Supergeometry in Effective QFTs

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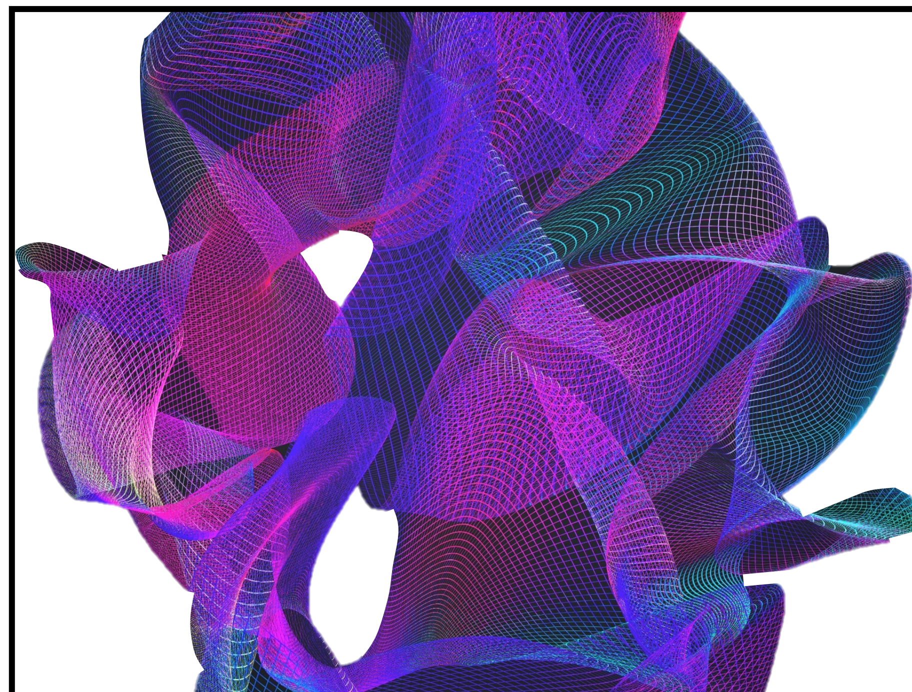
Based on arXiv:2307.01126
with Prof Apostolos Pilaftsis



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Motivation

<u>Disclaimer</u>: Supergeometry $\neq$ supersymmetry

- A theory with fermions and bosons with no extra symmetry
- Use VDW formalism [Vilkovisky (1984), DeWitt (1985)]
- Applications to geometric EFTs [Alonso, Jenkins, Manohar (2016), Cohen, Craig, Sutherland (2021), Talbert (2023), Assi, Helset, Manohar, Pagès, Shen (2023) ...]
- Solving the frame-dependence problem in cosmic inflation

[Burns, Karamitsos, Pilaftsis (2016), Falls, Herrero-Valea (2019), Finn, Karamitsos, Pilaftsis (2020) ..]

The Set-Up

- Field-space supermanifold of dimension (N|8M) in 4D spacetime

[DeWitt (2012)]

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$$\mathbf{\Phi} \;\equiv\; \{\Phi^{\alpha}\} \;=\; \left(\phi^{A}\,,\, \psi^{X}\,,\, \overline{\psi}^{Y,\mathsf{T}}\right)^{\mathsf{T}}$$

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$$\mathcal{L} \;=\; \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi^\alpha\;{}_\alpha k_\beta(\boldsymbol{\Phi})\,\partial_\nu\Phi^\beta \;+\; \frac{i}{2}\zeta^\mu_\alpha(\boldsymbol{\Phi})\,\partial_\mu\Phi^\alpha \;-\; U(\boldsymbol{\Phi})$$

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where

$$\overleftarrow{\Sigma}_\mu = \frac{1}{D}\begin{pmatrix} \frac{\overleftarrow{\partial}}{\partial\gamma^\mu} & 0 \\ 0 & \Gamma_\mu \end{pmatrix}$$

The Set-Up (continued)

- Endow supermanifold with metric

$$ {}_{\alpha}G_{\beta} = \left({}_{\alpha}G_{\beta}\right)^{\mathsf{sT}} $$

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- Global metric found from vielbeins and local metric

