Corfu BSM workshop, 30th August 2023

Potential positivity bounds

Tevong You

2308.06226 Davighi, Melville, Mimasu, **TY**

Corfu BSM workshop, 30th August 2023

Positively light Higgs

Tevong You

2308.06226 Davighi, Melville, Mimasu, **TY**

Outline

Our idea here of positivity bounds on scalar potentials for fine-tuned ratios of operator coefficients is very simple. I will focus on the *context* of applying it to the Higgs.

Outline

Naturalness

Effective Field Theory (EFT)

Positivity bounds on EFT

Positivity bounds on scalar potentials

Positively light Higgs

Naturalness

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - naturalness after LHC

Example 2

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{\rm Coulomb}. \qquad \Delta E_{\rm Coulomb} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}.$$

Avoiding cancellation between “bare” mass and divergent self-energy in classical electrodynamics requires new physics around

$$e^2/(4\pi\varepsilon_0 m_e c^2) = 2.8 \times 10^{-13}\ \text{cm}$$

Indeed, the positron and quantum-mechanics appears just before!

$$\Delta E = \Delta E_{\rm Coulomb} + \Delta E_{\rm pair} = \frac{3\alpha}{4\pi} m_e c^2 \log\frac{\hbar}{m_e c r_e}$$

Naturalness

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - naturalness after LHC

Example 3

Divergence in pion mass: $$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi}\Lambda^2$$

Experimental value is $m_{\pi^\pm}^2 - m_{\pi_0}^2 \sim (35.5\,\mathrm{MeV})^2$

Expect new physics at $\Lambda \sim 850$ MeV to avoid fine-tuned cancellation.

ρ meson appears at 775 MeV!

Naturalness

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - naturalness after LHC

<u>Example 4</u>

Divergence in Kaons mass difference in a theory with only up, down, strange:

$$m_{K_L^0} - m_{K_S^0} =\simeq \frac{1}{16\pi^2} m_K f_K^2 G_F^2 \sin^2\theta_C \cos^2\theta_C \times \Lambda^2$$

Avoiding fine-tuned cancellation requires $\Lambda < 3$ GeV.

Gaillard & Lee in 1974 predicted the charm quark mass!

Naturalness

Take fine-tuning problems seriously.

e.g. 2205.05708 N. Craig - Snowmass review,
1307.7879 G. Giudice - naturalness after LHC

Higgs?

Higgs also has a quadratically divergent contribution to its mass

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2}\left(-6y_t^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda\right)$$

Avoiding fine-tuned cancellation requires $\Lambda < O(100)$ GeV??

As Λ is pushed to the TeV scale by null results, tuning is around 10% - 1%.

Note: in the SM the Higgs mass is a parameter to be measured, not calculated. What the quadratic divergence represents (independently of the choice of renormalisation scheme) is the fine-tuning in an underlying theory in which we expect the Higgs mass to be calculable.

Naturalness

Gauge theories have the quality we seek in a satisfying theory.

In contrast, everything to do with the Higgs in the SM is arbitrary; more like a parametrisation than an explanation of electroweak symmetry breaking.

We seek to better understand the origin of the Higgs in an underlying theory from which it emerges, where we can calculate its potential in terms of more fundamental principles (*c.f.* condensed-matter Higgs)

Avoiding fine-tuning in underlying theory = expect such new physics to appear close to the weak scale!

Naturalness

Take fine-tuning problems seriously, but new physics appears not to be at the weak scale.

Symmetry may not be the answer. Cosmological dynamics? UV-IR?

When past successful approaches fail, *we have a chance to learn something genuinely new.*

This talk: Perhaps we live in a fine-tuned corner of EFT parameter space where the Higgs *must* be light as a consequence!

Effective Field Theory

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \dots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

Effective Field Theory

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

Effective Field Theory

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

1960s point of view: renormalisability of a *finite* number of parameters is essential

Modern point of view: our QFTs are really EFTs - include *all* operators allowed by symmetries

Effective Field Theory

e.g. QED as an EFT includes Fermi theory and Euler-Heisenberg dimension-8 operators

$$\mathcal{L}^{EFT}_{QED} = \bar{\Psi} i \gamma^\mu D_\mu \Psi - m \bar{\Psi}\Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Fermi theory
(1933)

$$+ \sum_i \frac{c_6^{(i)}}{\Lambda^2} (\bar{\Psi} \Gamma \Psi)(\bar{\Psi} \Gamma \Psi) \qquad \Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}\}$$

Euler-Heisenberg
(1936)

$$+ \frac{c_8^{(1)}}{\Lambda^4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{c_8^{(2)}}{\Lambda^4} F_{\mu\nu} F^{\nu\rho} F_{\rho\lambda} F^{\lambda\mu} + \ldots$$

