UV/IR mixing, (non)renormalisation and naturalness from a string perspective

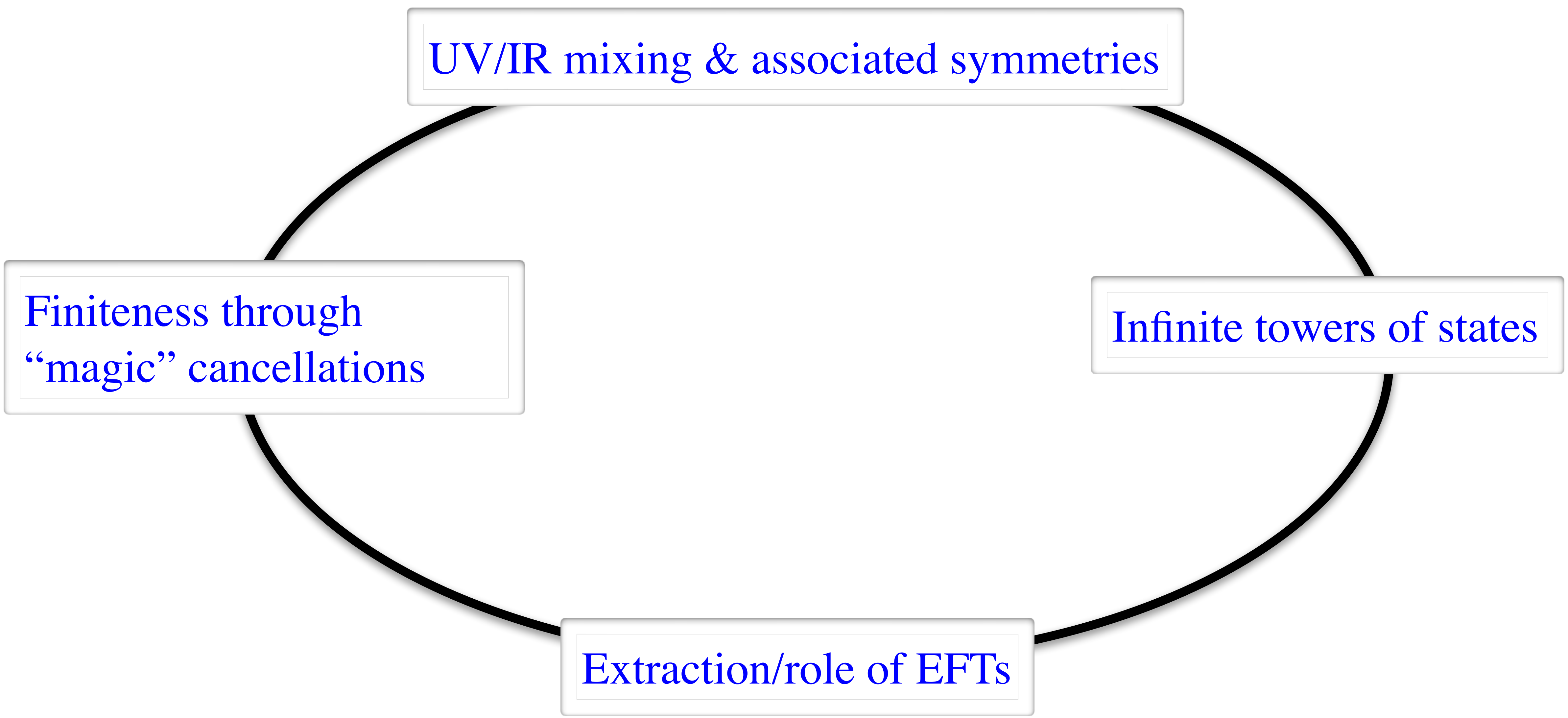
Steve Abel *(IPPP)*

Mainly based on …

- w/ Dienes and Nutricati — arXiv:2303.08534; arXiv:23MM.NNNNN; arXiv:YYMM.NNNNN
- w/ Keith Dienes arXiv:2106.04662
- w/ Dienes+Mavroudi *Phys.Rev.D* 97 (2018) 12, 126017 arXiv: 1712.06894
- w/ Stewart, *Phys.Rev.D* 96 (2017) 10, 106013 arXiv:1701.06629
- Aaronson, SAA, Mavroudi, *Phys. Rev. D* 95, (2016) 106001, arXiv:1612.05742
- SAA JHEP 1611 (2016) 085, arXiv:1609.01311
- w/ Dienes+Mavroudi *Phys.Rev. D* 91, (2015) 126014, arXiv:1502.03087

Motivation

This talk is dedicated to a circle of ideas that are all connected to exotic approaches to naturalness...



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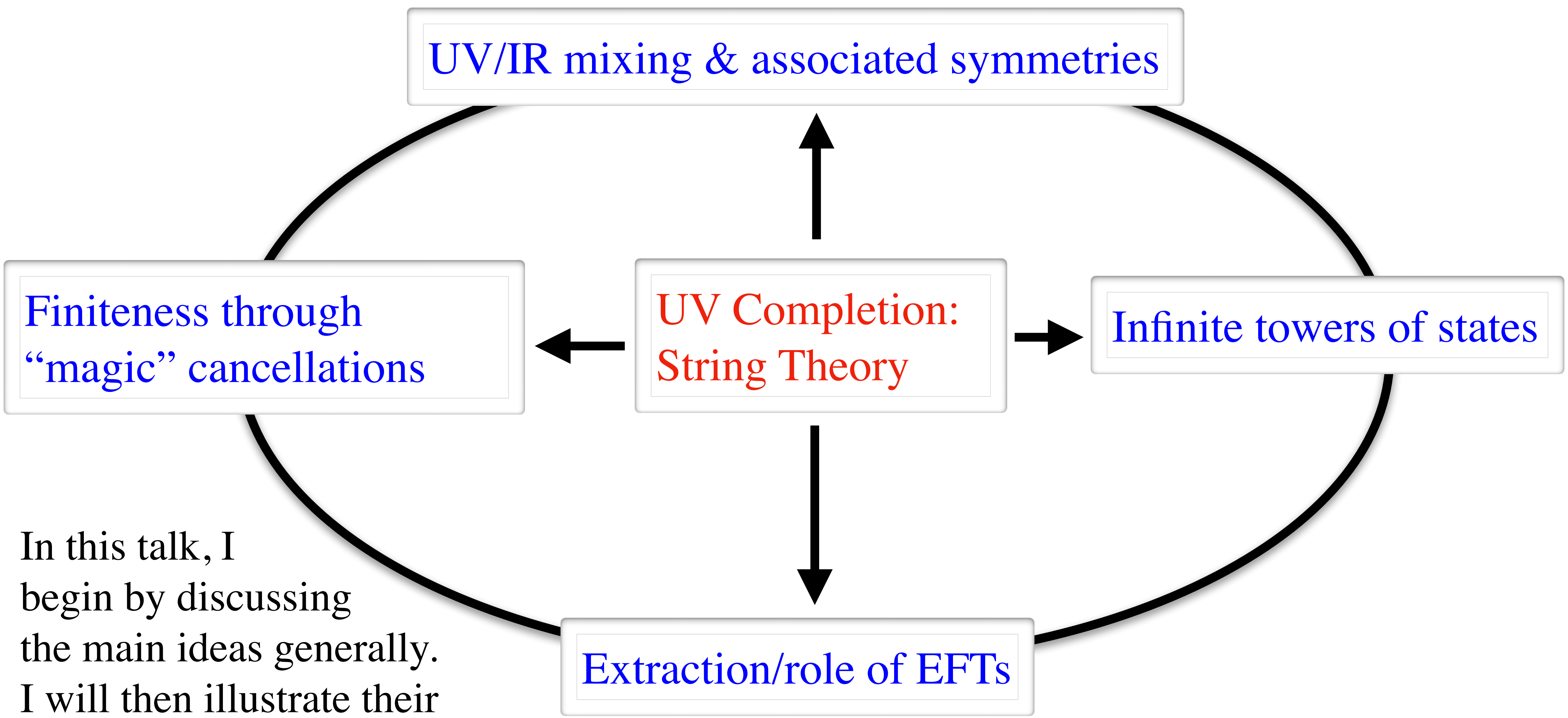


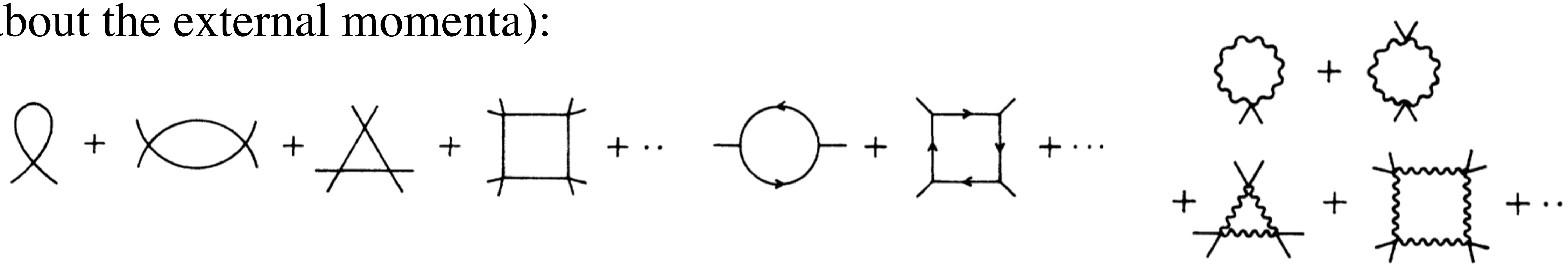
In this talk, I begin by discussing the main ideas generally. I will then illustrate their power by calculating the **Higgs mass** and **gauge-coupling running** in string theory --- all while keeping track of the full UV/IR string symmetries and complete towers of string states.

