Testing Nonlocal Cosmologies
from the Chiral/conformal Anomaly Effective Action

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Work in colaboration with
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Assuming a conformal phase of the early universe, we discuss the **conformal backreaction**, by studying the **anomaly effective action around flat space** and, in parallel, **the anomaly induced action in curved space**. Gravity is treated classically.

Anomalies included take the form both of conformal (parity-even) and of chiral (parity-odd) contributions. We show that both the anomaly induced actions in the **Riegert and the Fradkin-Vilkovisky gauge are inconsistent** starting at the level of 4-point functions. They agree with our CFT analysis only up to 3-point functions.

We show how correlators of the form **TTJJ**, predicted by the anomaly induced action, can be corrected in order to provide the correct expressions derived in free field theory realizations of the same correlators.

Crucial, in these derivations, is the possibility of solving the conformal Ward identities (CWIs) using CFT methods in momentum space, that we have extended from 3- to 4-point functions and define a powerful way to patch flat and curved spacetime derivations
e-Print: 2212.12779

In the case of parity odd anomalies of correlators such as **TJJ, AVV, AAA** and the gravitational chiral anomaly **J5TT**, we show how **the inclusion of a nonlocal exchange** (an anomaly pole) **and the CWIs** completely fix these correlation functions.
This shows the crucial role of such nonlocal interactions in the early universe.
2303.10710 [hep-th] and 2307.03038 [hep-th]

Refs

Three-Wave and Four-Wave Interactions in the 4*d* Einstein Gauss-Bonnet (EGB) and Lovelock Theories
e-Print: 2302.02103 [hep-th]

with Maglio, Mario Creti', S. Lionetti

Broken scale invariance and the regularization of a conformal sector in gravity with Wess-Zumino actions

Phys.Lett.B 843 (2023) 138003
e-Print: 2301.07460 [hep-th] with **Maglio, Creti'**

Topological corrections and conformal backreaction in the Einstein Gauss–Bonnet/Weyl theories of gravity at □=4*D*=4

Eur.Phys.J.C 82 (2022) 12, 1121
e-Print: 2203.04213 [hep-th]

with **Maglio, D. Theofilopoulos**

Einstein Gauss-Bonnet theories as ordinary, Wess-Zumino conformal anomaly actions

Phys.Lett.B 828 (2022) 137020. with M. Maglio
e-Print: 2201.07515

Conformal field theory in momentum space and anomaly actions in gravity: The analysis of three- and four-point function

Review of CWIs methods , Maglio, CC
Phys.Rept. 952 (2022) 1-95 e-Print: 2005.06873

The Gravitational Chiral Anomaly from Parity-Violating CFT's in Momentum Space

Stefano Lionetti, M. Maglio, C.C.

To appear soon

Momentum space methods in CFT allow to describe quite efficiently the correlators containing insertions of stress energy tensor (T) and/or axial vector currents, and affected by conformal and chiral anomalies. (TTT, TTJ5, J5JJ,TTTT)

Analysis have been performed up to 4-point functions (4T).

The hierarchy of the conformal Ward identities (CWIs) constraining such correlation functions have been investigated using both free field theory realizations and, nonperturbatively, using their CWIs

By this approach it has also been shown the inconsistency of anomaly induced actions in the Riegert and in the Fradkin-Vilkovisky beyond 3-point functions. Corrections identified for a specific correlator (TTJJ)

We will overview the methodology and the main results in this area, and the central role played by anomaly poles in determining the structure of these interactions.

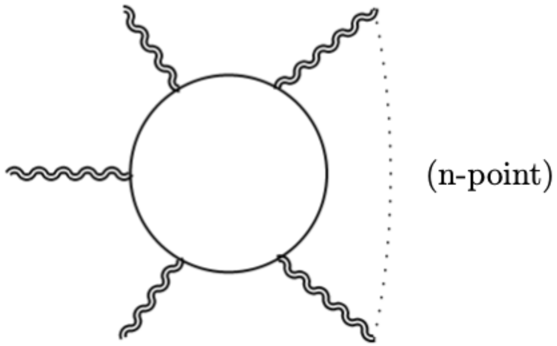
Yangian Symmetry in momentum space (Maglio, CC)

- *JHEP* 09 (2019) 107 e-Print: 1903.05047

On Some Hypergeometric Solutions of the Conformal Ward Identities of Scalar 4-point Functions in Momentum Space

Conformal Backreaction

$$\mathcal{S}(g) = \sum_n$$



The conformal backreaction
is the description of how a conformal matter sector modifies gravity. We can think of it as a functional expansion of correlation functions of n-graviton vertices.
We don't quantize gravity, which is purely external and classical

$$\langle T^{\mu_1\nu_1}(x_1)\dots T^{\mu_n\nu_n}(x_n)\rangle \equiv \frac{2}{\sqrt{g_1}}\cdots\frac{2}{\sqrt{g_n}}\frac{\delta^n\mathcal{S}(g)}{\delta g_{\mu_1\nu_1}(x_1)\delta g_{\mu_2\nu_2}(x_2)\dots\delta g_{\mu_n\nu_n}(x_n)}$$

This functional expansion,
Is the expansion of a classical effective action.
The "easiest" thing to do
Is to compute the first few
Contributions around flat space.

We can think of the effective action as generated by integrating out a conformal matter sector in a functional integral

$$\mathcal{Z}_B(g) = \mathcal{N} \int D\chi e^{-S_0(g,\chi)},$$

The bare partition function

$$S_0(g,\phi) \;\; = \;\; \frac{1}{2}\int d^dx\,\sqrt{-g}\,\left[g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \chi\,R\,\phi^2\right]$$

Simplest case: a conformal scalar sector

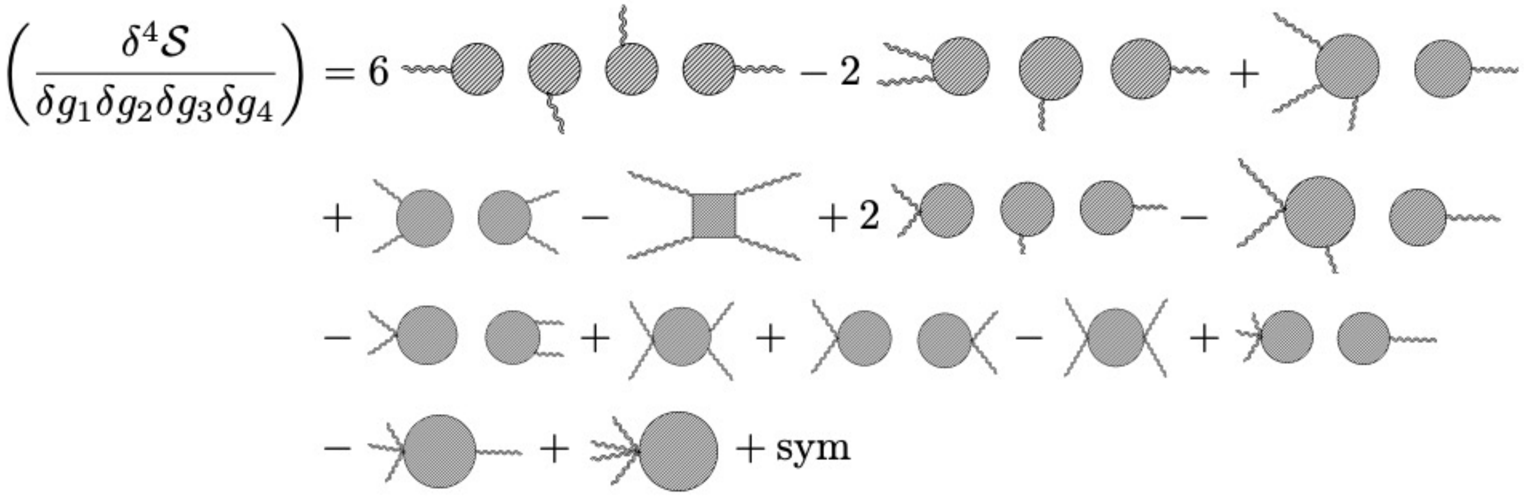
$$\begin{aligned} T^{\mu\nu}_{scalar} &\equiv \frac{2}{\sqrt{g}}\frac{\delta S_0}{\delta g_{\mu\nu}} \\ &= \nabla^\mu\chi\,\nabla^\nu\chi - \frac{1}{2}\,g^{\mu\nu}\,g^{\alpha\beta}\,\nabla_\alpha\chi\,\nabla_\beta\chi + \chi\left[g^{\mu\nu}\Box - \nabla^\mu\,\nabla^\nu + \frac{1}{2}\,g^{\mu\nu}\,R - R^{\mu\nu}\right]\chi^2, \end{aligned}$$

$$\chi(d) = \frac{1}{4}\frac{(d-2)}{(d-1)},$$

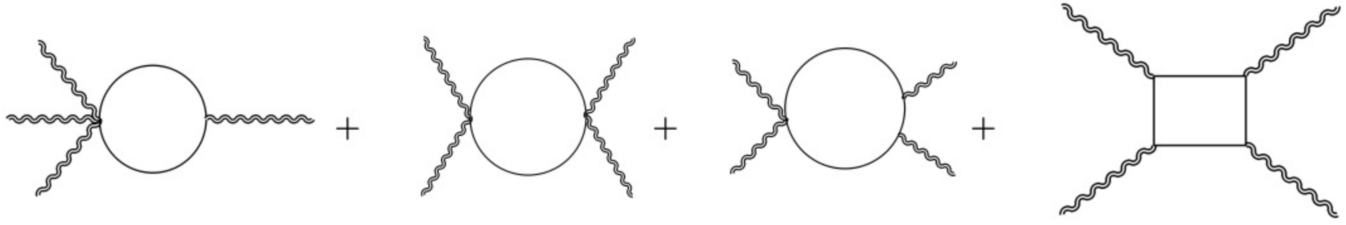
Conformally coupled scalar

$$e^{-\mathcal{S}_B(g)} = \mathcal{Z}_B(g) \leftrightarrow \mathcal{S}_B(g) = -\log \mathcal{Z}_B(g).$$