Effective Field Theory

e.g. QED as an EFT includes Fermi theory and Euler-Heisenberg dimension-8 operators

$$\mathcal{L}^{EFT}_{QED} = \bar{\Psi} i \gamma^\mu D_\mu \Psi - m \bar{\Psi}\Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Fermi theory (1933)

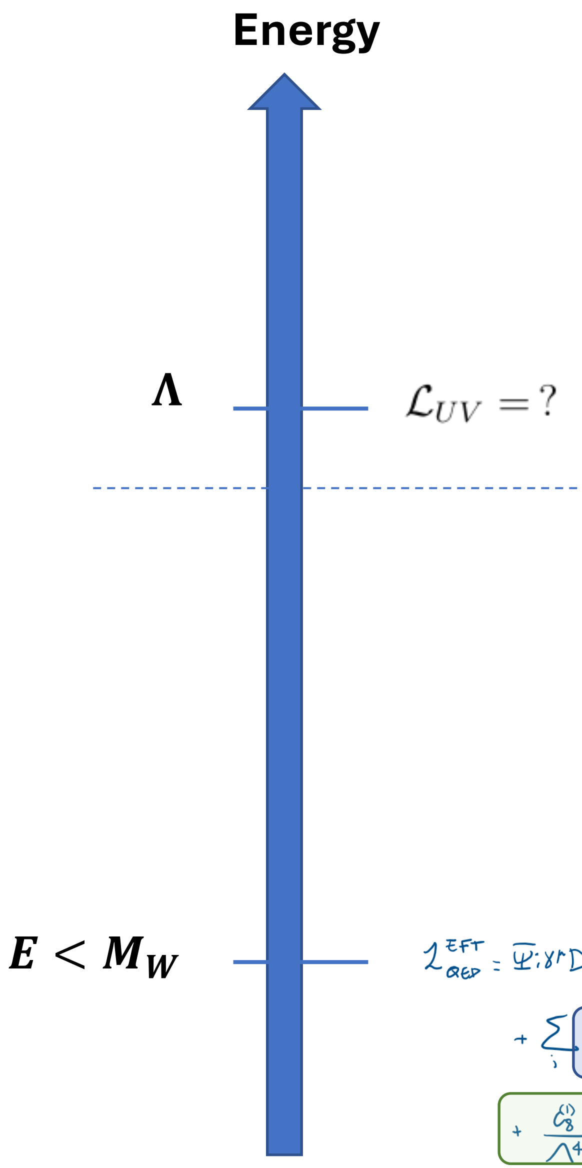
$$+ \sum_i \frac{c_6^{(i)}}{\Lambda^2} (\bar{\Psi} \Gamma \Psi)(\bar{\Psi} \Gamma \Psi) \qquad \Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}\}$$

Euler-Heisenberg (1936)

$$+ \frac{c_8^{(1)}}{\Lambda^4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{c_8^{(2)}}{\Lambda^4} F_{\mu\nu} F^{\nu\rho} F_{\rho\lambda} F^{\lambda\mu} + \ldots$$

Wilson coefficients generated by UV physics

Effective Field Theory

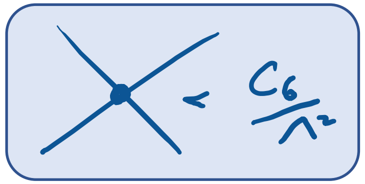


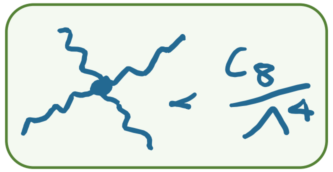
$$\mathcal{L}_{QED}^{EFT} = \bar{\Psi} i \gamma^\mu D_\mu \Psi - m \bar{\Psi}\Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \sum_i \frac{c_6^{(i)}}{\Lambda^2} (\bar{\Psi} \Gamma \Psi)(\bar{\Psi} \Gamma \Psi)$$

