SM in Weyl geometry and Weyl anomaly

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Corfu Summer Institute, 5 Sep 2023.

Based on: arXiv: arXiv:2309.11372, 2203.05381 (EPJC); 2104.15118 (EPJ C)

also: 1812.08613 (JHEP), 2007.14733, 2003.08516 (EPJC)

* work supported by grant # PCE-2020-2255 of the Ministry of Education and Research, Romania

• Outline:

- From scale symmetry to Weyl geometry (WG)
- Weyl quadratic gravity: gauge theory of scale invariance, Einstein gravity as broken phase.

- SM: natural, truly minimal embedding in Weyl geometry, including Einstein gravity.
  - origin of mass in non-metric geometry;
  - Higgs from "geometry", mass hierarchy solution,
  - inflation: Starobinsky-like (gauged R^2), accelerated expansion
  - Weyl anomaly, etc.

• Beyond SM and GR

- SM + its gauge symmetry and Higgs mechanism confirmed experimentally (LHC); origin of EW scale.
- Gravity: origin of Planck mass M_P of mass? Is it geometric? must go beyond Einstein gravity
- Mass hierarchy solution? seek an alternative to Susy/Sugra & Strings, using the gauge principle.

• why scale symmetry:

- SM with $m_{\rm Higgs}=0$ scale invariant; Early Universe or at short distances: EFTs are scale invariant.
- discrete (fractals, in Nature), global, local, gauged=Weyl gauge symmetry (WGS) =gauged dilatations (WGS $\Rightarrow$ quantum scale symmetry!)

• History:

- WGS: is a symmetry of Weyl geometry (WG)! first gauge theory (of scale invariance) 1918!
- WG $\Rightarrow$ Weyl $\tilde R^2$ gravity. Weyl thought it describes Gravity "+" Electromagnetism - soon disregarded!
- Einstein's critique: WG non-metric $\nabla_\mu g_{\alpha\beta}\neq 0$. btw: Einstein-Palatini quadratic gravity non-metric,too!
- we show: non-metricity not a problem but an advantage: the origin of mass!

- Global scale symmetry

$$x'_\mu = \rho\, x_\mu; \qquad \phi'(\rho\, x) = (1/\rho)\, \phi(x), \qquad \text{forbids} \quad \int d^4x\, m^2 \phi^2$$

- SM with Higgs ϕ of mass $m_\phi = 0$ is scale invariant [Bardeen 1995]
- no dim-ful couplings; scales generated from vev's, e.g. $M_P \sim \langle\sigma\rangle$. Broken by quantum corrections.
- Global symmetries broken by BH physics

[Kallosh, Linde, Susskind, hepth/9502069]

- Side remark: quantum scale symmetry (QSS):

- replace DR scale $\mu \to \sigma$ (dilaton) to keep scale invariance in $d{=}4-2\epsilon$; extra field! Different theory!
- QSS broken spontaneously; if $\langle\sigma\rangle \to \infty$ decouples, usual results (breaking by DR) recovered.
- at 1-loop [Englert et al 1976], Shaposhnikov 0809.3406; D.G. 1508.00595] and in SM [D.G., Z. Lalak, P. Olszewski, 1612.09120]]
- at 2-loops [D.G., Z. Lalak, P. Olszewski, 1608.05336]; 3-loops in SM [D.G. 1712.06024,Gretsch, Monin 1308.3863]
- protects a classical hierarchy $\phi \ll \sigma$; c-terms: ϕ^6/σ^2, ϕ^8/σ^4.... [D.G. 1508.00595, 1712.06024]
- Higgs-Gravity coupling: $\xi\, \phi^2 R \to$ tuning higgs selfcoupling $\beta_\lambda \sim \lambda(..) + \xi\, (...)$.

• Local scale/Weyl symmetry: L invariant under : $\hat{g}_{\mu\nu} = \Omega(x)^2\, g_{\mu\nu}, \quad \hat{\phi} = \frac{\phi}{\Omega(x)}, \quad \hat{\psi} = \frac{\psi}{\Omega(x)^{3/2}}$

- Include gravity (ϕ real):

$$L_0 = -\frac{1}{2}\sqrt{g}\left[\frac{1}{6}\phi^2 R + g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi\right], \qquad \Leftrightarrow \qquad L_0 = -\frac{1}{2}\sqrt{\hat{g}}\,M_P^2\,\hat{R} \quad \text{generated spontaneously}$$

$$\Omega^2 = \frac{\phi^2}{\langle\phi\rangle^2}, \qquad M_P^2 \equiv \frac{1}{6}\langle\hat{\phi}\rangle^2, \quad \text{“gauge fixing”}$$

Einstein frame: $M_p \sim \langle\phi\rangle$ then ϕ decouples $\Rightarrow$ Conformal SM:

[t'Hooft 1104.4543; 1410.6675; Bars, Steinhardt, Turok 1307.1848, Englert et al 1976]

But:

a) - has a negative kinetic term for ϕ; not general, linear in R.

b) - Fake conformal symmetry! - vanishing current. [Jackiw, Pi 2015],

c) - ϕ: compensator, added “ad-hoc” to enforce symmetry; ϕ and $M_p \sim \langle\phi\rangle$: no geometric origin.

d) - L_0 has a symmetry that its underlying geometry (connection) does not! consistent? Γ: not invariant.

$\Rightarrow$ We want to avoid these issues: $\Rightarrow$ gauged scale invariance.

- Gauged scale symmetry: $\hat{\omega}_\mu(x) = \omega_\mu(x) - \frac{1}{\alpha}\partial_\mu \ln \Omega(x)^2$, $\hat{g}_{\mu\nu}(x) = \Omega(x)^2 g_{\mu\nu}(x)$, $\hat{\phi}(x) = \frac{\phi(x)}{\Omega(x)}$ $(*)$

- Weyl geometry: equiv classes $(g_{\mu\nu}, \omega_\mu)$

$\tilde{\nabla}_\mu g_{\alpha\beta} = -\alpha\, \omega_\mu\, g_{\alpha\beta}$, $\Rightarrow$ $\widehat{\tilde{\nabla}}'_\lambda g_{\mu\nu} = 0$, $\widehat{\tilde{\nabla}} = \tilde{\nabla}\big|_{\partial_\lambda \to \partial_\lambda + \text{charge} \times \alpha \times \omega_\lambda}$

$\Rightarrow \tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + (\alpha/2)\left(\delta^\rho_\mu\, \omega_\nu + \delta^\rho_\nu\, \omega_\mu - g_{\mu\nu}\, \omega^\rho\right)$ inv of (*);

$\Rightarrow \tilde{R} = R - 3\,\alpha \nabla_\mu \omega^\mu - 3/2\, \alpha^2 \omega^\mu \omega_\mu$; $\hat{\tilde{R}} = \frac{\tilde{R}}{\Omega^2}$ (!)