It is better to define the effective action as the log of the bare partition function



$\mathcal{S}_4[g] =$



$$\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}T^{\mu_3\nu_3}T^{\mu_4\nu_4}\rangle = 16\Bigg(- \left\langle \frac{\delta S_0}{\delta g_1}\frac{\delta S_0}{\delta g_2}\frac{\delta S_0}{\delta g_3}\frac{\delta S_0}{\delta g_4}\right\rangle_c + \left\langle \frac{\delta S_0}{\delta g_3}\frac{\delta S_0}{\delta g_4}\frac{\delta^2 S_0}{\delta g_1 \delta g_2}\right\rangle_c - \left\langle \frac{\delta^2 S_0}{\delta g_1\delta g_4}\frac{\delta^2 S_0}{\delta g_2\delta g_3}\right\rangle_c - \left\langle \frac{\delta S_0}{\delta g_4}\frac{\delta^3 S_0}{\delta g_1\delta g_2\delta g_3}\right\rangle_c + \left\langle \frac{\delta^4 S_0}{\delta g_1\delta g_2\delta g_3\delta g_4}\right\rangle_c + \text{sym}\Bigg).$$

Dimensional counting can be performed in two ways: either by considering the canonical dimensions of the fields

$$x_\mu \to \lambda x_\mu \qquad g_{\mu\nu} \to g_{\mu\nu} \qquad \phi \to \lambda^{(1-d/2)}\phi.$$

or by the response under a Weyl transformation

$$g_{\mu\nu} \to \lambda^2 g_{\mu\nu}, \qquad x_\mu \to x_\mu \qquad \sqrt{g} \to \lambda^d \sqrt{g} \qquad \phi \to \lambda^{1-d/2}\phi.$$

Notice that the coordinates are inhert, while we keep the same canonical dimensions for the fields.

Anomalous dimensions start developing as we switch on a marginal interaction

For instance, a $\lambda_0\phi^4/4!$ potential

$$\begin{aligned} \Delta_\phi &= \frac{d-2}{2} + \frac{\lambda_0^2}{12(4\pi)^4} + O(\lambda_0^3) \\ \beta(\lambda_0) &= -2\epsilon\lambda_0 + 3\frac{\lambda_0^2}{(4\pi)^2} + O(\lambda_0^3), \end{aligned}$$

The simplest case is to leave aside such corrections, and consider only a free field theory interacting with External gravity. This is what we would call a “free field theory” realization.

We don't need gravity in order to define the correlation functions of n-point functions, but it is convenient to do so.

In ordinary CFT's there is no running, the structure is rigid and the scaling dimensions are fixed. In this case the correlaton functions are fixed, modulo few general constants that are typical of a specific CFT

On he other hand, we have theories like QCD where, for instance around the light cone,
The field develop anomalous dimensions (DIS, parton distributions, factorization and perturbative evolution)

At a conformal fixed point the structure of the correlators is constrained. Away from these points,
We need a case by case computation to describe these theories.

There are theories that sit "in between" Theories with a "generalized conformal structure"

$$\langle O_\Delta(x) O_\Delta(0)\rangle = \frac{\tilde{c}_\Delta(\tilde{g})}{x^{2\Delta}}$$

Two-point function of the energy-momentum tensor and generalised conformal structure

Delle Rose, Skenderis,CC

In general, in a gravitational theory, we require diffeomorphism invariance of the action,
i.e. conservation of the stress energy tensor.

However, we could consider a class of metrics that **allow conformal Killing vectors**.
Having CKV is a special property of the metric and we **restrict** ourselves to such class of metrics.

We consider a conformal sector, in our case a free field theory realization of a CFT made of
- conformally coupled scalars,
- a fermion sector
- a spin 1 sector at d=4

Coupled to an external metric that supports CKVs.
We keep the multiplicities of such fields arbitrary. **So these combined realizations have 3 independent constants**

If the metric is selected in such a way to allow conformal Killing vectors, then the diffeomorphisms $x^\mu \to x'^\mu = x^\mu + \epsilon^{(K)}(x)$ induce a simple rescaling of the infinitesimal distance, under a local rescaling with $\sigma(x)$

$$(ds')^2 = e^{2\sigma(x)}(ds)^2.$$

Under these special diffeomorphisms, the metric undergoes only a local dilatation

This require that $\sigma(x)$ and the same vectors are related

$$\nabla_\mu \epsilon_\nu^{(K)} + \nabla_\nu \epsilon_\mu^{(K)} = 2\sigma\delta_{\mu\nu} \qquad \sigma = \frac{1}{4}\nabla \cdot \epsilon^{(K)}.$$

$$\langle J_c^{\mu(K)} \rangle \equiv \epsilon_\nu^{(K)} \langle T^{\mu\nu} \rangle$$

We define the conformal currents using such CKV and perform a quantum average

that differentiated give

$$\nabla_\mu \langle J_c^{\mu(K)} \rangle = \frac{1}{2}\left(\nabla_\mu \epsilon_\nu^{(K)} + \nabla_\nu \epsilon_\mu^{(K)}\right)\langle T^{\mu\nu}\rangle + \epsilon_\nu^{(K)} \nabla_\mu \langle T^{\mu\nu} \rangle.$$

$$\sqrt{g}\nabla_\mu \langle J_c^{\mu(K)} \rangle = \frac{\delta}{\delta\sigma}\langle \mathcal{S}_{cl} \rangle + \epsilon_\nu^{(K)} \nabla_\mu \langle \frac{\delta}{\delta g_{\mu\nu}} \mathcal{S}_{cl} \rangle.$$

The conservation of the conformal currents

$$\sqrt{g}\nabla_\mu \langle J_c^{\mu(K)} \rangle = 0,$$

requires that

$$\frac{\delta}{\delta\sigma}\mathcal{S}_{cl} = 0,$$

plus the ordinary diffeomorphism invariance,

$$\nabla_\mu \langle \frac{\delta}{\delta g_{\mu\nu}} \mathcal{S}_{cl} \rangle = 0.$$

$$\frac{\delta}{\delta\sigma}\langle\mathcal{S}_{cl}\rangle = \sqrt{g}\langle T^\mu_\mu\rangle = \sqrt{g}\mathcal{A},$$

where $\mathcal{A}$ is the anomaly functional. In general we need to consider the anomalous terms

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = b_1\,E_4 + b_2\,C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + b_3\,\nabla^2 R + b_4\,F^{\mu\nu}F_{\mu\nu} + f_1\,\varepsilon^{\mu\nu\rho\sigma}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma} + f_2\,\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}.$$

Parity even + Parity odd contributions.

We bring the example of a specific correlation function, TTJ5, discussed in

The Gravitational Chiral Anomaly from Parity-Violating CFT's in Momentum Space

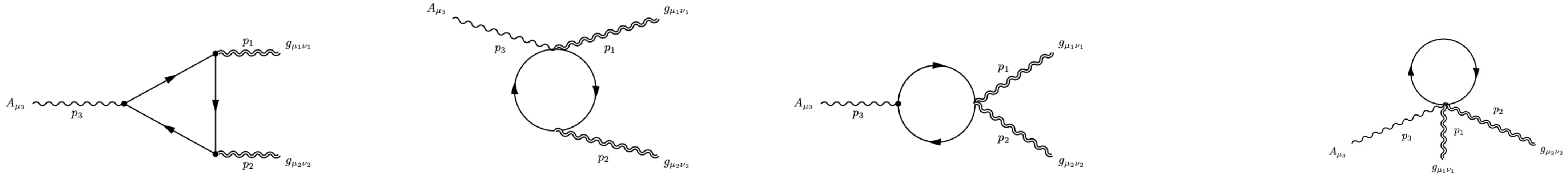
Stefano Lionetti, M. Maglio, CC

We illustrate how the Conformal Ward Identities (CWI) of Conformal Field theories (CFT's) in momentum space, and the anomaly, completely determine the structure of the $\langle TTJ_5\rangle$

$$\langle T^{\mu_1\nu_1}(x_1)\, T^{\mu_2\nu_2}(x_2)\, J_5^{\mu_3}(x_3)\rangle \equiv 4\, \frac{\delta^3 Z}{\delta g_{\mu_1\nu_1}(x_1)\delta g_{\mu_2\nu_2}(x_2)\delta A_{\mu_3}(x_3)}\Bigg|_{\substack{g=\delta;\\ A=0;}}$$

Gravitational chiral anomaly

Delbourgo-Salam



$$\nabla_\mu \langle J_5^\mu \rangle = a_1\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_2\, \varepsilon^{\mu\nu\rho\sigma} R^{\alpha\beta}{}_{\mu\nu} R_{\alpha\beta\rho\sigma},$$