Layout

- A bit of field theory origami (toy model)
- Modular invariance — the ultimate UV/IR mixer
- Extracting EFT's: e.g. Calculating the Higgs mass
- Conclusions

1. A bit of field theory origami:

Let's start our story by examining the one-loop CW effective potential in field theory (and similar amplitudes where we don't care about the external momenta):



giving …

$$\Lambda(\phi) = \sum_n \int \frac{d^4k}{(2\pi)^4} (-1)^F \log(k^2 + M_n^2)$$

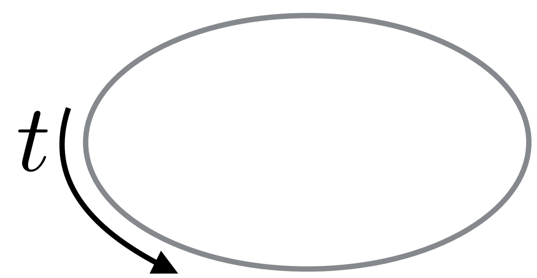
$$= \frac{1}{2} M_{UV}^4 + \frac{M_{UV}^2}{32\pi^2} \mathrm{Str} M^2 - \frac{1}{64\pi^2} \mathrm{Str} M^4 \log\left(c \frac{M^2}{M_{UV}^2} \right)$$

where masses can be functions of the Higgs ϕ and we are forced to put in a cut-off.

Note there was previously no equivalent to the CW potential in string theory

It turns out that the best way to connect this to our UV/IR mixing ideas is through the ***Schwinger worldline formalism, t.***

Purely field-theoretic formalism: *the* “Schwinger proper time”, t, is in some sense the total “length” of the worldline around the bubble. Total one-loop diagram is then the *integral* of the amplitude over all possible *t*.



$$\Lambda = \sum_n \int \frac{d^4k}{(2\pi)^4} (-1)^F \log(k^2 + M_n^2) = \sum_n \int \frac{d^4k}{(2\pi)^4} \int \frac{dt}{t} (-1)^F e^{-t(k^2+M_n^2)}$$

$$= \sum_n \int_{M_{UV}^{-2}}^{\infty} \frac{dt}{t^3} (-1)^F e^{-t\, M_n^2}$$

Can identify a “particle partition function” as a weighted sum over the spectral density:

$$Z(t) = \mathrm{Str}\left[t^{-2} e^{-tM^2}\right]$$

Performing the integral of *Z(t)* indeed gives the effective potential

$$\Lambda(\phi) \;=\; \frac{1}{2}M_{UV}^4 + \frac{M_{UV}^2}{32\pi^2}\mathrm{Str}M^2 - \frac{1}{64\pi^2}\mathrm{Str}M^4\log\left(c\frac{M^2}{M_{UV}^2}\right)$$

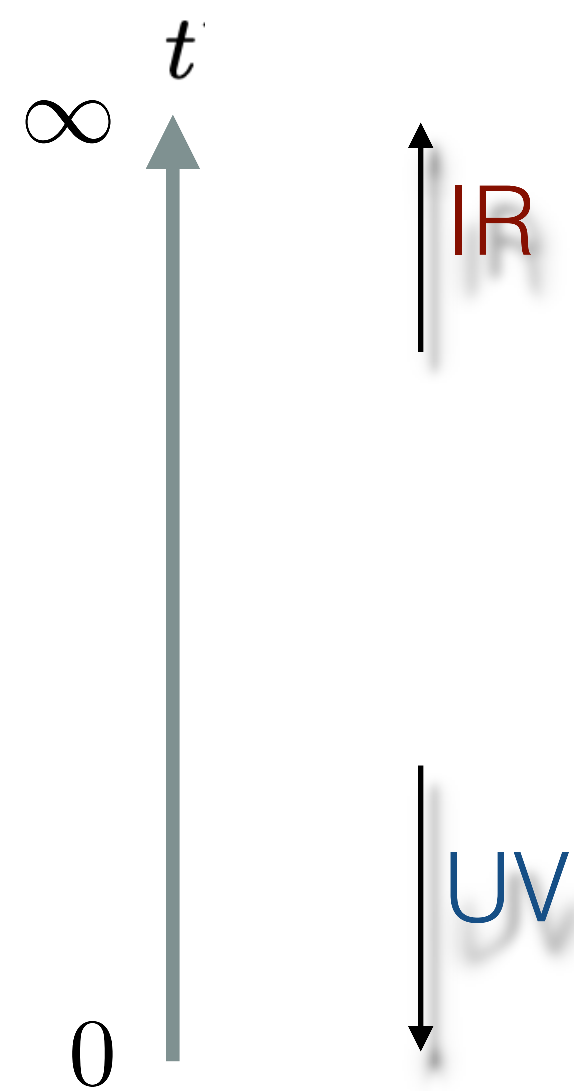
From which we can infer the running Higgs mass-squared from the double derivative:

$$m_\phi^2 \;=\; \frac{M_{\rm UV}^2}{32\pi^2}\,\mathrm{Str}\;\partial_\phi^2 M^2 \;-\; \mathrm{Str}\;\partial_\phi^2\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{M_{\rm UV}^2}\right)\right]$$

This is the origin of the unfortunate naturalness problem associated with the Higgs mass. It is associated with the quadratic UV divergence in the EFT.

We thus have an alternative worldline picture of the integral (which remember depends purely on the mass-spectrum):

$$\Lambda = \int_0^\infty \frac{dt}{t} Z(t) \qquad\qquad Z(t) = \mathrm{Str}\left[t^{-2} e^{-tM^2}\right]$$

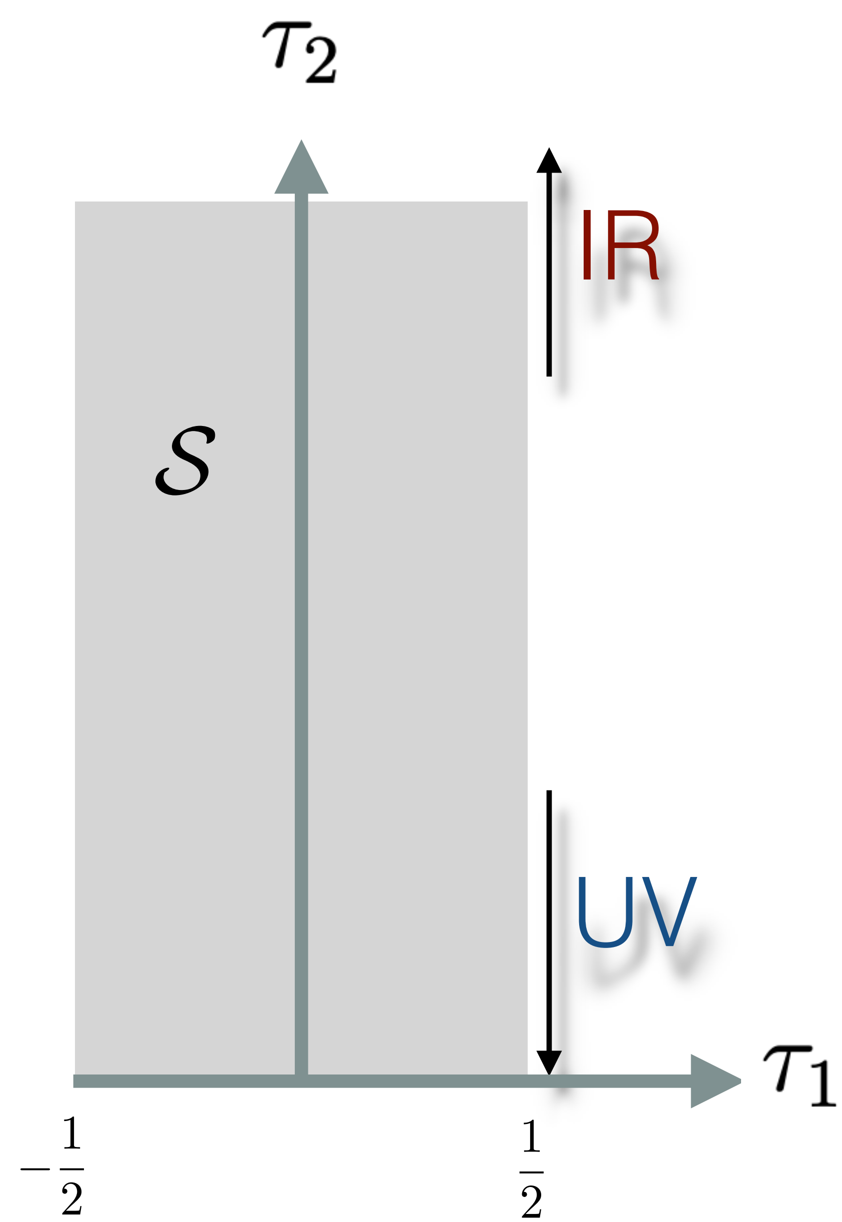


We thus have an alternative worldline picture of the integral (which remember depends purely on the mass-spectrum):

$$\Lambda = \int_0^\infty \frac{d\tau_2}{\tau_2} \int_{-1/2}^{1/2} d\tau_1 \, Z(\pi\alpha'\tau_2)$$

