Extending the SM with Vector-Like Quarks: consequences for CKM unitarity and CP violation

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Corfu, August 31, 2023

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Outline

- Motivation
- Solving the Cabibbo Angle Anomaly (s_{14}-dominance limit)
- Phenomenology
- Weak-Basis Invariants in VLQ extensions (enhancing CP violation)

Work based on:

F. J. Botella, G. C. Branco, M. N. Rebelo, J. I. Silva-Marcos, J. F. Bastos - *Decays of the Heavy Top and New Insights on ε_K in a one-VLQ Minimal Solution to the CKM Unitarity Problem* **[2111.15401]**.

F. Albergaria, G.C. Branco, J. F. Bastos, J. I. Silva-Marcos - *CP-odd and CP-even Weak-Basis Invariants in the Presence of Vector-Like Quarks* **[2210.14248]**

Motivation

A fourth chiral generation of quarks is ruled out. However, extensions with VLQs where the LH and RH fields transform in the same way under the SM gauge group are still allowed.

VLQs take part in many models from GUTs, to the Nelson-Barr solutions to the strong CP problem.
They have a rich phenomenology that can be used to try to explain several types of anomalies/ tensions.

Cabibbo Angle Anomaly (CAA): The independent determinations of $|V_{us}|$ (semi-leptonic kaon decays), the ratio $|V_{us}/V_{ud}|$(kaon and pion leptonic decays) and $|V_{ud}|$ (β decays) are not in agreement with each other within the framework of the CKM unitary of SM (discrepancy of $\sim 3\sigma$). These values fit best to the relation

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta^2, \quad \Delta \approx 0.04$$

Extensions with VLQs iso-singlets are natural candidates to explain this anomaly because they introduce deviations to CKM unitarity.

CP Violation: The introduction of VLQs leads to larger mass matrices and, in principle, more physical phases which could lead to the enhancement of CP violation in the quark sector.

Solving the CAA with an up-type VLQ iso-singlet

Introducing a $Q = 2/3$ VLQ iso-singlet $T = T_L + T_R$ with mass m_T leads to:

- $$-\mathcal{L}_Y = y^u_{ij}\overline{Q'_{iL}}\tilde{\Phi}u'_{jR} + y^u_{i4}\overline{Q'_{iL}}\tilde{\Phi}T'_R + y^d_{ij}\overline{Q'_{iL}}\Phi d'_{jR} + \underbrace{M^u_i\overline{T'}_L u'_{iR} + M^u_4\overline{T'}_L T'_R}_{\textbf{Bare mass terms}} + h.c.$$

- $$-\mathcal{L}_m = \begin{pmatrix}\overline{u}'_L & \overline{T}'_L\end{pmatrix}\mathcal{M}_u\begin{pmatrix}u'_R \\ T'_R\end{pmatrix} + \overline{d}'_L m_d d'_R + h.c.$$

$$m_{u,d} = v y_{u,d}$$

$$\mathcal{M}_u = \underbrace{\begin{pmatrix} m_u \\ M_u \end{pmatrix}}_{4}\begin{matrix}\}3 \\ \}1\end{matrix}$$

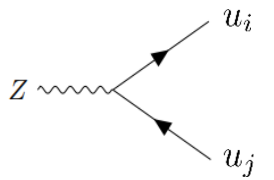
Both T_L and T_R have the same quantum numbers as u_R. In a general WB, the EW currents preserve their SM form.

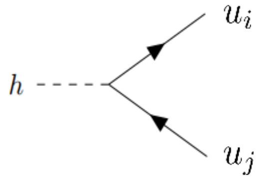
Major changes arise when going to the physical basis where we get **non-unitary mixing** and **tree-level FCNCs**:

- $$\mathcal{L}_W = -\frac{g}{\sqrt{2}}W^+\begin{pmatrix}\overline{u}_L & \overline{T}_L\end{pmatrix}V_{\text{CKM}}\,\gamma_\mu d_L + h.c. \longrightarrow$$ Non-Unitary 4x3 CKM matrix

- $$\mathcal{L}_Z = \frac{g}{2c_W}Z^\mu\left[\begin{pmatrix}\overline{u}_L & \overline{T}_L\end{pmatrix}F_u\gamma_\mu\begin{pmatrix}u_L \\ T_L\end{pmatrix} - \overline{d}_L\gamma_\mu d_L - 2s^2_W J^\mu_{\text{EM}}\right]$$

- $$-\mathcal{L}_h = \frac{h}{v}\left[\begin{pmatrix}\overline{u}_L & \overline{T}_L\end{pmatrix}F_u\begin{pmatrix}D_u & 0 \\ 0 & m_T\end{pmatrix}\begin{pmatrix}u_R \\ T_R\end{pmatrix} + \overline{d}_L D_d d_R\right] + h.c.$$





Tree-Level FCNCs

Solving the CAA with an up-type VLQ iso-singlet

The mixing can be parametrized as:

