

APPLICATIONS OF THE TUNNELING POTENTIAL FORMALISM

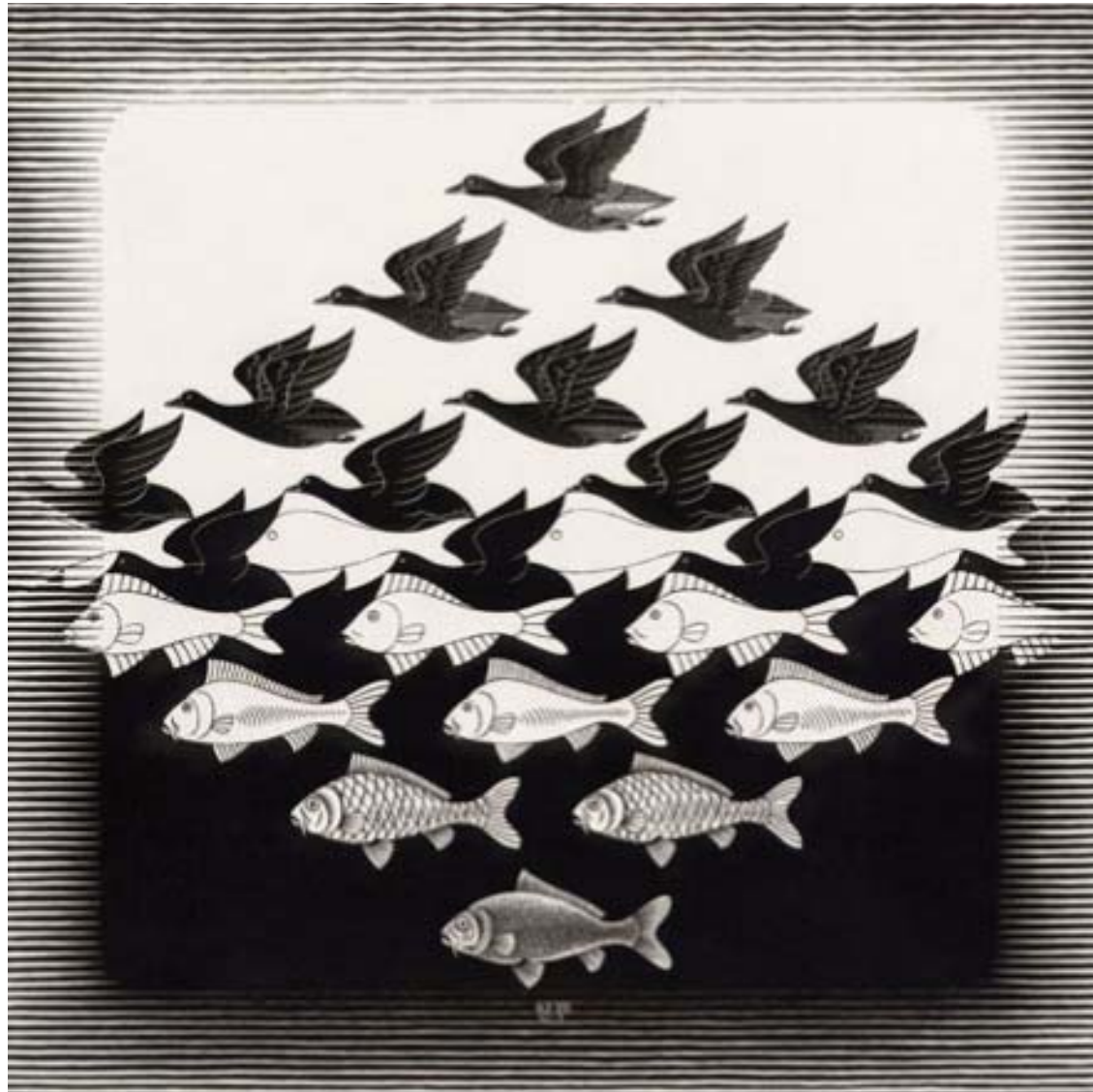
6/9/2023
Corfu



J.R.Espinosa

1. TUNNELING POTENTIAL FORMALISM

JRE'1805

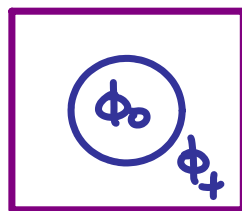
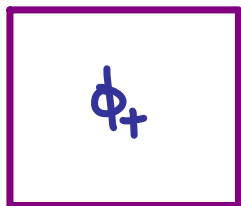
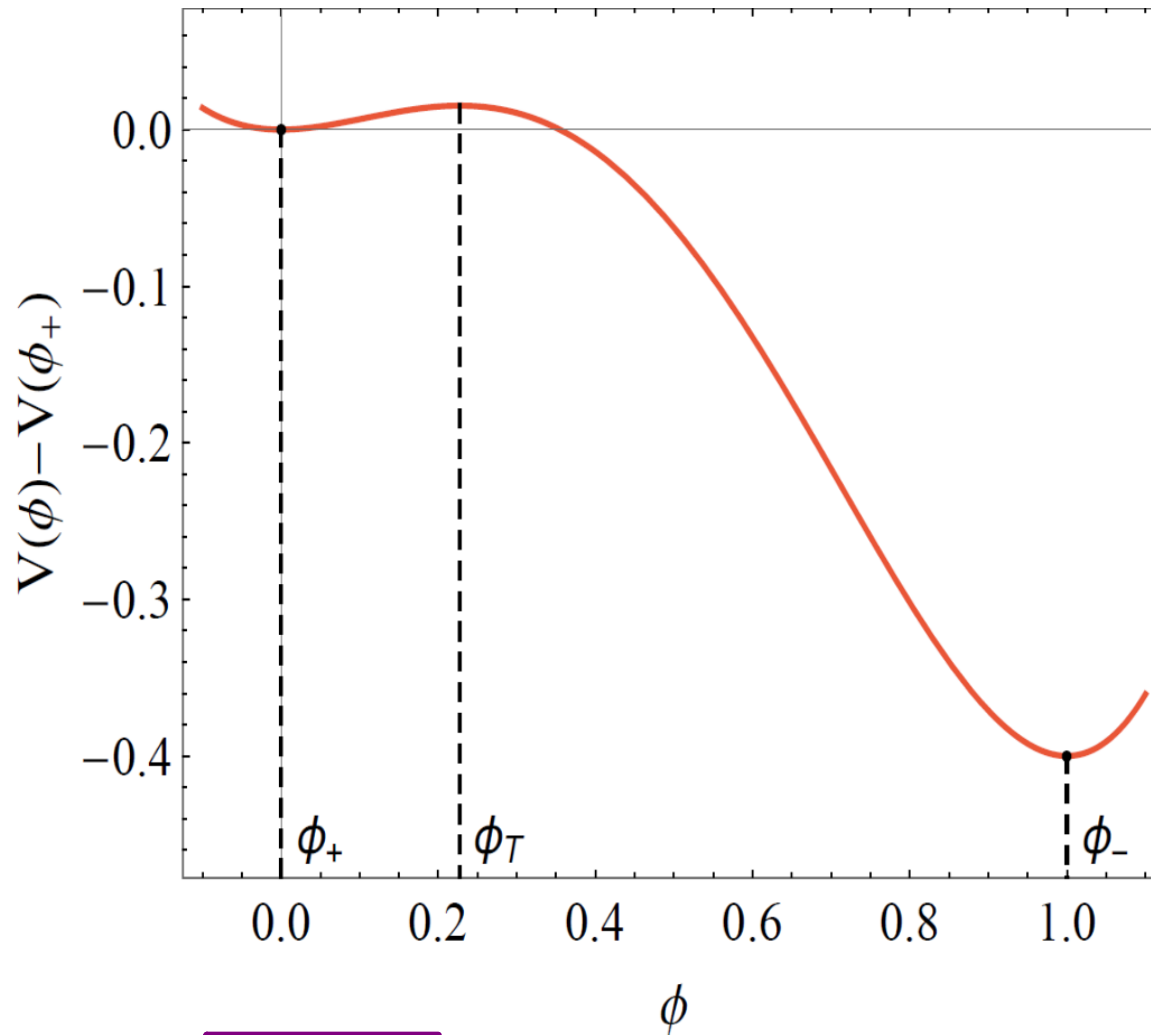


WHY VACUUM DECAY ?

- BSM/SM in false vacuum
- Early universe phase transitions
- Vacuum decay triggered by inflation
- How the string vacuum landscape is populated

⋮

THE PROBLEM



$$I/N = A e^{-S}$$

calculate this

EUCLIDEAN FORMALISM

P. Frampton '76, Coleman '77

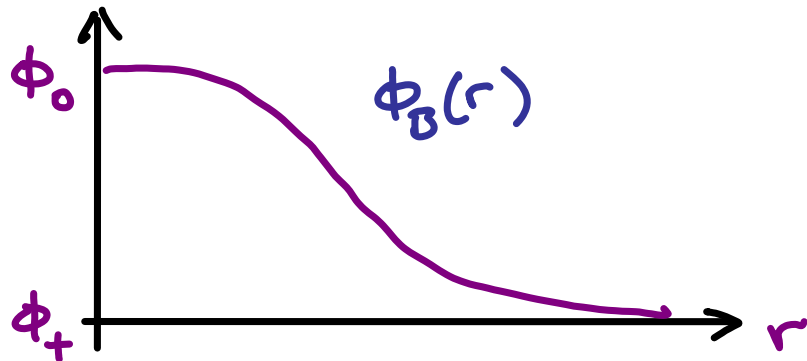
Euclidean bounce $\phi_B(r)$, $O(4)$ -sym, extremizes

$$S_E = 2\pi^2 \int_0^\infty dr r^3 \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right]$$