With an eye towards an eventual connection to string theory, let's define a dimensionless Schwinger parameter $\tau_2 = t/\pi\alpha'$

Also let's introduce a dummy variable and enlarge our region of integration with it:



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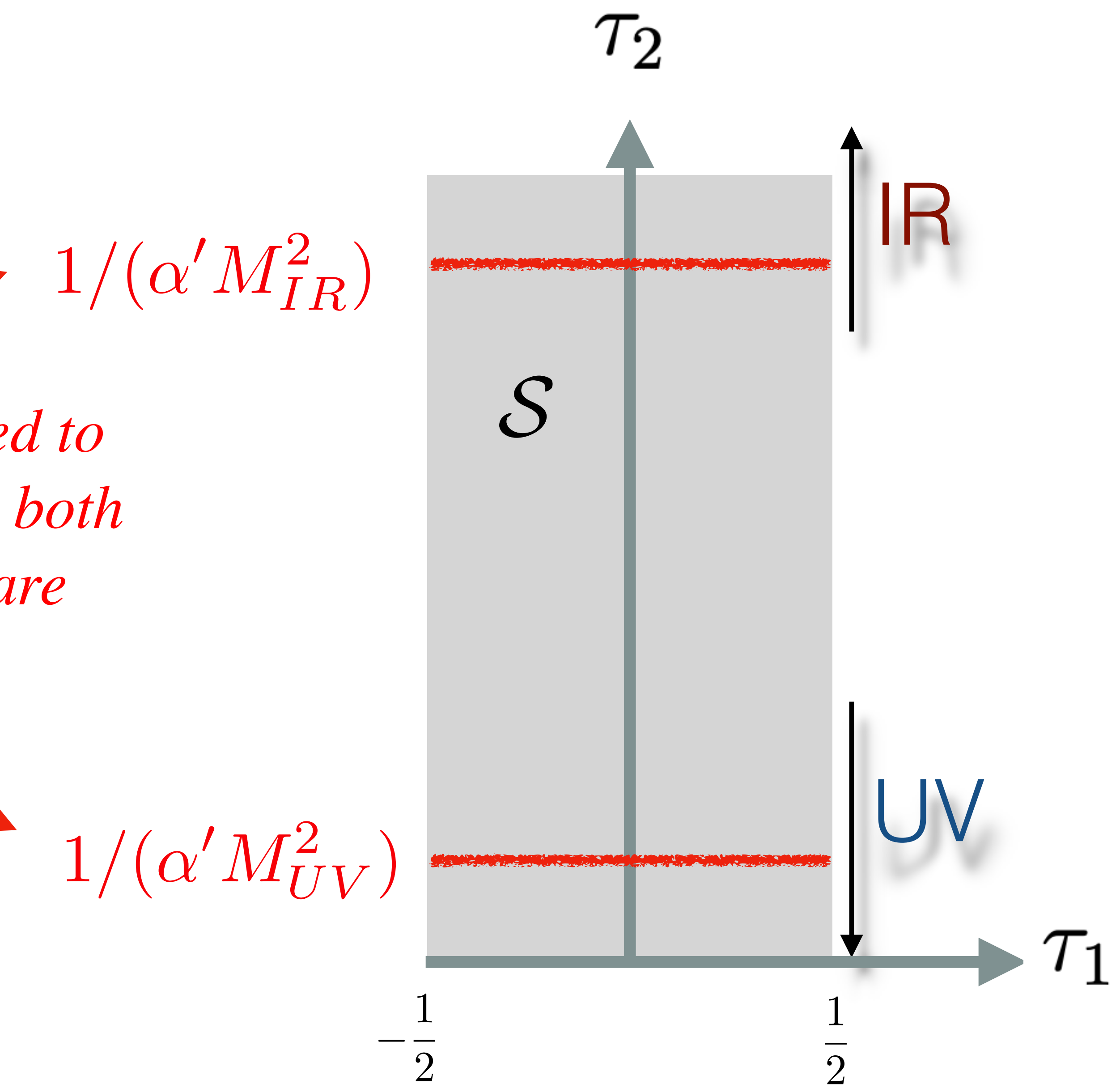


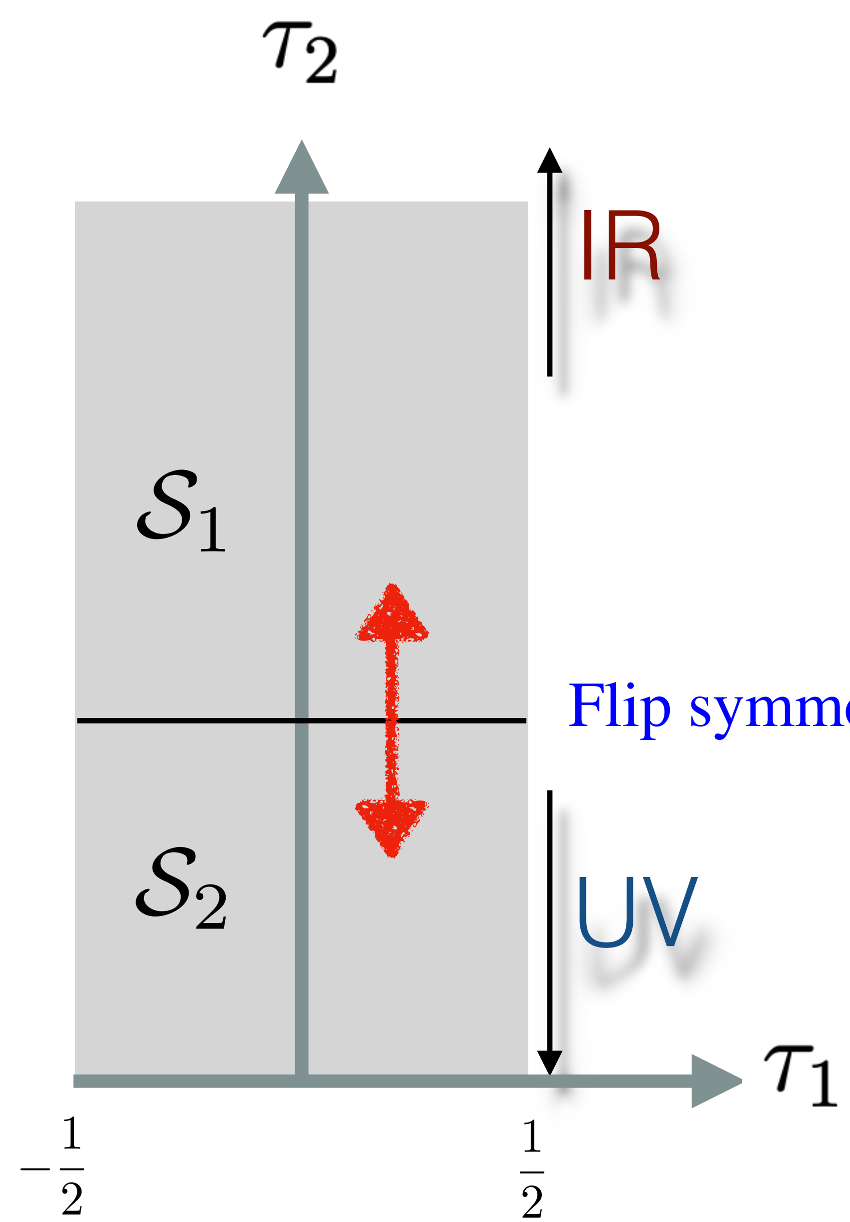
Note that whenever we need to make it finite we may need both UV and IR cut-offs which are independent.

OK, thus far this is all field theory. Why am I doing this?
Suppose our theory had an exact symmetry under $\tau_2 \to 1/\tau_2$

- c.f. SAA+Dondi, 2019

This symmetry is clearly not field-theoretic! But let's pursue it anyway.

- What effects would this have?
- How could we interpret this?



Flip symmetry across τ_2=1line.

