On three-generation super no-scale models in heterotic string theory

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Introduction

The Standard Model of particle physics is very successful in interpreting experimental results as of today. However, the Standard Model is not considered as a complete theory as it leaves a number of unanswered questions including the hierarchy problem, neutrino masses, dark matter and does not include gravity.

Supersymmetry is an extension of the Standard Model that has been extensively studied. It can help to resolve some of these issues, e.g. provide a technical solution to the hierarchy problem.

However, experiments have not provided any signs of SUSY yet.

String Theory

String theory is our best candidate for a consistent theory of quantum gravity that incorporates gauge interactions including the Standard Model of Particle Physics.

String phenomenology focuses on the construction and study of phenomenological features of string derived gauge models. These include extensions of the SM or GUTs that comprise the SM. The research in this field has yielded low energy effective models with realistic characteristics, including the $SU(3)^3$, flipped $SU(5) \times U(1)$, Pati-Salam models. All these models exhibit $N = 1$ space-time supersymmetry.

Non-supersymmetric strings

Space-time supersymmetry is not required for consistency in string theory.

From the early days of the first string revolution it was known that heterotic strings in 10D comprise both the supersymmetric $E_8 \times E_8$ and $SO(32)$ models and the non-supersymmetric tachyon free $SO(16) \times SO(16)$ theory.

However, non-supersymmetric string models face serious challenges:
- Tachyon instabilities.
- The cosmological constant does not vanish.

Recent developments provide some interesting solutions to these issues.

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* see e.g. S. Abel, K. R. Dienes and E. Mavroudi (2015,2017) , J. R. and I. Florakis (2016,2017) , Y. Sugawara, T. Wada (2016) , A. Lukas, Z. Lalak and E. E. Svanes (2015) , S.G. Nibbelink, O. Loukas, A. Mütter, E. Parr, P. K. S. Vaudrevange (2017), Faraggi et all (2020) , T. Coudarchet, E. Dudas, H. Partouche (2021)...

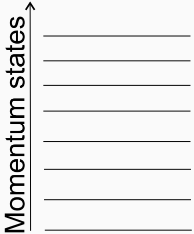
Coordinate dependent compactifications

Supersymmetry may be spontaneously broken within a string theory setup admitting an exact worldsheet description via coordinate-dependent compactifications which essentially realise the stringy analogue of the Scherk-Schwarz mechanism. A (minimal) implementation of a stringy Scherk–Schwartz mechanism requires an extra dimension X^5 and a conserved charge Q. Upon compactification

$$\Phi\left(X^5 + 2\pi R\right) = e^{iQ}\Phi\left(X^5\right)$$

we obtain a shifted tower of Kaluza–Klein states for charged fields, starting at $M_{KK} = \frac{|Q|}{2\pi R}$

$$\Phi(X^5) = e^{\frac{i\,QX^5}{2\pi R}} \sum_{n\in Z} \Phi_n\, e^{i\,nX^5/R}$$



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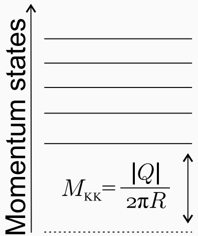

Coordinate dependent compactifications

Choosing

$$Q = \text{Fermion Number}$$

Leads to different masses for fermions-bosons (lying in the same supermultiplet) and thus to spontaneous breaking of supersymmetry.

SUSY breaking related to the compactification radius $M \sim \frac{1}{R}$

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see e.g.
J. Scherk and J. H. Schwarz (1978,1979) , R. Rohm (1984) , C. Kounnas and M. Porrati (1988) , S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner (1989) , C. Kounnas and B. Rostand (1990) , C. Kounnas, H. Partouche (2017)

Gravitino mass

We focus on compactifications of the six internal dimensions in three separate two-tori parametrised by the $T^{(i)}, U^{(i)}$, $i = 1, 2, 3$ moduli. For simplicity, we will assume that the Scherk–Schwartz mechanism is realised utilising the first torus $T^{(1)} = T_1^{(1)} + iT_2^{(1)}, U^{(1)} = U_1^{(1)} + iU_2^{(1)}$.