$\Rightarrow \tilde{D}_\mu \phi = (\partial_\mu - \alpha/2\, \omega_\mu)\, \phi \Rightarrow \hat{\tilde{D}}_\mu \hat{\phi} = \frac{1}{\Omega}\, \tilde{D}_\mu \phi$;

- Riemannian geometry $(g_{\mu\nu})$

$\nabla_\mu g_{\alpha\beta} = 0$

$\Gamma^\rho_{\mu\nu}$ = Levi-Civita; ∇_μ with Γ

$\Rightarrow F_{\mu\nu} = \tilde{\nabla}_\mu \omega_\nu - \tilde{\nabla}_\nu \omega_\mu = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ inv (*). Also $\alpha\, \omega_\mu \sim \tilde{\Gamma}_\mu - \Gamma_\mu$ deviation from Levi-Civita.

$\Rightarrow$ if $\omega_\mu \to 0$: $\tilde{\Gamma} \to \Gamma$, Weyl geometry $\to$ Riemannian; $\tilde{R} \to R$, Weyl tensor $\tilde{C}_{\mu\nu\rho\sigma} \to C_{\mu\nu\rho\sigma}$

$\Rightarrow$ All invariants of (*) [no matter]: $\sqrt{g}\, \tilde{R}^2$, $\sqrt{g}\, F^2_{\mu\nu}$, $\sqrt{g}\, \tilde{C}^2_{\mu\nu\alpha\beta}$; no higher dim ops (no scale!)

$\tilde{X}$ (X) notation in Weyl (Riemannian) geometry

• Weyl quadratic action $\Rightarrow$ Einstein gravity + massive ω_μ [D.G. arXiv:2203.05381, 2104.15118, 1812.08613]

$$\mathcal{L}_0 = \sqrt{g}\,\Big[\frac{1}{4!}\frac{1}{\xi^2}\tilde{R}^2 - \frac{1}{4}F_{\mu\nu}^2\Big] = \sqrt{g}\,\Big[\frac{1}{4!}\frac{1}{\xi^2}\big(-2\phi_0^2\,\tilde{R} - \phi_0^4\big) - \frac{1}{4}F_{\mu\nu}^2\Big], \qquad \text{eq. motion } \phi_0^2 = -\tilde{R}.$$

Riemannian notation: $$\mathcal{L}_0 = \sqrt{g}\,\Big\{\frac{-1}{2\,\xi^2}\Big[\frac{1}{6}\,\phi_0^2\,R + (\partial_\mu\phi_0)^2\Big] - \frac{\phi_0^4}{4!\,\xi^2} + \frac{\alpha^2}{8\,\xi^2}\,\phi_0^2\,\Big[\omega_\mu - \frac{1}{\alpha}\partial_\mu \ln\phi_0^2\Big]^2 - \frac{1}{4}F_{\mu\nu}^2\Big\}.$$

If $\langle\phi_0\rangle \neq 0$, "gauge fixing": $\Omega^2 = \phi_0^2/\langle\hat{\phi}_0\rangle^2 \;\Rightarrow\; \hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu},\; \langle\hat{\phi}_0\rangle^2 = 6\,\xi^2\,M_P^2,\; \hat{\omega}_\mu = \omega_\mu - \frac{1}{\alpha}\,\partial_\mu\ln\phi_0^2$

$$\mathcal{L}_0 = \sqrt{\hat{g}}\,\Big[-\frac{1}{2}M_p^2\hat{R} + \frac{3}{4}M_p^2\,\alpha^2\,\hat{\omega}_\mu\hat{\omega}^\mu - \frac{1}{4}\hat{F}_{\mu\nu}^2 - \Lambda\,M_p^2\Big], \qquad M_p^2 \equiv \frac{\langle\phi_0^2\rangle}{6\,\xi^2}, \quad \Lambda \equiv \frac{1}{4}\langle\phi\rangle^2.$$

$\Rightarrow$ Einstein-Proca action & M_p, Λ, m_ω by Stueckelberg mechanism. ω_μ massive $\to$ it decouples!

$\Rightarrow$ $\tilde{\Gamma} \to \Gamma$ then WG $\to$ Riemannian. Spontaneous breaking (# dof constant), non-trivial.

$\Rightarrow$ Einstein gravity=broken phase of Weyl action. Metricity restored below $m_\omega \sim M_p$! $\alpha \ll 1$? No ghost!

• Weyl quadratic gravity $\Rightarrow$ Einstein gravity + massive ω_μ

$\Rightarrow$ we have: $\nabla^\mu J_\mu = 0$; $J_\mu = \alpha/(2\xi^2)\ \phi_0 \left[\partial_\mu - (\alpha/2)\,\omega_\mu\right] \phi_0$; if ϕ_0 constant $\nabla_\mu \omega^\mu = 0$ gauge fixing.

$\Rightarrow$ if ω_μ not dynamical: $\omega_\mu \sim (1/\alpha)\,\partial_\mu(\ln\phi_0^2)$, $J_\mu = 0$ (metric case) $\Rightarrow$ local scale symmetry

- ϕ_0 is part of $\tilde{R}^2$: then $M_p^2 \sim \langle\phi_0\rangle^2/\xi^2$, $\Lambda \sim \langle\phi_0\rangle^2$, $m_\omega^2 \sim \alpha^2 M_p$: have (non-metric) geometric origin.

