,$$

$$\text{where} \quad \Lambda'_{\phi_1} = \frac{1}{2}(\lambda_{31})v^2, \quad \Lambda'_{\phi_2} = \frac{1}{2}(\lambda_{23})v^2.$$

Masses and mixing angles

- **The CP-odd neutral inert fields**
  The pair of inert pseudo-scalar gauge eigenstates, A_1^0, A_2^0, are rotated by

$$R_{\theta_a} = \begin{pmatrix} \cos\theta_a & \sin\theta_a \\ -\sin\theta_a & \cos\theta_a \end{pmatrix}, \text{with} \quad \tan 2\theta_a = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda''_{\phi_1} - \mu_2^2 + \Lambda''_{\phi_2}},$$

into the mass eigenstates, A_1, A_2, with squared masses

$$m_{A_1}^2 = (-\mu_1^2 + \Lambda''_{\phi_1})\cos^2\theta_a + (-\mu_2^2 + \Lambda''_{\phi_2})\sin^2\theta_a - 2\mu_{12}^2 \sin\theta_a \cos\theta_a,$$
$$m_{A_2}^2 = (-\mu_1^2 + \Lambda''_{\phi_1})\sin^2\theta_a + (-\mu_2^2 + \Lambda''_{\phi_2})\cos^2\theta_a + 2\mu_{12}^2 \sin\theta_a \cos\theta_a,$$
$$\text{where} \quad \Lambda''_{\phi_1} = \frac{1}{2}(\lambda_{31} + \lambda'_{31} - 2\lambda_3)v^2, \quad \Lambda''_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} - 2\lambda_2)v^2.$$

Dependent parameters in terms of input parameters

$$\Lambda_{\phi_2} = \frac{v^2 g_{H_1 H_1 h}}{4(\sin^2\theta_h + n\cos^2\theta_h)},$$

$$\Lambda'_{\phi_2} = \frac{2\mu_{12}^2}{(1-n)\tan 2\theta_c} + \mu_2^2,$$

$$\Lambda''_{\phi_2} = \frac{2\mu_{12}^2}{(1-n)\tan 2\theta_a} + \mu_2^2,$$

$$\mu_2^2 = \Lambda_{\phi_2} - \frac{m_{H_1}^2 + m_{H_2}^2}{1+n},$$

$$\mu_{12}^2 = \frac{1}{2}\sqrt{(m_{H_1}^2 - m_{H_2}^2)^2 - (-1+n)^2(\Lambda_{\phi_2} - \mu_2^2)^2},$$

$$\lambda_2 = \frac{1}{2v^2}(\Lambda_{\phi_2} - \Lambda''_{\phi_2}),$$

$$\lambda_{23} = \frac{2}{v^2}\Lambda'_{\phi_2},$$

$$\lambda'_{23} = \frac{1}{v^2}(\Lambda_{\phi_2} + \Lambda''_{\phi_2} - 2\Lambda'_{\phi_2})$$