At tree level the gravitino receives a mass, e.g.

$$m_{3/2} = \frac{|U^{(1)}|}{\sqrt{T_2^{(1)} U_2^{(1)}}} = \frac{1}{R_1}\,,$$

for a square torus: $T = \imath R_1 R_2, U = \imath R_2/R_1$.

Moreover, at tree level, all $T^{(i)}, U^{(i)}$ moduli remain massless.

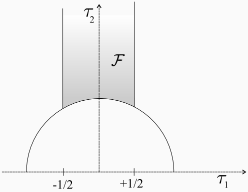
At $R_1 \to \infty$ we have $m_{3/2} = 0$ and the supersymmetry is restored.

One loop potential

The effective potential at one loop, as a function moduli $t_I = T^{(i)}, U^{(i)}$, is obtained by integrating the string partition function $Z(\tau_1, \tau_2; t_I)$ over the worldsheet torus Σ_1

$$V_{\rm one-loop}(t_I) = -\frac{1}{2(2\pi)^4} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^3}\, Z(\tau, \bar{\tau}; t_I)\,,$$

where $\mathcal{F}$ is the fundamental domain .



For given values of the moduli

$$Z = \sum_{\substack{n\in\mathbb{Z}/2 \\ n\geq -1/2}} \sum_{m\in\mathbb{Z}} Z_{n,m}\, q_{\rm r}^n\, q_{\rm i}^m = \sum_{\substack{n\in\mathbb{Z}/2 \\ n\geq -1/2}} \left[\sum_{m=-[n]-1}^{[n]+2} Z_{n,m}\, q_{\rm i}^m \right] q_{\rm r}^n\,.$$

where $q_{\rm r} = e^{-2\pi\tau_2}$ and $q_{\rm i} = e^{2\pi i\tau_1}$.

One loop partition function

$$Z = \frac{1}{\eta^2\bar\eta^2}\,\frac{1}{2^4}\sum_{\substack{h_1,h_2,H,H'\\ g_1,g_2,G,G'}}\frac{1}{2^3}\sum_{\substack{a,k,\rho\\ b,\ell,\sigma}}\frac{1}{2^3}\sum_{\substack{H_1,H_2,H_3\\ G_1,G_2,G_3}}(-1)^{a+b+HG+H'G'+\Phi}$$

$$\times\ \frac{\vartheta[^a_b]}{\eta}\,\frac{\vartheta[^{a+h_1}_{b+g_1}]}{\eta}\,\frac{\vartheta[^{a+h_2}_{b+g_2}]}{\eta}\,\frac{\vartheta[^{a-h_1-h_2}_{b-g_1-g_2}]}{\eta}$$

$$\times\ \frac{\bar\vartheta[^k_\ell]^3}{\bar\eta^3}\,\frac{\bar\vartheta[^{k+H'}_{\ell+G'}]}{\bar\eta}\,\frac{\bar\vartheta[^{k-H'}_{\ell-G'}]}{\bar\eta}\,\frac{\bar\vartheta[^{k+h_1}_{\ell+g_1}]}{\bar\eta}\,\frac{\bar\vartheta[^{k+h_2}_{\ell+g_2}]}{\bar\eta}\,\frac{\bar\vartheta[^{k-h_1-h_2}_{\ell-g_1-g_2}]}{\bar\eta}$$

$$\times\ \frac{\bar\vartheta[^{\rho+H'}_{\sigma+G'}]}{\bar\eta}\,\frac{\bar\vartheta[^{\rho-H'}_{\sigma-G'}]}{\bar\eta}\,\frac{\bar\vartheta[^\rho_\sigma]^2}{\bar\eta^2}\,\frac{\bar\vartheta[^{\rho+H}_{\sigma+G}]^4}{\bar\eta^4}$$

$$\times\ \frac{\Gamma^{(1)}_{2,2}[^{H_1}_{G_1}|^{h_1}_{g_1}](T^{(1)},U^{(1)})}{\eta^2\bar\eta^2}\,\frac{\Gamma^{(2)}_{2,2}[^{H_2}_{G_2}|^{h_2}_{g_2}](T^{(2)},U^{(2)})}{\eta^2\bar\eta^2}\,\frac{\Gamma^{(3)}_{2,2}[^{H_3}_{G_3}|^{h_1+h_2}_{g_1+g_2}](T^{(3)},U^{(3)})}{\eta^2\bar\eta^2}\,,$$

where $T^{(i)} = T^{(i)}_1 + iT^{(i)}_2$, $U^{(i)} = U^{(i)}_1 + iU^{(i)}_2$ are the moduli of the three two tori, $\eta(\tau)$ is the Dedekind eta function and $\vartheta[^\alpha_\beta](\tau)$ stand for the Jacobi theta functions.