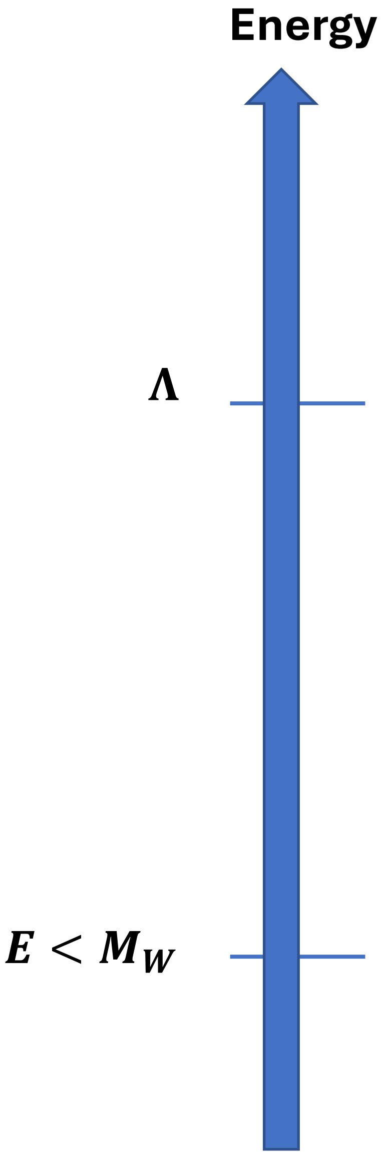
$\Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$

$$+ \frac{c_8^{(1)}}{\Lambda^4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{c_8^{(2)}}{\Lambda^4} F_{\mu\nu} F^{\nu\rho} F_{\rho\lambda} F^{\lambda\mu} + \ldots$$





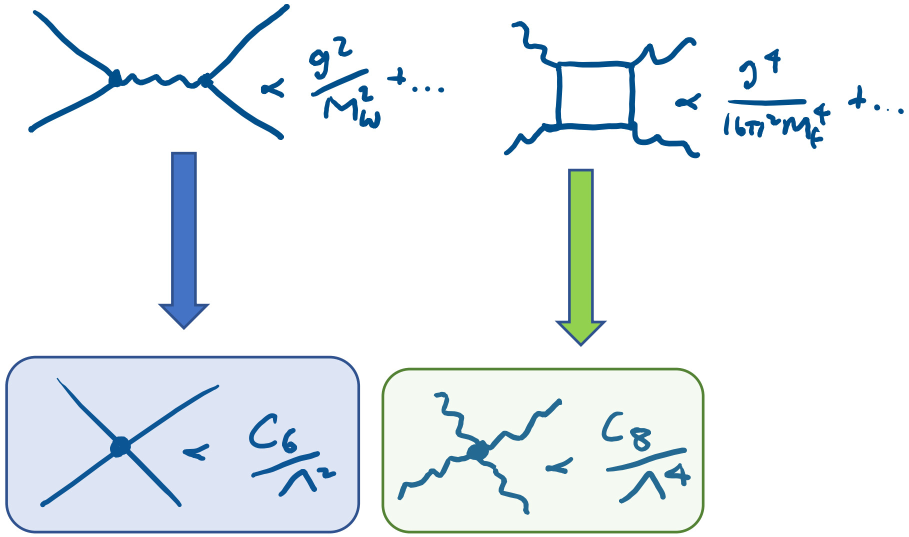
Effective Field Theory



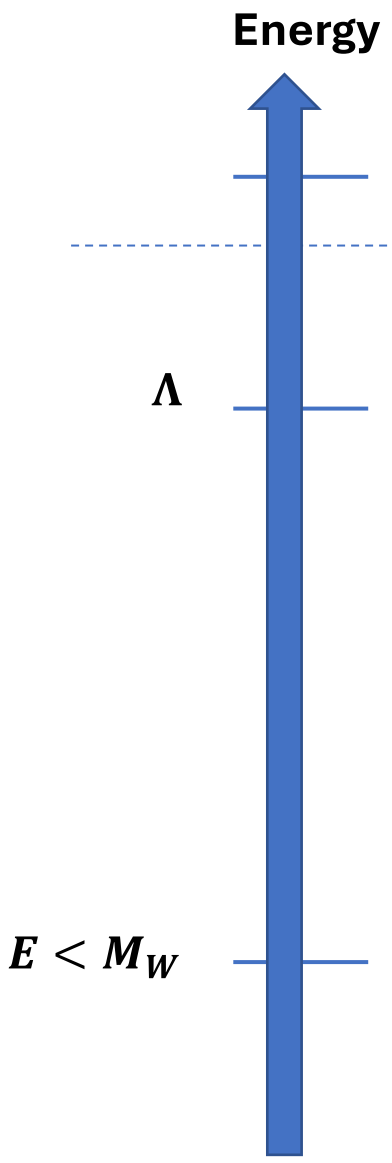
$$\mathcal{L}_{SM} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y \quad ,$$

