Standard Model & extra dimensions
UV sensitivity / Higgs mass / vacuum energy

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CORFU23 - Workshop on the Standard Model and Beyond

Carlo Branchina ,VB , Filippo Contino , Phys.Rev.D 108 (2023) 4, 045007 , *arXiv:2304.08040*
Carlo Branchina , VB , Filippo Contino , Arcangelo Pernace , *arXiv: 2308.16548*

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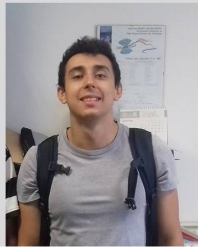

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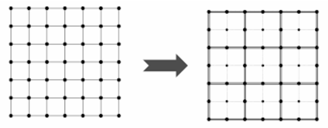
Plan of the talk

- Wilson's lesson.
- Higher dimensional theories. KK modes
- Sherk-Schwarz. Non trivial boundary conditions
- Vacuum Energy : the appearance of UV-sensitive terms
- Dark Dimension

Wilson's Lesson

What is the Wilson's lesson all about?



Theory at Λ $\quad\rightarrow\quad$ Theory at $\Lambda/2$ $\quad\rightarrow\quad$ ...

$$S_\Lambda \quad\rightarrow\quad S_{\Lambda/2} \quad\rightarrow\quad \ldots$$

Effective Field Theory paradigm

Any QFT is an Effective Field Theory

Steven Weinberg - Third Law of Progress in Theoretical Physics : you may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry

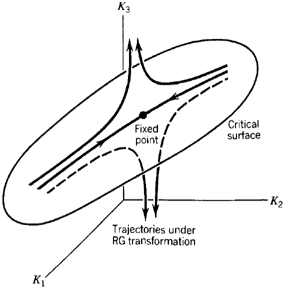
Weinberg - Laws of Progress in Theoretical Physics

This paper (Gellman-Low) is one of the most important ever published in quantum field theory ... This paper has a strange quality. It gives conclusions which are enormously powerful ... The input seems incommensurate with the output. The paper seems to violate

First Law of Progress in Theoretical Physics : Conservation of Information. Another way of expressing this law is : You will get nowhere by churning equations.

Second Law of Progress in Theoretical Physics : Do not trust arguments based on the lowest order of perturbation theory

RG flow

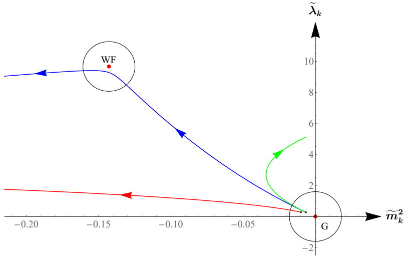


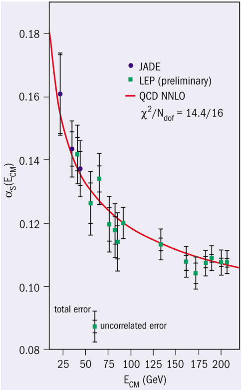
Renormalized theory: defined around a fixed point (critical surface)

... For theories in any dimesion: ..., $d = 3$, $d = 4$, ...

$d = 3$ dimensions : Wilson-Fisher

$d = 4$ dimensions : AF





Also for theories with $d > 4$ dimesions

in particular

Theories with compact extra dimensions: $d = 4 + n$

Field Theories with compact extra dimensions :

$$d = 4 + n$$

- Field Theories with compact extra dimensions are ubiquitous
- Typically approached as $4D$ theories with infinite towers of states:

$$m_n = f_n \, m_{\text{tow}}$$

- Surprising UV-softness :

Towers contribute $\sim m_{\text{tow}}^4$

to Vacuum Energy / Effective Potential

How is it possible?