Twisted/shifted lattices

The Scherk–Schwarz breaking is implemented utilising orbifold shifts parametrised by $G_i, H_i, i = 1, 2, 3$

$$\Gamma_{2,2}\left[{}^{H_i}_{G_i}\middle|{}^{h}_{g}\right](T,U) = \begin{cases} \left|\frac{2\eta^3}{\vartheta\left[{}^{1-h}_{1-g}\right]}\right|^2 & , \ (H_i, G_i) = (0,0) \text{ or } (H_i, G_i) = (h, g) \\ \Gamma^{\text{shift}}_{2,2}\left[{}^{H_i}_{G_i}\right](T,U) & , \ h = g = 0 \\ 0 & , \ \text{otherwise} \end{cases}$$

$$\Gamma^{\text{shift}}_{2,2}\left[{}^{H_i}_{G_i}\right](T,U) = \sum_{\substack{m_1,m_2 \\ n_1,n_2}} (-1)^{G_i(m_1+n_2)}\, q^{\frac{1}{4}|P_L|^2}\, \bar{q}^{\frac{1}{4}|P_R|^2} \,,$$

with

$$P_L = \frac{m_2 + \frac{H_i}{2} - Um_1 + T(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2 U_2}} \,,$$

$$P_R = \frac{m_2 + \frac{H_i}{2} - Um_1 + \bar{T}(n_1 + \frac{H_i}{2} + Un_2)}{\sqrt{T_2 U_2}} \,.$$

One loop potential: Large volume limit

The asymptotic behaviour of the one loop potential is

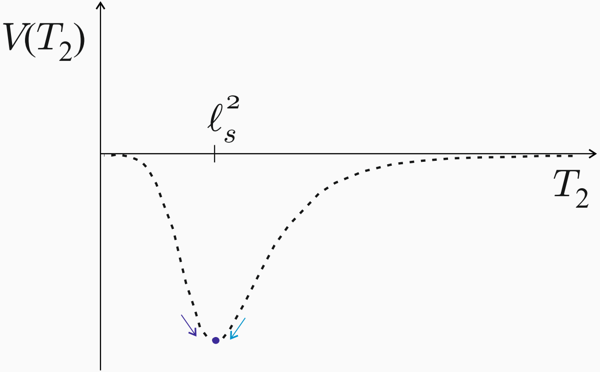
$$\lim_{T_2 \gg 1} V_{\text{one-loop}}(T,U) = -\frac{(n_B - n_F)}{2^4 \pi^7 T_2^2} \sum_{m_1, m_2 \in \mathbb{Z}} \frac{U_2^3}{\left| m_1 + \frac{1}{2} + U\, m_2 \right|^6} + \mathcal{O}\left(e^{-\sqrt{2\pi T_2}}\right)$$

$$\lim_{T_2 \gg 1} V_{\text{one-loop}}(T,U) = \xi\, \frac{(n_B - n_F)}{T_2^2} + \text{exponentially supressed}$$

where ξ is a constant and $T_2 = R^2$ for a square torus.
n_B, n_F stand for the number of massless bosonic and fermionic degrees of freedom respectively.

Cosmological constant is exponentially small for large R for models with fermion-boson degeneracy $n_B = n_F$, the “super-no-scale models”, termed so by Costas Kounnas an excellent physicist and great friend who passed away last year.