We have a chiral current coupled to gravity

This current, in a field theory realization can be bilinear in the fermions (the usual axial vector current, but could also be a Chern-Simons current

$$J^\lambda_{5f} = \bar\psi \gamma_5 \gamma^\lambda \psi$$

$$J^\lambda_{CS} = \epsilon^{\lambda\mu\nu\rho} A_\mu \partial_\nu A_\rho,$$

$$\exp\left\{i\,\mathcal{S}[g]\right\} \equiv \int [d\Phi]\exp\left\{i\,\mathrm{S}_0[\Phi,g]\right\}$$

$$\langle T^{\mu\nu}(x)\rangle = \frac{2}{\sqrt{-g(x)}}\frac{\delta Z}{\delta g_{\mu\nu}(x)}\bigg|_{g=\delta}, \qquad \langle J_5^{\mu}(x)\rangle = \frac{1}{\sqrt{-g(x)}}\frac{\delta Z}{\delta A_{\mu}(x)}\bigg|_{A=0}$$

$$\langle T^{\mu_1\nu_1}(x_1)\,T^{\mu_2\nu_2}(x_2)\,J_5^{\mu_3}(x_3)\rangle \equiv \frac{2^2}{\sqrt{-g(x_1)}\sqrt{-g(x_2)}\sqrt{-g(x_3)}}\frac{\delta^3 Z}{\delta g_{\mu_1\nu_1}(x_1)\delta g_{\mu_2\nu_2}(x_2)\delta A_{\mu_3}(x_3)}\bigg|_{\substack{g=\delta;\\ A=0;}}.$$

- **Diffeomorhism invariance**

We start from diffeomorhism invariance.
Under a diffeomorphism the fields transform with a Lie derivative

$$\begin{aligned}\delta g_{\mu\nu} &= \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu\\ \delta A_\mu &= \xi^\nu\nabla_\nu A_\mu + \nabla_\mu\xi^\nu A_\nu.\end{aligned}$$

obtaining

$$\nabla^\mu\langle T_{\mu\nu}\rangle - F_{\nu\mu}\langle J_5^\mu\rangle + A_\nu\nabla_\mu\langle J_5^\mu\rangle = 0.$$

$$\begin{aligned}0 = \delta_\xi Z &= \int d^dx\left[(\nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu)\frac{\delta}{\delta g_{\mu\nu}} + (\xi^\nu\nabla_\nu A_\mu + \nabla_\mu\xi^\nu A_\nu)\frac{}{\delta A_\mu}\right]Z\\ &= \int d^dx\sqrt{-g}\,\xi^\nu\left[-\nabla^\mu\langle T_{\mu\nu}(x)\rangle + \nabla_\nu A_\mu\langle J_5^\mu(x)\rangle - \nabla_\mu\left(A_\nu\langle J_5^\mu(x)\rangle\right)\right]\end{aligned}$$

- **Gauge invariance**

$$\delta g_{\mu\nu} = 0, \qquad \delta A_\mu = \partial_\mu \alpha.$$

$$\nabla_\alpha \langle J_5^\alpha \rangle = a_1\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_2\, \varepsilon^{\mu\nu\rho\sigma} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma},$$

Assume that there is an anomaly

We now apply two functional derivatives with respect to the metric to such equation, and perform the limit $g_{\mu\nu} \to \delta_{\mu\nu}$ and $A_\mu \to 0$ as above. After going to the momentum space, we derive the relation

$$p_{3\mu_3} \langle T^{\mu_1\nu_1}(p_1) T^{\mu_2\nu_2}(p_2) J_5^{\mu_3}(p_3) \rangle = 4\, i\, a_2\, (p_1 \cdot p_2) \left\{ \left[\varepsilon^{\nu_1\nu_2 p_1 p_2} \left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2} p_2^{\mu_1}}{p_1 \cdot p_2} \right) + (\mu_1 \leftrightarrow \nu_1) \right] + (\mu_2 \leftrightarrow \nu_2) \right\}.$$

- **Weyl invariance**

The action of a Weyl transformation with a dilaton $\sigma(x)$ on the fields, acting as a parameter is

$$\delta g_{\mu\nu} = 2 g_{\mu\nu} \sigma,$$
$$\delta A_\mu = 0.$$

$$\frac{\delta}{\delta\sigma}\langle \mathcal{S}_{cl}\rangle = \sqrt{g}\langle T^{\mu}_{\mu}\rangle = \sqrt{g}\mathcal{A},$$

Weyl invariance

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = b_1\, E_4 + b_2\, C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + b_3\,\nabla^2 R + b_4\, F^{\mu\nu}F_{\mu\nu} + f_1\,\varepsilon^{\mu\nu\rho\sigma}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma} + f_2\,\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}.$$

After applying the functional derivatives $\frac{\delta}{\delta g}\frac{\delta}{\delta A}$ and performing the limit $g_{\mu\nu}\to\delta_{\mu\nu}$ and $A_\mu\to 0$, the anomalous terms do not survive. Going to momentum space, we obtain the constraint

$$g_{\mu_i\nu_i}\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = 0, \qquad i = 1,2$$

For Weyl invariance, we need to go in a local frame, introduce the conforma CKVs
And the conformal currents. For the dilatations:

$$J^{\mu}_{K} = K_\nu T^{\mu\nu},$$

$$K^{(D)}_{\mu}(x) \equiv x_\mu\,, \qquad \partial\cdot K^{(D)} = d, \qquad \partial_\mu K_\nu + \partial_\nu K_\mu = \frac{2}{d}\eta_{\mu\nu}\,(\partial\cdot K)\,.$$

while for the special conformal transformation they are given by

$$K^{(S)\kappa}_{\mu}(x) \equiv 2x^\kappa x_\mu - x^2\delta^\kappa{}_\mu\,, \qquad \partial\cdot K^{(S)\,\kappa}(x) = (2d)\,x^\kappa\,, \qquad \kappa = 1,\ldots,d.$$

This exercise allows us to derive the special and the dilatation CWIs

special

$$
\begin{aligned}
0 = &\sum_{i=1}^{3}\left[2x_i^\kappa\left(\Delta_i + x_i^\alpha\frac{\partial}{\partial x_i^\alpha}\right) - x_i^2\delta^{\kappa\alpha}\frac{\partial}{\partial x_i^\alpha}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle \\
&+ 2\left[\delta^{\kappa\mu_1}x_{1\alpha} - \delta^\kappa_\alpha x_1^{\mu_1}\right]\langle T^{\alpha\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle + 2\left[\delta^{\kappa\nu_1}x_{1\alpha} - \delta^\kappa_\alpha x_1^{\nu_1}\right]\langle T^{\mu_1\alpha}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle \\
&+ 2\left[\delta^{\kappa\mu_2}x_{2\alpha} - \delta^\kappa_\alpha x_2^{\mu_2}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\alpha\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle + 2\left[\delta^{\kappa\nu_2}x_{2\alpha} - \delta^\kappa_\alpha x_2^{\nu_2}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\alpha}(x_2)J_5^{\mu_3}(x_3)\rangle \\
&+ 2\left[\delta^{\kappa\mu_3}x_{3\alpha} - \delta^\kappa_\alpha x_3^{\mu_3}\right]\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\alpha}(x_3)\rangle\,,
\end{aligned}
\tag{47}
$$

$$
0 = \left[\sum_i x_i^\mu\,\partial_\mu^{(x_i)} + 3d - 1\right]\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)J_5^{\mu_3}(x_3)\rangle,
$$

dilatation

$$
\begin{aligned}
0 = &\,\mathcal{K}^\kappa\langle T^{\mu_1\nu_1}(p_1)\,T^{\mu_2\nu_2}(p_2)\,J_5^{\mu_3}(p_3)\rangle \\
= &\sum_{j=1}^{2}\left(2(\Delta_j - d)\frac{\partial}{\partial p_{j\kappa}} - 2p_j^\alpha\frac{\partial}{\partial p_j^\alpha}\frac{\partial}{\partial p_{j\kappa}} + (p_j)^\kappa\frac{\partial}{\partial p_j^\alpha}\frac{\partial}{\partial p_{j\alpha}}\right)\langle T^{\mu_1\nu_1}(p_1)\,T^{\mu_2\nu_2}(p_2)\,J_5^{\mu_3}(p_3)\rangle \\
&+ 4\left(\delta^{\kappa(\mu_1}\frac{\partial}{\partial p_1^{\alpha_1}} - \delta^\kappa_{\alpha_1}\delta^{(\mu_1}_\lambda\frac{\partial}{\partial p_{1\lambda}}\right)\left\langle T^{\nu_1)\alpha_1}(p_1)\,T^{\mu_2\nu_2}(p_2)\,J_5^{\mu_3}(p_3)\right\rangle \\
&+ 4\left(\delta^{\kappa(\mu_2}\frac{\partial}{\partial p_2^{\alpha_2}} - \delta^\kappa_{\alpha_2}\delta^{(\mu_2}_\lambda\frac{\partial}{\partial p_{2\lambda}}\right)\left\langle T^{\nu_2)\alpha_2}(p_2)\,T^{\mu_1\nu_1}(p_1)\,J_5^{\mu_3}(p_3)\right\rangle.
\end{aligned}
\tag{48}
$$

In momentum Space

$$
\left(\sum_{i=1}^{3}\Delta_i - 2d - \sum_{i=1}^{2}p_i^\mu\frac{\partial}{\partial p_i^\mu}\right)\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = 0.
$$

The correlato is reconstructed by extending a methodology developed by Bzowski, McFadden and Skenderis to The parity odd sector

Decomposition of the correlator

$$T^{\mu_i\nu_i}(p_i) = t^{\mu_i\nu_i}(p_i) + t^{\mu_i\nu_i}_{loc}(p_i),$$
$$J_5^{\mu_i}(p_i) = j_5^{\mu_i}(p_i) + j^{\mu_i}_{5\,loc}(p_i),$$