$$\mathcal{L}_m = \bar{Q}_L i\gamma^\mu D^L_\mu Q_L + \bar{q}_R i\gamma^\mu D^R_\mu q_R + \bar{L}_L i\gamma^\mu D^L_\mu L_L + \bar{l}_R i\gamma^\mu D^R_\mu l_R$$
$$\mathcal{L}_G = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}$$
$$\mathcal{L}_H = (D^L_\mu\phi)^\dagger(D^{L\mu}\phi) - V(\phi)$$
$$\mathcal{L}_Y = y_d\bar{Q}_L\phi q^d_R + y_u\bar{Q}_L\phi^c q^u_R + y_L\bar{L}_L\phi l_R + \text{h.c.} \quad ,$$

$$\mathcal{L}^{EFT}_{QED} = \bar{\Psi} i\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+ \sum_i \frac{C_6^{(i)}}{\Lambda^2}(\bar{\Psi}\Gamma\Psi)(\bar{\Psi}\Gamma\Psi) \qquad \Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$$
$$+ \frac{C_8^{(1)}}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{C_8^{(2)}}{\Lambda^4}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} + \ldots$$



Effective Field Theory



$$\mathcal{L}_{UV} = ?$$

$$\mathcal{L}_{SM}^{\text{EFT}} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \frac{c_8}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

$$\begin{aligned}
\mathcal{L}_m &= \bar{Q}_L i\gamma^\mu D_\mu^L Q_L + \bar{q}_R i\gamma^\mu D_\mu^R q_R + \bar{L}_L i\gamma^\mu D_\mu^L L_L + \bar{l}_R i\gamma^\mu D_\mu^R l_R \\
\mathcal{L}_G &= -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} \\
\mathcal{L}_H &= (D_\mu^L \phi)^\dagger (D^{L\mu}\phi) - V(\phi) \\
\mathcal{L}_Y &= y_d \bar{Q}_L \phi q_R^d + y_u \bar{Q}_L \phi^c q_R^u + y_L \bar{L}_L \phi l_R + \text{h.c.} \qquad ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{QED}^{EFT} &= \bar{\Psi} i\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&+ \sum_i \frac{c_6^{(i)}}{\Lambda^2}(\bar{\Psi}\Gamma\Psi)(\bar{\Psi}\Gamma\Psi) \qquad \Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\} \\
&+ \frac{c_8^{(1)}}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{c_8^{(2)}}{\Lambda^4}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} + \ldots
\end{aligned}$$

Effective Field Theory

Energy

$\mathcal{L}_{UV} = ?$

Λ

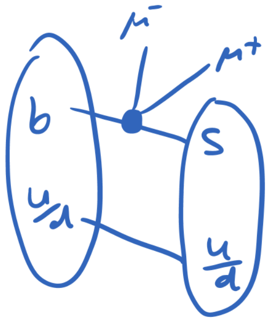
$$\mathcal{L}_{SM}^{\text{EFT}} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \frac{c_8}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

$$\mathcal{L}_m = \bar{Q}_L i\gamma^\mu D_\mu^L Q_L + \bar{q}_R i\gamma^\mu D_\mu^R q_R + \bar{L}_L i\gamma^\mu D_\mu^L L_L + \bar{l}_R i\gamma^\mu D_\mu^R l_R$$

$$\mathcal{L}_G = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu}$$

$$\mathcal{L}_H = (D_\mu^L \phi)^\dagger (D^{L\mu}\phi) - V(\phi)$$

$$\mathcal{L}_Y = y_d \bar{Q}_L \phi q_R^d + y_u \bar{Q}_L \phi^c q_R^u + y_L \bar{L}_L \phi l_R + \text{h.c.} \quad ,$$



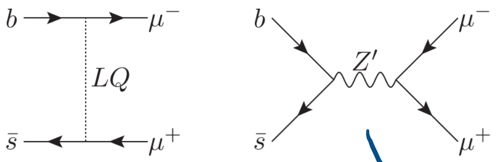
$E < M_W$

$$\mathcal{L}_{QED}^{EFT} = \bar{\Psi} i\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+ \sum_i \frac{c_6^{(i)}}{\Lambda^2}(\bar{\Psi}\Gamma\Psi)(\bar{\Psi}\Gamma\Psi) \qquad \Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$$
$$+ \frac{c_8^{(1)}}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{c_8^{(2)}}{\Lambda^4}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} + \ldots$$