Example : Scherk-Schwarz

5D SUSY theory defined on the multiply connected spacetime $\mathcal{M}^4 \times S^1$

- Different R-charges for superpartners $(i = b, f)$

$$\Psi_i(x, z + 2\pi R) = e^{2i\pi R q_i}\Psi_i(x, z) \;\Rightarrow\; \Psi_i(x, z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \psi_{i,n}(x) e^{i(\frac{n}{R}+q_i)z}$$

$\int dz\, \mathcal{S}_{(5)} \to \mathcal{S}_{(4)} \leftarrow$ infinite tower of $4D$ KK fields, $m^2_{i,n} \propto \left(\frac{n}{R} + q_i\right)^2$

- 4D mismatch in the masses of the superpartners :
  effective 4D non-local soft SUSY breaking

Higgs field ϕ : ϕ_0 , or $4D$ brane field , or . . .

Effective 4D quadratic operator

$$M^2_{i,n}(\phi) = m^2_i(\phi) + \left(\frac{n}{R} + q_i\right)^2$$

One-loop Higgs Effective Potential (4D calculation)

$$V_{1l}^{(4)}(\phi) = \frac{1}{2}\sum_a \sum_{i_a} (-1)^{\delta_{i_a,f_a}} \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \log\left(p^2 + M_a^2(\phi) + \left(\frac{n}{R} + q_{i_a}\right)^2\right)$$

One way of doing the calculation (not the only one)*:
Perform (first) the infinite sum; (then) integrate in d^4p with a cutoff Λ

Delgado, Pomarol, Quiros

Each tower contributes :

$$V_{1l}^{(4)}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

* Other methods, "Proper time" (Antoniadis, Benakli, Quiros), "Pauli-Villars" (Contino, Pilo), Thick brane (Delgado, von Gersdorf, John, Quiros), all give the same result.

One-loop Higgs Effective Potential (4D calculation)

Let's have a closer look to this result

From each tower the Higgs Potential receives the contribution :

$$V_{1l}^{(4)}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

- Power UV-sensitivity through $m \Longrightarrow$ canceled by SUSY
- No UV-sensitivity through q

$\Longrightarrow$ Finite Higgs potential

Old Times $\sim$ 2000

- Finite Higgs effective potential!
- Finite Higgs mass!
- KK regularization

... Heated debate! ...

Criticism : sum $[-L, L] \rightarrow$ UV-sensitive terms   Ghilencea, Nilles/Kim

Countercriticism   Delgado, v.Gersdoff, John, Quiros/Contino, Pilo/Barbieri, Hall, Nomura/Masiero, Scrucca, Silvestrini

- L does NOT respect $5D$ symmetries (Lorentz, SUSY): “spurious”
- Thick brane & Pauli-Villars: (apparently) safer derivation

Debate closed in favour of UV-insensitiveness ... but ...

4D Higgs Effective Potential from the 5D side

$$\mathcal{S}_{(5)} = \int dz\, d^4x \left(\frac{1}{2}\partial_a\widehat{\Phi}\,\partial^a\widehat{\Phi} + \partial_a\widehat{\chi}\,\partial^a\widehat{\chi}^\dagger + \frac{m_\Phi^2}{2}\widehat{\Phi}^2 + m_\chi^2\,\widehat{\chi}\widehat{\chi}^\dagger + \frac{\widehat{\lambda}}{4!}\widehat{\Phi}^4 + \frac{\widehat{g}}{2}\widehat{\Phi}^2\widehat{\chi}\widehat{\chi}^\dagger \right)$$

$$\widehat{\Phi}(x, z+2\pi R) = \widehat{\Phi}(x,z) \quad ; \quad \widehat{\chi}(x, z+2\pi R) = e^{2i\pi R\, q}\,\widehat{\chi}(x,z)$$

$$V_{1l}^{(5)}(\widehat{\Phi}) = \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\widehat{\lambda}}{2}\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}} + \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \left(\frac{n}{R}+q\right)^2 + m_\chi^2 + \frac{\widehat{g}}{2}\widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}}$$

$$\widehat{p} = (p_1, p_2, p_3, p_4, p_5 = \tfrac{n}{R}) = (p, p_5 = \tfrac{n}{R}) \to \mathrm{Tr}_5 = \tfrac{1}{2\pi R}\textstyle\sum_n \int \frac{d^4p}{(2\pi)^4}$$

$$\mathrm{Tr}_5 \equiv \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R} \right)' \equiv \frac{1}{2\pi R}\sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \quad ; \quad C_\Lambda^n \equiv \sqrt{\Lambda^2 - \frac{n^2}{R^2}}$$