Longitudinal terms due to the anomaly
+ transverse terms

$$t^{\mu_i\nu_i}(p_i) = \Pi^{\mu_i\nu_i}_{\alpha_i\beta_i}(p_i)\, T^{\alpha_i\beta_i}(p_i),$$

$$j_5^{\mu_i}(p_i) = \pi^{\mu_i}_{\alpha_i}(p_i)\, J_5^{\alpha_i}(p_i),$$

Appearance of an anomaly pole in J5

$$t^{\mu_i\nu_i}_{loc}(p_i) = \Sigma^{\mu_i\nu_i}_{\alpha_i\beta_i}(p)\, T^{\alpha_i\beta_i}(p_i),$$

$$j^{\mu_i}_{5\,loc}(p_i) = \frac{p_i^{\mu_i}\, p_{i\,\alpha_i}}{p_i^2}\, J_5^{\alpha_i}(p_i),$$

$$\pi^{\mu}_{\alpha} = \delta^{\mu}_{\alpha} - \frac{p^{\mu}p_{\alpha}}{p^2},$$

$$\Pi^{\mu\nu}_{\alpha\beta} = \frac{1}{2}\left(\pi^{\mu}_{\alpha}\pi^{\nu}_{\beta} + \pi^{\mu}_{\beta}\pi^{\nu}_{\alpha}\right) - \frac{1}{d-1}\pi^{\mu\nu}\pi_{\alpha\beta},$$

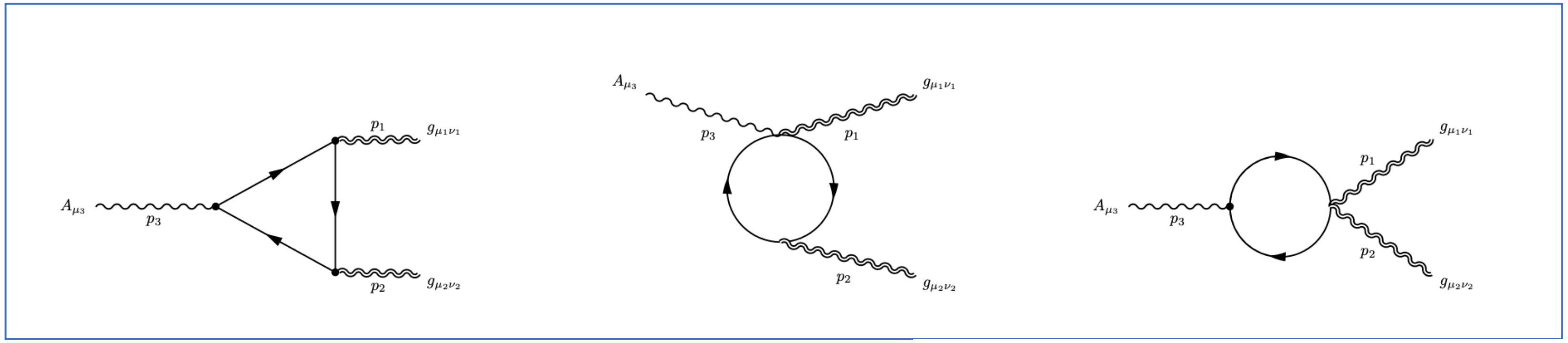
$$\Sigma^{\mu_i\nu_i}_{\alpha_i\beta_i} = \frac{p_{i\,\beta_i}}{p_i^2}\left[2\delta^{(\nu_i}_{\alpha_i}p_i^{\mu_i)} - \frac{p_{i\alpha_i}}{(d-1)}\left(\delta^{\mu_i\nu_i} + (d-2)\frac{p_i^{\mu_i}p_i^{\nu_i}}{p_i^2}\right)\right] + \frac{\pi^{\mu_i\nu_i}(p_i)}{(d-1)}\delta_{\alpha_i\beta_i}.$$

$$\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}J_5^{\mu_3}\rangle = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \left\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}j^{\mu_3}_{5\,loc}\right\rangle = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \left\langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j^{\mu_3}_{5\,loc}\right\rangle.$$

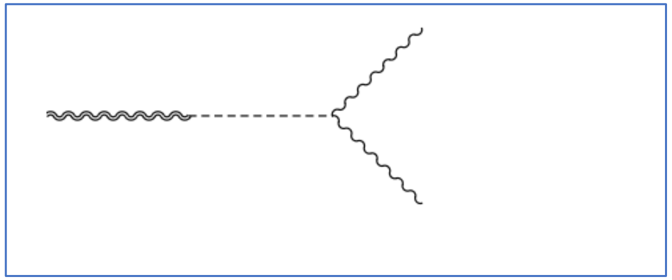
reconstruction

$$\langle t^{\mu_1\nu_1} t^{\mu_2\nu_2} j_{5\,loc}^{\mu_3} \rangle = p_3^{\mu_3}\, \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\, \Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\, \varepsilon^{\alpha_1\alpha_2 p_1 p_2} \left(F_1\, g^{\beta_1\beta_2} + F_2\, p_1^{\beta_2} p_2^{\beta_1} \right)$$

Longitudinal terms coming from the anomaly deetermined
By the anomaly



$$F_1 = \frac{16 i a_2 (p_1 \cdot p_2)}{p_3^2}, \qquad F_2 = -\frac{16 i a_2}{p_3^2}.$$



The pole, from the perturbative perspective, is fue to the exchange of a
Pseudoscalar mode (a collinear fermion antifermion pair)

$$\langle t^{\mu_1\nu_1} t^{\mu_2\nu_2} j_{5\,loc}^{\mu_3} \rangle = 4 i a_2 \frac{p_3^{\mu_3}}{p_3^2} (p_1 \cdot p_2) \left\{ \left[\varepsilon^{\nu_1\nu_2 p_1 p_2} \left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2} p_2^{\mu_1}}{p_1 \cdot p_2} \right) + (\mu_1 \leftrightarrow \nu_1) \right] + (\mu_2 \leftrightarrow \nu_2) \right\}.$$

Transverse terms built only by the symmetries of the correlator

$$\langle t^{\mu_1\nu_1}(p_1)\, t^{\mu_2\nu_2}(p_2)\, j_5^{\mu_3}(p_3)\rangle = \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\,\Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\,\pi^{\mu_3}_{\alpha_3}(p_3)\, X^{\alpha_1\beta_1\alpha_2\beta_2\alpha_3}$$

$$\begin{aligned}\langle t^{\mu_1\nu_1}(p_1) t^{\mu_2\nu_2}(p_2)\, j_5^{\mu_3}(p_3)\rangle = \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\,\Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\,\pi^{\mu_3}_{\alpha_3}(p_3)\Big[& \\ A_1\varepsilon^{p_1\alpha_1\alpha_2\alpha_3}p_2^{\beta_1}p_3^{\beta_2} - A_1(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}p_2^{\beta_1}p_3^{\beta_2} & \\ + A_2\varepsilon^{p_1\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2} - A_2(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2} & \\ + A_3\varepsilon^{p_1p_2\alpha_1\alpha_2}p_2^{\beta_1}p_3^{\beta_2}p_1^{\alpha_3} + A_4\varepsilon^{p_1p_2\alpha_1\alpha_2}\delta^{\beta_1\beta_2}p_1^{\alpha_3}\Big] &\end{aligned}$$

We derive a set of differential equations that can be solved in terms of 1 constant: the coefficient of the gravitational anomaly

Equations are split into primary and secondary CWIs

$$0 = K_{31}A_1, \qquad 0 = K_{32}A_1 + \frac{2}{p_1^2}\left(p_1\frac{\partial}{\partial p_1} - 4\right)A_1(p_1 \leftrightarrow p_2)$$

$$0 = K_{31}A_2 + 4A_1, \qquad 0 = K_{32}A_2 + \frac{2}{p_1^2}\left(p_1\frac{\partial}{\partial p_1} - 4\right)A_2(p_1 \leftrightarrow p_2) + 4A_1$$