$\Rightarrow$ Non-metric geometry as the origin of mass! [D.G. 2203.05381 [hep-th]]

- Other terms: Weyl tensor ($\tilde{C}_{\mu\nu\rho\sigma}$) does not change the result:

$$L_C = \frac{\sqrt{g}}{\eta}\,\tilde{C}_{\mu\nu\rho\sigma}\,\tilde{C}^{\mu\nu\rho\sigma} = \frac{\sqrt{g}}{\eta}\Big[C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma} + \frac{3}{2}\,\alpha^2\,F_{\mu\nu}^2\Big]$$

Weyl geometry Riemannian geometry

$\Rightarrow$ Weyl's theory: gauge theory of scale inv, an embedding of Einstein gravity. Renormalizable [Stelle 1979]

• SM Higgs in Weyl gravity/geometry:

$$\mathcal{L}=\sqrt{g}\,\Big\{\frac{1}{4!\,\xi^2}\,\tilde R^2-\frac{1}{4}\,F_{\mu\nu}^2\;-\frac{1}{12}\,\xi_1\,h^2\,\tilde R\;+\frac{1}{2}\,(\tilde D_\mu h)^2-\frac{\lambda}{4!}\,h^4\Big\},\qquad \tilde R^2\rightarrow -2\phi_0^2\,\tilde R-\phi_0^4$$

$$=\;\sqrt{g}\,\Big\{-\frac{1}{12}\underbrace{\Big[\frac{1}{\xi^2}\phi_0^2+\xi_1\,h^2\Big]}_{=\,6\,\rho^2}\tilde R-\frac{1}{4}\,F_{\mu\nu}^2+\frac{1}{2}\,(\tilde D_\mu h)^2-\frac{1}{4!}\,\Big[\lambda\,h^4+\frac{1}{\xi^2}\phi_0^4\Big]\Big\},\qquad \tilde D_\mu h=(\partial_\mu-\alpha/2\,\omega_\mu)h.$$

Riemannian notation:

$$\mathcal{L}=\sqrt{g}\,\Big\{-\frac{1}{2}\,\Big[\rho^2\,R+6\,(\partial_\mu\rho)^2\Big]+\frac{3}{4}\,\rho^2\,\Big(\omega_\mu-\frac{1}{\alpha^2}\,\partial_\mu\ln\rho^2\Big)^2-\frac{1}{4}\,F_{\mu\nu}^2+\frac{1}{2}(\tilde D_\mu h)^2-V(h,\rho)\Big\}.$$

Stueckelberg: radial direction $\ln\rho$ eaten by ω_μ. Next (*): $\quad \Omega=\frac{\rho^2}{\langle\hat\rho\rangle},\ \langle\hat\rho\rangle=M_P,\ \hat\omega_\mu=\omega_\mu-\frac{1}{\alpha^2}\partial_\mu\ln\rho^2$

⇒ Einstein-Proca: (ω_μ) :
$$\mathcal{L}=\sqrt{\hat g}\,\Big\{-\frac{1}{2}\,M_P^2\,\hat R+\frac{3}{4}\,\alpha^2\,M_p^2\;\hat\omega_\mu\hat\omega^\mu-\frac{1}{4}\,\hat F_{\mu\nu}^2+\frac{1}{2}\,(\hat{\tilde D}_\mu\hat h)^2-V\Big]\Big\},$$

Unitarity gauge: $\hat{h} \to M_p\sqrt{6}\sinh\frac{\sigma}{M_p\sqrt{6}}, \qquad \hat{\omega}_\mu \to \hat{\omega}_\mu + \partial_\mu \ln\cosh^2\frac{\sigma}{M_p\sqrt{6}}$

$$\Rightarrow \quad \mathcal{L} = \sqrt{\hat{g}}\,\Big\{ -\frac{1}{2}M_p^2\,\hat{R} + \frac{3}{4}\alpha^2\,M_p^2\;\hat{\omega}_\mu\hat{\omega}^\mu\cosh^2\frac{\sigma}{M_p\sqrt{6}} \;-\frac{1}{4}\hat{F}_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - V(\sigma)\Big\},$$

where $$V(\sigma) \;=\; V_0\,\Big\{\,\xi^2\Big[1-\xi_1\sinh^2\frac{\sigma}{M_p\sqrt{6}}\Big]^2 + \lambda\sinh^4\frac{\sigma}{M_p\sqrt{6}}\Big\},$$

$\Rightarrow$ Higgs coupling to ω_μ: $\qquad \mathcal{L} \sim \sqrt{\hat{g}}\;(1/8)\;\alpha^2\,\sigma^2\,\hat{\omega}_\mu\hat{}\omega^\mu + ...$

- Higgs from Weyl vector fusion: $\omega_\mu\omega_\mu \to \sigma\sigma \Rightarrow$ Higgs has a (non-metric) geometric origin!.
- Matter (higgs) creation from geometry (ω_μ is "geometric"). Irreversible processes [I. Prigogine et al, 1986]

• Palatini quadratic gravity [D.G. arxiv:2003.08516; 2007.14733]

- Palatini approach to gravity due to Einstein (1925): $\tilde{\Gamma}$ unknown, fixed by eqs of motion (action).
- $\tilde{\Gamma}$ independent of $g_{\mu\nu} \Rightarrow$ invariant of (*); define $\omega_\mu = (1/2)(\tilde{\Gamma}_\mu - \Gamma_\mu)$. $\qquad \tilde{R} = R(\tilde{\Gamma}, g)$.
- same $V(\sigma)$ but $\xi_1 \to 4\,\xi_1$, $\lambda \to 16\lambda$, (different non-metricity) but many more operators.