$$\ddot{\phi} + \frac{3}{r} \dot{\phi} = \frac{\partial V}{\partial \phi}$$

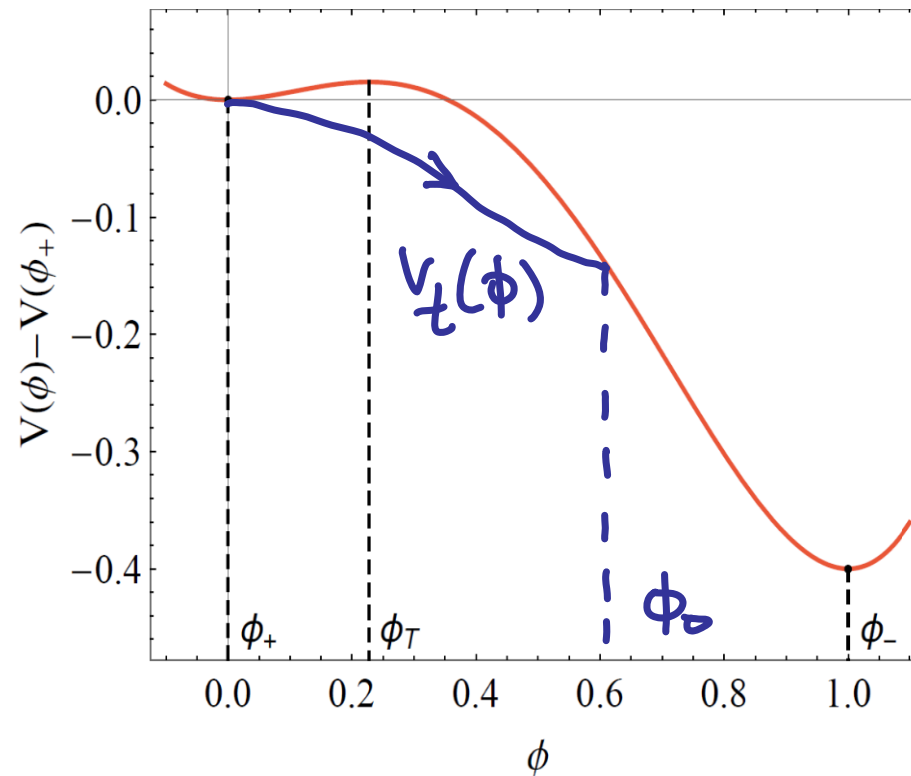
with $\phi(0) = \phi_0$ $\dot{\phi}(0) = 0$ $\phi(\infty) = \phi_+$



$$S = \Delta S_E \equiv S_E[\phi_B] - S_E[\phi_+]$$

TUNNELING POTENTIAL FORMALISM

JRE '1805



$$S[V_t] = 54\pi^2 \int_{\phi_+}^{\phi_0} \frac{(V - V_t)^2}{(-V_t')^3} d\phi$$

$$S = \text{Min}_{V_t} S[V_t]$$

$\phi_0 = \phi(0)$, Euclidean

DE-EUCLIDEANIZE

JRE'1805

⇒ Get rid of Euclidean quantities in terms of v & v_t ones

LINK: $v_t = v - \frac{1}{2} \dot{\phi}^2$

⇒ $\dot{\phi} = -\sqrt{2(v-v_t)}$ $\ddot{\phi} = \frac{d}{d\phi}(-\sqrt{2(v-v_t)}) \dot{\phi} = v' - v_t'$

EOM: $\ddot{\phi} + \frac{3}{r} \dot{\phi} = v'$ ⇒ $r = \frac{3\sqrt{2(v-v_t)}}{-v_t'}$

$\frac{d}{dr}(\dots)$ ⇒ $(4v_t' - 3v')v_t' + 6(v-v_t)v_t'' = 0$

Action gives this EOM for v_t

PROPERTIES & APPS OF V_t -FORM.

JRE'1805

★ Field can be a distraction.

V_t on same footing as V

★ V_t monotonic \Rightarrow Easy to approximate

★ $S[V_t]$ minimized ($\phi_0(r)$ saddle-point)

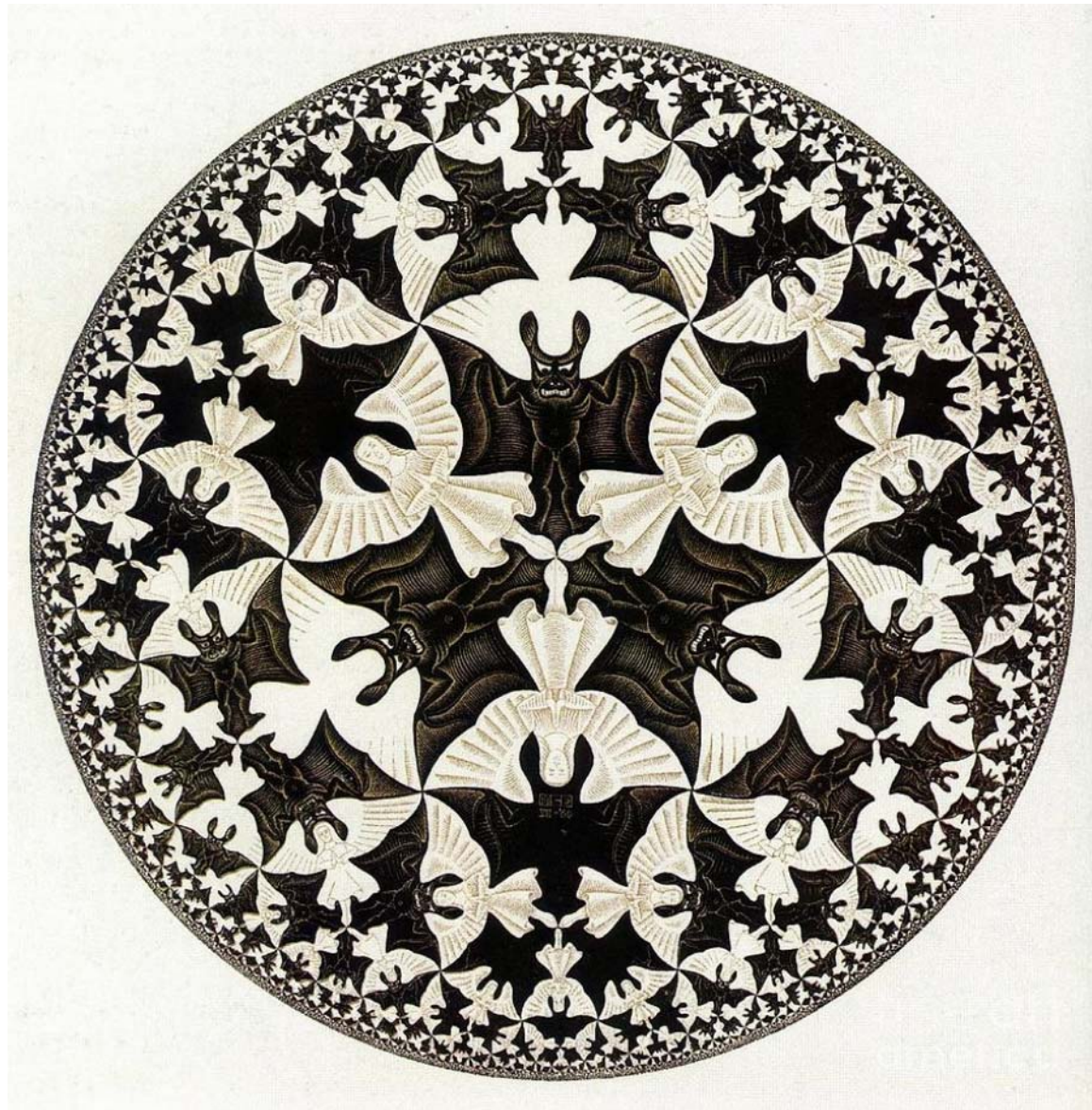
\Rightarrow Good for numerics

\Rightarrow Useful for multi-field case JRE, Konstandin'1811

★ Generalizes to any dimension

$d=3$ applicable to finite T phase transitions

2. GRAVITY CORRECTIONS



JRE '1808
JRE, Huertas,
Fortin '2106
JRE, Fortin '2211

WHY ?

Gravity relevant if tunneling involves

$$\Delta\phi \lesssim m_p \quad \text{or} \quad \Delta V \lesssim m_p^4 \quad \text{or} \quad |V_+| \lesssim m_p^4$$

Note $\Delta\phi \lesssim m_p$ already in SM...

When relevant :

Need to include reaction of the metric.

Gravity can cause *qualitative* changes

EUCLIDEAN FORMALISM W/GRAVITY

Coleman, De Luccia '80

Assuming $O(4)$

Euclidean bounce $\phi_B(\xi)$ and metric function $f(\xi)$

$$ds^2 = d\xi^2 + f^2(\xi) d\Omega_3^2$$

↑ line-element S^3

$$\Rightarrow \ddot{\phi} + \frac{3\dot{f}}{f} \dot{\phi} = \frac{\partial V}{\partial \phi}$$

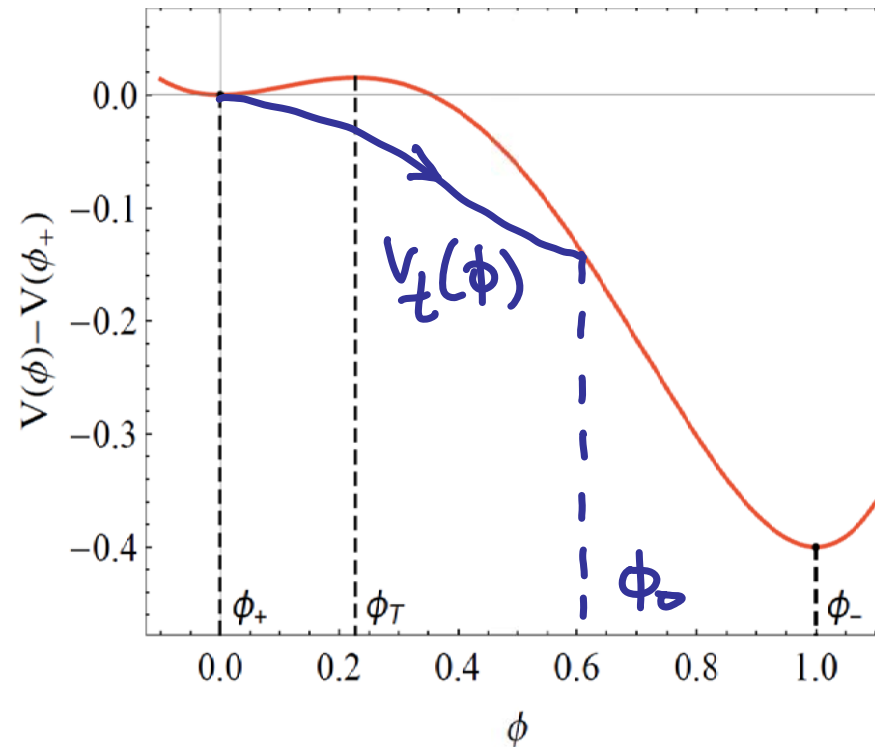
$$\dot{f}^2 = 1 + \frac{\kappa f^2}{3} \left(\frac{1}{2} \dot{\phi}^2 - V \right)$$

$$S = \Delta S_E = S_E[\phi_B, f_B] - S_E[\phi_+, f_+]$$

$$\kappa \equiv 1/m_P^2$$

V_t FORMALISM w/ GRAVITY

JRE'1808



$$S[V_t] = \frac{6\pi^2}{k^2} \int_{\phi_+}^{\phi_0} \frac{(D + V_t')^2}{D V_t^2} d\phi = \Delta S_E$$

with

$$D = \sqrt{(V_t')^2 + 6k(V - V_t)V_t} \quad k \equiv 1/m_p^2$$

DE-EUCLIDEANIZE

JRE'1808

⇒ Get rid of Eucl. quant. in terms of v & v_t

$$v_t = v - \frac{1}{2} \dot{\phi}^2$$

$$\Rightarrow \dot{\phi} = -\sqrt{2(v-v_t)} \quad \ddot{\phi} = \frac{d}{d\phi} \left(-\sqrt{2(v-v_t)} \right) \dot{\phi} = v' - v_t'$$

$$\text{EOMs:} \quad \ddot{\phi} + \frac{3\dot{\phi}}{\rho} \dot{\phi} = \frac{\partial V}{\partial \phi} \quad \dot{\phi}^2 = 1 + \frac{\kappa \rho^2}{3} \left(\frac{1}{2} \dot{\phi}^2 - v \right)$$