primary

$$\begin{aligned}
0 &= -2p_1\frac{\partial}{\partial p_1}A_1 + 2p_2\frac{\partial}{\partial p_2}A_1(p_1 \leftrightarrow p_2) \\
0 &= -\left(p_1^2 - p_2^2 + p_3^2\right)A_1 + \left(-p_1^2 + p_2^2 + p_3^2\right)A_1(p_1 \leftrightarrow p_2) - 2p_1\frac{\partial}{\partial p_1}A_2 + 2p_2\frac{\partial}{\partial p_2}A_2(p_1 \leftrightarrow p_2) \\
&\quad + 2A_2 - 2A_2(p_1 \leftrightarrow p_2) \\
0 &= -\frac{2p_2^3}{p_3^2}\frac{\partial}{\partial p_2}A_1(p_1 \leftrightarrow p_2) - 2\left(\frac{p_2^2 + p_3^2}{p_3^2}\right)p_2\frac{\partial}{\partial p_2}A_1 + \left(-\frac{2p_2^2}{p_3^2} + \frac{p_3^2 - p_2^2 - p_1^2}{p_1^2}\right)p_1\frac{\partial}{\partial p_1}A_1 \\
&\quad + 2p_2^2\left(\frac{p_3^2 - p_1^2}{p_3^2 p_1}\right)\frac{\partial}{\partial p_1}A_1(p_1 \leftrightarrow p_2) - 4p_2^2\left(\frac{2}{p_1^2} + \frac{1}{p_3^2}\right)A_1(p_1 \leftrightarrow p_2) + 4\left(\frac{p_1^2 + p_2^2 - p_3^2}{p_1^2} - \frac{p_2^2}{p_3^2}\right)A_1 \\
&\quad - \frac{2}{p_1}\frac{\partial}{\partial p_1}A_2 + \frac{8}{p_1^2}A_2 - \frac{64iap_2^2}{p_3^2} \\
0 &= -\left(\frac{p_1^2 + p_2^2 - p_3^2}{p_3^2}\right)p_1\frac{\partial}{\partial p_1}A_1 - \frac{\left(p_1^2 - 2p_3^2\right)\left(p_1^2 + p_2^2 - p_3^2\right)}{p_1 p_3^2}\frac{\partial}{\partial p_1}A_1(p_1 \leftrightarrow p_2) - \left(\frac{p_1^2 + p_2^2 - p_3^2}{p_3^2}\right)p_2\frac{\partial}{\partial p_2}A_1 \\
&\quad - \left(\frac{p_1^2 + p_2^2 - 3p_3^2}{p_3^2}\right)p_2\frac{\partial}{\partial p_2}A_1(p_1 \leftrightarrow p_2) - 2\left(\frac{p_1^2 + p_2^2 - 2p_3^2}{p_3^2} + 4\frac{p_1^2 + p_2^2 - p_3^2}{p_1^2}\right)A_1(p_1 \leftrightarrow p_2) \\
&\quad - 2\left(\frac{p_1^2 + p_2^2 - 2p_3^2}{p_3^2}\right)A_1 + \frac{2}{p_1}\frac{\partial}{\partial p_1}A_2(p_1 \leftrightarrow p_2) - \frac{8}{p_1^2}A_2(p_1 \leftrightarrow p_2) - \frac{32ia\left(p_1^2 + p_2^2 - p_3^2\right)}{p_3^2} \\
0 &= \frac{2p_1}{p_3^2}\frac{\partial}{\partial p_1}A_1 + 2\left(\frac{p_1^2 - p_3^2}{p_3^2 p_1}\right)\frac{\partial}{\partial p_1}A_1(p_1 \leftrightarrow p_2) + \frac{2p_2}{p_3^2}\frac{\partial}{\partial p_2}A_1 + \frac{2p_2}{p_3^2}\frac{\partial}{\partial p_2}A_1(p_1 \leftrightarrow p_2) \\
&\quad + 4\left(\frac{2}{p_1^2} + \frac{1}{p_3^2}\right)A_1(p_1 \leftrightarrow p_2) + \frac{4}{p_3^2}A_1 + \frac{64ia}{p_3^2} \\
0 &= -\frac{2p_1}{p_3^2}\frac{\partial}{\partial p_1}A_2 + 2\left(\frac{p_3^2 - p_1^2}{p_3^2 p_1}\right)\frac{\partial}{\partial p_1}A_2(p_1 \leftrightarrow p_2) - \frac{2p_2}{p_3^2}\frac{\partial}{\partial p_2}A_2 - \frac{2p_2}{p_3^2}\frac{\partial}{\partial p_2}A_2(p_1 \leftrightarrow p_2) - \frac{8}{p_1^2}A_2(p_1 \leftrightarrow p_2) \\
&\quad + \frac{32ia\left(p_1^2 + p_2^2 - p_3^2\right)}{p_3^2}
\end{aligned} \tag{104}$$

The solution:

$$\begin{aligned}
A_1 &= -4\,i\,a_2\,p_2^2\,I_{5\{2,1,1\}} \\
A_2 &= -8\,i\,a_2\,p_2^2\left(p_3^2\,I_{4\{2,1,0\}} - 1\right) \\
A_3 &= 0 \\
A_4 &= 0.
\end{aligned}$$

+ the pole

Parametric integrals of Bessel functions (3K)

(generalized hypergeometric functions)

$$I_{\alpha\{\beta_1\beta_2\beta_3\}}\left(p_1, p_2, p_3\right) = \int dx x^\alpha \prod_{j=1}^{3} p_j^{\beta_j} K_{\beta_j}\left(p_j x\right)$$

$$K_\nu(x) = \frac{\pi}{2}\frac{I_{-\nu}(x) - I_\nu(x)}{\sin(\nu\pi)}, \qquad \nu \notin \mathbb{Z} \qquad I_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(\nu+1+k)}\left(\frac{x}{2}\right)^{2k}$$

(154)

$$A_1 = -4\, i\, a_2\, p_2^2\, I_{5\{2,1,1\}}$$
$$A_2 = -8\, i\, a_2\, p_2^2\left(p_3^2\, I_{4\{2,1,0\}} - 1\right)$$
$$A_3 = 0$$
$$A_4 = 0.$$

$$I_{5\{2,1,1\}} = \frac{i}{4} p_1 p_2 p_3 \left(p_1 \frac{\partial}{\partial p_1} - 1\right) \frac{\partial^3}{\partial p_1 \partial p_2 \partial p_3} C_0(p_1^2, p_2^2, p_3^2)$$
$$I_{4\{2,1,0\}} = -\frac{i}{4} p_1 p_2 \left(p_1 \frac{\partial}{\partial p_1} - 1\right) \frac{\partial^2}{\partial p_1 \partial p_2} C_0(p_1^2, p_2^2, p_3^2)$$

Rewritten in temrs of master Integrals B0 and C0

$$\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}J_5^{\mu_3}\rangle = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\rangle$$

The anomalous longitudinal part is given by

$$\langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\rangle = 4ia_2\frac{p_3^{\mu_3}}{p_3^2}(p_1\cdot p_2)\left\{\left[\varepsilon^{\nu_1\nu_2p_1p_2}\left(g^{\mu_1\mu_2}-\frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right)+(\mu_1\leftrightarrow\nu_1)\right]+(\mu_2\leftrightarrow\nu_2)\right\} \tag{150}$$

while the transverse-traceless part is

$$\langle t^{\mu_1\nu_1}(p_1)\,t^{\mu_2\nu_2}(p_2)\,j_5^{\mu_3}(p_3)\rangle = \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\,\Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\,\pi^{\mu_3}_{\alpha_3}(p_3)\Big[$$
$$A_1\varepsilon^{p_1\alpha_1\alpha_2\alpha_3}p_2^{\beta_1}p_3^{\beta_2}-A_1(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}p_2^{\beta_1}p_3^{\beta_2}+A_2\varepsilon^{p_1\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2}-A_2(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2}\Big]$$

$$A_1=\frac{gp_2^2}{24\pi^2\lambda^4}\left\{A_{11}+A_{12}\log\left(\frac{p_1^2}{p_2^2}\right)+A_{13}\log\left(\frac{p_1^2}{p_3^2}\right)+A_{14}\,C_0(p_1^2,p_2^2,p_3^2)\right\},$$

$$A_2=\frac{gp_2^2}{48\pi^2\lambda^3}\left\{A_{21}+A_{22}\log\left(\frac{p_1^2}{p_2^2}\right)+A_{23}\log\left(\frac{p_1^2}{p_3^2}\right)+A_{24}\,C_0(p_1^2,p_2^2,p_3^2)\right\},$$

$$A_3=0,$$

$$A_4=0$$

In perfect agreement with The perturbative analysis

Whats we learn: The entire gravitational anomaly interaction can be derived just from 2 requirements

1. The inclusion of an anomaly pole to solve the longitudinals ector

2. The conformal Ward identities

We can immediately write down the gravitational effective action that accounts for this interaction

$$S_{anom} \sim \int d^4x\, d^4y\, \partial_\lambda B^\lambda \, \frac{1}{\square}(x,y) R\tilde{R}(y) + \dots$$

Notice that in our derivation we have not identified a specific current, but just required that the current has an anomaly of a certain form

What kind of currents are possible ?

Similar analysis (non perturbative like this)
Have been presented for the AVV/AAA correlator and the T5JJ correlator, with similar results.

CFT Correlators and CP-Violating Trace Anomalies

e-Print: EPJ-C, in press 2307.03038 [hep-th]

Parity-odd 3-point functions from CFT in momentum space and the chiral anomaly

Eur.Phys.J.C 83 (2023) 6, 502 e-Print: 2303.10710 [hep-th]

Lionetti, Maglio, C C

$$\mathcal{S}_{TJJ} \int d^4x d^4y R^{(1)}(x) \,\square^{-1}(x,y) F_A F_A(y) + \ldots$$

TJJ parity even

$$\mathcal{S}_{J_5 JJ} = \int d^4x d^4y \partial \cdot B \,\square^{-1}(x,y) F_A \tilde{F}_A(y) + \ldots$$

AVV

$$\mathcal{S}_{JJT} = f_2 \int d^4x' \sqrt{g(x')} \int d^4x \sqrt{g(x)} R(x) \square^{-1}_{x,x'} F\tilde{F}(x')$$

TJJ parity odd

$$\mathcal{S}_{TTT} = f_1 \int d^4x' \sqrt{g(x')} \int d^4x \sqrt{g(x)} R(x) \square^{-1}_{x,x'} R\tilde{R}(x').$$

TTT parity odd

DISCRETE DUALITY of Electrodynamics

The Maxwell equations in the absence of charges and currents satisfy the duality symmetry ($E \to B$ and $B \to -E$). The symmetry can be viewed as a special case of a continuous symmetry

the discrete case, then the action flips sign since ($F^2 \to -\tilde{F}^2$).
In general, the infinitesimal variation of the action takes the form

$$\delta_\beta \mathcal{S}_{cl} = -\beta \int d^4x \partial_\mu \left(\tilde{F}^{\mu\nu} A_\nu \right) .$$

The current corresponding to the infinitesimal symmetry

$$\delta F^{\mu\nu} = \beta \tilde{F}^{\mu\nu}$$

where $\delta\beta$ is an infinitesimal $SO(2)$ rotation. Its finite form

$$\begin{pmatrix} E \\ B \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} E \\ B \end{pmatrix}$$