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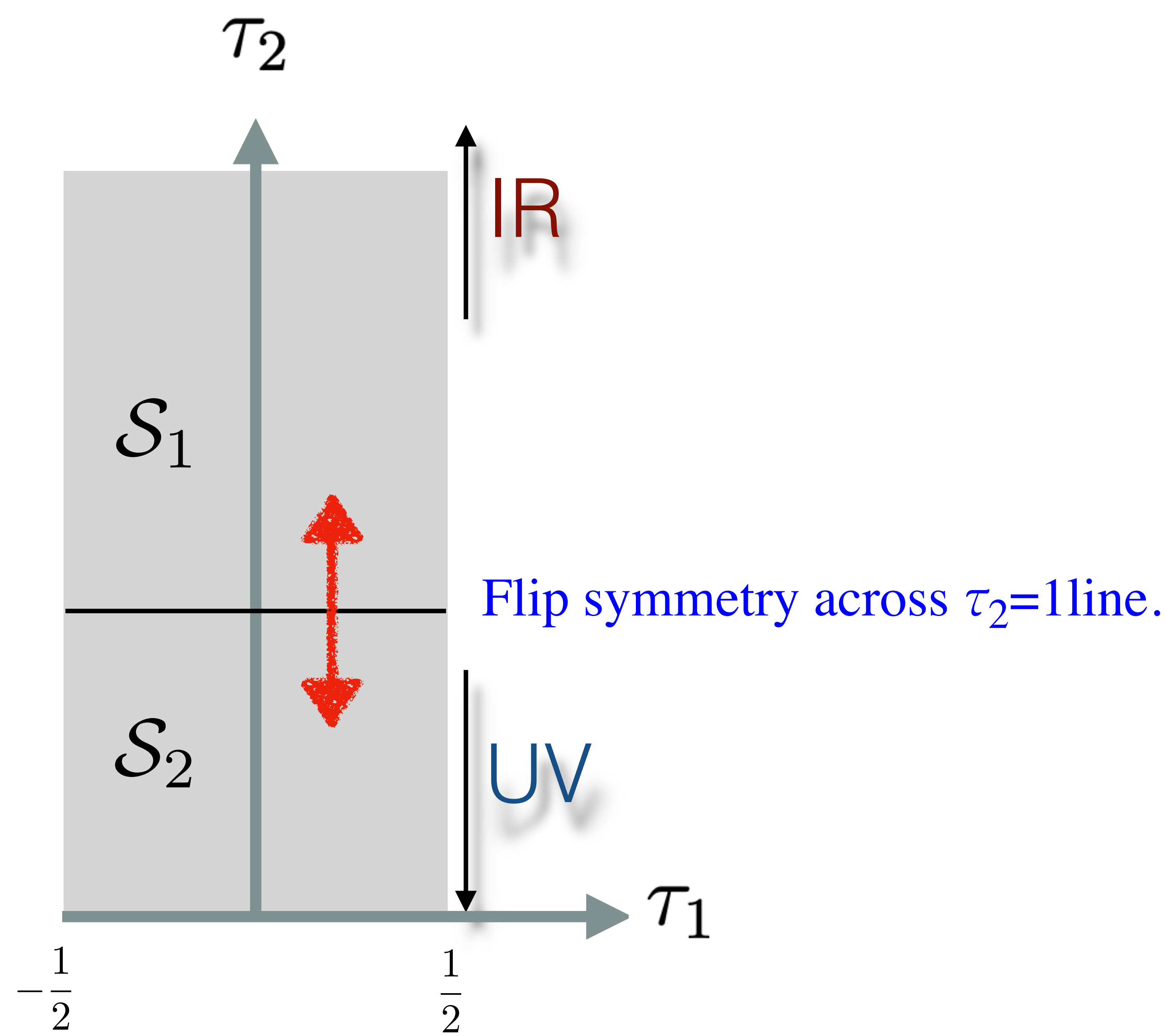
Suppose our theory had an exact symmetry under $\tau_2 \to 1/\tau_2$

- c.f. SAA+Dondi, 2019

Physics from S_1 and S_2 integration regions becomes identical.

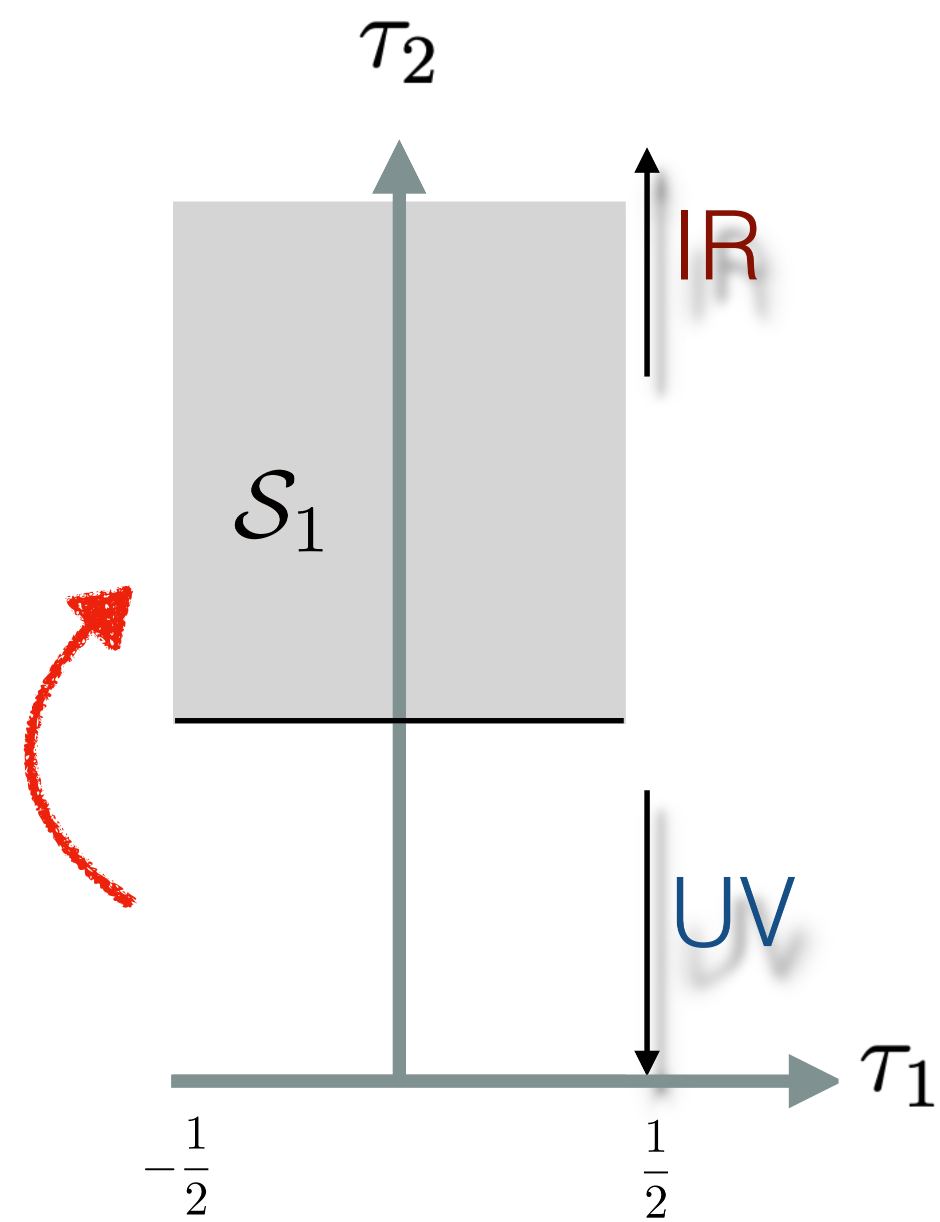
- S_1 and S_2 provide redundant descriptions of the same physics
- Thus, UV divergence must be the same as IR divergence, likewise attributable to same underlying physics.



The symmetry $\tau_2 \to 1/\tau_2$ is a redundancy in the description and we should remove it (and a spurious factor of 2) by folding along the axis of symmetry:

What does this folding imply for UV vs IR?

- There is no longer a notion of increasingly UV or IR "directions" $\to$ all directionality is lost. "Non-orientable"
- The two divergences (UV and IR) have been folded on top of each other
- Thus, ***there is only one divergence.*** You can call it UV or IR according to your choice/convention $\to$ meaningless distinction!



Interesting to see what happens if we try to implement this symmetry?

$$\Lambda = \int_{\mathcal{S}} \frac{d\tau_1 d\tau_2}{\tau_2} Z(\pi\alpha'\tau_2)$$

- Strip is already invariant
- Measure is already invariant
- Thus all that remains is to make partition function invariant in such a theory!

$$Z(\pi\alpha'\tau_2) = Z(\pi\alpha'/\tau_2)$$

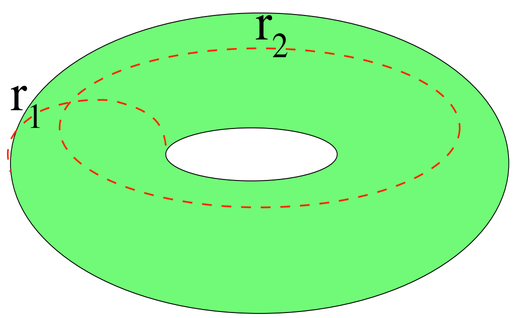
- But this symmetry is very hard to arrange in a particle theory because recall all we have to play with in Z is the spectrum of masses. Seems to require an *infinite tower of states*. For example

$$Z = \frac{1}{\tau_2^2} \sum_{\vec{n}\in\mathbb{Z}^8} e^{-\pi\vec{n}^2\tau_2} = \tau_2^2 \sum_{\vec{n}\in\mathbb{Z}^8} e^{-\pi\vec{n}^2/\tau_2}$$

by Poisson resummation, but this implies an insanely tuned spectrum $M_{\vec{n}}^2 = \vec{n}^2/\alpha'$

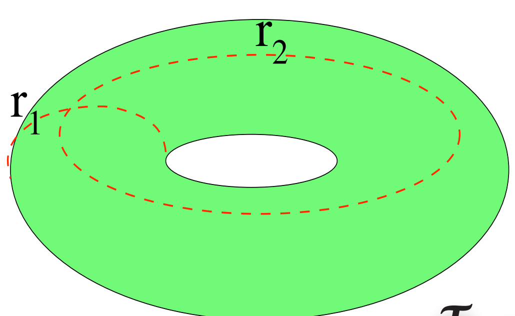
2. Modular invariance

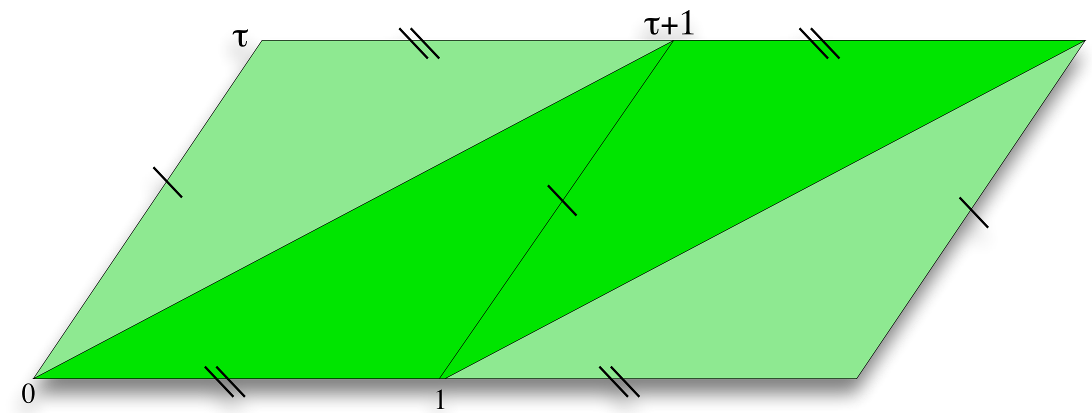
The miracle of string theory can be put down to such insane tuning being inherent *because* the finiteness of the theory is guaranteed (just like in our toy example) by a symmetry, namely modular invariance. Let us revisit the cosmological constant …