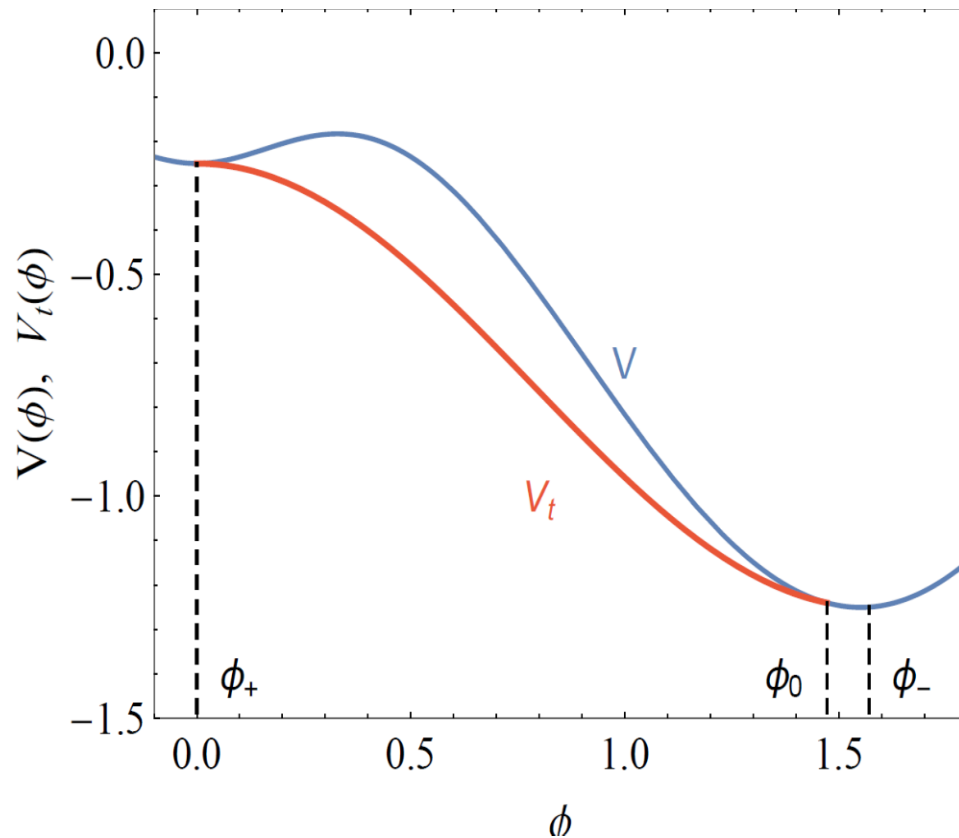
$$\Rightarrow \rho = \frac{3\sqrt{2(v-v_t)}}{D}$$

$$\frac{d}{d\xi}(\dots) \Rightarrow 0 = (4v_t' - 3v') v_t' + 6(v - v_t) \left[v_t'' + \kappa(3v - 2v_t) \right]$$

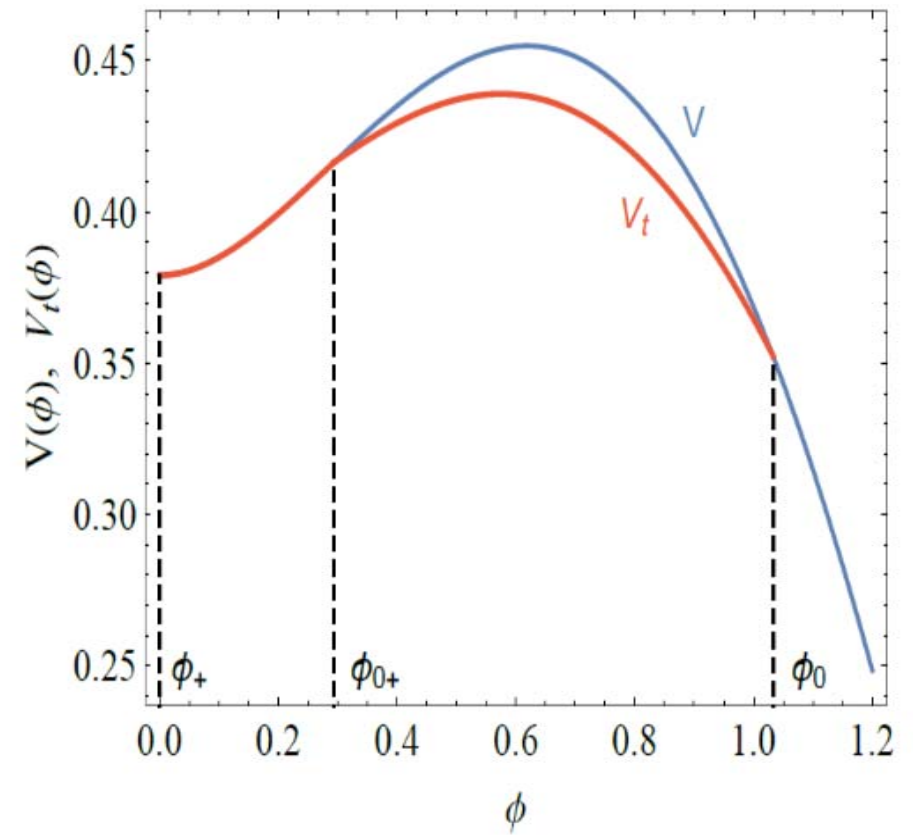
Action gives this EOM for v_t

PROPERTIES & APPS OF V_t -FORM.

Universal formula, valid for AdS, Minkowski or dS



Minkowski or AdS



dS

(Th. int: Brown, Weinberg, 0706)

PROPERTIES & APPS OF V_t -FORM.

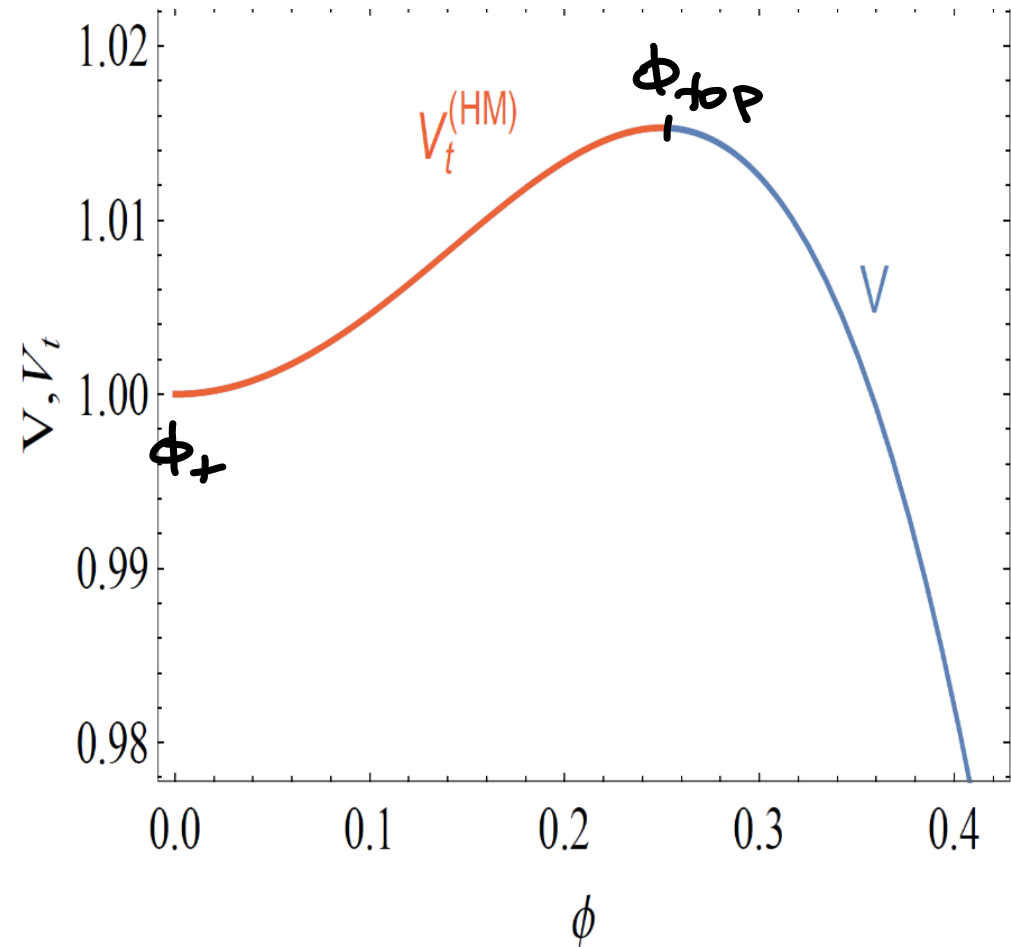
For dS, $V_+ \uparrow \Leftrightarrow$ no CdL bounce

Hawking-Moss decay '82

with rate

$$S_{HM} = \frac{24\pi^2}{\kappa^2} \left(\frac{1}{V_+} - \frac{1}{V_{top}} \right)$$

$S[V_t]$ reproduces this ✓



PROPERTIES & APPS OF V_t -FORM.