Effective Field Theory

Energy

$$\mathcal{L}_{UV} = ?$$



Λ

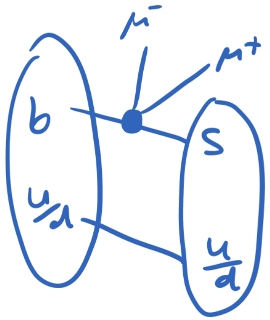
$$\mathcal{L}_{SM}^{\text{EFT}} = \mathcal{L}_m + \mathcal{L}_g + \mathcal{L}_h + \mathcal{L}_y + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \frac{c_8}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

$$\mathcal{L}_m = \bar{Q}_L i\gamma^\mu D^L_\mu Q_L + \bar{q}_R i\gamma^\mu D^R_\mu q_R + \bar{L}_L i\gamma^\mu D^L_\mu L_L + \bar{l}_R i\gamma^\mu D^R_\mu l_R$$

$$\mathcal{L}_G = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}$$

$$\mathcal{L}_H = (D^L_\mu\phi)^\dagger(D^{L\mu}\phi) - V(\phi)$$

$$\mathcal{L}_Y = y_d\bar{Q}_L\phi q^d_R + y_u\bar{Q}_L\phi^c q^u_R + y_L\bar{L}_L\phi l_R + \text{h.c.} \quad ,$$



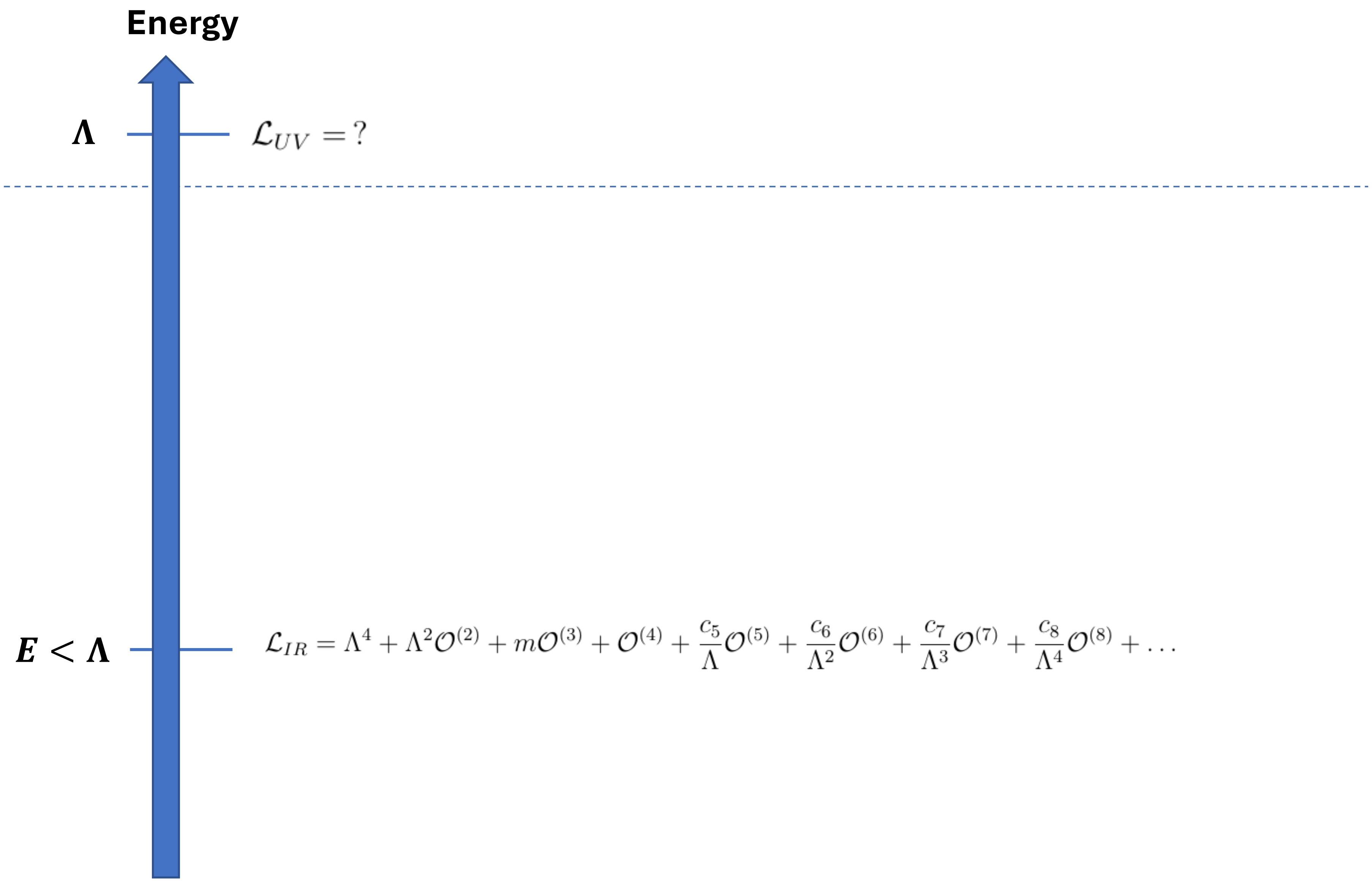
$E < M_W$

$$\mathcal{L}^{EFT}_{QED} = \bar{\Psi} i\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