Closed string theory instead maps out a torus:

We are only interested in the shape: the torus can be mapped to parallelogram in complex plane with consequently only a single complex parameter, τ, but the theory is invariant under modular transformations:

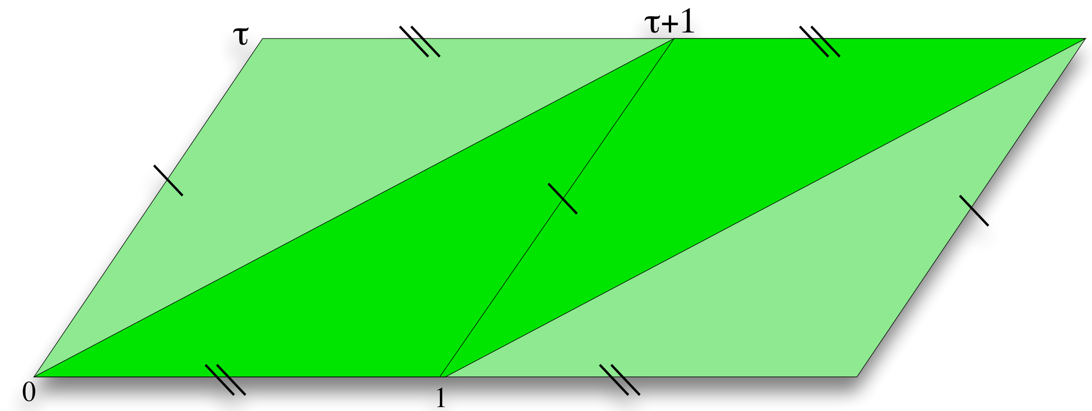




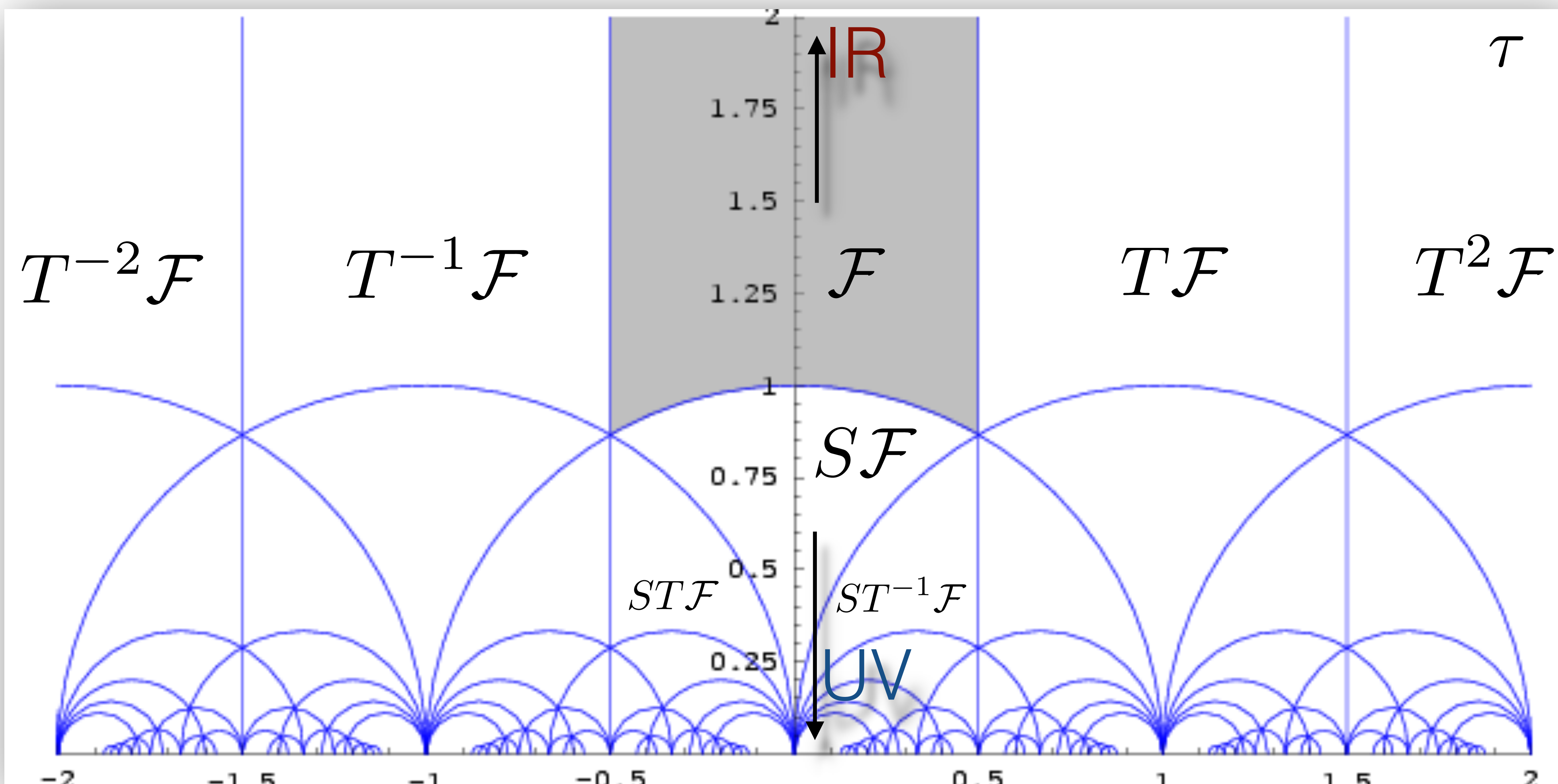
$\tau \rightarrow \tau + 1$ redefines torus :

$\tau \rightarrow -1/\tau$ swops σ_1 and σ_2 and just reorients torus

$T:\ \boldsymbol{\tau} \to \boldsymbol{\tau}+1$ redefines torus :



$S:\ \boldsymbol{\tau} \to -1/\boldsymbol{\tau}$ swops σ_1 and σ_2 and just reorients torus

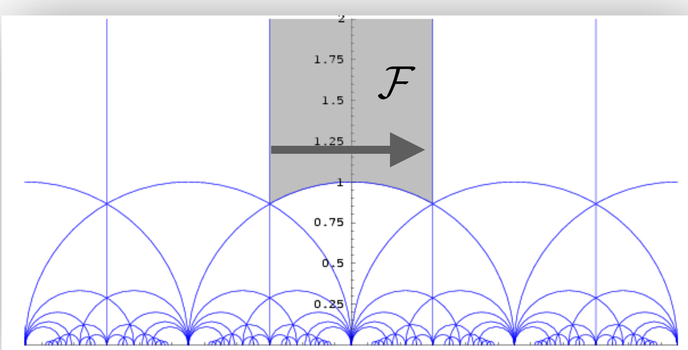


So then we simply have to integrate over all inequivalent tori, i.e. over the complex τ, with the string partition function Z(τ) in place of the particle partition function Z(*t*)

A bit of notation: $\tau = \tau_1 + i\tau_2$ $\left(\mathcal{M}^2 = \frac{1}{4\pi^2\alpha'} = \frac{M_s^2}{4\pi^2}\right)$

$$\Lambda \equiv -\tfrac{1}{2}\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^2} Z(\tau) \qquad q = e^{2\pi i\tau}$$

$$= -\frac{1}{2}\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^{\frac{D}{2}+1}} \sum_{m,n} a_{mn}\overline{q}^m q^n$$



Counts physical (level matched) states weighted by statistics at each level (spectral density)

$$\approx -\frac{1}{2}\mathcal{M}^D \int_{M_{UV}^{-2}}^{\mu_{IR}^{-2}} \frac{d\tau_2}{{\tau_2}^{\frac{D}{2}+1}} \sum_n a_{nn} e^{-\pi\tau_2\alpha' M_n^2}$$