- Auxiliary Unitary Matrix: $\mathcal{V} = \underbrace{\begin{pmatrix} A_u \\ B_u \end{pmatrix}}_{4} \begin{matrix} \}3 \\ \}1 \end{matrix}$
- Non-Unitary 4x3 CKM Matrix: $V_{\text{CKM}} = A_u^\dagger$
- Matrix Controlling FCNCs: $F_u = A_u^\dagger A_u$

$$\mathcal{V}^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24}e^{-i\delta_{24}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}e^{i\delta_{24}} & 0 & c_{24} \end{pmatrix} \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 \\ 0 & -s_{23} & c_{23} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} & 0 \\ 0 & 1 & 0 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\theta_{ij} \in [0, \pi/2]$

$\delta_{ij} \in [0, 2\pi]$

We have for the first row of the mixing: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - s_{14}^2 \xRightarrow{\text{CAA}} s_{14} \approx 0.04 \sim \lambda^2$

General solution and its parameter space analyzed in **Branco et al. [2103.13409]** for $m_T > 1$ TeV

At first glance, a “minimal” solution to the CAA could be: $s_{14} \approx 0.04$ $\quad s_{24}, s_{34} = 0$ $\quad \delta_{14}, \delta_{24} \longrightarrow$ Factored out

We study this case in **Botella et al. [2111.15401].**

Phenomenology: the s_{14} - dominance limit

In the limit $s_{24}, s_{34} = 0$ we have:

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13}c_{14} & s_{12}c_{13}c_{14} & s_{13}c_{14}e^{-i\delta} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} & -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} & c_{13}c_{23} \\ -c_{12}c_{13}s_{14} & -s_{12}c_{13}s_{14} & -s_{13}s_{14}e^{-i\delta} \end{pmatrix} \qquad F_u = \begin{pmatrix} c_{14}^2 & 0 & 0 & -s_{14}c_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}c_{14} & 0 & 0 & s_{14}^2 \end{pmatrix}$$

This limit, with $s_{14} \sim \lambda^2$ is interesting phenomenologically:

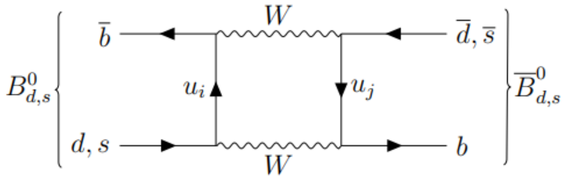
Sirunyan et al. (CMS) [1708.02510]

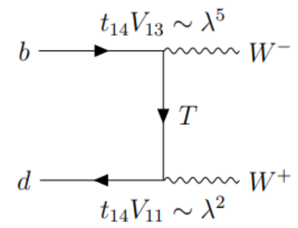
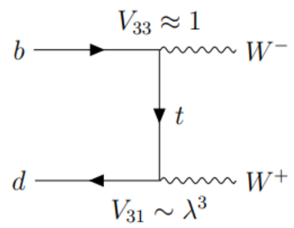
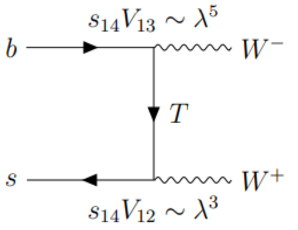
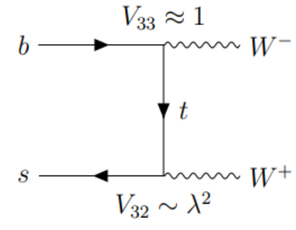
- The VLQ decays predominantly to the first generation: $\text{Br}\left(T \to dW^+\right) + \text{Br}\left(T \to uZ\right) + \text{Br}\left(T \to uh\right) \simeq 1 \implies m_T > 0.685\ \text{TeV}$

  Typically searches assume: $\text{Br}\left(T \to bW^+\right) + \text{Br}\left(T \to tZ\right) + \text{Br}\left(T \to th\right) \simeq 1 \implies m_T > 1 - 1.3\ \text{TeV}$

- The NP contributions to $B^0_{d,s} - \overline{B}^0_{d,s}$ mixing are suppressed

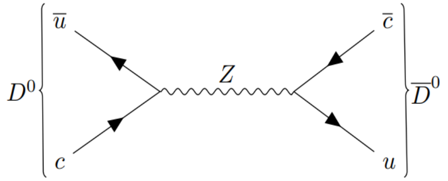
$B^0_{d,s} - \overline{B}^0_{d,s}$:



	NP	SM	NP/SM
$B^0_d - \overline{B}^0_d$:			$\sim\lambda^4$
$B^0_s - \overline{B}^0_s$:			$\sim\lambda^6$

- No NP contribution to $D^0 - \overline{D}^0$ in the s_{14} – dominance limit:

$D^0 - \overline{D}^0$:

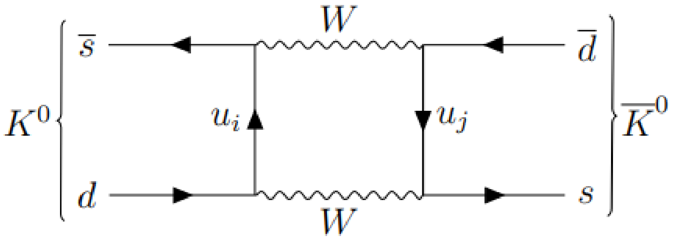


$\sim \left|F^u_{12}\right|^2 = s^2_{14}s^2_{24}$

- No NP contribution to $K_L \to \pi^0\overline{\nu}\nu$ or ε'/ε, since $\text{Im}\left(V_{Td}V^*_{Ts}\right) = 0$

The kaon system imposes the most stringent constraints.