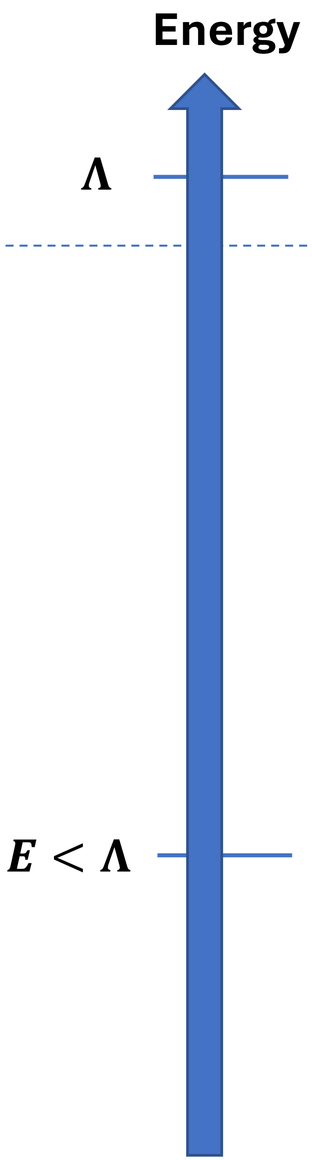
$$+ \sum_i \frac{c_6^{(i)}}{\Lambda^2}(\bar{\Psi}\Gamma\Psi)(\bar{\Psi}\Gamma\Psi) \qquad \Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$$

$$+ \frac{c_8^{(1)}}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{c_8^{(2)}}{\Lambda^4}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} + \ldots$$