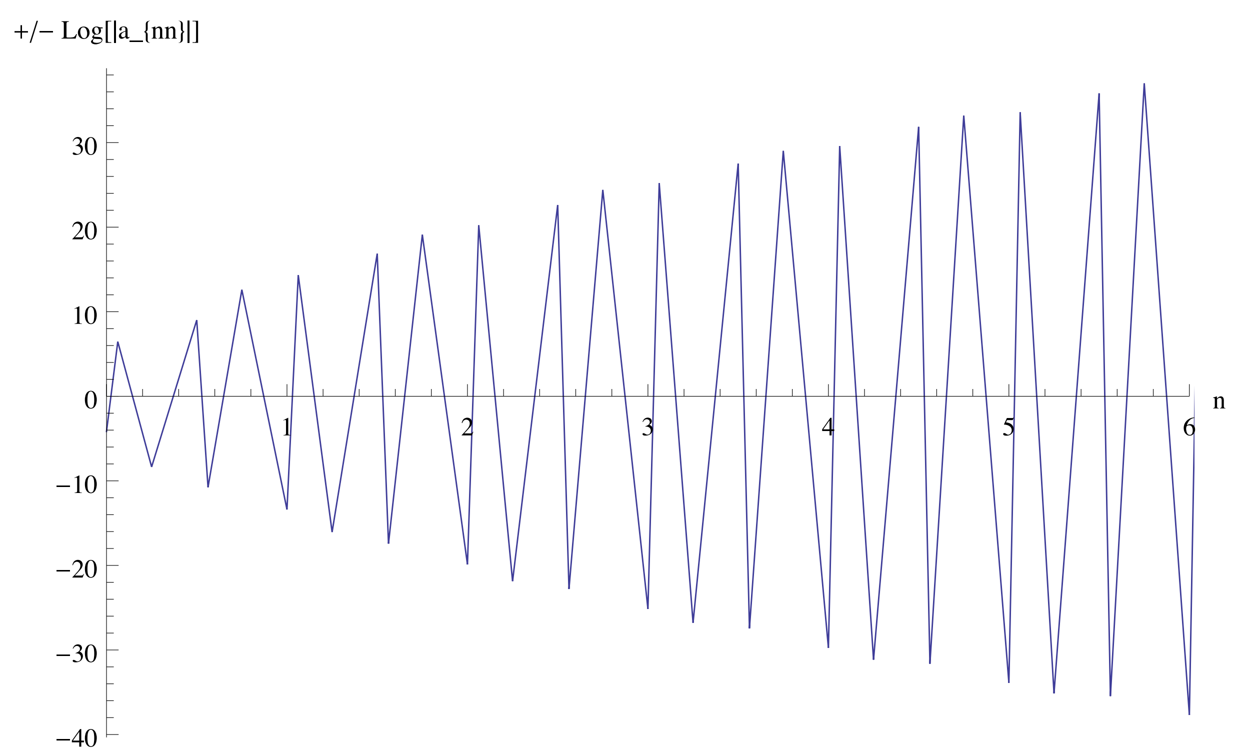
Note that string th. indeed yields particle EFT with $t \equiv \pi\tau_2\alpha'$
But normally we cheat by putting a cut-off and breaking modular invariance!

Crucial observation. Unlike in SUSY, UV-completeness means modular invariance always satisfied! Because of this fact one can find an equivalent to the Coleman Weinberg potential generically, as a supertrace over the infinite tower of ***physical masses***, without ever specifying the theory. Much more natural for what we want to do: even looks similar to the field theory:

- Dienes, Misaligned SUSY, 1994
- Kutasov, Seiberg, 1994
- Dienes, Moshe, Myers 1995

$$\Lambda = \frac{1}{24}\mathcal{M}^2 \mathrm{STr} M^2$$

But note this definitely is *not* a normal field theory object — this supertrace is over the *infinite* string tower of physical states!! e.g. in non-supersymmetric models …



- This crazy spectrum has finite Λ !! !
- Note: modular invariance is so constraining that it has rendered the contribution from *everything* in terms of just level-matched physical states.

This way of understanding the integral can give us amazing insights: to see why let us summarise the (Rankin-Selberg) result that gives it: for any modular invariant function, *F*, we have

$$\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} F(\tau,\overline{\tau}) = \frac{\pi}{3} \operatorname*{Res}_{s=1} \int_0^\infty d\tau_2\, \tau_2^{s-2} \left[\int_{-1/2}^{1/2} d\tau_1\, F(\tau,\overline{\tau}) \right]$$

Inverse Mellin transform

$$= \frac{\pi}{3} \lim_{\tau_2\to 0} \int_{-1/2}^{1/2} d\tau_1\, F(\tau,\overline{\tau}) \; = \frac{\pi}{3} \lim_{\tau_2\to 0} g(\tau_2)$$

where in this case $g(\tau_2) = -\frac{\mathcal{M}^4}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \mathcal{Z}(\tau)$

$$= -\frac{\mathcal{M}^4}{2} \tau_2^{-1} \mathrm{Str} e^{-\pi\tau_2\alpha' M^2}$$

is the particle-like partition function: the limit looks superficially like it should diverge!

So the incredible fact that this infinite supertrace is finite can be put down to the fact that the spectral density functions behaves as follows as $\tau_2 \to 0$:

$$g(\tau_2) \sim \tau_2^{-1} \mathrm{Str}\,(e^{-\pi\tau_2\alpha' M^2}) \longrightarrow c_0$$

In other words ***Str(1)=0 EVEN IF SUSY BROKEN!*** This explains why there is no term that is quartic in the string scale. This looks like the sort of magic cancellation we would like to solve the hierarchy problem.

Note we have a natural definition of a stringy supertrace appropriate for theories with infinite towers of states:

$$\mathrm{Str}\, X \equiv \lim_{y\to 0} \sum_{\substack{\text{physical}\\ \text{states}}} (-1)^F X e^{-yM^2/M_s^2}$$

The supertrace relation Str*(1)=0* is thus an example of a ***magic cancellation*** that runs across the entire string spectrum, suppressing divergences and/or ensuring the finiteness of string amplitudes relative to naive QFT expectations.

It turns out the Str*(1)=0* relation is just the tip of the iceberg!

- SAA, Dienes, Nutricati to appear

$$\langle X\rangle = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z_X(\tau) \qquad Z_X = \sum_{nm} a_{mn} X\, \overline{q}^m q^n$$

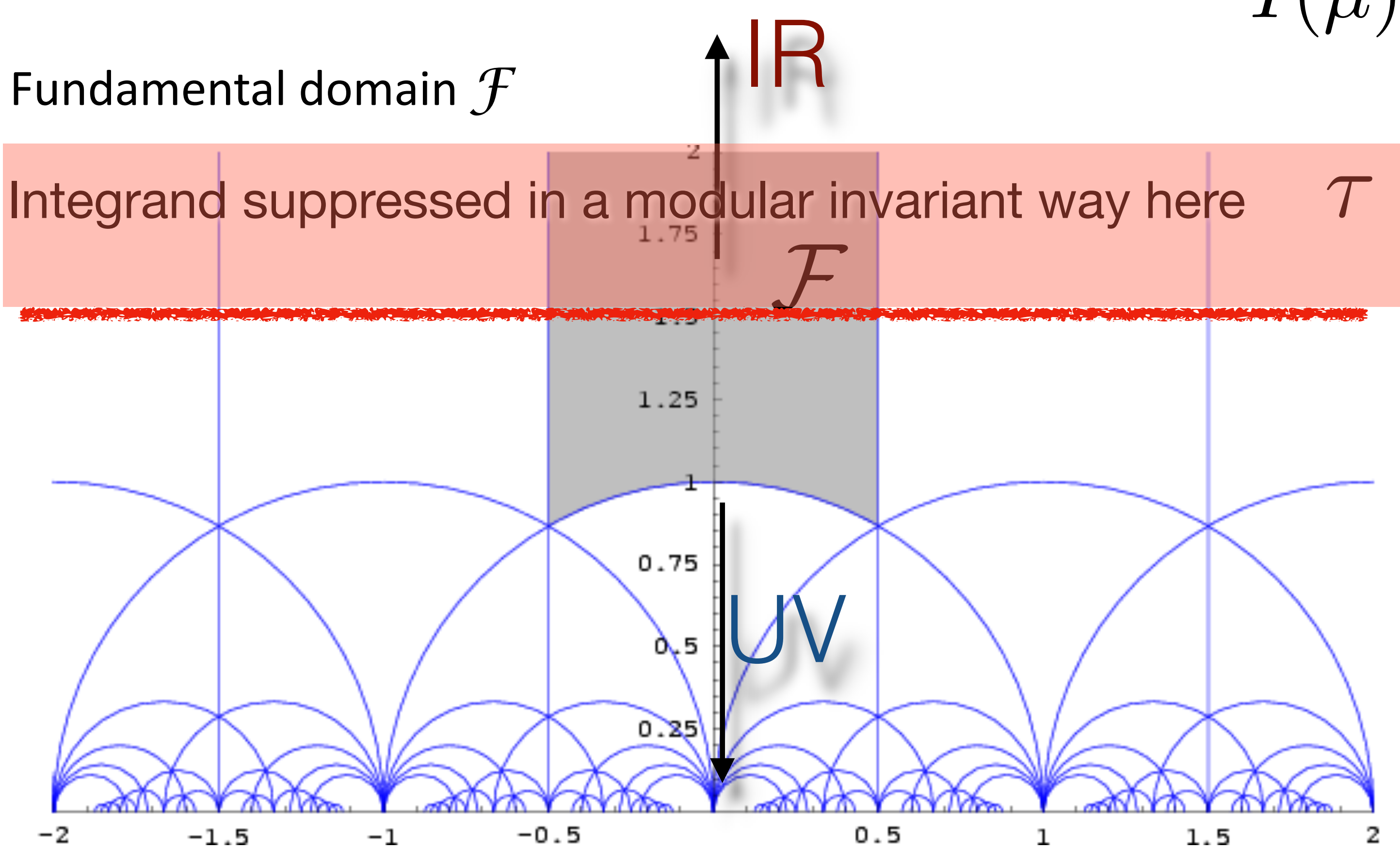
where X is *any* insertion of operators into the sum that respects modular invariance and is a function only of τ_2. (NB: it doesn't have to correspond to any physical amplitude). We know the integral is finite as it can always be integrated over the fundamental domain (modulo IR divergences which we can handle) and hence …

$$\mathrm{Str}\, X \ = \ 0 \qquad \langle X\rangle \ = \ -\frac{1}{12\mathcal{M}^2}\,\mathrm{Str}(XM^2)$$