$K^0 - \overline{K}^0$:



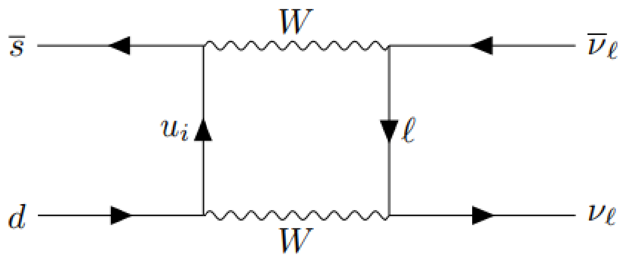
$$\left.\begin{array}{r} |\varepsilon_K^{\text{exp}}| \simeq (2.228 \pm 0.011) \times 10^{-3} \\ |\varepsilon_K^{\text{SM}}| \simeq (2.16 \pm 0.18) \times 10^{-3} \end{array}\right\} \; |\varepsilon_K^{\text{NP}}| \approx (0.68 \pm 1.80) \times 10^{-4} \implies \begin{array}{c} s_{24} \neq 0 \\ \sin(\delta_{24} - \delta_{14}) \neq 0 \end{array}$$

SM prediction from **Brod et al. [1911.06822]**

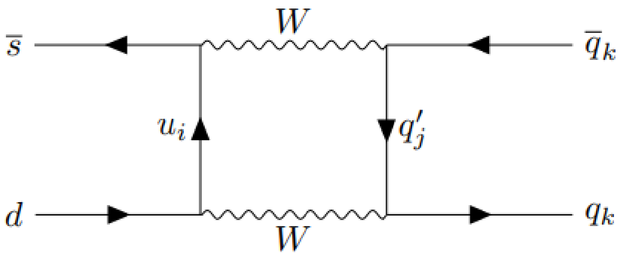
$\Downarrow$

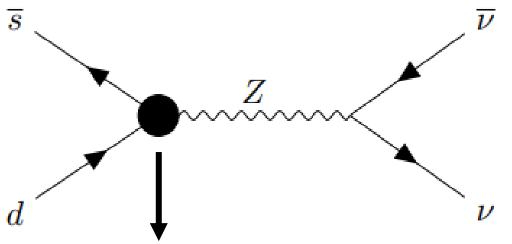
Use instead $s_{24}, s_{34} \ll s_{14}$

$K^+ \to \pi^+ \nu \overline{\nu}$:

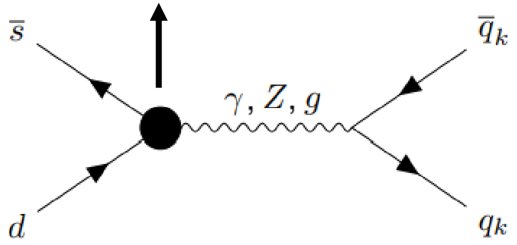


$K_L \to \pi\pi$:



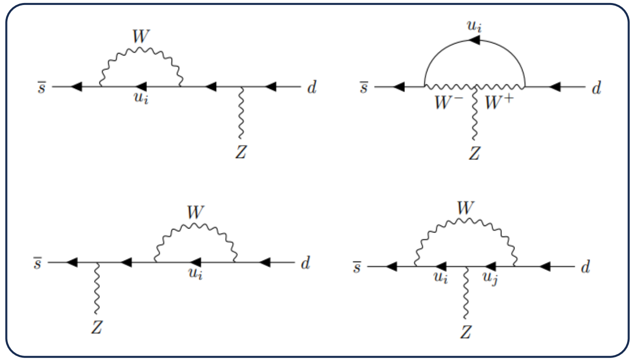


Penguin Loops



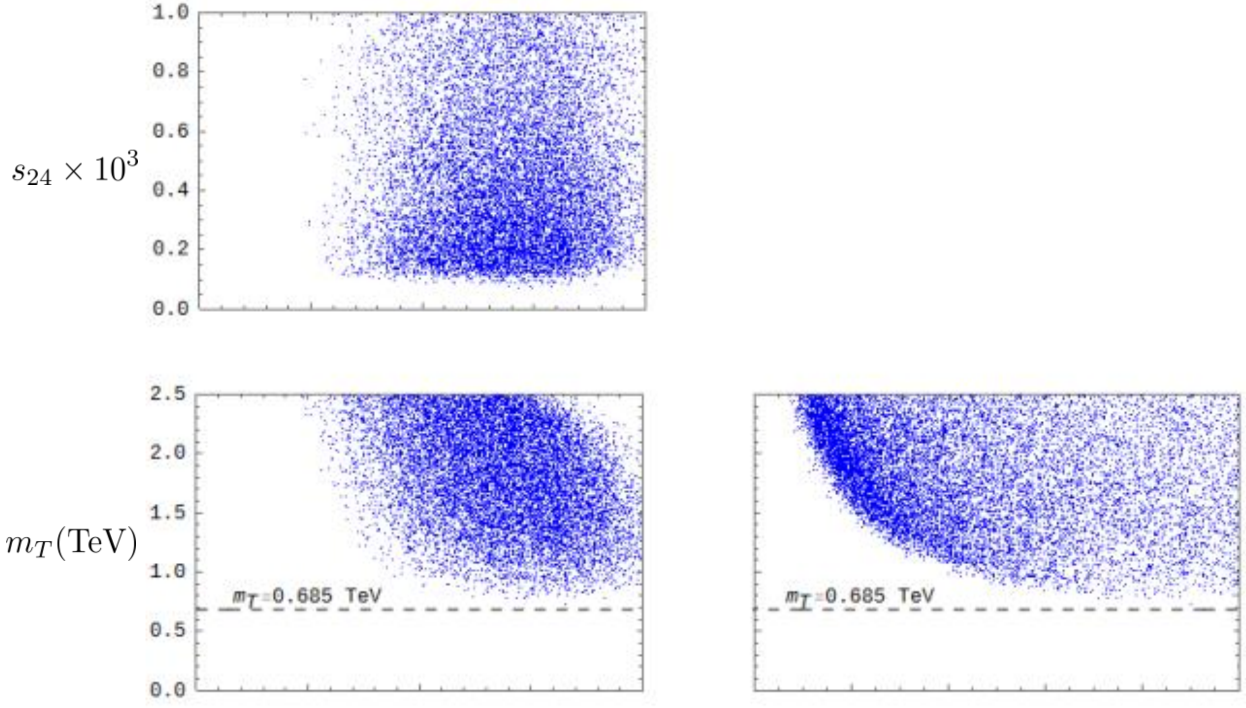
$$\frac{\text{Br}\,(K^+ \to \pi^+ \overline{\nu}\nu)_{\text{exp}}}{\text{Br}\,(K^+ \to \pi^+ \overline{\nu}\nu)_{\text{SM}}} = \frac{(10.6^{+4.0}_{-3.4} \pm 0.9) \times 10^{-11}}{(8.4 \pm 0.1) \times 10^{-11}} = 1.26 \pm 0.51$$

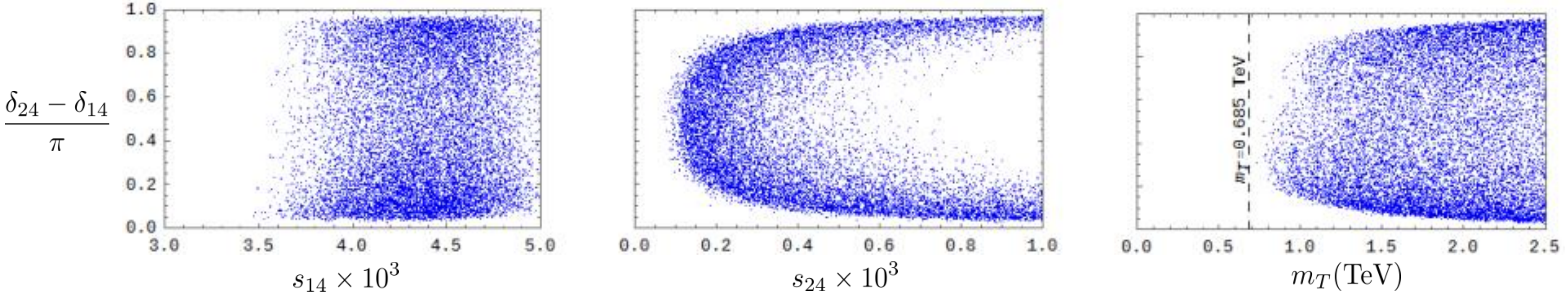
EW Penguin Loops



$$-4 \times 10^{-4} \lesssim \left(\frac{\epsilon'}{\epsilon}\right)^{\text{NP}}_{1\sigma} \lesssim 10 \times 10^{-4}$$

[Aebischer et al. 2005.0597]





Best-fit (s_{14}-dom.): $\sqrt{\chi^2} \simeq 2.25$ $\quad m_T = 1477$ GeV

$\theta_{12} = 0.22579,\quad \theta_{13} = 0.0038275,\quad \theta_{23} = 0.039524,$

$\theta_{14} = 0.045334,\quad \theta_{24} = 7.412 \times 10^{-4},\quad \theta_{34} = 2.346 \times 10^{-4},$

$\delta = 0.382\pi,\quad \delta_{14} = 1.872\pi,\quad \delta_{24} = 1.979\pi.$

$$\chi^2 = \sum_{i,j} \left(\frac{|V_{ij}| - |V_{ij}|_c}{\sigma_{ij}} \right)^2 + \left(\frac{\gamma - \gamma_c}{\sigma_\gamma} \right)^2 + \left(\frac{\left|\varepsilon_K^{\mathrm{NP}}\right| - \left|\varepsilon_K^{\mathrm{NP}}\right|_c}{\sigma_\varepsilon} \right)^2$$
$$+ \left(\frac{(k/k_{\mathrm{SM}}) - (k/k_{\mathrm{SM}})_c}{\sigma_{k/k_{\mathrm{SM}}}} \right)^2 + \left(\frac{(\varepsilon'/\varepsilon)^{\mathrm{NP}} - (\varepsilon'/\varepsilon)_c^{\mathrm{NP}}}{\sigma_{\varepsilon'/\varepsilon}} \right)^2$$

$$k \equiv \mathrm{Br}\left(K^+ \to \pi^+ \overline{\nu}\nu\right)$$

Constraints:

- $\left(\Delta m_K^{\mathrm{NP}}\right)_{\mathrm{SD}} < \Delta m_K^{\mathrm{exp}}$
- $\sqrt{\chi^2} < 3$
- $m_T \in [0.685, 2.5]\mathrm{TeV}$
- $s_{24}, s_{34} \in [0, 0.001]$

Weak-Basis Invariants (WBIs) – Standard Model

Weak-basis invariant quantities can relate the parameters in any WB to physical quantities (masses and mixing).

WBIs remain unchanged under weak-basis transformations (WBTs) which leave EW currents flavor-diagonal.

- WBTs in the SM: $Q'_L \to W_L Q'_L, \quad u'_R \to W^u_R u'_R, \quad d'_R \to W^d_R d'_R$ $\qquad W_L, W_R^{u,d} \longrightarrow$ 3x3 unitary matrices
- Hermitian "building blocks":
$$(h_u)^n = (m_u m_u^\dagger)^n \to W_L^\dagger h_u^n W_L$$
$$(h_d)^n = (m_d m_d^\dagger)^n \to W_L^\dagger h_d^n W_L$$
- Moduli of CKM determined by CP-even WBIs: $\mathrm{tr}\left(h_u^n h_d^m\right) = \sum_{\alpha,i=1}^{3} m_{u_\alpha}^{2n} m_{d_i}^{2m} \left|V_{\alpha i}\right|^2$
- CP violating is determined by a single CP-odd WBI:
$$\mathrm{tr}[h_u, h_d]^3 = 6i(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\ I_{\mathrm{CP}}$$
$$I_{\mathrm{CP}} = \mathrm{Im}\left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*\right) = s_{12} s_{13} s_{23} c_{12} c_{13}^2 c_{23} \sin\delta$$
- The masses are determined by:
$$\mathrm{tr}\left(h_q\right) = m_1^2 + m_2^2 + m_3^2 \qquad \chi\left(h_q\right) = m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2 \qquad \det\left(h_q\right) = m_1^2 m_2^2 m_3^2$$

WBIs – Extension with up-type VLQ iso-singlet

This case is studied by us in **Albergaria et al. [2210.14248]**. Main take-aways:

- WBTs with one up-type VLQ iso-singlet: $Q'_L \to W_L Q'_L, \quad T'_L \to e^{i\varphi} T'_L, \quad d'_R \to W^d_R d'_R, \quad \begin{pmatrix} u'_R \\ T'_R \end{pmatrix} \to \underset{\text{4x4 unitary}}{\underset{\downarrow}{\mathcal{W}^u_R}} \begin{pmatrix} u'_R \\ T'_R \end{pmatrix}$

- Hermitian "building blocks" (all transforming as $H \to W_L^\dagger H W_L$):

$$h_d^n = (m_d m_d^\dagger)^n \qquad h_u^n = (m_u m_u^\dagger)^n \qquad h_u^{(n)} = m_u (m_u^\dagger m_u + M_u^\dagger M_u)^{n-1} m_u^\dagger$$

- Moduli of CKM determined by CP-even WBIs: $\operatorname{tr}\left(h_u^{(n)} h_d^m\right) = \sum_{\alpha,i=1}^{3} m_{u_\alpha}^{2n} m_{d_i}^{2m} \left|V_{\alpha i}\right|^2$

- More CP violating phases imply more independent CP-odd WBI. The WBI of lowest mass dimension is:

$$\operatorname{tr}\left(\left[h_u, h_d\right] h_u^{(2)}\right) = 2i \sum_{i=1}^{3} \sum_{\alpha,\beta=1}^{4} m_{d_i}^2 m_{u_\alpha}^4 m_{u_\beta}^2 \operatorname{Im}\left(F^u_{\alpha\beta} V^*_{\alpha i} V_{\beta i}\right) \sim M^8$$