- SM Fermions in Weyl gravity/geometry:

$$L_\psi = \frac{i}{2}\sqrt{g}\,\overline{\psi}\,\gamma^a\,e_a^\mu\,\underbrace{\Big[\partial_\mu + \frac{1}{2}\,s_\mu^{ab}\,\sigma_{ab}\Big]}_{\nabla_\mu}\psi + \text{h.c.}\qquad \text{spin connection (Riemann):}\quad s_\mu^{ab} = -e^{\lambda b}\,(\partial_\mu e_\lambda^a - \Gamma^\nu_{\mu\lambda}\,e_\nu^a).$$

\- Weyl spin connection: $\tilde{s}_\mu^{ab} = s_\mu^{ab}\Big|_{\partial_\lambda\to\partial_\lambda+(\text{charge})\,\alpha\,\omega_\lambda}$, $\qquad s_\mu^{ab}\to\tilde{s}_\mu^{ab} = s_\mu^{ab} + (1/2)\,\alpha\,(e_\mu^a\,e^{\nu b} - e_\mu^b\,e^{\nu a})\,\omega_\nu.$

$\Rightarrow$ $\tilde{s}_\mu^{ab}$ is Weyl gauge invariant (like $\tilde{\Gamma}$). Lagrangian: replace $\partial_\lambda\psi\to\partial_\lambda + d_\psi\,\alpha\,\omega_\lambda$. Then ω_μ cancels out:

$$L_\psi = \frac{i}{2}\sqrt{g}\,\overline{\psi}\,\gamma^a\,e_a^\mu\Big[\partial_\mu + d_\psi\,\alpha\,\omega_\mu + \frac{1}{2}\,\tilde{s}_\mu^{ab}\,\sigma_{ab}\Big]\psi = \frac{i}{2}\sqrt{g}\,\overline{\psi}\,\gamma^a\,e_a^\mu\Big[\partial_\mu + \frac{1}{2}\,s_\mu^{ab}\,\sigma_{ab}\Big]\psi + \text{h.c.}$$

In SM: $$\mathcal{L}_\psi = \frac{i}{2}\sqrt{g}\,\overline{\psi}\,\gamma^a\,e_a^\mu\,\Big[\partial_\mu - i g\,\vec{T}\hat{\vec{A}}_\mu - i\,Y\,g'\hat{B}_\mu + \frac{1}{2}\,s_\mu^{ab}\,\sigma_{ab}\Big]\,\psi + \text{h.c.}$$

$\Rightarrow \mathcal{L}_\psi$ as in SM in Riemannian geometry, no coupling to ω_μ! Yukawa interactions: invariant [Kugo 1977]

\- Note: if $\text{U}(1)_Y\times\text{D}(1)$ kinetic mixing: $\hat{B}_\mu = B'_\mu - \omega'_\mu\tan\tilde{\chi}$; coupling $\propto Y$. ω_μ anomaly free & massive!

• SM Gauge bosons:

$$\mathcal{L}_b = -\frac{1}{4}\sqrt{g}\, F_{\mu\nu}\, F_{\rho\sigma}\, g^{\mu\rho}\, g^{\nu\sigma},$$

with $F_{\mu\nu} = \tilde{\nabla}_\mu A_\nu - \tilde{\nabla}_\nu A_\mu ... = \partial_\mu A_\nu - \partial_\nu A_\mu ...$, symmetric $\tilde{\Gamma}$. $\mathcal{L}_b$ invariant under (*) for $\hat{A}_\mu = A_\mu$.
$\Rightarrow$ Action similar to (pseudo)Riemannian case. $\Rightarrow$ only SM Higgs sector changes!

$\Rightarrow$ SM in Weyl geometry: minimal embedding, no new dof's beyond SM & WG. Higgs coupling to ω_μ.

- $m_\omega \sim \alpha M_p$ can be light, few TeV for $\alpha \ll 1$. Current bound on non-metricity: few TeV! [Latorre, Y. Lobo]

- Higgs mass quantum corrections: $\delta m_\sigma^2 \propto m_\omega^2$. Light m_ω: solution to mass hierarchy!

- above m_ω symmetry restored; no scale, no counterterm; quantum scale invariance necessary!

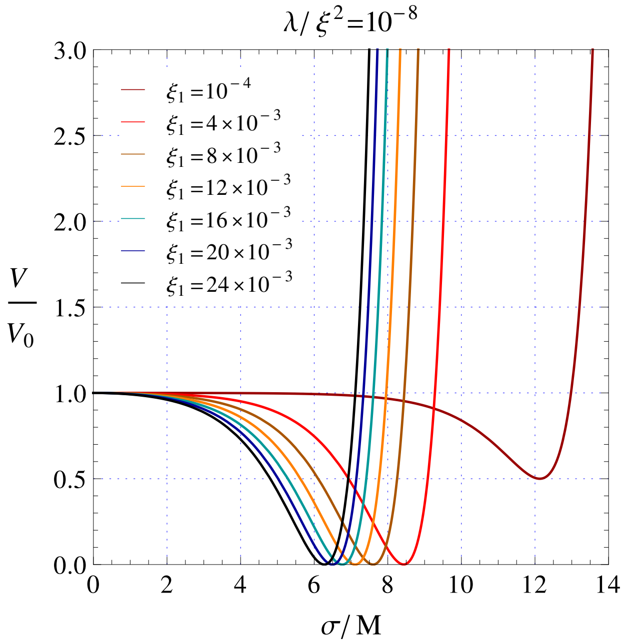
[D.G. 2203.05381, 2104.15118]

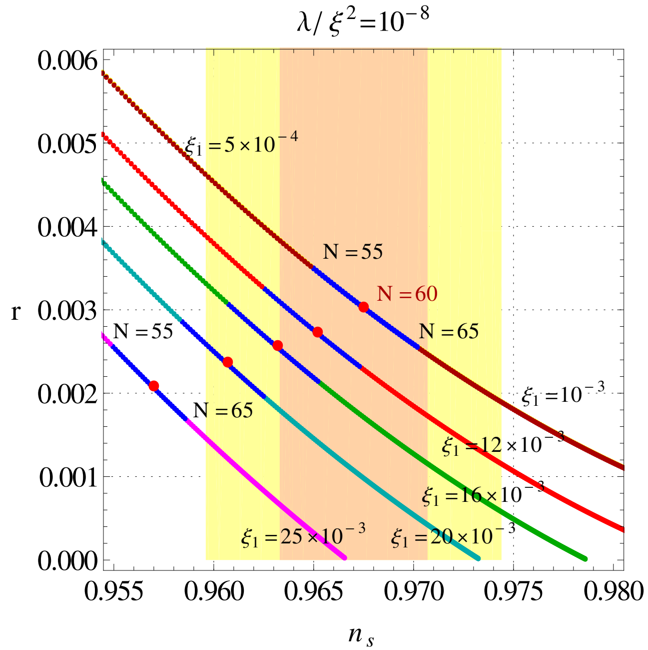
• Non-metricity (solid state physics): d=0 defects: metric anomalies/point defects: missing/extra atoms
- destroys crystalline structure, modify local notion of length, described by non-metric (Weyl) geometry.