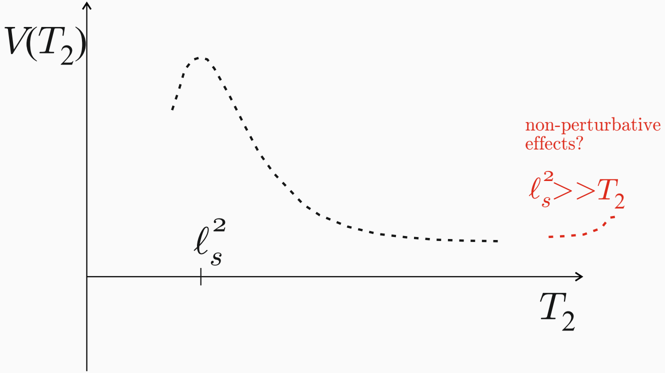
The self-dual points under the duality symmetry $T_2 \to 1/T_2$ correspond to extrema of the potential $V(T_2) = V\left(\ell_s^2/T_2\right)$. Typically, the potential is of the form



This leads to SUSY breaking at the string scale.

Shape of the potential

It has been shown that there exist models with potentials of the form



This brings the possibility of supersymmetry breaking at a low scale provided a mechanism for modulus stabilization at $T_2 \gg \ell_s^2$.

Free fermionic formulation of the heterotic string

The heterotic string is a hybrid construction that combines the 10-dimensional superstring with the 26-dimensional bosonic string.

In the free fermionic formulation of the heterotic string all world-sheet bosonic coordinates are fermionized (except the ones associated with 4D space-time). World-sheet supersymmetry is preserved as it is non-linearly realized among left moving fermions. In the standard notation the fermionic coordinates in the light-cone gauge are:

$$\begin{array}{lll|l|l} \text{Left:} & \psi^\mu, \chi^{1,\dots,6}, & & y^{1,\dots,6}, \omega^{1,\dots,6} & \\ \text{Right:} & & & \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, & \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \quad \bar{\phi}^{1,\dots,8} \end{array}$$

In this framework a model is defined by a set of basis vectors which encode the parallel transport properties of the fermionic fields along the non-contractible loops of the world-sheet torus, and a set of phases associated with generalised GSO projections (GGSO).

The non-supersymmetric Pati–Salam model

Based on "Lepton Number as the Fourth Color", J. C. Pati and A. Salam (1974)

$$\text{Gauge symmetry}: SU(4)\times SU(2)_L \times SU(2)_R$$

SM Fermions:

$$F_L(\mathbf{4},\mathbf{2},\mathbf{1}) = Q(\mathbf{3},\mathbf{2},-1/6) + L(\mathbf{1},\mathbf{2},1/2)\,,$$
$$\overline{F}_R(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) = u^c(\overline{\mathbf{3}},\mathbf{1},2/3) + d^c(\overline{\mathbf{3}},\mathbf{1},-1/3) + e^c(\mathbf{1},\mathbf{1},-1) + \nu^c(\mathbf{1},\mathbf{1},0)$$

Extra triplets: $(\mathbf{6},\mathbf{1},\mathbf{1})$

Pati-Salam Higgs scalars: $H\,(\mathbf{4},\mathbf{1},\mathbf{2})$

SM Higgs scalars:

$$h\,(\mathbf{1},\mathbf{2},\mathbf{2}) = H_u\left(\mathbf{1},\mathbf{2},+\frac{1}{2}\right) + H_d\left(\mathbf{1},\mathbf{2},-\frac{1}{2}\right)$$

Pati–Salam string models

A class of Pati-Salam models can be generated by the basis

$$\beta_1 = \mathbf{1} = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\},$$
$$\beta_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\},$$
$$\beta_3 = T_1 = \{y^{12}, \omega^{12} | \bar{y}^{12}, \bar{\omega}^{12}\},$$
$$\beta_4 = T_2 = \{y^{34}, \omega^{34} | \bar{y}^{34}, \bar{\omega}^{34}\}$$
$$\beta_5 = T_3 = \{y^{56}, \omega^{56} | \bar{y}^{56}, \bar{\omega}^{56}\},$$
$$\beta_6 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1}\},$$
$$\beta_7 = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{2}\},$$
$$\beta_8 = z_1 = \{\bar{\phi}^{1,\dots,4}\},\ \beta_9 = z_2 = \{\bar{\phi}^{5,\dots,8}\},\ \beta_{10} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\},$$

and a set of $10(10-1)/2+1 = 46$ GGSO phases $c\begin{bmatrix}\beta_i\\ \beta_j\end{bmatrix} = \pm 1$.