- The up-type masses are determined by:

$$\operatorname{tr}\left(\mathcal{H}_u\right) = m_u^2 + m_c^2 + m_t^2 + m_T^2 \qquad \chi_1\left(\mathcal{H}_u\right) = m_u^2 m_c^2 + m_u^2 m_t^2 + m_u^2 m_T^2 + m_c^2 m_t^2 + m_c^2 m_T^2 + m_t^2 m_T^2$$

$$\det\left(\mathcal{H}_u\right) = m_u^2 m_c^2 m_t^2 m_T^2 \qquad \chi_2\left(\mathcal{H}_u\right) = m_u^2 m_c^2 m_t^2 + m_u^2 m_c^2 m_T^2 + m_u^2 m_t^2 m_T^2 + m_c^2 m_t^2 m_T^2$$

$$\mathcal{H}_u = \mathcal{M}_u^\dagger M_u$$

WBIs – Enhancement of CP Violation with VLQs

CP violation should depend on dimensionless quantities such as

$$\mathcal{I}_{\text{SM}} = \text{tr}\left[y_u y_u^\dagger, y_d y_d^\dagger\right]^3 = \frac{\text{tr}\,\overbrace{[h_u, h_d]^3}^{\sim M^{12}}}{v^{12}} \sim 10^{-25} \qquad \mathcal{I}_{\text{VLQ}} = \frac{\text{tr}\left(\overbrace{[h_u, h_d]\, h_u^{(2)}}^{\sim M^{8}}\right)}{v^6 m_T^2} \qquad \mathcal{I}'_{\text{VLQ}} = \frac{\text{tr}\left(\overbrace{[h_u^2, h_d]\, h_u^{(2)}}^{\sim M^{10}}\right)}{v^8 m_T^2}$$

Some CP-odd dimensionless WBIs from VLQ extensions can be significantly larger than the SM one:

Best-fit (s_{14}-dom.): $\sqrt{\chi^2} \simeq 2.25$ $\quad m_T = 1477$ GeV

$\theta_{12} = 0.22579,\quad \theta_{13} = 0.0038275,\quad \theta_{23} = 0.039524,$

$\theta_{14} = 0.045334,\quad \theta_{24} = 7.412 \times 10^{-4},\quad \theta_{34} = 2.346 \times 10^{-4},$

$\delta = 0.382\pi,\quad \delta_{14} = 1.872\pi,\quad \delta_{24} = 1.979\pi.$

$$\mathcal{I}_{\text{VLQ}} = \frac{\text{tr}\left([h_u, h_d]\, h_u^{(2)}\right)}{v^6 m_T^2} \simeq 2.02 \times 10^{-10} \qquad \mathcal{I}'_{\text{VLQ}} = \frac{\text{tr}\left([h_u^2, h_d]\, h_u^{(2)}\right)}{v^8 m_T^2} \simeq 1.16 \times 10^{-10}$$

Important for Baryogenesis??

In these extensions we can even obtain CP violation in the limit of extremely high energies (extreme chiral limit) where $m_u = m_c = m_d = m_s = 0$ and $\mathcal{I}_{\text{SM}} = 0$ (also pointed out in **del Aguila et al. [hep-ph/9703410]**).

$$\text{tr}\left([h_u, h_d]\, h_u^{(2)}\right) = 2i\; m_b^2 m_t^2 m_T^2 (m_T^2 - m_t^2) I_{\text{ECL}}$$

$$I_{\text{ECL}} = c_{23} c_{14}^2 c_{24}^2 c_{34} s_{23} s_{24} s_{34} \sin\delta_{\text{ECL}}$$

$$V_{\text{CKM}} = \begin{pmatrix} c_{14} & 0 & 0 \\ -s_{14} s_{24} & c_{23} c_{24} & s_{23} c_{24} \\ -s_{14} c_{24} s_{34} e^{i\delta_{\text{CL}}} & -s_{23} c_{34} - c_{23} s_{24} s_{34} e^{i\delta_{\text{CL}}} & c_{23} c_{34} - s_{23} s_{24} s_{34} e^{i\delta_{\text{CL}}} \\ -s_{14} c_{24} c_{34} e^{i\delta_{\text{CL}}} & s_{23} s_{34} - c_{23} s_{24} c_{34} e^{i\delta_{\text{CL}}} & -c_{23} s_{34} - s_{23} s_{24} c_{34} e^{i\delta_{\text{CL}}} \end{pmatrix}$$