We cannot introduce any hierarchy between the different components of the loop momentum when calculating the Higgs Effective Potential

4D Effective Potential from the 5D Effective Potential

Fourier expansion of $\widehat{\chi}(x,z)$ (similarly for $\widehat{\Phi}$)

$$\widehat{\chi}(x,z) = \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R}\right)' \widehat{\chi}_{n,p}\, e^{i\left(p\cdot x + \left(\frac{n}{R}+q\right)z\right)}$$

$$\widehat{\chi}(x,z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \chi_n^\Lambda(x)\, e^{i\left(\frac{n}{R}+q\right)z}; \quad \chi_n^\Lambda(x) \equiv \frac{1}{\sqrt{2\pi R}} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \widehat{\chi}_{n,p}\, e^{ip\cdot x}$$

Performing z integration $\to$ effective $4D$ theory with $\phi = \phi_0$

$$V_{1l}^{(4)}(\phi) = \frac{1}{2} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \left(\log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\lambda}{2}\phi^2}{p^2 + \frac{n^2}{R^2}} + \log \frac{p^2 + \left(\frac{n}{R}+q\right)^2 + m_\chi^2 + \frac{g}{2}\phi^2}{p^2 + \frac{n^2}{R^2}} \right)$$

$$\lambda \equiv \frac{\widehat{\lambda}}{2\pi R} \quad ; \quad g \equiv \frac{\widehat{g}}{2\pi R} \quad ; \quad \widehat{\Phi} = \frac{1}{\sqrt{2\pi R}}\,\phi$$

$$\boxed{V_{1l}^{(4)}(\phi) = 2\pi R\, V_{1l}^{(5)}(\widehat{\Phi})}$$

UV-sensitivity and non-trivial topology

$$V_{1l}(\phi) = \frac{5m^2 + 3q^2}{180\pi^2} R\Lambda^3 - \frac{35m^4 + 14m^2q^2 + 3q^4}{840\pi^2} R\Lambda + \frac{m^5 R}{60\pi} - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

New q-dependent UV-sensitive terms:

- Not canceled by SUSY! $\propto \boxed{(q_b^2 - q_f^2)}\; m^2(\phi)\,\Lambda$
- Topological origin
- Absent for $q = 0$ and for $q_b = q_f$. But : (1) $q \neq 0$ in multiply connected space ; (2) $q_b \neq q_f$ for SUSY breaking

UV-sensitive terms solely due to the non-trivial topology of space

Alternatively : Infinite sum & Smooth cut

Typical argument: cut on sum $\rightarrow$ spurious "divergences" ... But ...

$$V_{1l}(\phi) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\int \frac{d^4p}{(2\pi)^4}\log\left(\frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{p^2+\frac{n^2}{R^2}}\right)e^{-\frac{p^2+\frac{n^2}{R^2}}{\Lambda^2}}$$

$\Rightarrow$ Same result is found

UV-sensitive terms are NOT due to the sharp cut of the sum!
They come from a correct treatment of $\widehat{p}$ asymptotics

So ... why do "Proper time", "Thick brane" and "Pauli-Villars"

give UV-insensitive results ?

Secret liaison between proper time , thick brane & PV

Thick brane: $\sum_{n=-\infty}^{\infty} \int^{(\Lambda)} \frac{d^4p}{(2\pi)^4} \frac{e^{-\frac{\left(\frac{n}{R}+q\right)^2}{\Lambda^2}}}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ Delgado, von Gersdor, John, Quiros

Pauli-Villars: $\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{(\Lambda R)^4}{(\Lambda R)^4+p^2+(\frac{n}{R}+q)^2} \frac{1}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ Contino, Pilo