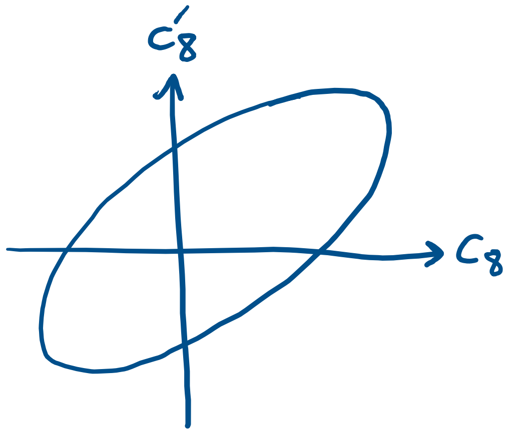
EFT Positivity Bounds



EFT Positivity Bounds

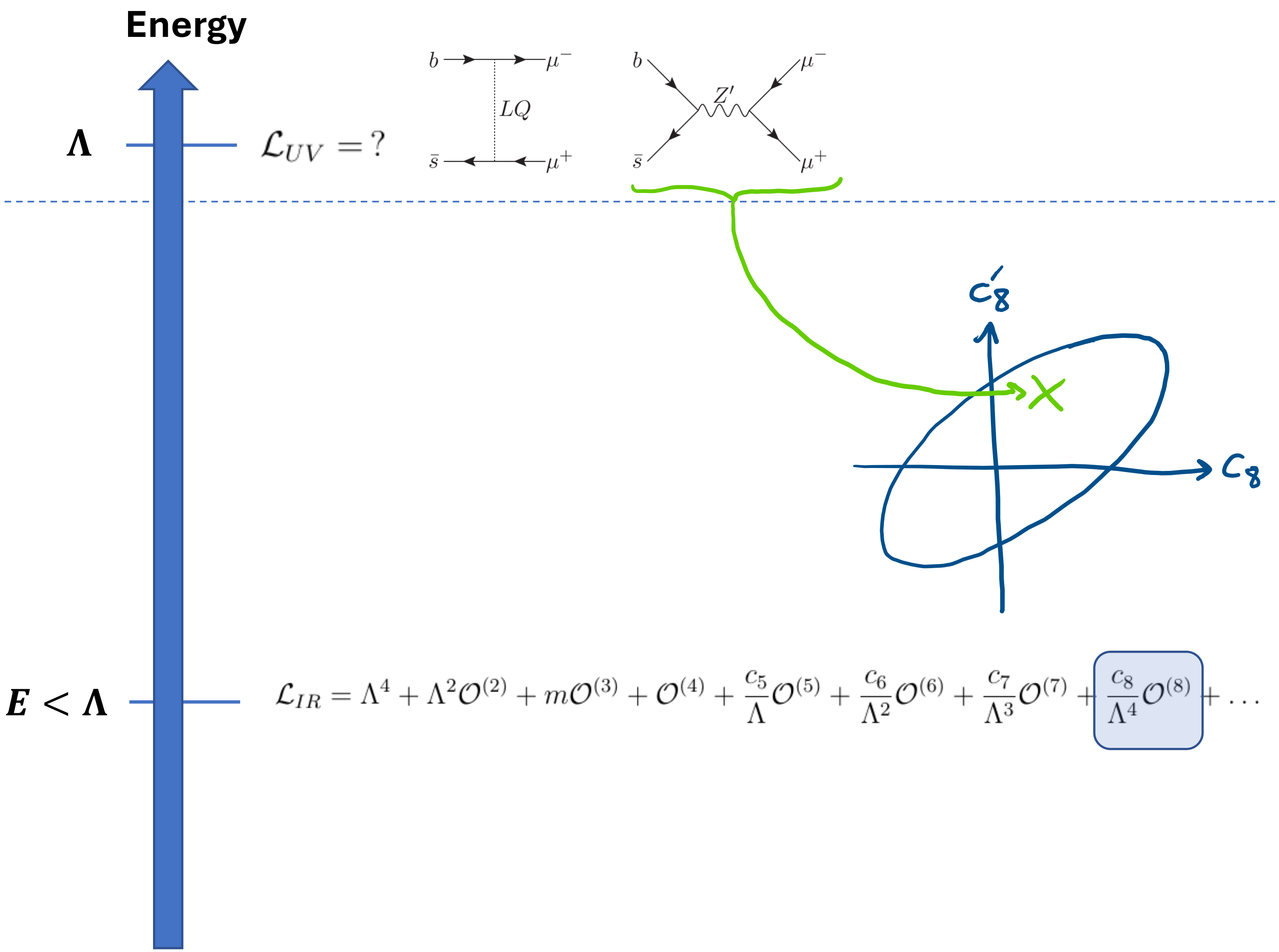


$$\mathcal{L}_{UV} = ?$$



$$\mathcal{L}_{IR} = \Lambda^4 + \Lambda^2\mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \boxed{\frac{c_8}{\Lambda^4}\mathcal{O}^{(8)}} + \ldots$$