This class compises $2^{46} \approx 7 \times 10^{13}$ models.

Gauge group:

$$G = SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3 \times SU(2)^4 \times SO(8)$$

Pati–Salam string models

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Phenomenological criteria

(a) Absence of physical tachyons in the string spectrum

(b) Existence of complete chiral fermion generations

(c) Existence of Pati–Salam and SM symmetry breaking scalar Higgs fields

(d) Absence of observable gauge group enhancements

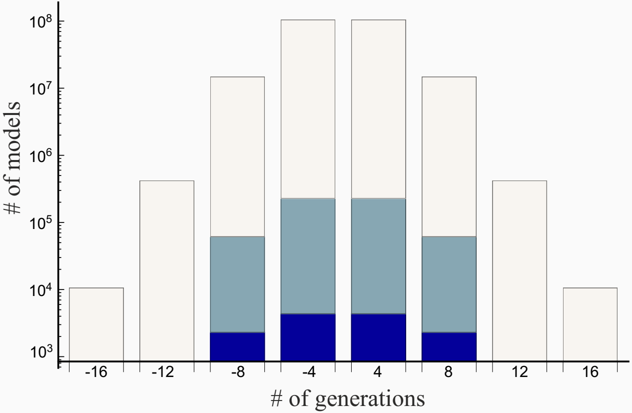
(e) Vector-like fractionally charged exotic states

(f) Consistency with the Scherk–Schwarz SUSY breaking

(g) Compliance with the super-no-scale condition, that is translated to equality of the fermionic and bosonic degrees of freedom

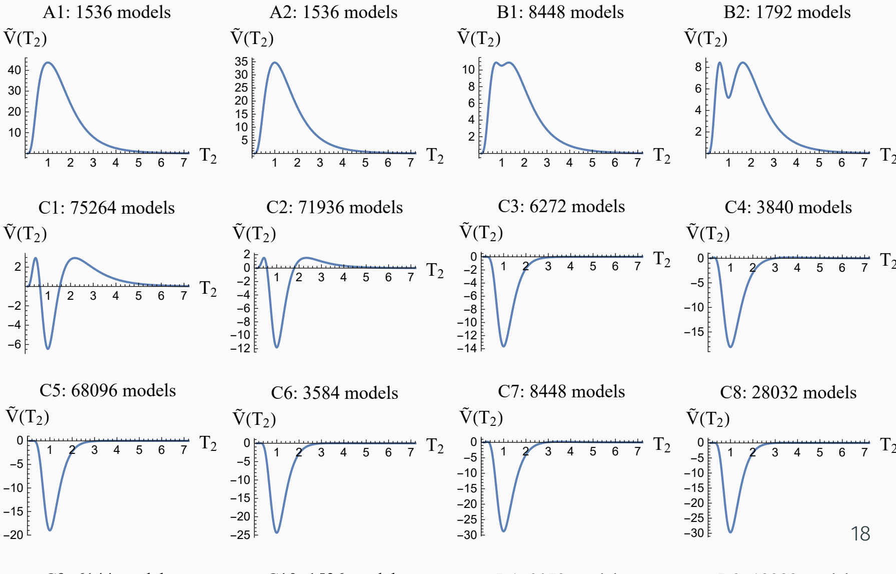
Phenomenologically promising Pati–Salam string models

A comprehensive computer scan over the full parameter space (1.7×10^{10} models) yields



Light shaded bars: (a)-(c) 2.4×10^8 *models, Medium shaded bars (a)-(g)* 5.6×10^5 *models, Dark shading bars:* 1.4×10^4 *models*