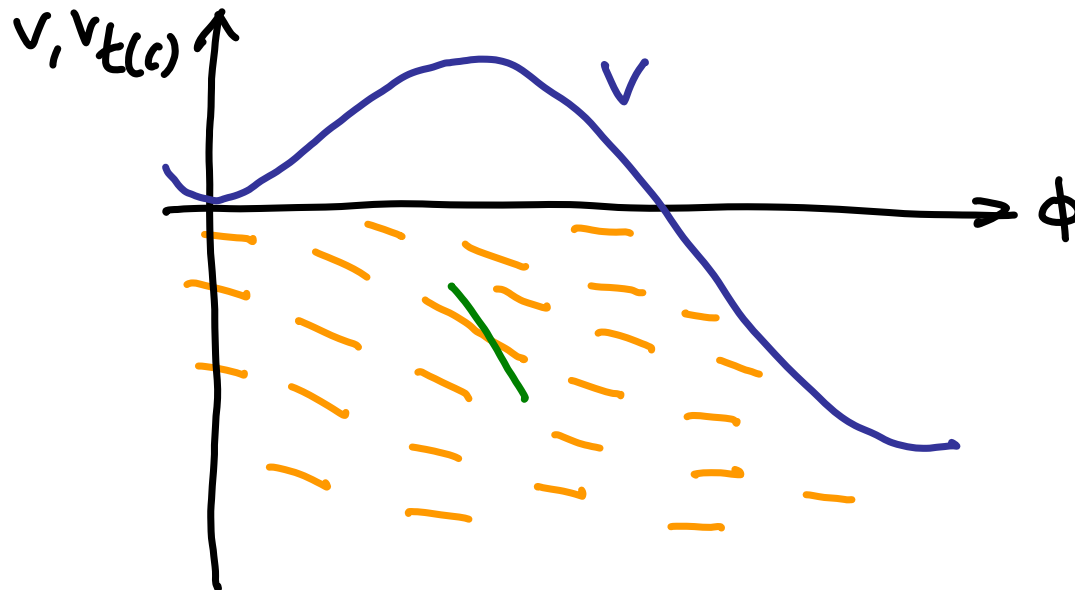
For AdS, Minkowski ($v_+ \leq 0$)

gravitational quenching of decay possible

Coleman, De Luccia '80

$$D = \sqrt{\underbrace{(V_t')^2}_{\hat{0}} + 6\kappa \underbrace{(V - V_t) V_t}_{\hat{0}}} \quad \text{must be real} \quad \curvearrowright$$

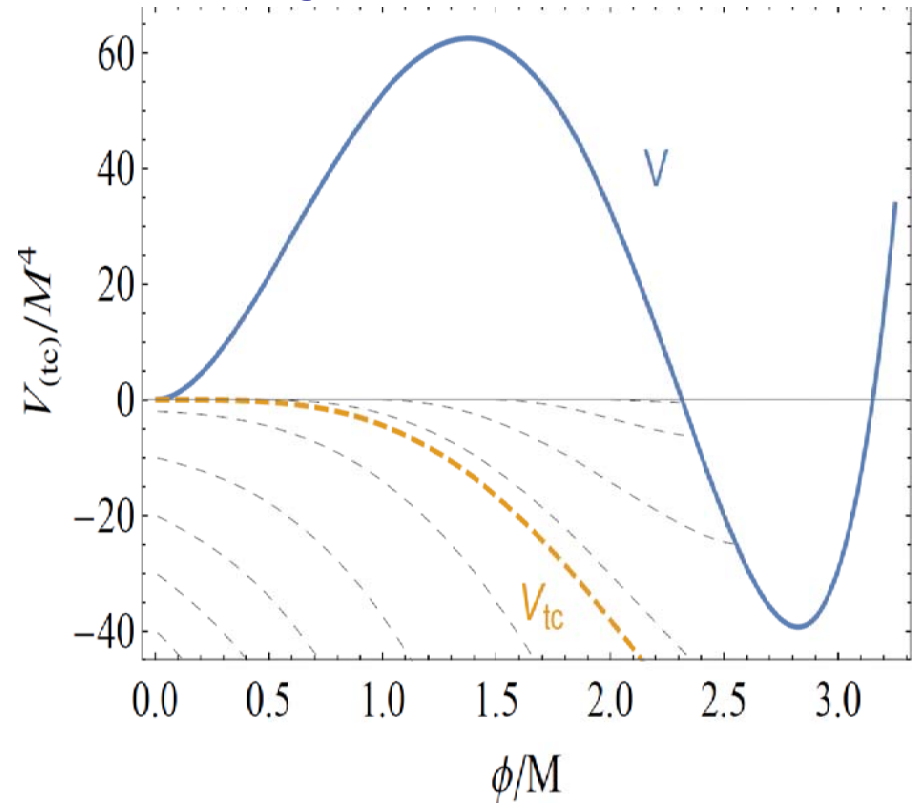
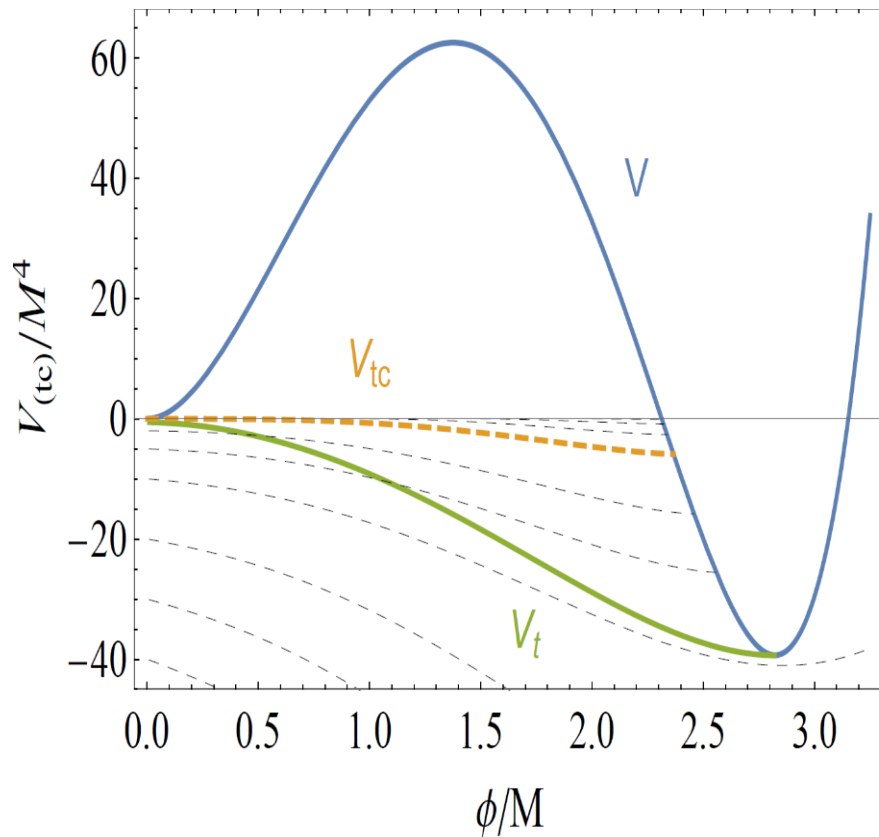
$$|V_t'| > |V_{tc}'| \equiv \sqrt{6\kappa (V - V_{tc}) (-V_{tc})}$$