[Finn, Karamitsos, Pilaftsis (2021), VG, Finn, Karamitsos, Pilaftsis (2022)]

$$
{}_{\alpha}G_{\beta} = {}_{\alpha}e^{a} \; {}_{a}H_{b} \; {}^{b}e^{\mathsf{sT}}_{\beta}
$$

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$$ {}_\alpha G_\beta = {}_\alpha e^a \, {}_a H_b \, {}^b e^{\mathsf{sT}}_\beta $$

$$ {}_a H_b \equiv \begin{pmatrix} \mathbf{1}_N & 0 & 0 \\ 0 & 0 & \mathbf{1}_{4M} \\ 0 & -\mathbf{1}_{4M} & 0 \end{pmatrix} $$

No-Go Theorem

- Flat field space can **always** be reparametrized into canonical Cartesian form

$$\mathcal{L} = -\frac{1}{2}\boldsymbol{h}(\phi)\,\overline{\boldsymbol{\psi}}\gamma^{\mu}\boldsymbol{\psi}(\partial_{\mu}\phi) + \frac{i}{2}\boldsymbol{g}(\phi)\left[\overline{\boldsymbol{\psi}}\gamma^{\mu}(\partial_{\mu}\boldsymbol{\psi}) - (\partial_{\mu}\overline{\boldsymbol{\psi}})\gamma^{\mu}\boldsymbol{\psi}\right]$$

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- ϕ acts as external parameter in the fermionic sector

No-Go Theorem (continued)

- Flatness confirmed by vanishing of Riemann tensor

$$
{}_{\alpha}G_{\beta} = \begin{pmatrix} k - \frac{1}{2}\overline{\boldsymbol{\psi}}\left(\boldsymbol{g}' - i\boldsymbol{h}\right)\boldsymbol{g}^{-1}\left(\boldsymbol{g}' + i\boldsymbol{h}\right)\boldsymbol{\psi} & -\frac{1}{2}\overline{\boldsymbol{\psi}}\left(\boldsymbol{g}' - i\boldsymbol{h}\right) & \frac{1}{2}\boldsymbol{\psi}^{\mathsf{T}}\left(\boldsymbol{g}'^{\mathsf{T}} + i\boldsymbol{h}^{\mathsf{T}}\right) \\ \frac{1}{2}\left(\boldsymbol{g}'^{\mathsf{T}} - i\boldsymbol{h}^{\mathsf{T}}\right)\overline{\boldsymbol{\psi}}^{\mathsf{T}} & 0 & \boldsymbol{g}^{\mathsf{T}}1_4 \\ -\frac{1}{2}\left(\boldsymbol{g}' + i\boldsymbol{h}\right)\boldsymbol{\psi} & -\boldsymbol{g}1_4 & 0 \end{pmatrix}
$$

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$$
\dashrightarrow \quad R^{\alpha}{}_{\beta\gamma\delta} = 0
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[VG, Pilaftsis (2023)]

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$$

$\dashrightarrow$ $R^{\alpha}{}_{\beta\gamma\delta} \;=\; 0$ [VG, Pilaftsis (2023)]

Take home message:

Non-zero fermionic curvature effects cannot be generated if ζ^{μ}_{α} depends **linearly** on ψ and $\overline{\psi}$

Model I

- A 2D factorizable model

$$\mathcal{L}_{\mathrm{I}} = \frac{1}{2}k\,(\partial_\mu\phi)\,(\partial^\mu\phi) + \frac{i}{2}\left(g_0 + g_1\overline{\psi}\psi\right)\left[\overline{\psi}\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\gamma^\mu\psi\right]$$

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- FS metric

$$\boldsymbol{G} = \begin{pmatrix} k + b^{\mathsf{T}}(d^{-1})^{\mathsf{T}}a^{\mathsf{T}} - a\,d^{-1}\,b & -a & b^{\mathsf{T}} \\ a^{\mathsf{T}} & 0 & d^{\mathsf{T}} \\ -b & -d & 0 \end{pmatrix}$$

$$a = \frac{1}{2}\overline{\psi}\left(g_0' + g_1'\overline{\psi}\psi\right)$$

$$b = \frac{1}{2}\left(g_0' + g_1'\overline{\psi}\psi\right)\psi$$

$$d = \left(g_0 + g_1\overline{\psi}\psi\right)1_2 + g_1\psi\overline{\psi}$$

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- Ricci scalar is fermion dependent

$$R = \frac{4g_1}{g_0^2} + \left(\frac{2g_1g_0'g_1'}{g_0^3k} - \frac{2g_1^2g_0'^2}{g_0^4k} - \frac{g_1'^2}{2g_0^2k}\right)(\overline{\psi}\psi)^2$$