It is affected by the same gravitational chiral anomaly

Pasti, Sorokin, Tonin

(Galaverni and Gionti) (see Del Rio, Navarro-Salas)

$$J^\mu = \tilde{F}^{\mu\nu} A_\nu - F^{\mu\nu} \tilde{A}_\nu$$

whose conserved charge is gauge invariant

$$Q_5 = \int d^3x \left(A \cdot \nabla \times A - \tilde{A} \cdot \nabla \times \tilde{A} \right)$$

$$Q_5 = \int d^3x \left(B \cdot A - E \cdot \tilde{A} \right)$$

$E = -\nabla \times \tilde{A}$, that coincides with the optical helicity

$$\mathcal{H}_{fluid} = \int d^3x \vec{v} \cdot \nabla \times \vec{v}$$

Linking number of the velocity field

Chiral anomaly (See Frohlich and Boyarsky)

Anomalous magnetohydrodynamic

$$\frac{d(n_L - n_R)}{dt} = \frac{2\alpha}{\pi}\frac{1}{V}\int d^3x\, E\cdot B = -\frac{\alpha}{\pi}\frac{d\mathcal{H}}{dt}\,,$$

$$\mathcal{H}(t) = \frac{1}{V}\int_V d^3x\, A\cdot B\,,$$

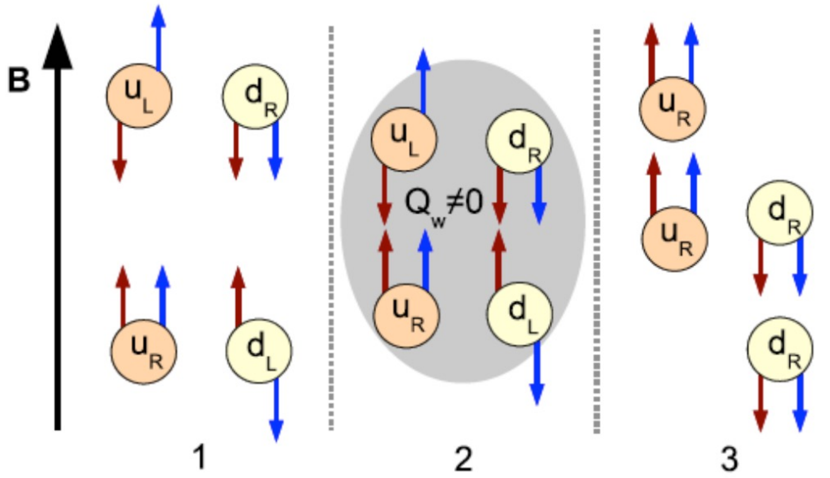
Magnetic helicity

$$\frac{d\mathcal{H}}{dt} = -\frac{2}{V}\int_V d^3x\, E\cdot B\,.$$

$$\Delta\mu \equiv \mu_L - \mu_R,$$

$$n_{L,R} = \frac{1}{2V}\int d^3x\, \psi^\dagger(1\pm\gamma_5)\psi$$

(from Kharzeev)



$$J_{5f}^{\lambda} = \bar{\psi}\gamma_5\gamma^{\lambda}\psi$$

The conformal backreaction in the presence of gravity, for chiral interactions, is entirely determined by an anomaly pole+ conformal symmetry.

But, the same is true for a Chern Simons current . Our analysis does not depend on the current

$$J_{CS}^{\lambda} = \epsilon^{\lambda\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho},$$

Gravitational anomalies induced **by Chern-Simons currents**. Do they have anomaly poles?

Yes, they do. This result is implicit already in an old analysis by Dolgov. Kriplovich, Vainshstein, Zakharov 80's, but somehow not noticed in the literature.

In our recent paper we show that the perturbative analysis of Dolgov et al is associated also with a sum rule associated to such excitations.

$$\begin{aligned}
\int_{4m^2}^{\infty} ds\, \Delta_{AVV}(s,m) &= 2\, d_{AVV} \\
\int_{4m^2}^{\infty} ds\, \Delta_{J_fTT}(s,m) &= \frac{2}{3}\, d_{J_fTT} \\
\int_{4m^2}^{\infty} ds\, \Delta_{J_{CS}TT}(s,m) &= \frac{14}{45}\, d_{J_{CS}TT},
\end{aligned}$$

Sum rule
(Lionetti, Maglio, CC)

$$\begin{aligned}
\langle 0|J_f^{\mu}|\gamma\gamma\rangle &= f_1(q^2)\frac{q^{\mu}}{q^2}F_{\kappa\lambda}\tilde{F}^{\kappa\lambda} \\
\langle 0|J_f^{\mu}||gg\rangle &= f_2(q^2)\frac{q^{\mu}}{q^2}R_{\kappa\lambda\rho\sigma}\tilde{R}^{\kappa\lambda\rho\sigma} \\
\langle 0|J_{CS}^{\mu}|gg\rangle &= f_3(q^2)\frac{q^{\mu}}{q^2}R_{\kappa\lambda\rho\sigma}\tilde{R}^{\kappa\lambda\rho\sigma},
\end{aligned}$$

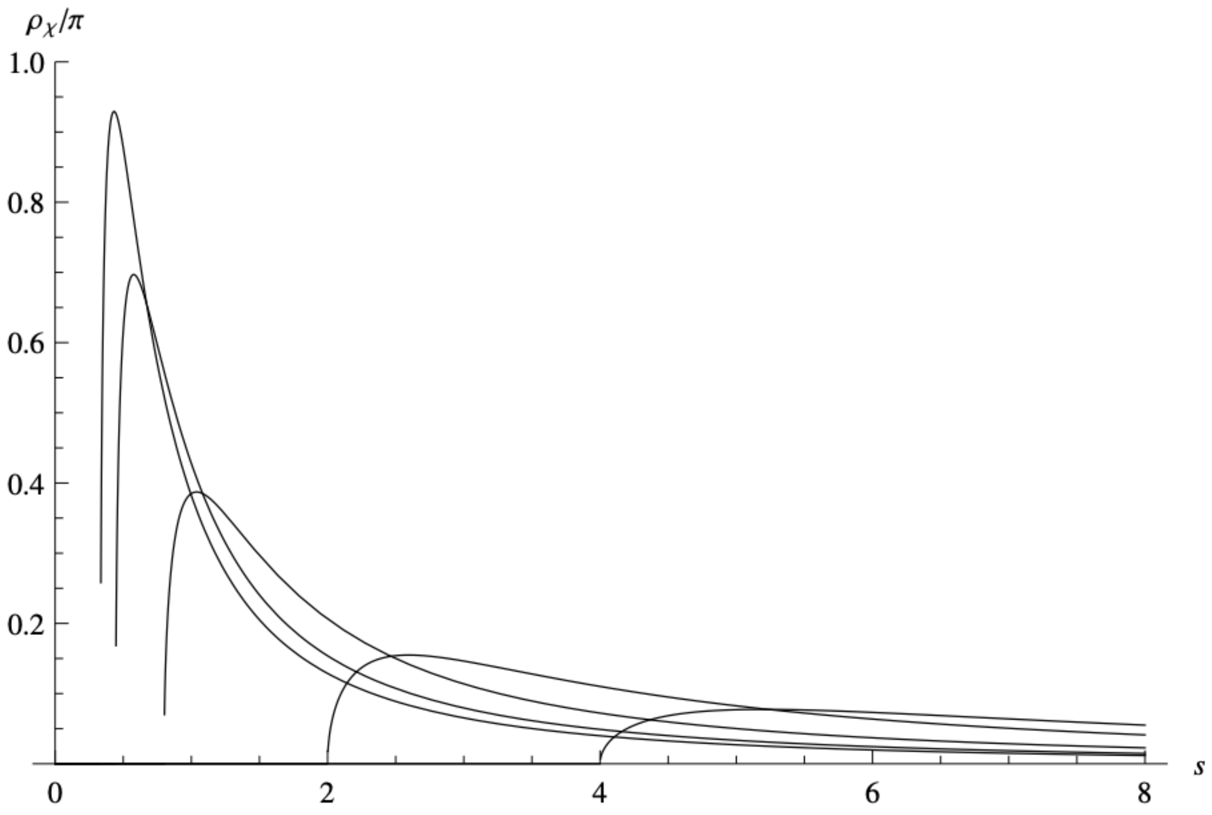
$$\lim_{m\to 0}\Delta(q^2,m) \propto \delta(q^2)$$