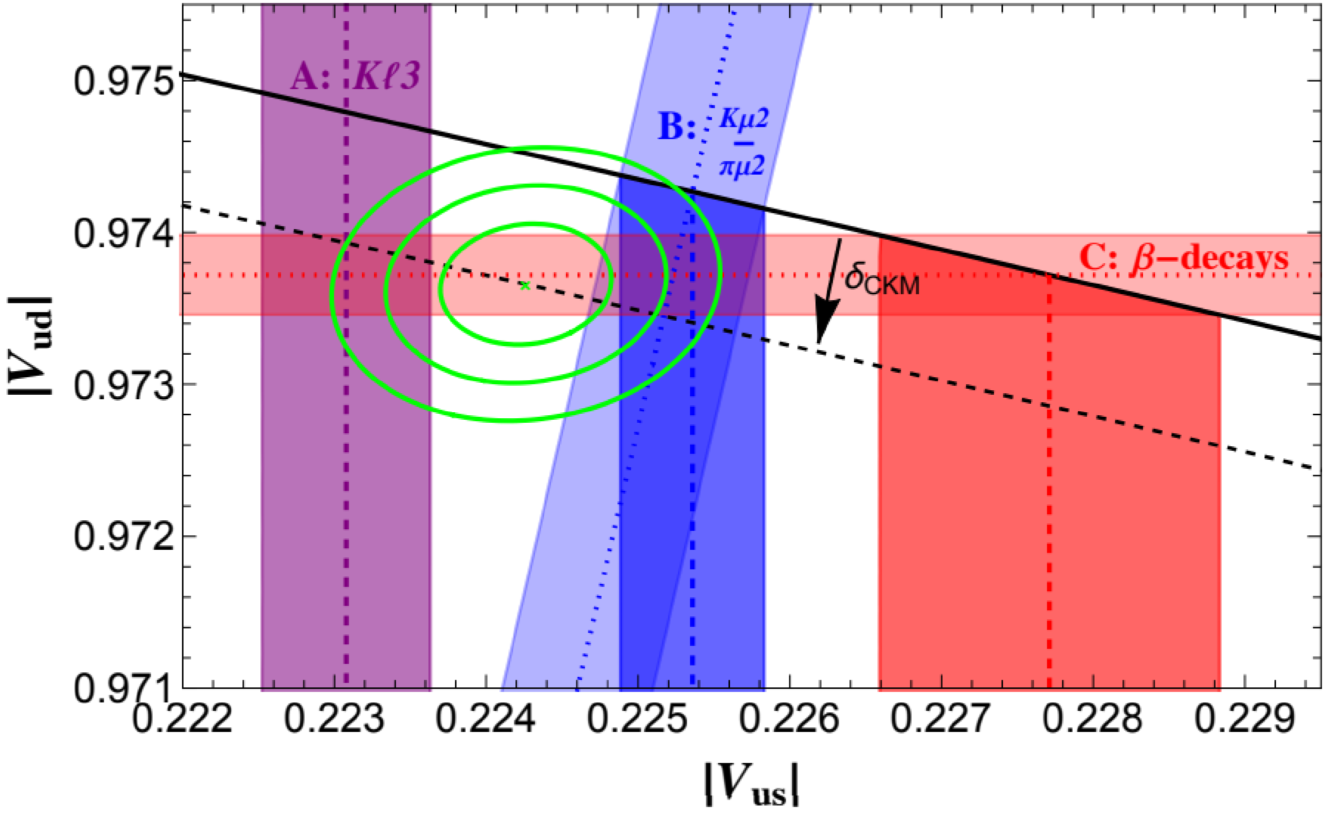
Summary/Conclusions

- Extension with VLQs can provide very simple solutions to the CAA.
- The s_{14}-dominance limit is particularly safe in relation to a large variety of EWPO and is related to an unusual decay pattern for the VLQ.
- The assumption of dominant decays of VLQs to the first generation has been largely overlooked but should be more seriously considered.
- The introduction of VLQs to the theory is capable of significantly enhancing CP violation in the quark sector and even achieve CP violation at very high energies. This is also a consequence of CKM non-unitarity.

Thank You!

CAA

B. Belfatto and S. Trifinopoulos [2302.14097]



From unitarity:

$|V_{us}|_A = 0.22308(55)$

$|V_{us}|_B = 0.22536(47)$

$|V_{us}|_C = 0.2277(11)$

$|V_{us}|_{A+B} = 0.22440(51)$ vs $|V_{us}|_C = 0.2277(11)$ — CAA1: $\sim 2.7\sigma$

$|V_{us}|_A = 0.22308(55)$ vs $|V_{us}|_B = 0.22536(47)$ — CAA2: $\sim 3.1\sigma$