Model II

- A 4D non-factorizable model

$$\mathcal{L}_{\mathrm{II}} = \frac{i}{2}\left[\overline{\psi}\gamma^{\mu}(\partial_{\mu}\psi) - (\partial_{\mu}\overline{\psi})\gamma^{\mu}\psi\right] + \frac{i}{2}\overline{\psi}\gamma^{\mu}\psi\left[\overline{\psi}(\partial_{\mu}\psi) - (\partial_{\mu}\overline{\psi})\psi\right]$$

Model II

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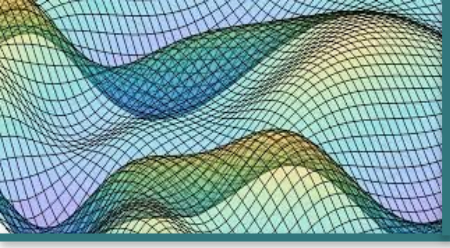
- FS metric

$$\boldsymbol{G} \equiv \{_{\alpha}G_{\beta}\} = \begin{pmatrix} 0 & d^{\mathsf{T}} \\ -d & 0 \end{pmatrix} \qquad d = 1_4 + \frac{1}{4}(\overline{\psi}\gamma^{\mu}\psi)\,\gamma_{\mu} + \frac{1}{4}\gamma^{\mu}\,\psi\overline{\psi}\,\gamma_{\mu}$$



Model II

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$$\mathcal{L}_{\rm II} = \frac{i}{2}\left[\overline{\psi}\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\gamma^\mu\psi\right] + \frac{i}{2}\overline{\psi}\gamma^\mu\psi\left[\overline{\psi}(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\psi\right]$$

- FS metric

$$\boldsymbol{G} \equiv \{_\alpha G_\beta\} = \begin{pmatrix} 0 & d^{\mathsf{T}} \\ -d & 0 \end{pmatrix} \qquad d = 1_4 + \frac{1}{4}(\overline{\psi}\gamma^\mu\psi)\,\gamma_\mu + \frac{1}{4}\gamma^\mu\,\psi\overline{\psi}\,\gamma_\mu$$

- Richer structure of Ricci scalar

$$R = -8 + 2(\overline{\psi}\psi) + \frac{23}{8}(\overline{\psi}\psi)^2 + \frac{9}{8}(\overline{\psi}\gamma_5\psi)^2 + \frac{5}{4}(\overline{\psi}\gamma_\mu\psi)(\overline{\psi}\gamma^\mu\psi) - \frac{29}{12}(\overline{\psi}\psi)^3 + \frac{7}{16}(\overline{\psi}\psi)^4$$

Supervertices

- Mixed ST and FS rank-2 tensor

$$ {}_{\alpha}\lambda^{\mu}_{\beta} \equiv \frac{1}{2}\left({}_{\alpha,}\zeta^{\mu}_{\beta} - (-1)^{\alpha+\beta+\alpha\beta}\, {}_{\beta,}\zeta^{\mu}_{\alpha} \right) $$

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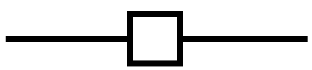
- Covariant inverse superpropagator



$$ S_{;\hat{\alpha}\hat{\beta}}\Big|_{\partial_{\mu}\Phi=0} \;=\; (-1)^{\alpha}\; {}_{\alpha}\lambda^{\mu}_{\beta}\, p^{\beta}_{\mu}\; \delta(p^{\alpha}+p^{\beta}) $$

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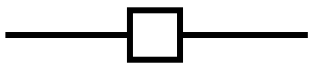
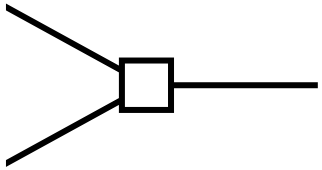
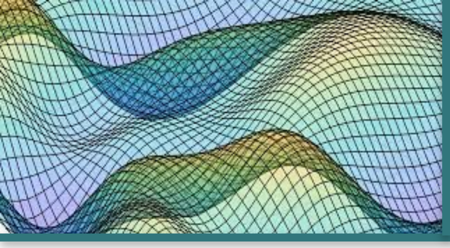


$$ S_{;\hat{\alpha}\hat{\beta}}\Big|_{\partial_\mu \Phi = 0} \;=\; (-1)^{\alpha}\; {}_{\alpha}\lambda^{\mu}_{\beta}\, p^{\beta}_{\mu}\;\; \delta(p^{\alpha} + p^{\beta}) $$