One-loop potentials



Three-generation models

Three generation models in the context of $Z_2 \times Z_2$ orbifolds can be obtained only utilising real fermions. To this end, we introduce additional vectors that separate internal fermions

$$v_1 = \mathbb{1} = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\},$$

$$v_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\} \quad , \quad v_{2+i} = e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}, i = 1, \dots, 6,$$

$$v_9 = b_1 = \{\chi^{34}, \chi^{56}, y^3, y^4, y^5, y^6 | \bar{y}^3, \bar{y}^4, \bar{y}^5, \bar{y}^6, \bar{\psi}^{1,\dots,5}, \bar{\eta}^1\},$$

$$v_{10} = b_2 = \{\chi^{12}, \chi^{56}, y^1, y^2, y^5, y^6 | \bar{y}^1, \bar{y}^2, \bar{y}^5, \bar{y}^6, \bar{\psi}^{1,\dots,5}, \bar{\eta}^2\},$$

$$v_{11} = z_1 = \{\bar{\phi}^{1,\dots,4}\},\ v_{12} = z_2 = \{\bar{\phi}^{5,\dots,8}\},\ v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\},$$

For generic choices of the GGSO projections this class comprises a huge number of $2^{\frac{13(13-1)}{2}+1} \sim 6 \times 10^{23}$ heterotic string models that exhibit

$$\mathcal{G} = SU(4) \times SU(2)_L \times SU(2)_R \times U(1)^3 \times SU(2)^4 \times SO(8)$$

gauge symmetry.

Large volume formula - Super-no-scale constraints

Depending on the implementation of the Scherk–Schwarz mechanism new constraints arise. In the simplest case, the large volume limit of the effective potential can be expressed as follows

$$V_{\rm eff} = -\frac{63}{2(2\pi)^4\, T_2^2} \left[\frac{1}{2} \sum_{H_2, G_2 \in \mathbb{Z}_2} (-1)^{H_2} C\left[{}^{0\,,\,H_2}_{1\,,\,G_2}\right] E^\star_\infty(3;U) \right. \\ \left. + \;\; \frac{1}{8} \sum_{G_2 \in \mathbb{Z}_2} C\left[{}^{0\,,\,1}_{1\,,\,G_2}\right] E^\star_\infty(3;2U) \right] + \dots$$

where $E^\star_\infty(s;z)$ is the zero weight, completed, non-holomorphic Eisenstein series and the ellipses denote exponentially suppressed terms.

A careful examination of this formula, shows that in order to obtain an exponentially suppressed contribution at large T_2, the coefficients of both Eisenstein series vanish independently.

Super-no-scale conditions

The last constraint can be expressed as

$$\Sigma(H_2) \equiv \frac{1}{4} \sum_{G_1, G_2 = 0,1} C\left[{}^{0}_{G_1}\, {}^{,\, H_2}_{,\, G_2}\right] \quad , \ H_2 = 0, 1$$

The first condition

$$\Sigma(0) = n_B - n_F = 0$$

is associated with the full massless spectrum of the theory accompanied with a tower of states that tend to become massless at the limit $T_2 \to \infty$, $M^2(\Gamma^{(1)}_{2,2}) = |m_2 - U m_1|^2 / T_2 U_2$. This is the "super no-scale" condition known in the string literature. The second condition

$$\Sigma(1) = 0$$

is non-trivial. It refers to a subset of massive states, arising from shifted lattice, that also become massless at the $T_2 \to \infty$ limit, $M^2(\Gamma^{(1)}_{2,2}) = |m_2 + \frac{1}{2} - U m_1|^2 / T_2 U_2$.

Three generation models

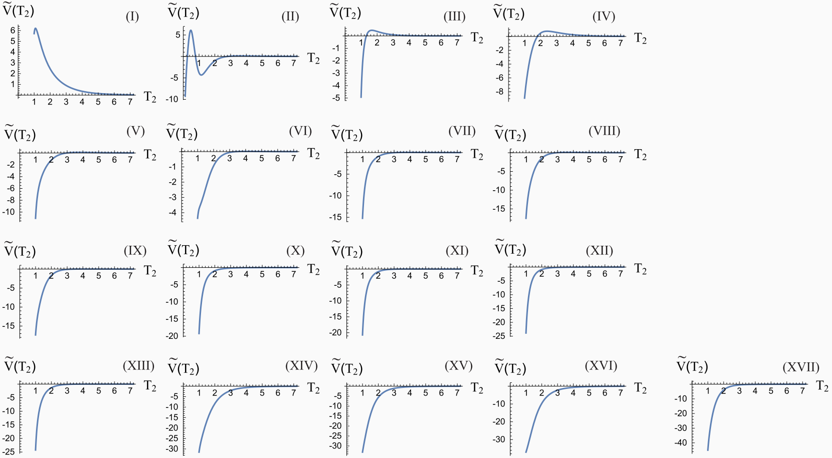
We have performed a detailed investigation of the model parameter space utilising a computer assisted two-stage scan procedure: we first perform a (random) scan and identify $SO(10)$ configurations compatible with our search criteria. Next, we consider all possible offspring Pati–Salam models generated by each (fertile) $SO(10)$ configuration and the related GGSO projection phases and check their compatibility with the aforementioned criteria.