3. Extracting an EFT?

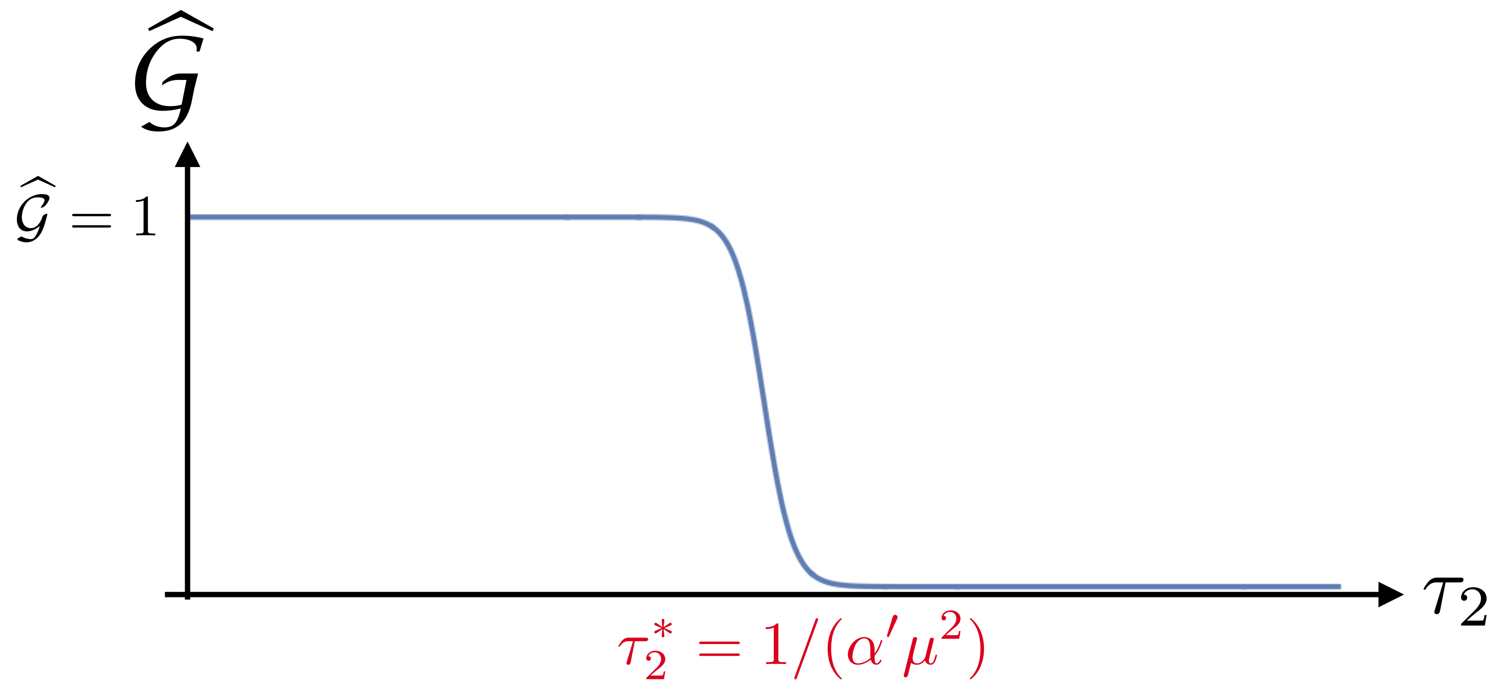
We must abandon the idea of *choosing* the EFT and see how an EFT *emerges* from string theory renormalisation by defining it with a modular invariant "Wilsonian" cut-off instead:

$$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$$



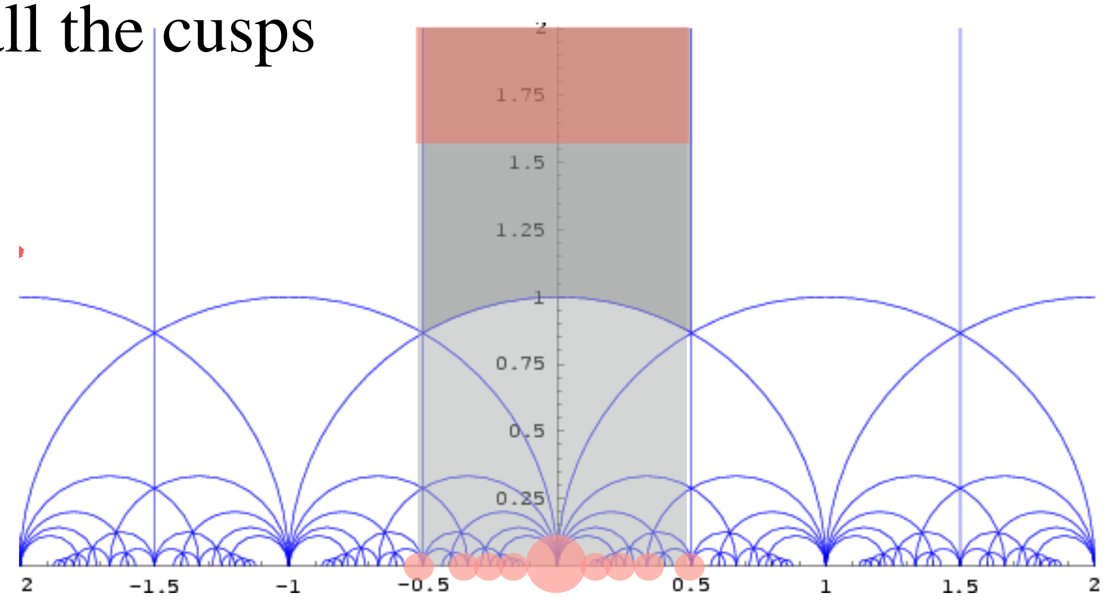
Required properties of Wilsonian regulator, $\widehat{\mathcal{G}}$: $$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$$

- a) Is itself a modular function
- b) Should look like this ….



- c) Remember, our goal is to write everything as a supertrace which ultimately means an integral over the critical strip … all the cusps are quenched equally. In other words: all the cusps are image IR cusps, implying…

$$\tau_2^* \equiv 1/\tau_2^* \implies \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau}) = \widehat{\mathcal{G}}(M_s^2/\mu,\tau,\overline{\tau})$$



- Adapted a circle partition function regulator of

(Kiritsis, Kounnas)

The result is a smooth modular invariant stringy Coleman-Weinberg potential

Infinite sum of Bessel functions, which pile up to yield the following CW potential:

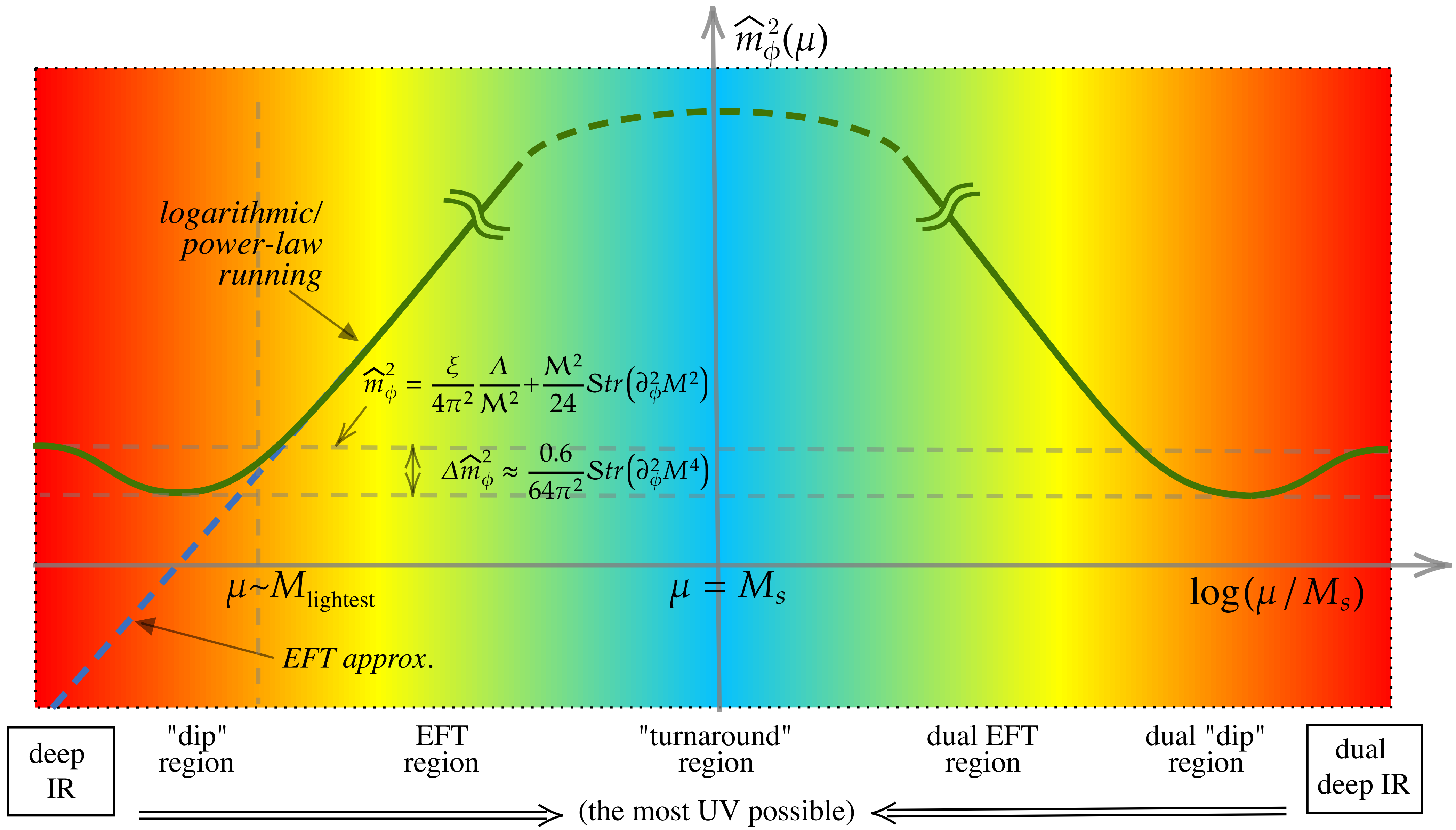
$$\widehat{\Lambda}(\mu,\phi) = \frac{1}{24}\mathcal{M}^2\,\mathrm{Str}\,M^2 - c'\,\underset{M\gtrsim\mu}{\mathrm{Str}}\,M^2\mu^2 \quad - \underset{0\leq M\lesssim\mu}{\mathrm{Str}}\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{\mu^2}\right)+c''\mu^4\right]$$