- Covariant three-vertex



$$ S_{;\hat{\alpha}\hat{\beta}\hat{\gamma}}\Big|_{\partial_\mu \Phi = 0} \;=\; \Big((-1)^{\alpha}\, {}_{\alpha}\lambda^{\mu}_{\beta;\gamma}\, p^{\beta}_{\mu} \;+\; (-1)^{\alpha+\beta\gamma}\, {}_{\alpha}\lambda^{\mu}_{\gamma;\beta}\, p^{\gamma}_{\mu} \Big)\, \delta(p^{\alpha} + p^{\beta} + p^{\gamma}) $$



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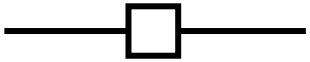
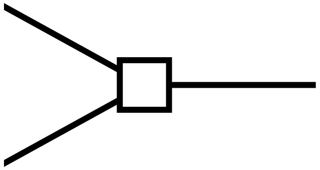


$$ S_{;\hat\alpha\hat\beta}\Big|_{\partial_\mu\Phi=0} = (-1)^\alpha\,{}_\alpha\lambda^\mu_\beta\,p^\beta_\mu\;\delta(p^\alpha+p^\beta) $$

- Covariant three-vertex

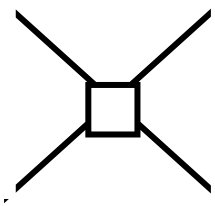


$$ S_{;\hat\alpha\hat\beta\hat\gamma}\Big|_{\partial_\mu\Phi=0} = \Big((-1)^\alpha\,{}_\alpha\lambda^\mu_{\beta;\gamma}\,p^\beta_\mu + (-1)^{\alpha+\beta\gamma}\,{}_\alpha\lambda^\mu_{\gamma;\beta}\,p^\gamma_\mu\Big)\,\delta(p^\alpha+p^\beta+p^\gamma) $$

Note: $\neq 0$ unlike purely bosonic

Supervertices (continued)

- Covariant four-vertex



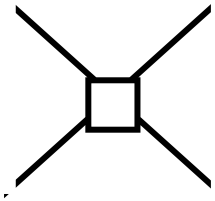
$$S_{;\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}\Big|_{\partial_\mu \Phi = 0} = \Big((-1)^{\alpha}{}_{\alpha}\lambda^{\mu}_{\rho}\, R^{\rho}{}_{\beta\gamma\delta}\, p^{\delta}_{\mu} \;+\; (-1)^{\alpha}{}_{\alpha}\lambda^{\mu}_{\beta;\gamma\delta}\, p^{\beta}_{\mu}$$

$$+\; (-1)^{\alpha+\beta\gamma}{}_{\alpha}\lambda^{\mu}_{\gamma;\beta\delta}\, p^{\gamma}_{\mu} \;+\; (-1)^{\alpha+\delta(\beta+\gamma)}{}_{\alpha}\lambda^{\mu}_{\delta;\beta\gamma}\, p^{\delta}_{\mu} \Big)$$

$$\delta(p^{\alpha} + p^{\beta} + p^{\gamma} + p^{\delta})$$

Supervertices (continued)