GRAVITATIONAL QUENCHING

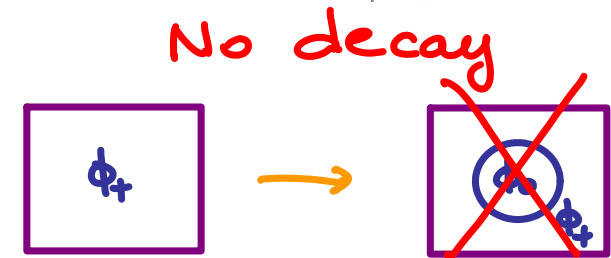
Weak grav. effects \longrightarrow

Strong grav. effects



Decays

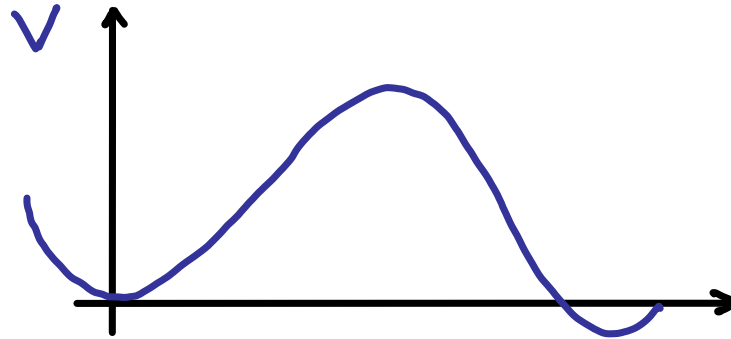
JRE '2005



NO EXIT

Positive Energy Theorem

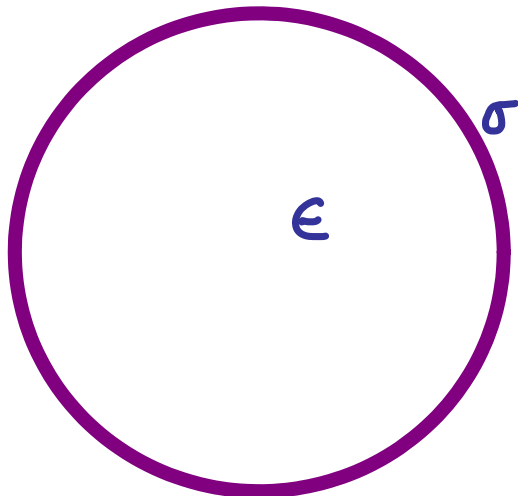
GRAVITATIONAL QUENCHING



$$V_- = -\epsilon$$

No gravity

$$E \sim -\epsilon R^3 + \sigma R^2 = 0$$



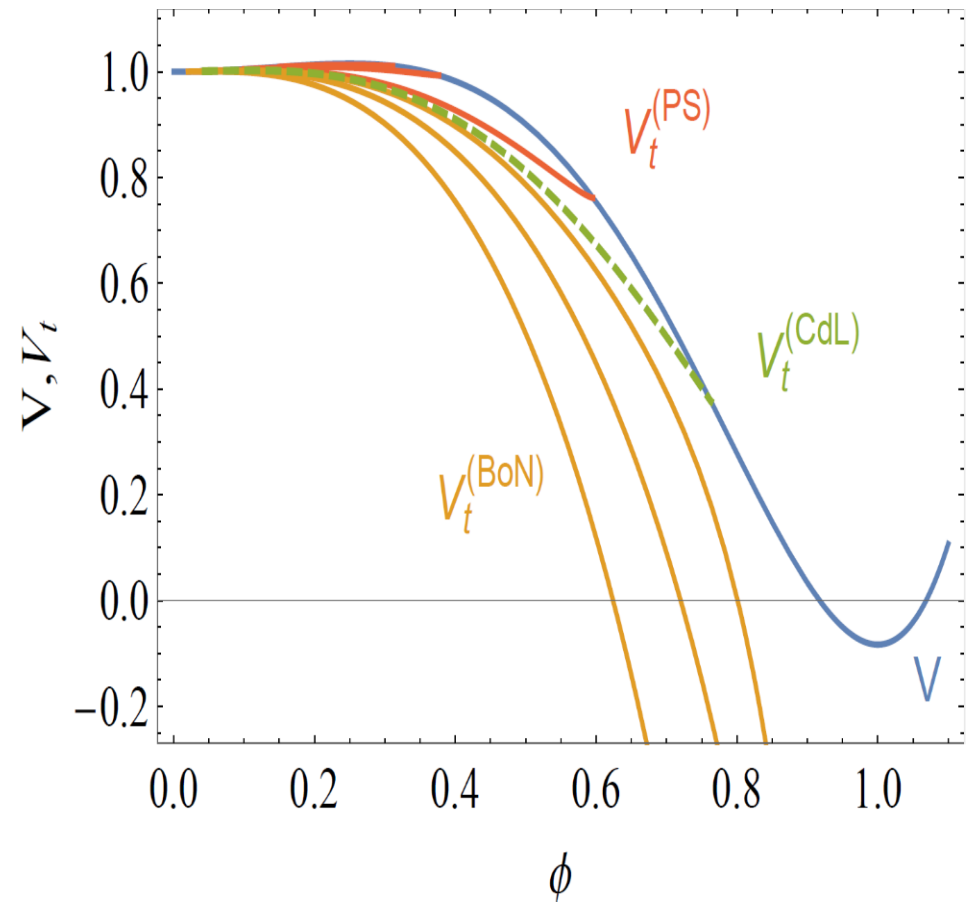
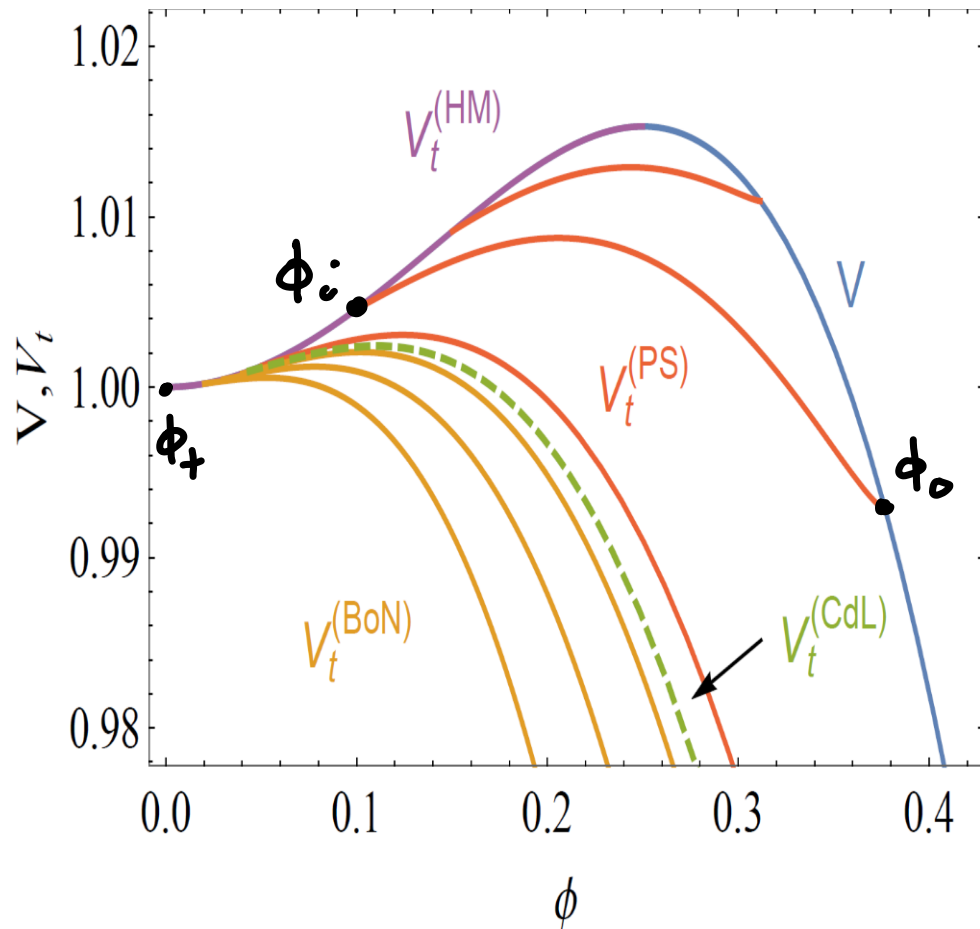
large $R \sim \sigma/\epsilon$



$E=0$ not guaranteed

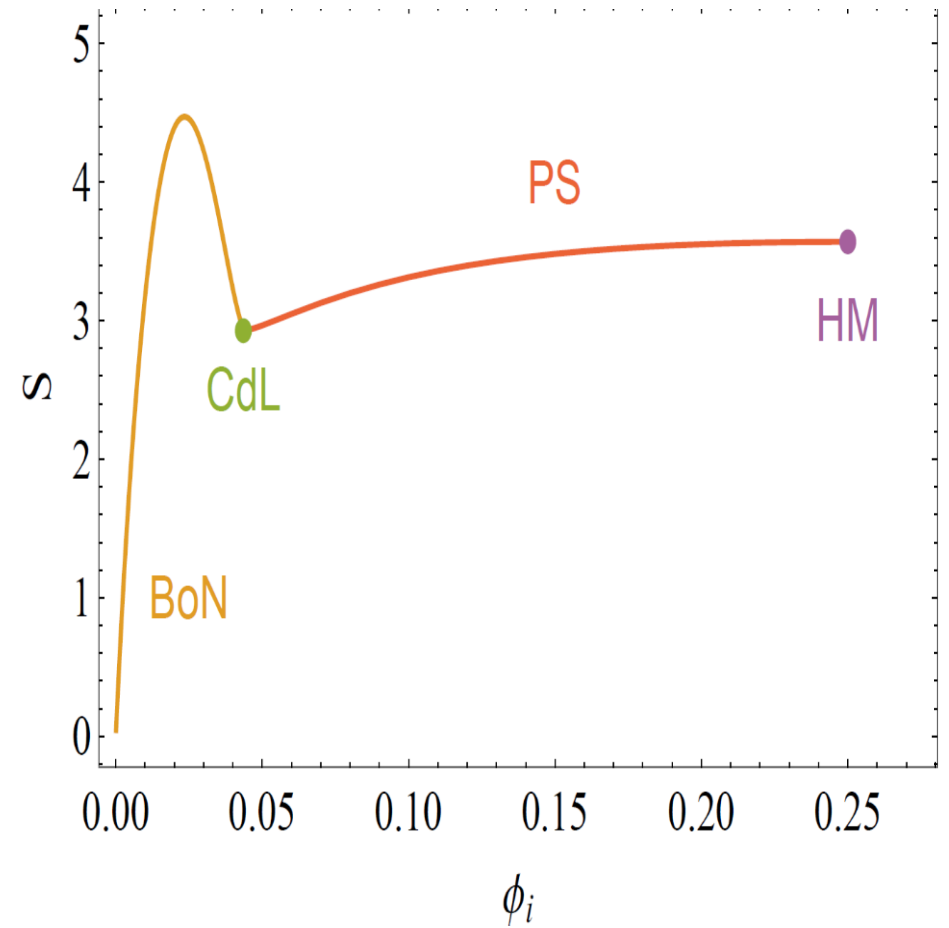
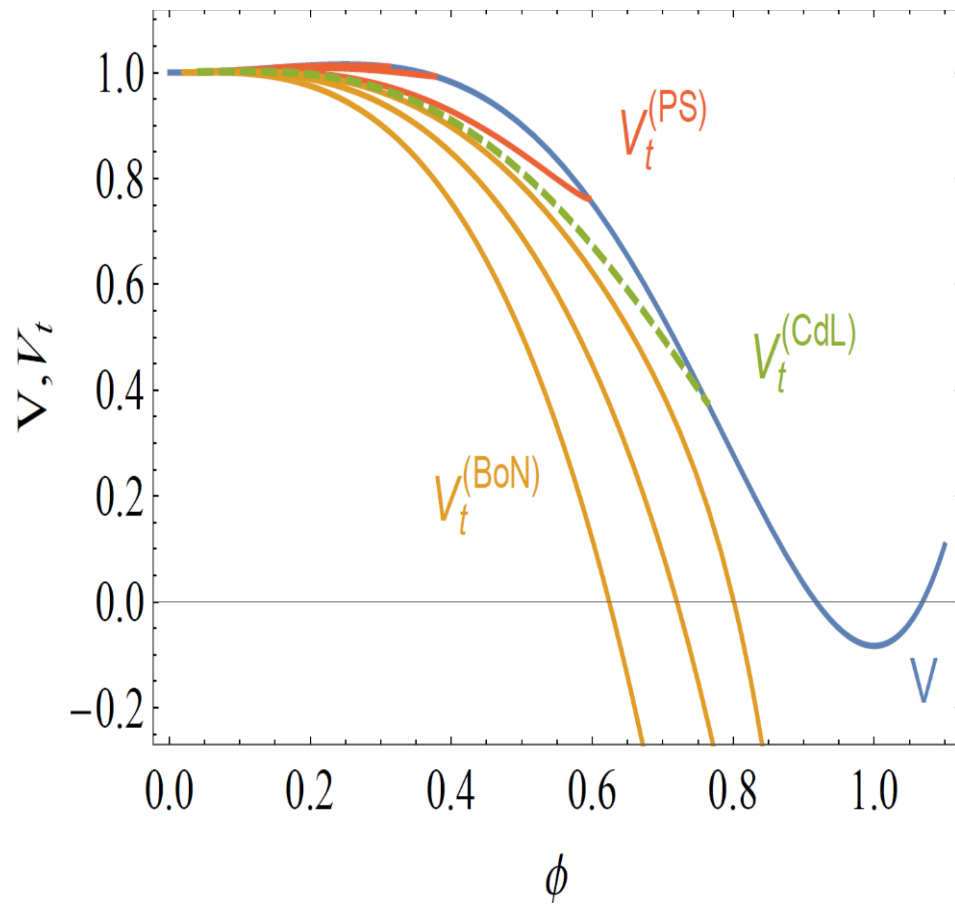
GLOBAL PICTURE (ds)

Family of solutions $V_t(\phi; \phi)$



PS = "pseudo-bounces"

GLOBAL PICTURE (dS)