$c = 2e^{2\gamma+1/2}$, $c' = 1/(96\pi^2)$, and $c'' = 7c'/10$.

Note the EFT adjusts automatically. Moreover the masses of Higgses are running:

$$\widehat{m}_\phi^2 = \frac{\xi}{4\pi^2}\frac{\widehat{\Lambda}(\mu)}{\mathcal{M}^2} + \partial_\phi^2\widehat{\Lambda}(\mu)$$

With this definition of RG, the Higgs mass begins at a UV value we can calculate, has RG running, maybe GUT breaking, EW and QCD phase transition, yada yada yada. But then it must eventually wind up at the exact same value in the IR. And everything is finite. Like this…



$$\lim_{\mu\to 0} \hat{m}_\phi^2(\mu) = \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} + \frac{1}{24}\mathcal{M}^2 \mathrm{Str}\,\partial_\phi^2 M^2$$

We can perform the same procedure for all the other couplings. e.g. the gauge couplings… e.g. in a model with 2 toroidal dimensions the threshold is the famous result of Dixon, Kaplunovsky and Louis. But note we get the entire energy dependence in Bessels!!

SAA, Dienes, Nutricati

$$
\begin{aligned}
\widehat{\Delta}_G \;=\; & \frac{-1}{1+a^2\rho}\Bigg\{ \log(cT_2U_2|\eta(T)\eta(U)|^4) \;+\; 2\log\sqrt{\rho}a \\
& + \frac{8}{\rho-1}\sum_{\gamma,\gamma'\in\Gamma_\infty\backslash\Gamma}\Bigg[\tilde{\mathcal{K}}_0^{(0,1)}\left(\frac{2\pi}{a\sqrt{\gamma\cdot T_2\gamma'\cdot U_2}}\right) \\
& - \frac{1}{\rho}\tilde{\mathcal{K}}_1^{(1,2)}\left(\frac{2\pi}{a\sqrt{\gamma\cdot T_2\gamma'\cdot U_2}}\right)\Bigg]\Bigg\}\,,
\end{aligned}
$$

where

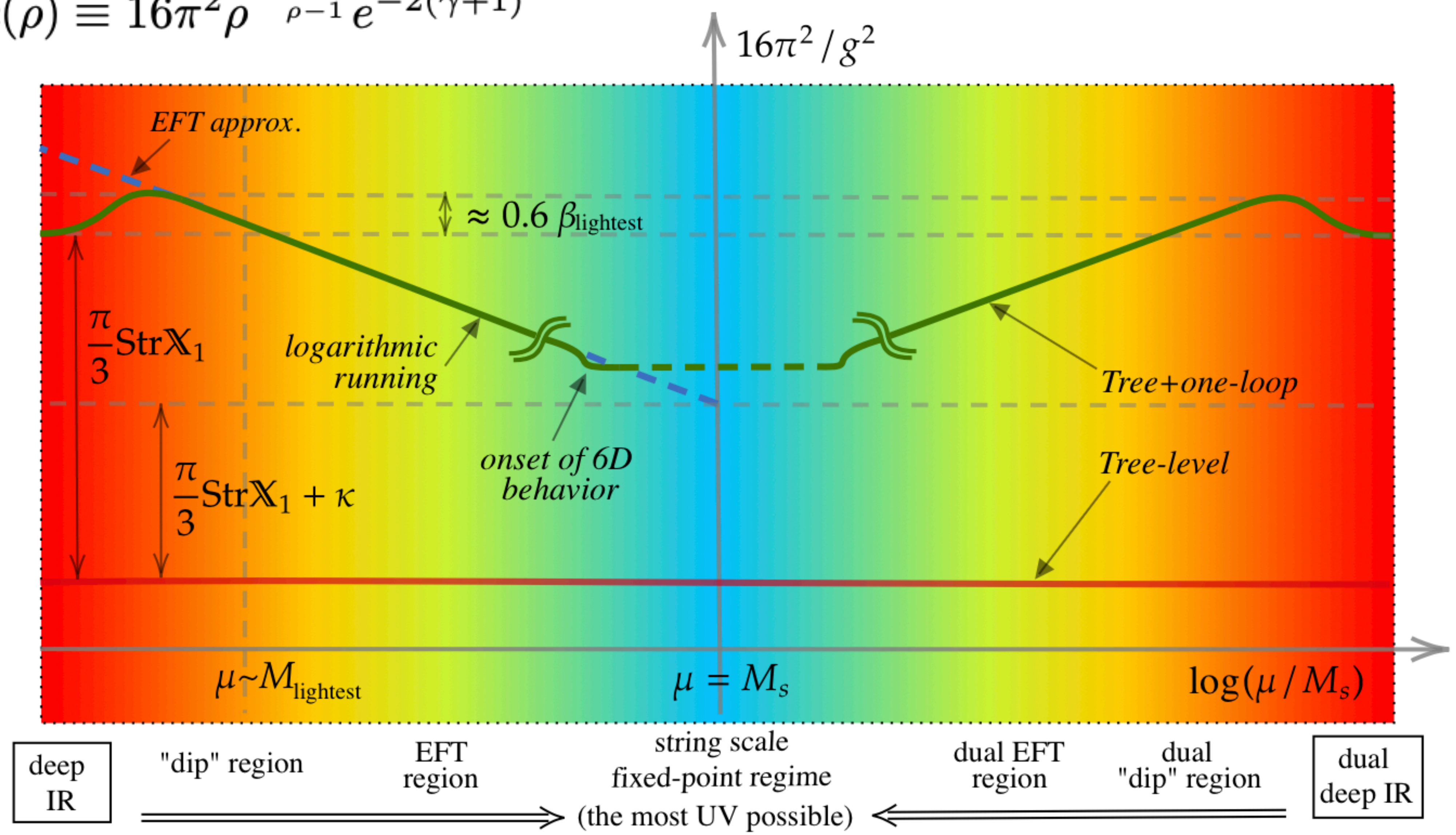
$$
\tilde{\mathcal{K}}_\nu^{(n,p)}(z,\rho) \;=\; \sum_{k,r=1}^{\infty}(krz)^n\Big(K_\nu(krz/\rho)-\rho^pK_\nu(krz)\Big)
$$

$$
c(\rho)\equiv 16\pi^2\rho^{-\frac{\rho+1}{\rho-1}}e^{-2(\gamma+1)}
$$

Again need asymptotics of infinite sums of Bessels:

$$\widehat{\Delta}_G \overset{a\to 0}{\approx} -\log(cT_2U_2|\eta(T)\eta(U)|^4) - 2\log\left(\frac{\mu}{M_s}\right)$$

where $c(\rho) \equiv 16\pi^2 \rho^{-\frac{\rho+1}{\rho-1}} e^{-2(\gamma+1)}$



Note there is no power law running even though 2 extra dimensions — and cannot be.

Conclusions

- We have developed a general supertrace formula for understanding the EFT in modular invariant theories.
- A modular invariant regulator provides a natural Wilsonian cut-off and definition of RG scale. Allows us to understand how an EFT emerges from UV/IR mixed UV complete theory.
- Relevant operators like the Higgs mass can be thought of as "running" to its cusp value: this is both the UV and IR asymptote.
- All hierarchy problems reduce to accidental symmetries in leading terms that appear as magic emergent cancellations in the IR.
- The Weak/Planck and cosmological constant hierarchy problems are connected
- A single stringy naturalness condition:

$$\mathrm{Str}\,\partial_\phi^2 M^2 \lesssim \frac{24}{\mathcal{M}^2} M_W^2$$

- Can be thought of as a solution to technical hierarchy problem (at one-loop)
- But beyond this and more generally Rankin-Selberg technique implies many non-SUSY supertrace relations beyond Str(1)=0.
- These relations have nothing to do with any SUSY.
- Such relations are preserved exactly (because modular invariance is exact).
- Seems a promising starting point for UV complete discussions of naturalness.

Thank you !!