- Covariant four-vertex



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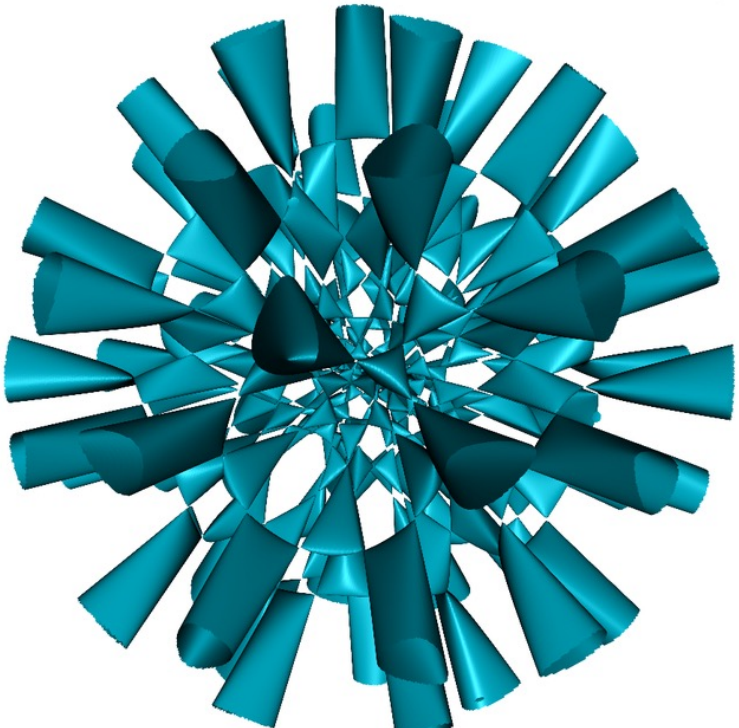
- Scalar contribution is additive

Summary and Outlook

- Fermionic curvature arises from non-linearity
- Unlike supergravity, curvature not real-valued
- Derived generalised expressions for covariant scalar-fermion vertices

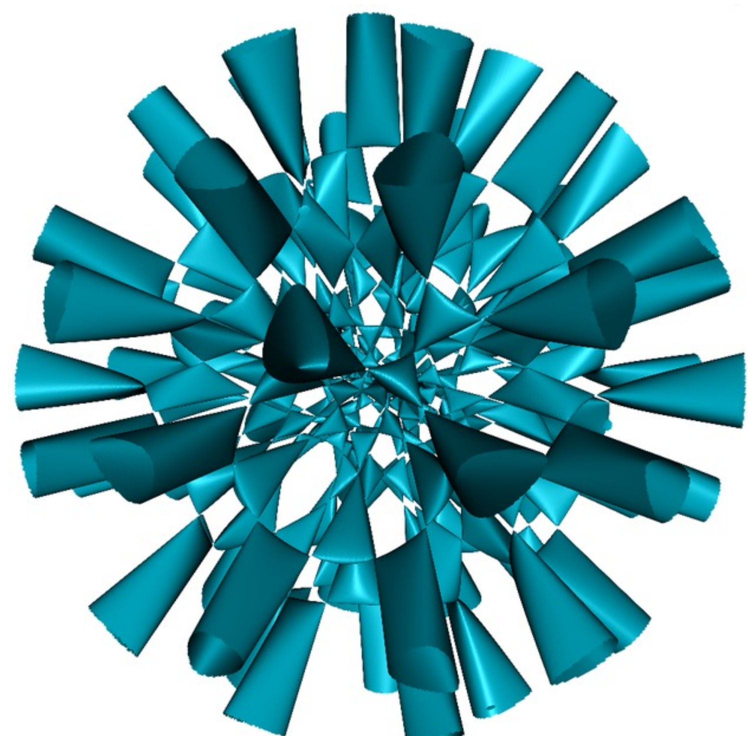


Summary and Outlook

- ❑ Fermionic curvature arises from non-linearity
- ❑ Unlike supergravity, curvature not real-valued
- ❑ Derived generalised expressions for covariant scalar-fermion vertices

What next?

- ❑ Compute higher loop effective actions
- ❑ Add symmetries
- ❑ Compute amplitudes



Thank you!

$$R_I = \frac{4g_1}{g_0^2} + \left(\frac{2g_1 g_0' g_1'}{g_0^3 k} - \frac{2g_1^2 g_0'^2}{g_0^4 k} - \frac{g_1'^2}{2g_0^2 k} \right) (\overline{\psi}\psi)^2$$

$$R_{II} = -8 + 2(\overline{\psi}\psi) + \frac{23}{8}(\overline{\psi}\psi)^2 + \frac{9}{8}(\overline{\psi}\gamma_5\psi)^2 + \frac{5}{4}(\overline{\psi}\gamma_\mu\psi)(\overline{\psi}\gamma^\mu\psi) - \frac{29}{12}(\overline{\psi}\psi)^3 + \frac{7}{16}(\overline{\psi}\psi)^4$$