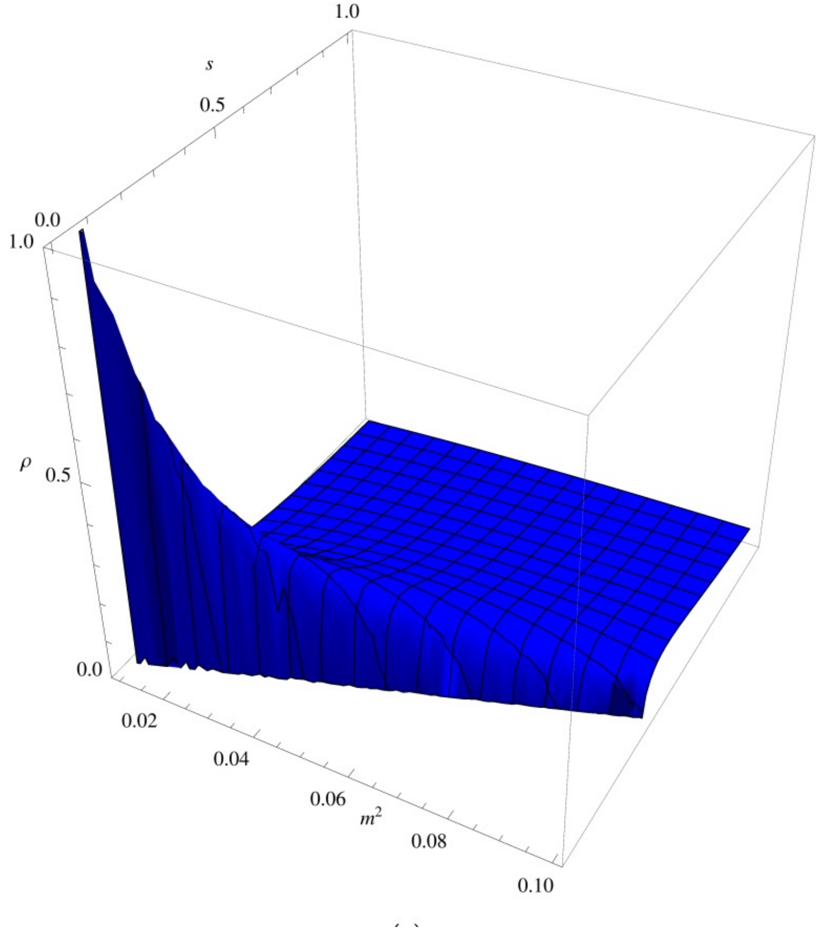
$$\begin{aligned}
\Delta_{AVV}(q^2,m) \equiv \mathrm{Im} f_1(q^2) &= \frac{d_{AVV}}{q^2}(1-v^2)\log\frac{1+v}{1-v} \\
\Delta_{J_fTT}(q^2,m) \equiv \mathrm{Im} f_2(q^2) &= \frac{d_{J_fTT}}{q^2}(1-v^2)^2\log\frac{1+v}{1-v} \\
\Delta_{J_{CS}TT}(q^2,m) \equiv \mathrm{Im} f_3(q^2) &= \frac{d_{J_{CS}TT}}{q^2}v^2(1-v^2)^2\log\frac{1+v}{1-v}
\end{aligned}$$

In Dolgov et al photons and gravitons are on-shell

with $v = \sqrt{1-4m^2/q^2}$ and d_{AVV}, d_{J_fTT} and $d_{J_{CS}TT}$ being the corresponding anomaly coefficients. Notice the different forms of Δ_{J_fTT} and $\Delta_{J_{CS}TT}(q^2,m)$ away from the conformal limit.

In the conformal limit we exchange the pole. Away from the conformal limit we have a sum rule fixed by the anomaly





Implications for the cosmology of the very early universe.

I will consider only the case of the Chern Simons current for the J5TT (orrelator chiral gravitational anomaly)

$$
\begin{aligned}
S_{pole} &= -\frac{c}{6}\int d^4x\, d^4y\, R^{(1)}(x)\,\Box^{-1}(x,y)\,F^a_{\alpha\beta}\,F^{a\,\alpha\beta} \\
&= \frac{1}{3}\,\frac{g^3}{16\pi^2}\left(-\frac{11}{3}\,C_A+\frac{2}{3}\,n_f\right)\int d^4x\, d^4y\, R^{(1)}(x)\,\Box^{-1}(x,y)\,F_{\alpha\beta}F^{\alpha\beta}
\end{aligned}
$$

$$
R^{(1)}_x \equiv \partial^x_\mu\,\partial^x_\nu\,h^{\mu\nu} - \Box\,h, \qquad h = \eta_{\mu\nu}\,h^{\mu\nu}
$$

$$
c = -2\,\frac{\beta(g)}{g}.
$$

$$
S_{anom}[g,A] = \frac{1}{8}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(E-\frac{2}{3}\Box R\right)_x \Delta_4^{-1}(x,x')\left[2b\,F+b'\left(E-\frac{2}{3}\Box R\right)+2\,c\,F_{\mu\nu}F^{\mu\nu}\right]_{x'}
$$

$$
F = C_{\lambda\mu\nu\rho}C^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3}
$$

$$
E = {}^*R_{\lambda\mu\nu\rho}\,{}^*R^{\lambda\mu\nu\rho} = R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2
$$

$$
\Delta_4 \equiv \nabla_\mu\left(\nabla^\mu\nabla^\nu + 2R^{\mu\nu} - \frac{2}{3}Rg^{\mu\nu}\right)\nabla_\nu = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu + \frac{1}{3}(\nabla^\mu R)\nabla_\mu - \frac{2}{3}R\Box\,.
$$

$4d$ **Einstein Gauss-Bonnet Gravity without a Dilaton** [1]

PoS Corfu 2022

[(a)]**Claudio Corianò**, [(a)(b)]**Mario Creti**, [(a)] **Stefano Lionetti**, [(c)]**Matteo Maria Maglio**, [(a)]**Riccardo Tommasi**

$$\Delta_4 \equiv \nabla_\mu \left(\nabla^\mu \nabla^\nu + 2R^{\mu\nu} - \frac{2}{3} R g^{\mu\nu} \right) \nabla_\nu = \Box^2 + 2R^{\mu\nu}\nabla_\mu \nabla_\nu + \frac{1}{3}(\nabla^\mu R)\nabla_\mu - \frac{2}{3} R \Box \, .$$

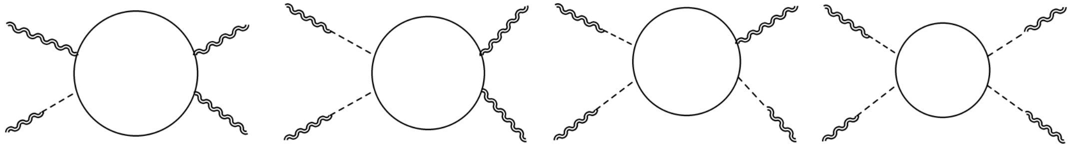
Paneitz operator

$$\sqrt{-g}\, \Delta_4 \chi_0 = \sqrt{-\bar{g}}\, \bar{\Delta}_4 \chi_0 ,$$

Weyl invariant if acting on conformal scalars (ie fields of vanishing scaling dimensions)

TTT in agreement with the free field theory realization and the general CFT derivation.
The general solution depends in 3 constants and can be matched by free feld theory (3 sectors)

$$
\begin{aligned}
&\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)T^{\mu_3\nu_3}(p_3)T^{\mu_4\nu_4}(\bar{p}_4)\rangle_{poles} = \\
&= \frac{\pi^{\mu_1\nu_1}(p_1)}{3}\,\langle T(p_1)T^{\mu_2\nu_2}(p_2)T^{\mu_3\nu_3}(p_3)T^{\mu_4\nu_4}(\bar{p}_4)\rangle_{anomaly} + (perm.) \\
&- \frac{\pi^{\mu_1\nu_1}(p_1)}{3}\frac{\pi^{\mu_2\nu_2}(p_2)}{3}\,\langle T(p_1)T(p_2)T^{\mu_3\nu_3}(p_3)T^{\mu_4\nu_4}(\bar{p}_4)\rangle_{anomaly} + (perm.) \\
&+ \frac{\pi^{\mu_1\nu_1}(p_1)}{3}\frac{\pi^{\mu_2\nu_2}(p_2)}{3}\frac{\pi^{\mu_3\nu_3}(p_2)}{3}\,\langle T(p_1)T(p_2)T(p_3)T^{\mu_4\nu_4}(\bar{p}_4)\rangle_{anomaly} + (perm.) \\
&- \frac{\pi^{\mu_1\nu_1}(p_1)}{3}\frac{\pi^{\mu_2\nu_2}(p_2)}{3}\frac{\pi^{\mu_3\nu_3}(p_3)}{3}\frac{\pi^{\mu_4\nu_4}(p_4)}{3}\,\langle T(p_1)T(p_2)T(p_3)T(\bar{p}_4)\rangle_{anomaly}\,.
\end{aligned}
$$



Maglio, Theofilopoulos, CC

Figure 3 The Weyl-variant contributions from $\mathcal{S}_A$ to the renormalized vertex for the 4T with the corresponding bilinear mixings in $d = 4$

$$
\begin{aligned}
\mathcal{S}_A = &\int d^4x_1 d^4x_2 \langle T\cdot h(x_1)T\cdot h(x_2)\rangle + \int d^4x_1 d^4x_2 d^4x_3 \langle T\cdot h(x_1)T\cdot h(x_2)T\cdot h(x_3)\rangle_{pole} \\
&+ \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \left(\langle T\cdot h(x_1)T\cdot h(x_2)T\cdot h(x_3)T\cdot h(x_4)\rangle_{pole} + \right. \\
&\qquad\qquad \left. + \langle T\cdot h(x_1)T\cdot h(x_2)T\cdot h(x_3)T\cdot h(x_4)\rangle_{0T}\right),
\end{aligned}
$$

But we realized that there is a new sector, Weyl invariant, appearing in the computaitons and necessary in order to have consistent coservation Wis.