[A. Roychowdhury, A. Gupta, 1601.06905]

• Weyl R^2-inflation

$$V = V_0 \left\{ \left[1 - \xi_1 \sinh^2 \frac{\sigma}{2\, M_p \sqrt{6}} \right]^2 + (\lambda/\xi^2)\ \sinh^4 \frac{\sigma}{2 M_p \sqrt{6}} \right\}$$





$$\lambda/\xi^2 \ll \xi_1^2 \ll 1: \quad \epsilon = \frac{M_p^2}{2} \frac{V'^2}{V^2} = \frac{\xi_1^2}{3} \sinh^2 \frac{2\sigma}{M_p\sqrt{6}} + \mathcal{O}(\xi_1^3); \quad \eta = M_p^2 \frac{V''}{V} = -\frac{2\xi_1}{3} \cosh \frac{2\sigma}{M_p\sqrt{6}} + \mathcal{O}(\xi_1^2)$$

$$\Rightarrow \quad r = 3\,(1-n_s)^2 - (16/3)\,\xi_1^2 + \mathcal{O}(\xi_1^3)$$

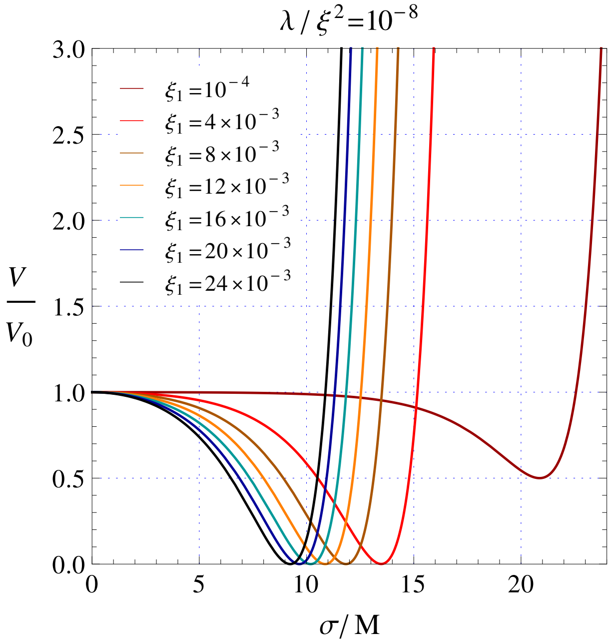
$0.002567 \leq r \leq 0.00303$ if $n_s = 0.9670 \pm 0.0037$; ($N = 60$) upper limit on r: Starobinsky: ($n_s \approx 0.968$)

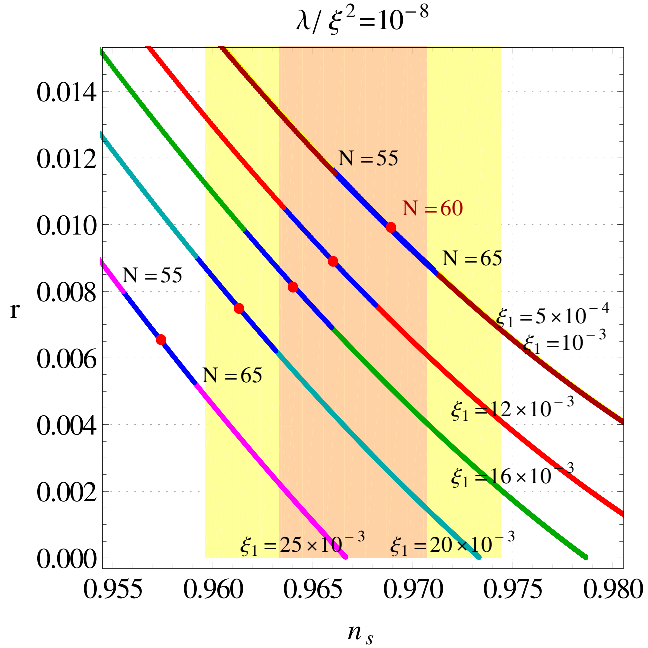
- Starobinsky: $R^2 + M_p R$. Weyl: $\tilde{R}^2 + h^2 \tilde{R} \Rightarrow$ similarity of $r(n_s)$. Gauged version of Starobinsky model!

[D.G. arxiv:2007.14733, 1906.11572; G. Ross, C. Hill, P, Ferreira, J. Noller 1906.03415]

• Palatini R^2-Inflation ($\theta = 4$)

$$V = V_0 \left\{ \left[1 - \theta\, \xi_1 \sinh^2 \frac{\sigma}{2\, M_p \sqrt{6\,\theta}} \right]^2 + (\lambda/\xi^2)\; \theta^2 \sinh^4 \frac{\sigma}{2 M_p \sqrt{6\,\theta}} \right\}$$





$$\lambda/\xi^2 \ll \xi_1^2 \ll 1: \quad \epsilon = \frac{M_p^2}{2} \frac{V'^2}{V^2} = \frac{\xi_1^2}{3}\, \theta \sinh^2 \frac{2\sigma}{M_p \sqrt{6\theta}} + \mathcal{O}(\xi_1^3); \quad \eta = M_p^2 \frac{V''}{V} = -\frac{2\xi_1}{3} \cosh \frac{2\sigma}{M_p \sqrt{6\theta}} + \mathcal{O}(\xi_1^2)$$