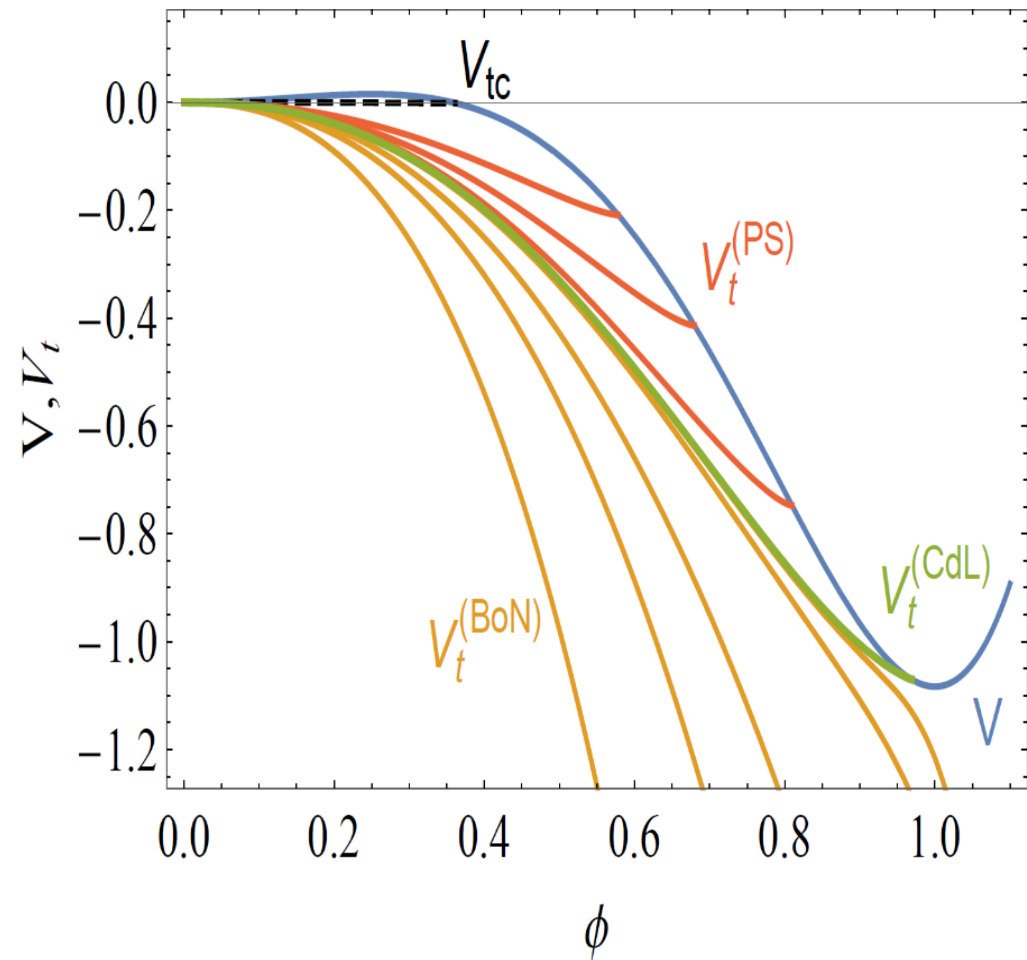
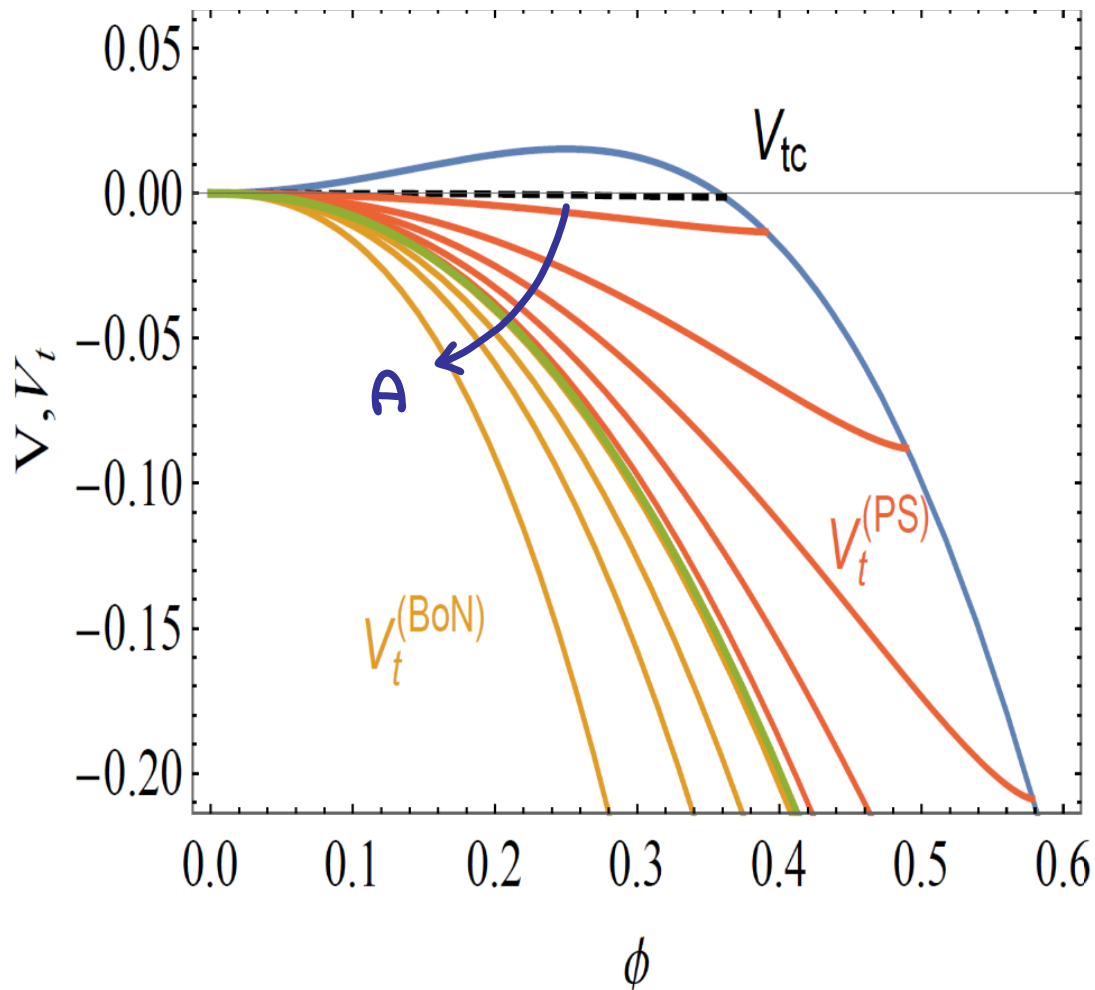


DECAY MENU

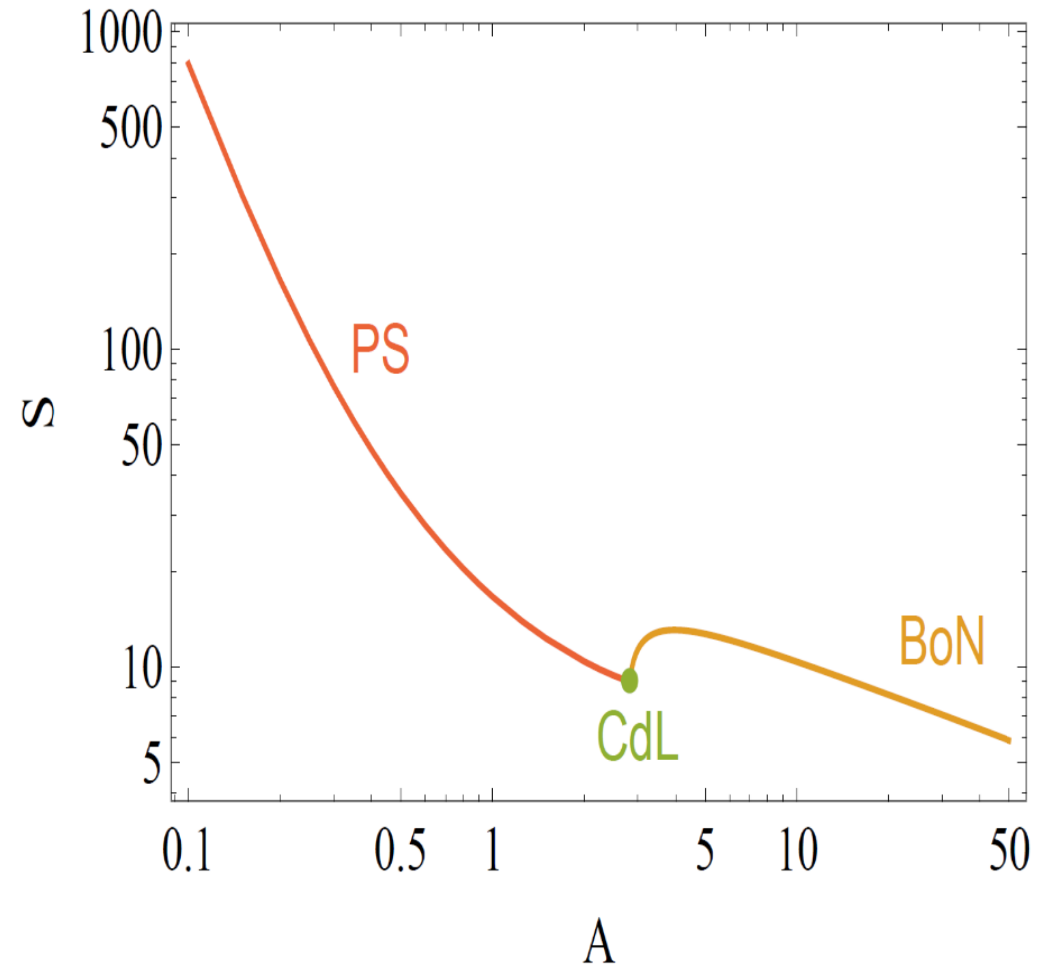
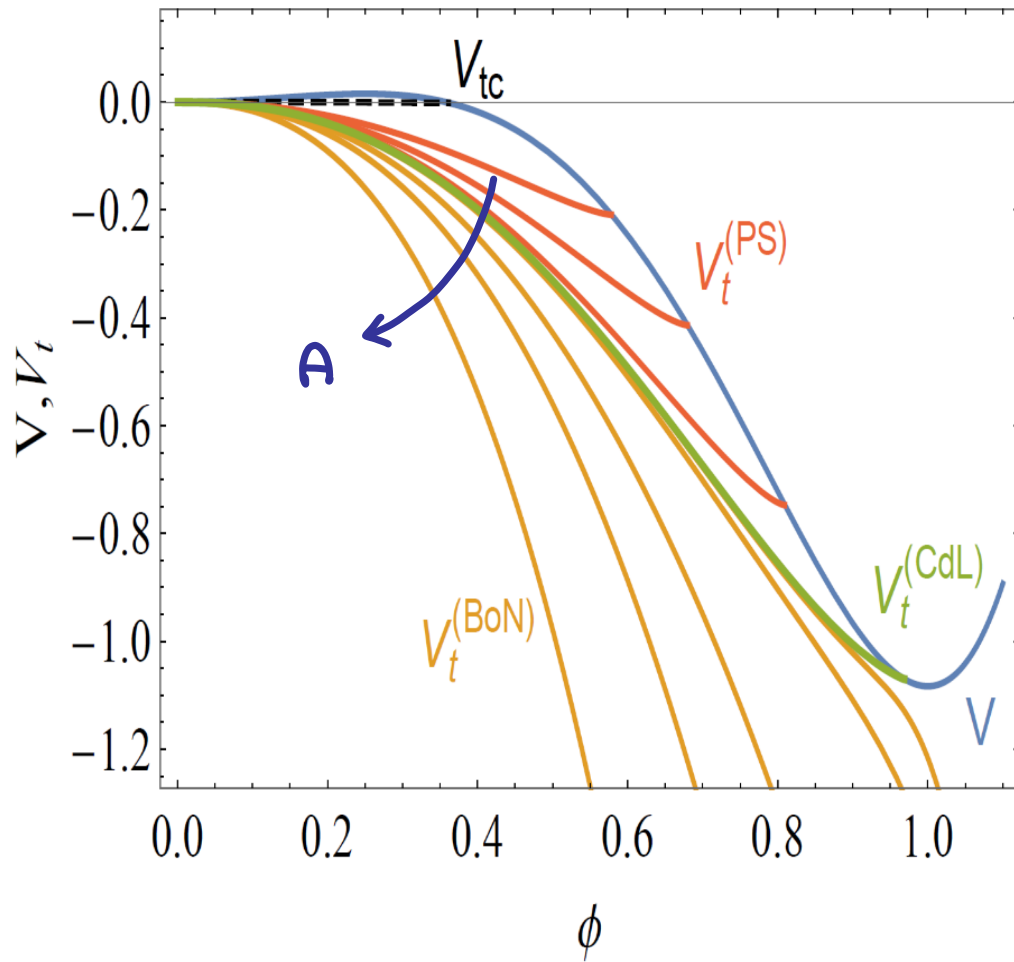
		Price
HM	homogeneous jump $\phi_+ \rightarrow \phi_B$	S_{HM}
		✓
PS	Tunneling bubble with central value ϕ_0 of choice	S_{PS}
		✓
cdL	Cheapest tunneling bubble	S_{cdL}
	Extra	
BoN	"Bubble of nothing"	S_{BoN}

GLOBAL PICTURE (Mink.)