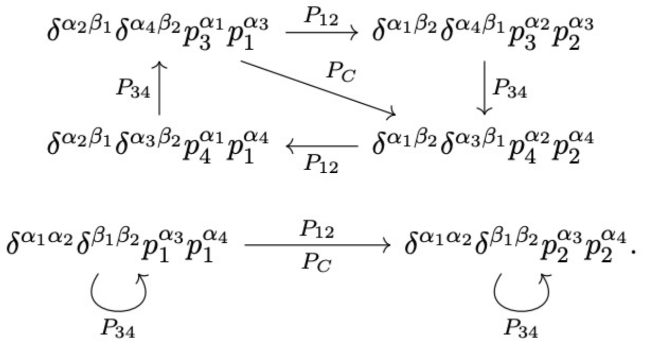
$$\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)J^{\mu_3}(p_3)J^{\mu_4}(p_4)\rangle$$
$$= \langle t^{\mu_1\nu_1}(p_1)t^{\mu_2\nu_2}(p_2)j^{\mu_3}(p_3)j^{\mu_4}(p_4)\rangle + \langle t^{\mu_1\nu_1}(p_1)t^{\mu_2\nu_2}(p_2)j^{\mu_3}(p_3)j^{\mu_4}(p_4)\rangle_{loc}$$

Classification of the orbits

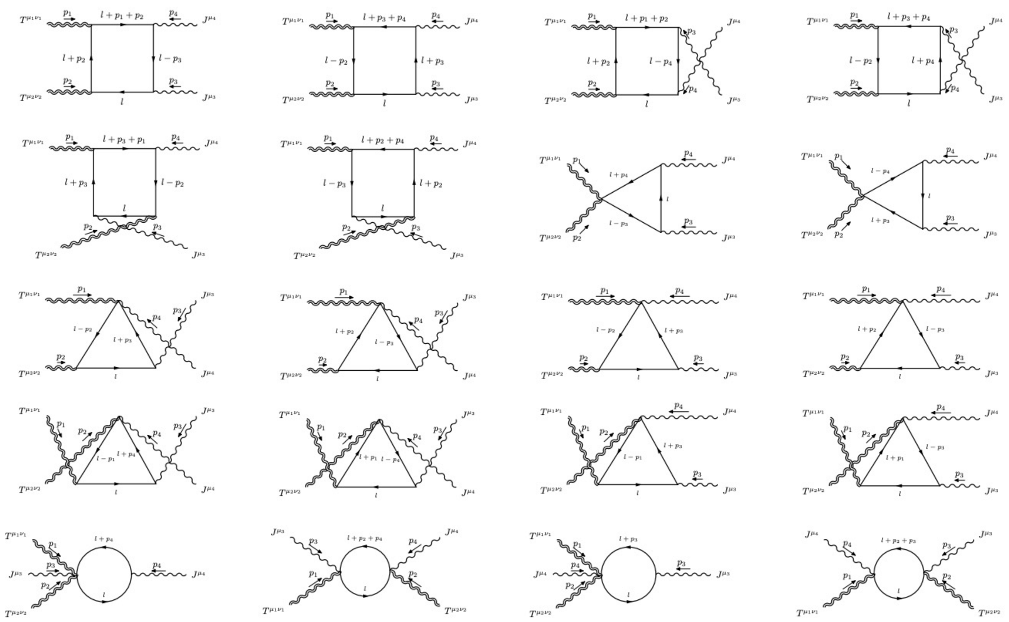
$$\delta_{\mu_i\nu_i}\,\langle t^{\mu_1\nu_1}(p_1)t^{\mu_2\nu_2}(p_2)j^{\mu_3}(p_3)j^{\mu_4}(p_4)\rangle = 0, \quad i=1,2\,,$$
$$p_{\mu_i}\,\langle t^{\mu_1\nu_1}(p_1)t^{\mu_2\nu_2}(p_2)j^{\mu_3}(p_3)j^{\mu_4}(p_4)\rangle = 0, \quad i=1,\dots,4\,,$$

Sector	# of tensor structures	# of orbits
$\delta\delta\delta$	3	2
$\delta\delta pp$	38	13
$\delta pppp$	73	21
$pppppp$	36	11
Total	150	47

$$\langle t^{\mu_1\nu_1}(p_1)t^{\mu_2\nu_2}(p_2)j^{\mu_3}(p_3)j^{\mu_4}(p_4)\rangle_{loc} = \langle t^{\mu_1\nu_1}_{loc}T^{\mu_2\nu_2}J^{\mu_3}J^{\mu_4}\rangle + \langle T^{\mu_1\nu_1}t^{\mu_2\nu_2}_{loc}J^{\mu_3}J^{\mu_4}\rangle - \langle t^{\mu_1\nu_1}_{loc}t^{\mu_2\nu_2}_{loc}J^{\mu_3}J^{\mu_4}\rangle$$
$$= \Bigg[\left(\mathcal{I}^{\mu_1\nu_1}_{\alpha_1}p_{1\,\beta_1} + \frac{\pi^{\mu_1\nu_1}(p_1)}{(d-1)}\delta_{\alpha_1\beta_1}\right)\delta^{\mu_2}_{\alpha_2}\delta^{\nu_2}_{\beta_2} + \left(\mathcal{I}^{\mu_2\nu_2}_{\alpha_2}p_{2\,\beta_2} + \frac{\pi^{\mu_2\nu_2}(p_2)}{(d-1)}\delta_{\alpha_2\beta_2}\right)\delta^{\mu_1}_{\alpha_1}\delta^{\nu_1}_{\beta_1}$$
$$- \left(\mathcal{I}^{\mu_1\nu_1}_{\alpha_1}p_{1\,\beta_1} + \frac{\pi^{\mu_1\nu_1}(p_1)}{(d-1)}\delta_{\alpha_1\beta_1}\right)\left(\mathcal{I}^{\mu_2\nu_2}_{\alpha_2}p_{2\,\beta_2} + \frac{\pi^{\mu_2\nu_2}(p_2)}{(d-1)}\delta_{\alpha_2\beta_2}\right)\Bigg]\langle T^{\alpha_1\beta_1}T^{\alpha_2\beta_2}J^{\mu_3}J^{\mu_4}\rangle\,. \qquad (4$$



Classification of the tensor structure is quite involved
Because of Lovelock identities
(tensor degeneracies for d=4)



Some contributons

Well define predicitons which are not reproduced by the Riegert action

$$
\begin{aligned}
&\langle T^{\mu_1\nu_1}(p_1)T^{\mu_2\nu_2}(p_2)J^{\mu_3}(p_3)J^{\mu_4}(p_4)\rangle_{anom} = \\
&= \frac{2\beta_C}{3}\Bigg\{ (\sqrt{-g}\,F^2)^{\mu_1\nu_1\mu_3\mu_4}(p_1,p_3,p_4)\,\frac{1}{p_2^2}\,[R]^{\mu_2\nu_2}(p_2) + (\sqrt{-g}\,F^2)^{\mu_2\nu_2\mu_3\mu_4}(p_2,p_3,p_4)\,\frac{1}{p_1^2}\,[R]^{\mu_1\nu_1}(p_1) \\
&+\ [F^2]^{\mu_3\mu_4}(p_3,p_4)\,\frac{1}{(p_1+p_2)^2}[R]^{\mu_1\nu_1\mu_2\nu_2}(p_1,p_2) + \frac{2}{3}[R]^{\mu_1\nu_1}(p_1)\frac{1}{p_1^2}\ [F^2]^{\mu_3\mu_4}(p_3,p_4)\ \frac{1}{p_2^2}\ [R]^{\mu_2\nu_2}(p_2) \\
&+ [F^2]^{\mu_3\mu_4}(p_3,p_4)\,\frac{1}{(p_1+p_2)^2}\left[[\Box_1]^{\mu_1\nu_1}(p_1,p_2)\frac{1}{p_2^2}[R]^{\mu_2\nu_2}(p_2) + [\Box_1]^{\mu_2\nu_2}(p_2,p_1)\frac{1}{p_1^2}[R]^{\mu_1\nu_1}(p_1)\right] \\
&- \frac{1}{6}\,[F^2]^{\mu_3\mu_4}(p_3,p_4)\ \frac{1}{(p_1+p_2)^2}\ \left[[R]^{\mu_1\nu_1}(p_1)\ \frac{1}{p_1^2}\ [R]^{\mu_2\nu_2}(p_2) + [R]^{\mu_2\nu_2}(p_2)\ \frac{1}{p_2^2}\ [R]^{\mu_1\nu_1}(p_1)\right]\Bigg\}\delta^{(4)}\left(\sum_{i=1}^{4} p_i\right)
\end{aligned}
$$

MISSING TERMS IN THE NONLOCAL ANOMALY-INDUCED ACTION

$$\mathcal{S}^{(2)}_{anom} = -\frac{\beta_C}{6}\int d^4x \int d^4x' \Big\{ (\sqrt{-g}\,F^2)^{(1)}_x \left(\frac{1}{\Box_0}\right)_{xx'} R^{(1)}_{x'} + F^2_x \left(\frac{1}{\Box_0}\right)_{xx'} R^{(2)}_{x'}$$
$$+ \int d^4x'' \Big[F^2_x \left(\frac{1}{\Box_0}\right)_{xx'} (\Box_1)_{x'} \left(\frac{1}{\Box_0}\right)_{x'x''} R^{(1)}_{x''} - \frac{1}{6} F^2_x \left(\frac{1}{\Box_0}\right)_{xx'} R^{(1)}_{x'} \left(\frac{1}{\Box_0}\right)_{x'x''} R^{(1)}_{x''}$$
$$+ \frac{1}{3} R^{(1)}_x \left(\frac{1}{\Box_0}\right)_{xx'} F^2_{x'} \left(\frac{1}{\Box_0}\right)_{x'x''} R^{(1)}_{x''} \Big] \Big\}.$$

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Four-point functions of gravitons and conserved currents of CFT in momentum space: testing the nonlocal action with the TTJJ

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Conclusions

The breaking of conformal symmetry is associated to the the propagation of massless effective states in the effective action.

For chiral anomalies, the interactions can be reconstructed by a combination of the Anomaly pole + CWIs. We have shown it in the case of the AVV, for the J5TT (work in preparation)

For parity breaking trace/conformal anomalies, we have also shown that
the reconstruction can also be based entirely on the selection of an anomaly pole to solve the CWIs.

We have used the TTJJ correlator to show that the anomaly induced actions either in the Riegert form or in the Fradkin-Vilkovisky form miss crucial Weyl invariant terms in order to be consistent with the CWIs
and identified such terms

Applicatons

Condensed Matter theory: application of this class of nonlocal actions in the context of topological Materials (via Luttinger formula)