$$\Rightarrow \qquad r = 3\theta\, (1 - n_s)^2 + \mathcal{O}(\xi_1^2)$$

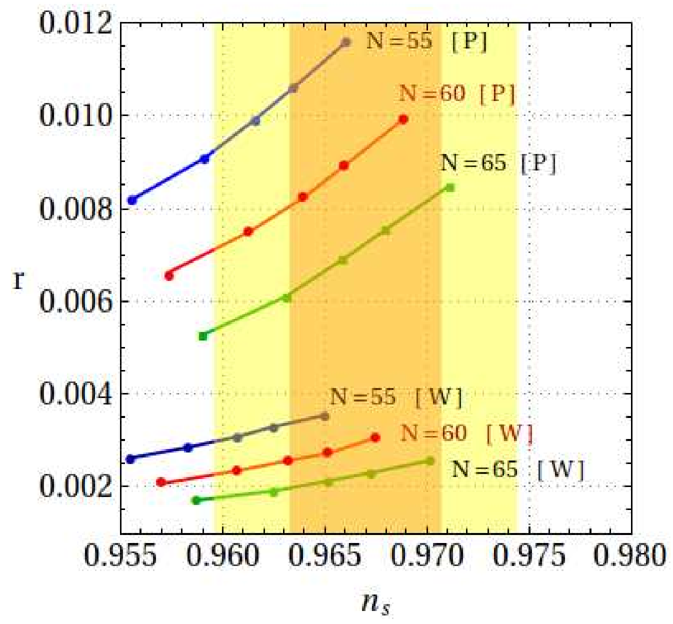
$0.00794 \leq r \leq 0.01002$ if $n_s = 0.9670 \pm 0.0037$; $(N = 60)$

[D.G. arxiv:2007.14733, 2003.08516]

• Weyl versus Palatini: R^2-inflation predictions [D.G. arXiv:2007.14733]

- tensor-to-scalar ratio r versus spectral index n_s with orange (yellow) values of n_s at 68% (95%) CL.
- the difference (θ) due to different non-metricity of these theories.
- such values of r reachable by future CMB experiments (0.0005 precision; LiteBIRD, CMB-S4).

⇒ One will be able test and discriminate Weyl vs Palatini model



• Weyl anomaly in Riemannian geometry:

- Weyl-invariant scalar field action W_s, coupled to gravity; $g_{\mu\nu}$ external field. At quantum level it generates:

$$W_d = \frac{1}{d-4}\int d^dx\sqrt{g}\,A(d), \qquad d = 4-2\epsilon,$$

$A(d)$ = higher derivative ops: $R\,\Box^{(d-4)/2}R,\quad R_{\mu\nu}\Box^{(d-4)/2}R^{\mu\nu},\quad R_{\mu\nu\rho\sigma}\Box^{(d-4)/2}R^{\mu\nu\rho\sigma}$, ...

$$W_c = -\frac{\mu^{d-4}}{(d-4)}\int d^dx\sqrt{g}\,\big(b\,C^2_{\alpha\beta\gamma\delta} + b'\,G\big), \qquad G \equiv R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2 \to E_4 \equiv \nabla_\mu V^\mu,\ d=4,$$

$$\begin{aligned}\frac{2}{\sqrt{g}}g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\int d^dx\sqrt{g}\,C^2_{\alpha\beta\gamma\delta} &= (d-4)\ \big(C^2_{\alpha\beta\gamma\delta} + \frac{2}{3}\Box R\big),\\ \frac{2}{\sqrt{g}}g_{\mu\nu}\frac{\delta}{\delta g_{\mu\nu}}\int d^dx\sqrt{g}\,G &= (d-4)\ G,\end{aligned}$$

$$T_\mu^\mu = \frac{-2}{\sqrt{g}}g_{\mu\nu}\frac{\delta(W_\phi + W_d + W_c)}{\delta g_{\mu\nu}}\Big|_{d=4} = \frac{-2}{\sqrt{g}}g_{\mu\nu}\frac{\delta W_c}{\delta g_{\mu\nu}}\Big|_{d\to 4} = b\,[\,C^2_{\mu\nu\rho\sigma} + (2/3)\Box R\,] + b'G \neq 0$$

$\Rightarrow$ Weyl anomaly.

• Weyl anomaly in Riemannian geometry:

$$d = 4 \qquad W_r = \frac{b}{2}\int d^4x\sqrt{g}\, C_{\mu\nu\rho\sigma}\ln(\Box/\mu^2)\, C^{\mu\nu\rho\sigma}, \qquad T^\mu_\mu \propto -g'_{\mu\nu}\delta W_r/\delta g'_{\mu\nu} \propto b\, C^2_{\mu\nu\rho\sigma}.$$

• Weyl gauge-invariant regularisation: $\mu \to \phi$ [ϕ dilaton]

$$W_c = -\frac{b}{d-4}\int d^dx\sqrt{g}\,\phi^{2(d-4)/(d-2)}\, C^2_{\mu\nu\rho\sigma}, \qquad W_r = \int d^4x\sqrt{g}\ \hat{C}_{\mu\nu\rho\sigma}\left(c_0 + c_1\ln\frac{\hat{\Box}}{\phi^2}\right)\hat{C}^{\mu\nu\rho\sigma}$$

⇒ Weyl invariance maintained for the Weyl term, anomaly recovered for $\phi \to \langle\phi\rangle = \mu$.

But Euler-Gauss-Bonnet anomaly (topological), since it is independent of μ! $G \propto \nabla_\mu(...)$ in $d = 4$ only.

Not Weyl invaraint in d dimensions.

• In Weyl geometry: $G \to \hat{G}$ is Weyl covariant in d dimensions!.