$|V_{us}|_A = 0.22308(55)$ vs $|V_{us}|_C = 0.2277(11)$ — $\sim 3.7\sigma$

Best fit: $\delta_{\rm CKM} \approx 1.7 \times 10^{-3}$

$$\delta_{\rm CKM} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2$$

Neutral Meson Mixings

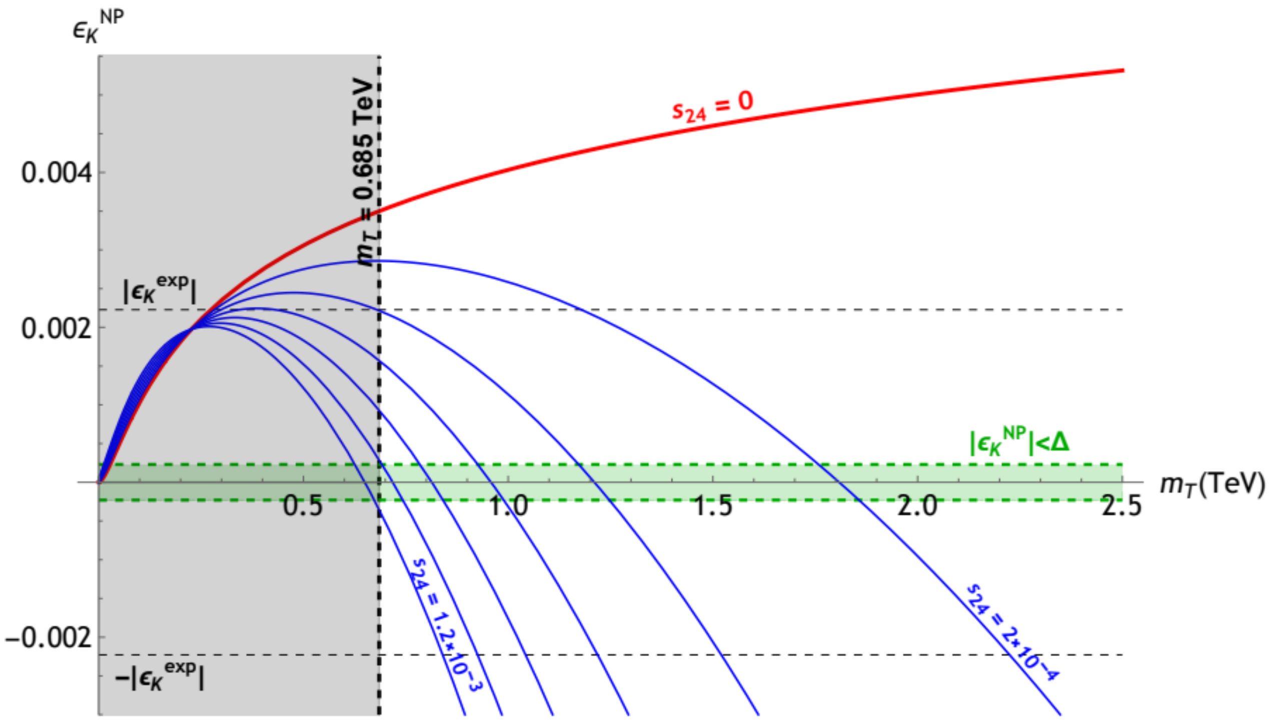
$$\Delta m_N^{\rm NP} \simeq \frac{G_F^2 M_W^2 m_N f_N^2 B_N}{6\pi^2} |2\eta_{cT}^N S_{cT}\lambda_c^N\lambda_T^N + 2\eta_{tT}^N S_{tT}\lambda_t^N\lambda_T^N + \eta_{TT}^N S_T(\lambda_T^N)^2|$$

$$|\epsilon_K^{\rm NP}| \simeq \frac{G_F^2 M_W^2 m_K f_K^2 B_K \kappa_\epsilon}{12\sqrt{2}\pi^2 \Delta m_K} \left|{\rm Im}\left[2\eta_{cT}^K S_{cT}\lambda_c^K\lambda_T^K + 2\eta_{tT}^K S_{tT}\lambda_t^K\lambda_T^K + \eta_{TT}^K S_T(\lambda_T^K)^2\right]\right| = \frac{G_F^2 M_W^2 m_K f_K^2 B_K \kappa_\epsilon}{12\sqrt{2}\pi^2 \Delta m_K}\mathcal{F}$$

$$\lambda_i^K = V_{id}V_{is}^*$$

$$\lambda_i^{B_d} = V_{id}V_{ib}^*$$

$$\lambda_i^{B_s} = V_{is}V_{ib}^*$$



- $\boldsymbol{s_{14} \sim \lambda^2, s_{24} = 0}$**:** $\mathcal{F} \approx 2\eta_{tT}^K S_{tT} s_{12} s_{14}^2 s_{13} s_{23} \sin\delta$
- $\boldsymbol{s_{14} \sim \lambda^2, s_{24} \sim \lambda^4}$**:**

$$\mathcal{F} \approx 2 s_{12} s_{14}^2 \left(\eta_{tT}^K S_{tT} s_{13} s_{23} \sin\delta - \eta_{TT}^K S_{TT} s_{14} s_{24} \sin\delta'\right)$$

$$\delta' = \delta_{24} - \delta_{14}$$

Kaon Decays

- $$\frac{\mathrm{Br}(K^+ \to \pi^+\overline{\nu}\nu)}{\mathrm{Br}(K^+ \to \pi^+\overline{\nu}\nu)_{\mathrm{SM}}} = \left|\frac{\lambda_c^K X^{\mathrm{NNL}}(x_c) + \lambda_t^K X(x_t) + \lambda_T^K X(x_T) + A_{ds}}{\lambda_c^K X^{\mathrm{NNL}}(x_c) + \lambda_t^K X(x_t)}\right|^2$$

$$A_{ds} = \sum_{i,j=c,t,T} V_{is}^* \left(F^u - I\right)_{ij} V_{jd} N(x_i, x_j) \simeq -\frac{x_T}{8} c_{14}^2 c_{24}^2 \lambda_T^K$$

$$N(x_i, x_j) = \frac{x_i x_j}{8}\left(\frac{\log x_i - \log x_j}{x_i - x_j}\right)$$

$$N(x_i, x_i) \equiv \lim_{x_j \to x_i} N(x_i, x_j) = \frac{x_i}{8}$$

- $$\left(\frac{\epsilon'}{\epsilon}\right)^{\mathrm{NP}} \simeq \tilde{F}(x_i)\mathrm{Im}(\lambda_T^K) \simeq -\tilde{F}(x_i) c_{12}^2 s_{14} s_{24} \sin\delta'$$

$$\tilde{F}(x_T) \equiv F(x_T) - \tfrac{x_T}{8}\left(P_X + P_Y + P_Z\right)$$

$$F(x_i) = P_0 + P_X X(x_i) + P_Y Y(x_i) + P_Z Z(x_i) + P_E E(x_i)$$