Proper Time: Antoniadis, Benakli

$$V_{1l}^{(4)}(\phi) = -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s} e^{-s\left(p^2+m^2+\left(\frac{n}{R}+q\right)^2\right)}$$

$$= -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \Gamma\left(0, \frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{\Lambda^2}\right)$$

In all cases a cut function of (p_5+q) instead of $p_5 (= \frac{n}{R})$

Equivalent to introduce a hierarchy between (p_1, p_2, p_3, p_4) and p_5

$\Rightarrow$ **Again : artificial wash-out of UV-sensitive terms**

Vacuum Energy and Dark Dimension

Montero, Vafa, Valenzuela

String theory (quantum gravity) Distance Conjecture

$\mu_{tow} \sim |\Lambda_{cc}|^{\alpha}$ with Λ_{cc} cosmological constant (times M_P^2)

One-loop string calculation gives : $\rho_4 \sim \mu_{tow}^4$ with $\mu_{tow} = M_s$ or m_{KK}

From which $\alpha \geq \frac{1}{4}$ $\Leftarrow$ Assumed as starting point for DD proposal

Experimental bounds on violations of $\frac{1}{r^2}$ Newton's law : $\mu_{tow} \geq 6.6$ meV

Energy scale associated to Λ_{cc} is of the same order : $\Lambda_{cc}^{1/4} \sim 2.31$ meV

$\Rightarrow$ $\alpha = 1/4$ $\Rightarrow$ "experimental value" of μ_{tow} :

$$\mu_{tow}^{exp} \sim 2.31\,\mathrm{meV} \quad \text{(order the neutrino scale)}$$

In principle possible $\mu_{tow} = M_s$, but above equation indicates that this option is "ruled out by experiments" since we know that physics around and above the neutrino scale is well described by effective field theories, and no sign of string excitations observed at these scales.

Vacuum Energy and Dark Dimension

They then conclude that the only possibility left is an "EFT decompactification scenario", with a Kaluza-Klein mass

$$m_{\mathrm{KK}} \sim \mu_{tow}^{exp} \sim 2.31\,\mathrm{meV}$$

This conclusion takes us from string theory to EFT, and is crucial to the formulation of the DD proposal.

When physics is described in terms of a string KK tower

original theory replaced by the corresponding higher dimensional EFT with compact extra dimensions

Vacuum Energy and Dark Dimension

Compactification with gravity $\widehat{g}_{MN} = \begin{pmatrix} e^{2\alpha\phi} g_{\mu\nu} - e^{2\beta\phi} A_\mu A_\nu & e^{2\beta\phi} A_\mu \\ e^{2\beta\phi} A_\nu & -e^{2\beta\phi} \end{pmatrix}$

Background configuration $g^0_{\mu\nu} = \eta_{\mu\nu}, A_\mu = 0, \phi = \phi_0$ (hereafter ϕ)

$$\rho_4 = \frac{5 \log \frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} - 2}{300\pi^2} e^{2\alpha\phi} R\Lambda^5 + \frac{5m^2 + 3q^2 e^{4\alpha\phi}}{180\pi^2} e^{2\alpha\phi} R\Lambda^3 - \frac{35m^4 + 14m^2 q^2 e^{4\alpha\phi} + 3q^4 e^{8\alpha\phi}}{840\pi^2} e^{2\alpha\phi} R\Lambda + \frac{m^5}{60\pi} e^{2\alpha\phi} R + \frac{3 \log \frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} + 2}{2880\pi^2 R^4} e^{10\alpha\phi} R\Lambda + R_4 + \mathcal{O}(\Lambda^{-1}) = 2\pi R e^{2\alpha\phi} \rho_5$$

$$R_4 = -\frac{x^2 \mathrm{Li}_3\left(r_b e^{-x}\right) + 3x \mathrm{Li}_4\left(r_b e^{-x}\right) + 3\mathrm{Li}_5\left(r_b e^{-x}\right) + 6\zeta(5)}{128\pi^6 R^4} e^{12\alpha\phi} + h.c.$$

$$r \equiv e^{2\pi i q R} \quad , \quad x \equiv 2\pi e^{-2\alpha\phi} R\sqrt{m^2} \implies R_4 \propto \frac{e^{12\alpha\phi}}{R^4} = m_{KK}^4$$