Family of solutions $\Psi_t(A; \phi)$



GLOBAL PICTURE (Mink.)



DECAY MENU

Price

V_{tc} Domain wall

∞

\downarrow

PS Tunneling bubble with
central value ϕ_0 of choice

S_{PS}

\downarrow

CdL Cheapest tunneling bubble

S_{CdL}

Extra

BoN "Bubble of nothing"

S_{BoN}

3. BUBBLE OF NOTHING DECAYS

w/ Blanco-Pillado, Huertas, Sousa

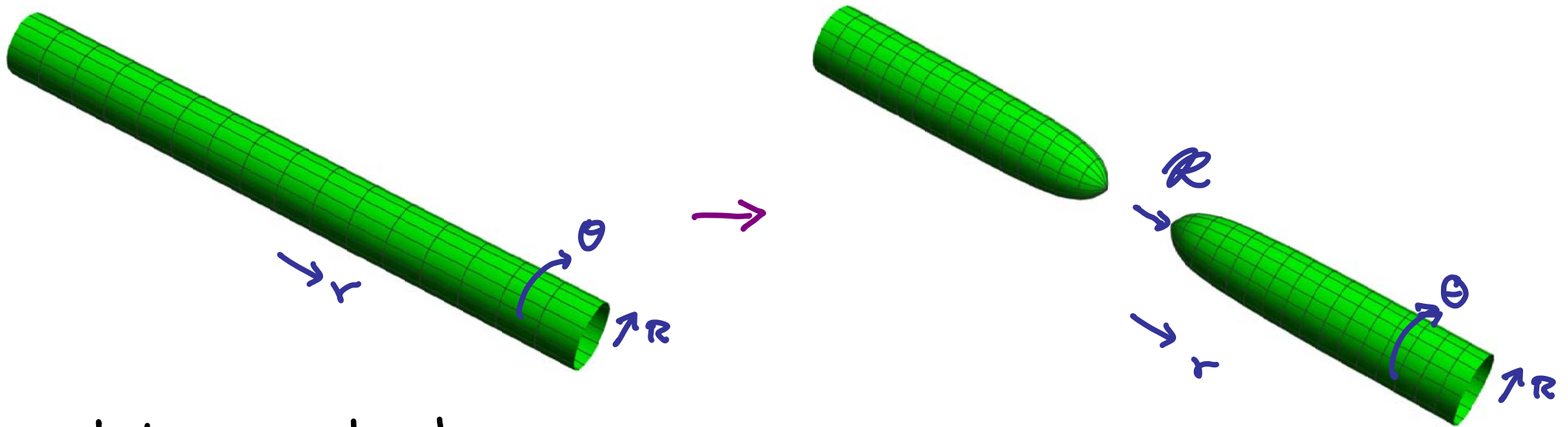


BUBBLE OF NOTHING DECAYS

Decays of spacetimes with compactified dim. like

5d KK ($M^4 \times S^1$)

Witten '82



Euclidean instanton:

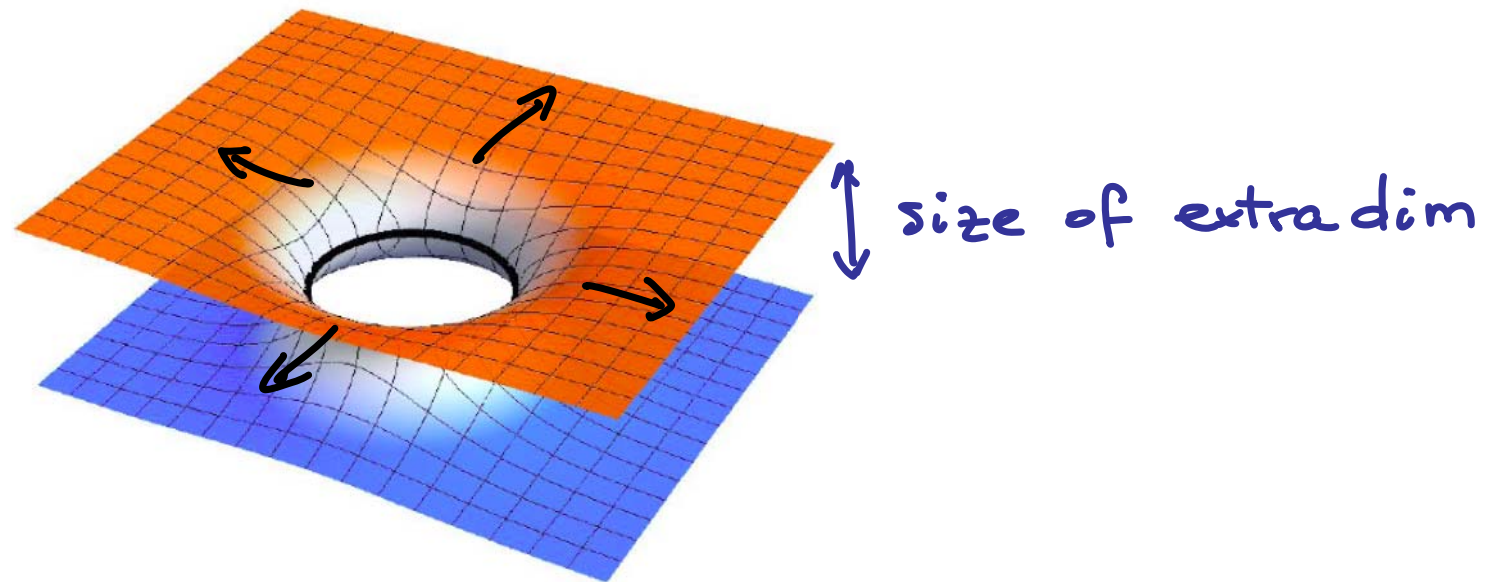
$$ds^2 = \frac{dr^2}{1 - R^2/r^2} + r^2 d\Omega_3^2 + \underbrace{R^2 \left(1 - \frac{R^2}{r^2}\right)}_{R(r)^2} d\theta^2$$

with $S = (\pi R m_p)^2$

$R(r)^2 \rightarrow 0$ at $r \rightarrow R = \mathcal{R}$

BUBBLE OF NOTHING DECAYS

End-product of tunneling process is a hole in space-time



It's smooth, w/o singularities, curvature not large
But it grows, eating the whole space

WHY ?

- Relevant for string vacua landscape
- B0Ns thought to be ultimate cause of decay of non-SUSY vacua in string compactifications
García-Etxebarria, Montero, Sousa, Valenzuela '2005

4d VIEW

Dine, Fox, Gorbatov '0405

5d \rightarrow 4d + Scalar ϕ (geometric modulus)

$$R^2 \left(1 - \frac{R^2}{r^2}\right) = R^2 e^{-2\sqrt{2\kappa/3} \phi} \quad \left\{ \begin{array}{ll} r \rightarrow R & \phi \rightarrow \infty \text{ B0N core} \\ r \rightarrow \infty & \phi \rightarrow 0 \text{ False vac} \end{array} \right.$$

and

$$ds_4^2 = d\xi^2 + g(\xi)^2 d\Omega_3^2 \quad \frac{d\xi}{dr} = \frac{1}{(1 - R^2/r^2)^{1/4}}$$

B0N reduces to a CdL problem, with

$$\phi(0) = \infty \quad \dot{\phi}(0) = -\infty \quad \phi(\infty) = \phi_+ = 0$$

Action reproduced (paying attention to boundary terms)