• Weyl geometry vs Weyl anomaly: In Weyl geometry: $\hat\nabla_\mu \hat R$, $\hat\nabla_\mu \hat R_{\alpha\beta}$, $\hat\Box R$, etc: Weyl covariant!

$$W_d = \frac{1}{d-4}\int d^dx\sqrt{g}\,A(d), \qquad d = 4-2\epsilon,$$

Weyl gauge invariant: $\hat R\,(\hat\nabla_\mu\hat\nabla^\mu)^{(d-4)/2}\hat R$, $\hat R_{\mu\nu}\,(\hat\nabla_\mu\hat\nabla^\mu)^{(d-4)/2}\hat R^{\mu\nu}$, $\hat R_{\mu\nu\rho\sigma}\,(\hat\nabla_\mu\hat\nabla^\mu)^{(d-4)/2}\hat R^{\mu\nu\rho\sigma}$, etc.

$$W_c = -\frac{1}{d-4}\int d^dx\sqrt{g}\left\{a_1\hat R^2 + b_1\hat F^2_{\mu\nu} + c_1\hat C^2_{\mu\nu\rho\sigma} + d_1\hat G\right\}\phi^{2(d-4)/(d-2)}, \quad \text{or} \quad \phi^{2(d-4)/(d-2)} \to |\hat R|^{(d-4)/2}.$$

$$T^\mu_\mu - \frac{1}{\alpha}\nabla_\mu J^\mu = 0, \qquad J_\mu \propto (\partial_\mu - \alpha\omega_\mu)\phi^2 = \hat\nabla_\mu\phi^2, \qquad \text{onshell: } J^\mu + \nabla_\sigma F^{\sigma\mu} = 0.$$

Renormalized action, $d=4$:

$$W_r = \int d^4x\sqrt{g}\,\Big\{\hat R\Big[a_0 + a_1\ln\frac{\hat\Box}{\phi^2}\Big]\hat R + \hat F_{\mu\nu}\Big[b_0 + b_1\ln\frac{\hat\Box}{\phi^2}\Big]\hat F^{\mu\nu} + \hat C_{\mu\nu\rho\sigma}\Big[c_0 + c_1\ln\frac{\hat\Box}{\phi^2}\Big]\hat C^{\mu\nu\rho\sigma}$$
$$+\ \hat R_{\mu\nu\rho\sigma}\Big[d_0 + d_1\ln\frac{\hat\Box}{\phi^2}\Big]\hat R^{\rho\sigma\mu\nu} - 4\hat R_{\mu\nu}\Big[d_0 + d_1\ln\frac{\hat\Box}{\phi^2}\Big]\hat R^{\nu\mu} + \hat R\Big[d_0 + d_1\ln\frac{\hat\Box}{\phi^2}\Big]\hat R\Big\},$$

$\Rightarrow$ Weyl gauge invariant! Weyl anomaly in the broken phase: $\phi \to \langle\phi\rangle = \mu$, ω_μ massive ($\to 0$), $\hat R \to R$...

- Conclusions:

- Weyl quadratic gravity action: viable gauge theory of scale invariance
- broken via Stueckelberg to Einstein-Proca action for ω_μ; ω_μ massive $>$ TeV
- Planck mass, m_ω, Λ: all of (non-metric/Weyl) geometric origin.

- SM in Weyl geometry: both geometry (i.e. connection) and action have this symmetry.
- Higgs coupling to ω: $\Delta L = \omega_\mu \omega^\mu h^2$: geometric origin of h: $\omega_\mu + \omega_\mu \to h + h$.
- Mass hierarchy solution: Weyl gauge symmetry protects m_h for light $m_\omega \sim$ TeV.

- Weyl anomaly: in Weyl geometry Euler-Gauss-Bonn term is Weyl covariant in d dimensions!
  - $\hat{G}\,\sqrt{g}$ and $\hat{C}^2_{\alpha\beta\gamma\delta}\sqrt{g}$ $+$ Weyl invariant regularisation: $\Rightarrow$ Weyl gauge symmetry maintained!
  - Weyl anomaly emerges in the broken phase only.

- Tests? a) Starobinsky-like inflation $0.00257 \leq r \leq 0.00303$. b) Higgs physics vs ω_μ.
  c) Gravitational waves d) ω_μ as dark matter

- Parallel transport for vector u_μ:

$$\hat{\omega}_\mu(x) = \omega_\mu(x) - \frac{1}{\alpha}\partial_\mu \ln\Omega(x)^2, \quad \hat{g}_{\mu\nu}(x) = \Omega(x)^2\, g_{\mu\nu}(x), \;\; \hat{\phi}(x) = \frac{\phi(x)}{\Omega(x)} \;\; \hat{A}_\mu = A_\mu, \quad \hat{u}^\mu = \Omega^{z_u} u^\mu$$

Parallel transport along $\gamma(\tau)$:

$$\frac{D\,u^\mu}{d\tau} = 0, \quad \text{where} \quad D \equiv dx^\lambda\, D_\lambda, \quad D_\lambda\, u^\mu = \tilde{\nabla}_\lambda u^\mu\Big\vert_{\partial_\lambda\to\partial_\lambda + z_u\,\alpha\,\omega_\lambda}, \qquad \tilde{\nabla}_\lambda u^\mu = \partial_\lambda u^\mu + \tilde{\Gamma}^\mu_{\lambda\rho}\, u^\rho,$$

and $x = x(\tau)$. Then the differential variation: $\quad d\,u^\mu = dx^\lambda\,\partial_\lambda u^\mu = -dx^\lambda\Big[z_u\,\alpha\,\omega_\lambda\,u^\mu + \tilde{\Gamma}^\mu_{\lambda\rho}\,u^\rho\Big],$

$$d\,\langle u, v\rangle = d\,\Big[u^\mu\, v^\nu\, g^{\mu\nu}\Big] = -\alpha\, dx^\lambda\,\omega_\lambda\, g_{\mu\nu}\Big[2 + (z_u + z_v)\Big]\, u^\mu\, v^\nu = -\alpha\, dx^\lambda\,\omega_\lambda\,\Big[2 + (z_u + z_v)\Big]\,\langle u, v\rangle$$

For the norm: $\quad d\ln|u|^2 = dx^\lambda\,\omega_\lambda\,(-\alpha)\,(1 + z_u), \quad \Rightarrow \quad |u|^2 = |u_0|^2\, e^{-\alpha\,(1+z_u)\,\int_{\gamma(\tau)}\omega_\lambda dx^\lambda}.$

WG: $\Rightarrow$ symmetric phase: no mass $\Rightarrow$ no clock rate $\Rightarrow$ no second clock effect & no experiment possible.

WG: $\Rightarrow$ broken phase: mass generated; metric theory below m_ω $\Rightarrow$ second clock effect suppressed by m_ω.

Ratio $|u|/|v|$ independent of units of length if $z_u = z_v$. Integrable geometry $\omega_\lambda = \partial_\lambda(\ldots)$ then $|u| = |u_0|$.