Light tower limit and Dark Dimension

SUSY case : dominant contribution $\rho_4 \sim (q_b^2 - q_f^2)\, e^{6\alpha\phi} R\Lambda^3 = m_{KK}^2 R\Lambda^3$

Non-SUSY case : dominant contribution $\rho_4 \sim e^{2\alpha\phi} R\Lambda^5 = m_{KK}^{\frac{2}{3}} \left(R^{\frac{1}{3}}\Lambda\right)^5$

Even in the light tower limit $\phi \to -\infty$, the UV-insensitive R_4 term cannot overthrow these dominating contributions.
No light tower regime where we can have $\rho_4 \sim m_{KK}^4$

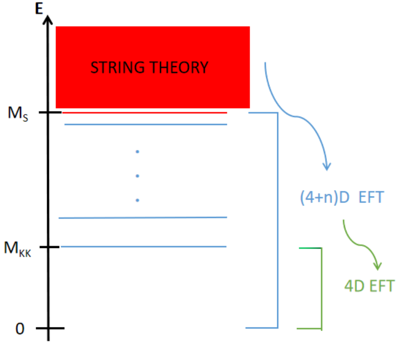
But this assumption ($\rho_4 \sim m_{KK}^4$) is crucial for Dark Dimension

In a theory with compact extra dimensions $\rho_4 \sim m_{KK}^4$ is untenable

From String Theory to 4D Effective Field Theory (SM)

String theory $\Rightarrow$ EFT : takes over at M_s

from M_s down to the "physical scales" : EFT heavy artilery



Summary & Conclusions

- Usual calculations mistreat the asymptotics of the loop momenta
- Correct treatment of the loop momenta asymptotics unveils the presence of UV-sensitive terms , missed in the usual calculations
- Interpretation of the $(4+n)$ D theory with compact extra dimensions as a $4D$ theory with an infinite number of fields needs to be taken with a grain of salt
- The UV-sensitive terms are of topological origin
- The idea that $\Lambda_{\rm cc}^{1/4} \sim 2.31$ meV $\Rightarrow$ 5^{th} dimension (compact) of size $\sim \mu m$ is untenable

Cutting tower modes in Swampland program

Cut in tower typical in Swampland: Species scale Λ_{sp} (e.g. emergence proposal)

Grimm, Palti, Valenzuela

Species scale $\Lambda_{sp} = \left(M_p^2 m_{_{KK}}\right)^{\frac{1}{3}}$: dominant depend on $|\phi|/M_p \lessgtr\sim 100$

- SUSY: $\rho_4 \sim (q_b^2 - q_f^2) M_p^2 m_{_{KK}}^3 \longrightarrow \rho_4 \sim -(q_b^2 - q_f^2) m^2 M_p^{2/3} m_{_{KK}}^{7/3}$
- Non-SUSY: $\rho_4 \sim M_p^{10/3} m_{_{KK}}^{7/3} \longrightarrow \rho_4 \sim m^5 m_{_{KK}}^{2/3}$

Global picture: EFTs with compact dimensions

Recent argument: EFT not applicable as it requires large hierarchy between last included and first excluded KK mode Burgess, Quevedo

$\rightarrow$ Rely on usual interpretation of KK modes as massive $4D$ states

- Start: $\mathcal{S}_\Lambda^{(5)}$ w/ "Wilsonian" mode expansion $\hat{p} \in [0, \Lambda]$
- Integrating out modes in $[k, \Lambda] \rightarrow \mathcal{S}_k^{(5)}$ k Wilsonian running scale

Due to $p_5 = n/R$ discreteness, p_5 eigenmodes contribution is stepwise

- For $k < 1/R$ no p_5 eigenmodes anymore: RG evolution becomes effectively of $4D$ type

It is **only in this sense** that the $4D$ theory emerges from the $5D$ one: **by no means it has an** *infinite* **tower of states**