★ 4d approach useful to study effect of $V(\phi)$

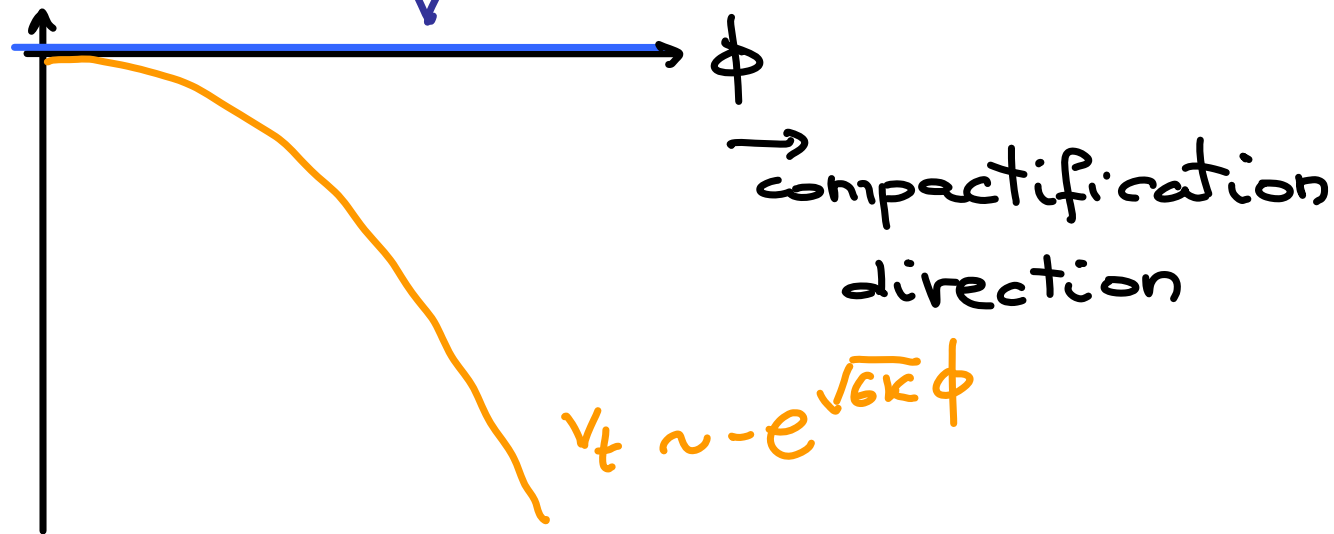
Draper, García-García, Lillard '2105

V_t FOR BONs

Blanco-Pillado, JRE, Huertas, Sousa

Witten's BoN

$$V_t = -6m_p^2 R^2 \sinh^3(\sqrt{2\kappa/3} \phi), \quad V=0$$

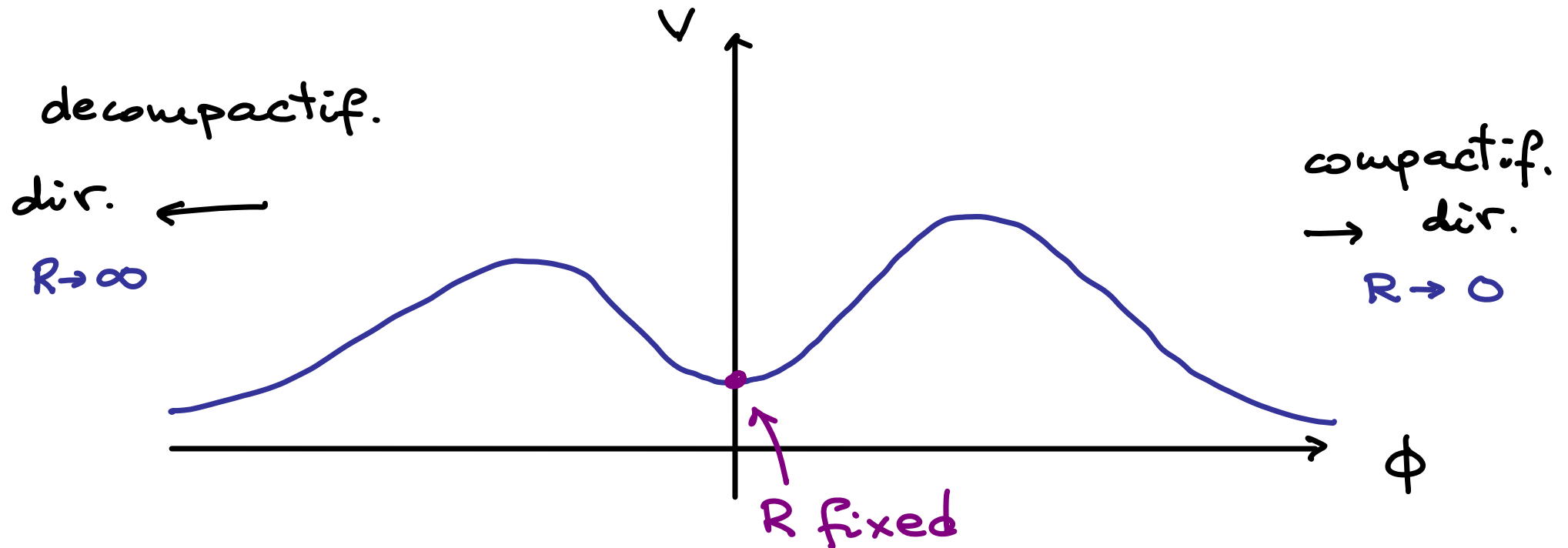


BoN tunneling to $-\infty$ AdS

$S[V_t]$ ✓ without additional boundary terms

★ Allows bottom-up approach to study which $V(\phi)$ admits BoN decays

BONS WITH $V(\phi)$?



★ Universal asymptotic behaviours at $\phi \rightarrow \infty$ via

$$0 = (4V_t' - 3V') V_t' + 6(V - V_t) [V_t'' + \kappa(3V - 2V_t)]$$

→ which V, V_t can be obtained from extra-d theories?

→ IS BON always the dominant decay?

$\phi \rightarrow \infty$ ASYMPTOTICS

Example: $M^4 \times S^1$

$$V_t \approx V_{tA}(A) e^{\sqrt{6k} \phi} + \dots \quad D \approx D_\infty(A) e^{\sqrt{8c/3} \phi} + \dots$$

UV input:

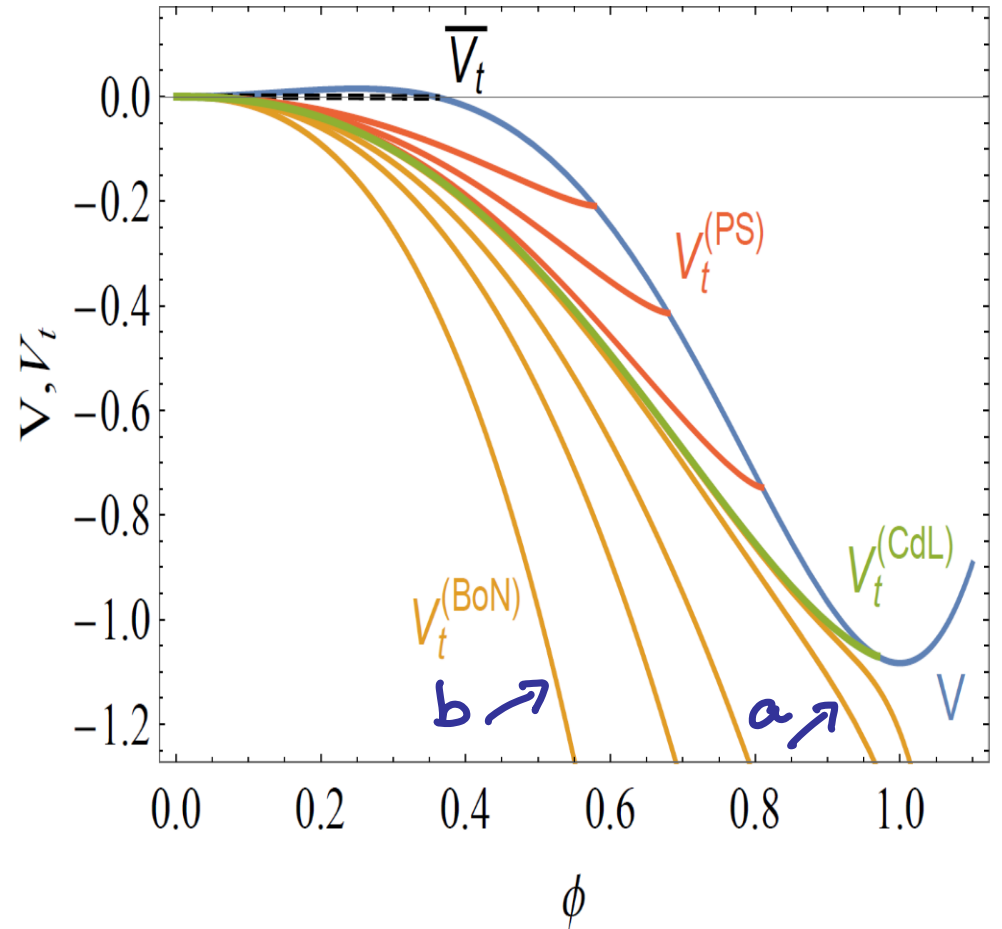
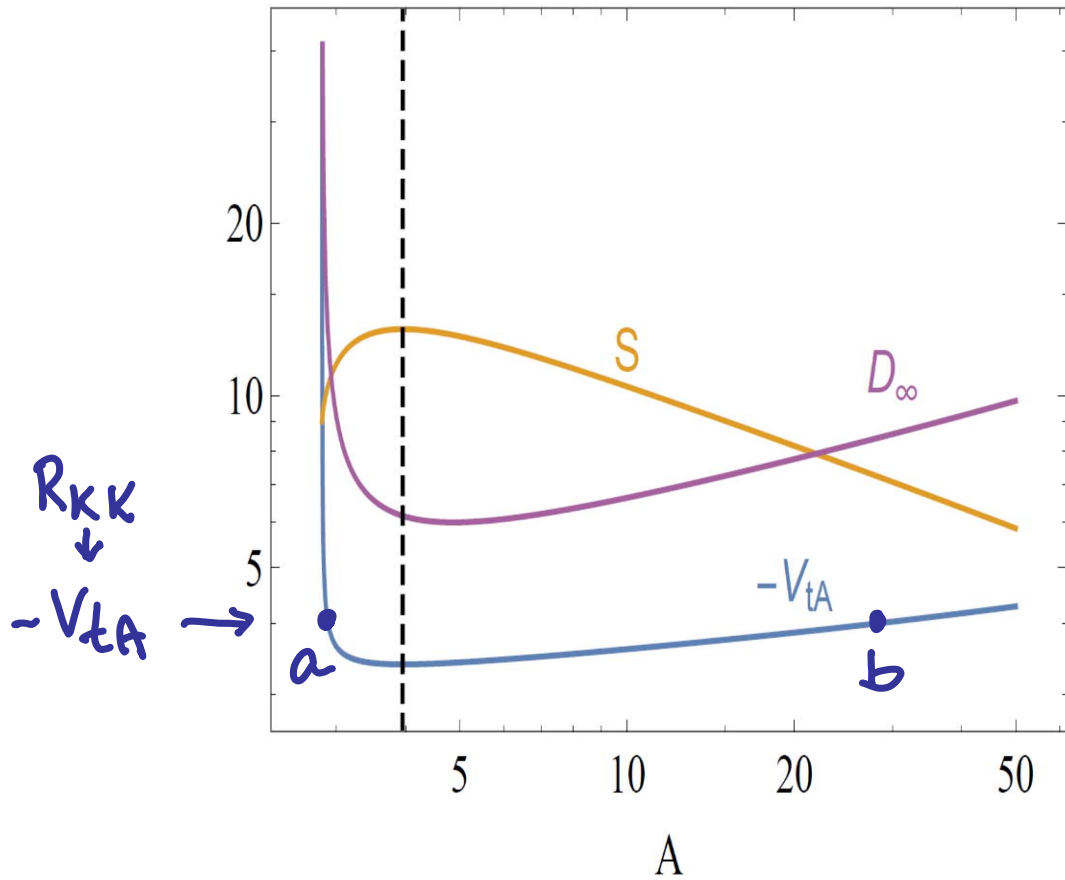
$$V_{tA} = \frac{C}{k R_{KK}^2}$$

$$D_\infty = \frac{C'}{R_{KK} \sqrt{k}}$$

BoN radius \swarrow \nearrow KK radius

For fixed $R_{KK} \Rightarrow$ fixed V_{tA}

WHICH V_t^{BoN} ?



Choose sol. with lowest S and D_∞ fixes R

If R_{KK} gives $-V_{tA} < -V_{tA}^{\min} \Rightarrow$ No BoN decay

CONCLUSIONS

Tunneling potential formalism simple and useful in

- Vacuum decay
- Finite T transitions
- Gravity effects on vacuum decays
- Bubble-of-nothing decays
- Q-balls w/Heeck, Sokharvili 2307
- Positive Energy theorems in AdS maxima w/Jinno
- Flat/curved Domain Walls w/C.Bachas, Z.Chen

More? Euclidean solutions \Rightarrow V_t solutions