• Weyl "photon" - photon mixing?: adding $U(1)_Y$ to $\mathcal{L}_0$; [source: SM fermions action]

$$\mathcal{L}_0 \to \mathcal{L}_1 = \sqrt{g}\,\Big\{ \frac{1}{4!\,\xi^2}\,\tilde{R}^2 - \frac{1}{4}\Big[F_{\mu\nu}^2 + 2\sin\chi\, F_{\mu\nu}\, F_y^{\mu\nu} + F_{y\,\mu\nu}^{\,2}\Big]\Big\}.$$

Re-do calculation, diagonalize mixing by:

$$\hat{\omega}_\mu \;=\; \gamma\,\omega'_\mu \sec\chi, \qquad\qquad \hat{B}_\mu = B'_\mu - \omega'_\mu\,\tan\chi,$$

Then

$$\mathcal{L}_1 = \sqrt{\hat{g}}\,\Big\{ -\frac{1}{2}\,M_p^2\,\hat{R} + \frac{3}{4}\,M_p^2\,\alpha^2\,\gamma^2 \sec\chi^2\;\omega'_\mu\omega'^\mu - \frac{1}{4}\,(F'^{\,2}_{\mu\nu} \;+\; F'^{\,2}_{y\,\mu\nu})\Big\},$$

- the photon after EWSB:

$$A_\mu = B'_\mu \cos\theta_w + A^3_\mu\,\sin\theta_w = \big[\hat{B}_\mu + \hat{\omega}_\mu \sin\chi\big]\,\cos\theta_w + \sin\theta_w A^3_\mu.$$

- gauge kinetic mixing $\to$ photon includes a small 'piece' of ω_μ, suppressed by $\sin\chi$

- Weyl was not completely wrong in trying to relate ω_μ to the photon - they can mix;

- mixing not forbidden by Coleman-Mandula: symmetry direct product $U(1)_Y \times D(1)$; both broken spontaneously.

• Higgs sector in Weyl geometry: $\tilde{D}_\mu H = \big[\partial_\mu - i\mathcal{A}_\mu - (1/2)\,\alpha\,\omega_\mu\big]\,H,$

$$\mathcal{L}_H = \sqrt{g}\,\Big\{\,\frac{\tilde{R}^2}{4!\,\xi^2} - \frac{\xi_1}{6}\,|H|^2\tilde{R} + |\tilde{D}_\mu H\,|^2 - \lambda\,|H|^4 - \frac{1}{4}\Big[F_{\mu\nu}^2 + 2\sin\chi\,F_{\mu\nu}\,F_y^{\mu\nu} + F_{y\,\mu\nu}^2\Big]\Big\}.$$

where $\mathcal{A}_\mu = (g/2)\,\vec{\sigma}.\vec{A}_\mu + (g'/2)\,B_\mu;$ $\vec{A}_\mu$ is the $SU(2)_L$ boson, B_μ is the $U(1)_Y$ boson.

- Potential:

$$\hat{V}(\sigma) \;=\; V_0\Big\{6\lambda\,\sinh^4\frac{\sigma}{M_p\sqrt{6}} + \xi^2\Big[1 - \xi_1\sinh^2\frac{\sigma}{M_p\sqrt{6}}\Big]^2\Big\}, \quad V_0 \equiv (3/4)M_p^4.$$

$$=\; \frac{1}{4}\Big[\lambda - \frac{1}{9}\xi_1\,\xi^2 + \frac{1}{6}\xi_1^2\,\xi^2\Big]\,\sigma^4 - \frac{1}{2}\,\xi_1\xi^2 M_p^2\,\sigma^2 + \frac{3}{2}\,\xi^2 M_p^4 + \mathcal{O}(\sigma^6/M_p^2).$$

if $\xi_1\xi^2 \ll 1$: $\quad \langle\sigma\rangle^2 = (\xi_1\,\xi^2/\lambda)\,M_p^2, \quad m_\sigma^2 = 2\,\xi_1\,\xi^2\,M_p^2,$

- Hierarchy using $\xi\sqrt{\xi_1} \sim 3.5\times 10^{-17}$, $\lambda \sim 0.12$ (SM). Hierarchy controlled by ξ of $\tilde{R}^2$ term!

$$m_Z^2 = \frac{1}{4}\,(g^2 + g'^2)\,\langle\sigma\rangle^2\Big\{1 + \frac{\langle\sigma\rangle^2}{18M_p^2}\Big[1 - \frac{3\,g'^2}{\alpha^2}\sin^2\chi\Big] + \mathcal{O}(\langle\sigma\rangle^4/M_p^4)\Big\}.$$

$\Rightarrow$ Part of Z mass due to Weyl geometry (mixing with ω_μ), beyond Einstein gravity/Riemannian geometry

- Precision constraints (Z mass):

$$\varepsilon \equiv \frac{\Delta m_Z}{m_{Z^0}} = -\frac{g'^2 \langle\sigma\rangle^2}{12\, M_p^2}\frac{\sin^2\chi}{\alpha^2} + \mathcal{O}\Big(\frac{\langle\sigma\rangle^4}{M_p^4}\Big) = -\frac{1}{8}\Big(\frac{\langle\sigma\rangle}{m_\omega}\Big)^2 (g'\,\tan\chi)^2 + \mathcal{O}\Big(\frac{\langle\sigma\rangle^4}{m_w^4}\Big).$$

- $\langle\sigma\rangle = 246.22$ GeV; at 68% CL, $\varepsilon = 2.3\times 10^{-5}$, then: $\alpha \geq 2.17\times 10^{-15}\sin\chi$.

- in terms of the mass: $\dfrac{m_\omega}{\text{TeV}} \geq 6.35 \times \tan\chi.$

- current bound on non-metricity scale $m_\omega \sim$ few TeV, then: $\tan\chi \leq 0.16$

$\Rightarrow$ the constraint from Z-mass is v. strong: e.g. effect of ω_μ to Δa_μ of muon magnetic moment:

$$\Delta a_\mu \sim \frac{1}{12\pi^2}\frac{m_\mu^2}{m_\omega^2}(g'\tan\chi)^2 = 2.56\times 10^{-13},$$

which is very small (cannot match the ongoing discrepancy).