In practice, this method allows us to effectively scan a big sample of 8.1×10^{12} models (almost one model in 10^4) of the full parameter space in about 10 days on a DELL PowerEdge R630 workstation with 32 GB of memory. It turns out that 8.8×10^6 models fulfil our phenomenological criteria. Out of these, about 0.1% meet the first super no-scale constraint $\Sigma(0) = 0$, while around 21% meet the second super no-scale constraint $\Sigma(1) = 0$.

Altogether, we have identified 174 three generation Pati–Salam models that comply with all requirements (one in 50 billions).

One-loop potential (3 generation models)

Based on the analysis of their partition functions the one-loop effective potentials of the 3-generation models fall into 17 distinct classes



The rescaled one-loop effective potential $\tilde{V}(T_2) = 2(2\pi)^4 V(T_2)$ for each of the 17 classes of 3-generation models satisfying all requirements.

Conclusions

We have shown the existence of non-supersymmetric heterotic string models with Pati-Salam gauge symmetry exhibiting interesting phenomenological characteristics:

• Chiral spectra with 3 generations, Pati–Salam and Standard Model breaking Higgs scalars.

• SUSY breaking via the Scherk–Schwarz mechanism at scales $M_{susy} \sim \frac{1}{R}$ that could be much smaller than M_{string}.

• Fulfil the $n_B = n_F$ ("super-no-scale") requirement that leads to exponentially small (and possibly positive) vacuum energy at the large volume limit.

• That has to be supplemented in the case of real fermions (necessary to obtain 3 generations) by some additional model dependent constraint.

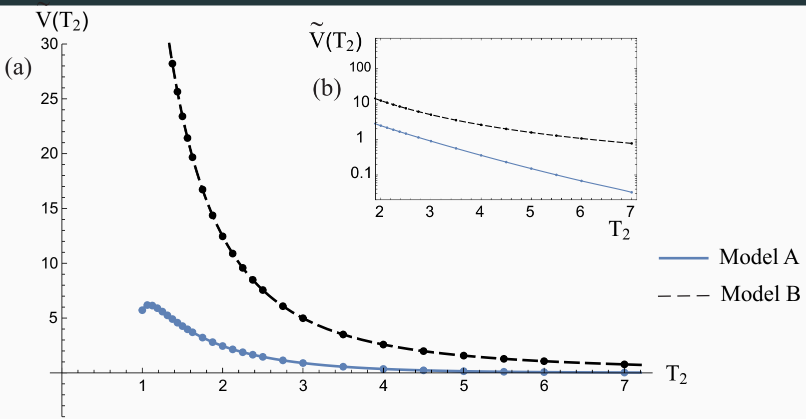
These developments bring us a few steps closer to the construction of a non-supersymmetric Standard Model from string theory.

Non-holomorphic Eisenstein series

$$E^*_\infty(s;z) = \frac{1}{2}\zeta^*(2s) \sum_{\substack{c,d\in Z \\ (2c,d)=1}} \frac{(\mathrm{Im} z)^s}{|2cz+d|^{2s}}\ ,\ \zeta^*(s) = \pi^{-s/2}\Gamma(s/2)\zeta(s).$$

where $(2c, d) = 1$ restricts the summation to coprime pairs.

One-loop potential (3 generation models)



Comparison between the rescaled one-loop effective potentials $\tilde{V}(T_2) = 2(2\pi)^4 V(T_2)$ of Models A and B in linear (a) and detail in semi-logarithmic (b) scale, showing the exponential suppression present in Model A as opposed to Model B.