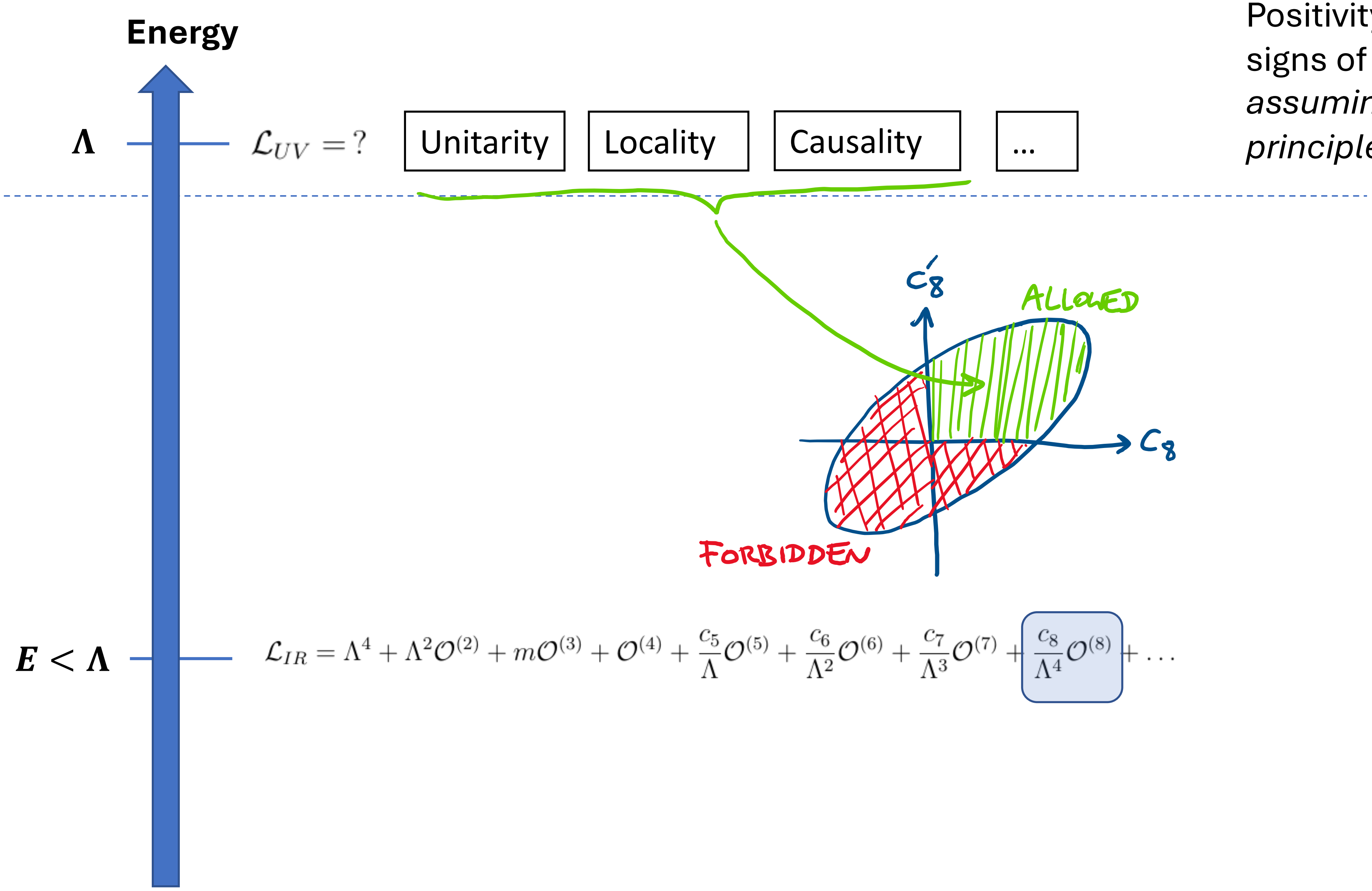
EFT Positivity Bounds



$$\mathcal{L}_{IR} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{c_5}{\Lambda}\mathcal{O}^{(5)} + \frac{c_6}{\Lambda^2}\mathcal{O}^{(6)} + \frac{c_7}{\Lambda^3}\mathcal{O}^{(7)} + \frac{c_8}{\Lambda^4}\mathcal{O}^{(8)} + \dots$$

Matching explicit UV models populates a subspace of Wilson coefficient space

EFT Positivity Bounds

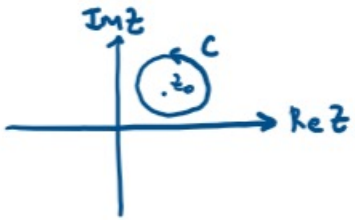


Positivity bounds forbid signs of Wilson coefficients *assuming only general principles* in the UV

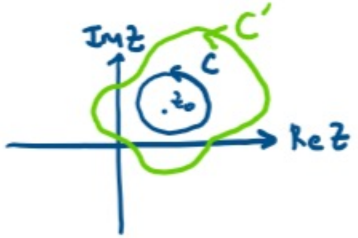
EFT Positivity Bounds

Contour integral isolates coefficient of simple pole

- $$f(z) = a_n (z-z_0)^n + \ldots + a_1 (z-z_0) + a_0 + \frac{b_1}{z-z_0} + \frac{b_2}{(z-z_0)^2} + \ldots$$



$$\Rightarrow \oint_C \frac{dz}{2\pi i} f(z) = b_1$$



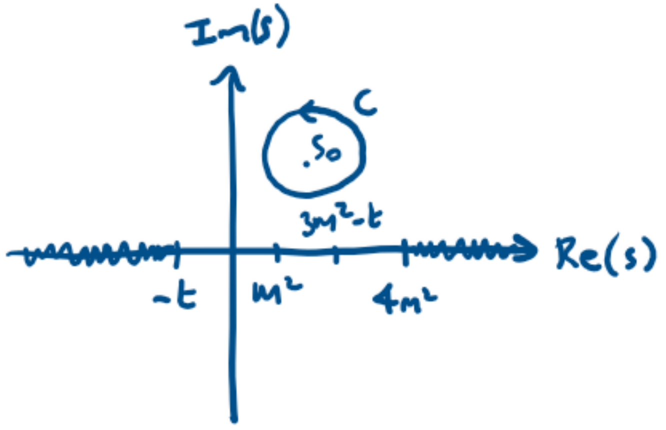
$$\Rightarrow \oint_{C'} \frac{dz}{2\pi i} f(z) = b_1$$

Analyticity allows contour deformation

EFT Positivity Bounds

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\partial_s^{(2N)} \hat{A}_{h_1h_2h_3h_4}(s_0,t) = \oint_C \frac{ds}{2\pi i} \frac{\hat{A}_{h_1h_2h_3h_4}(s,t)}{(s-s_0)^{2N+1}}$$

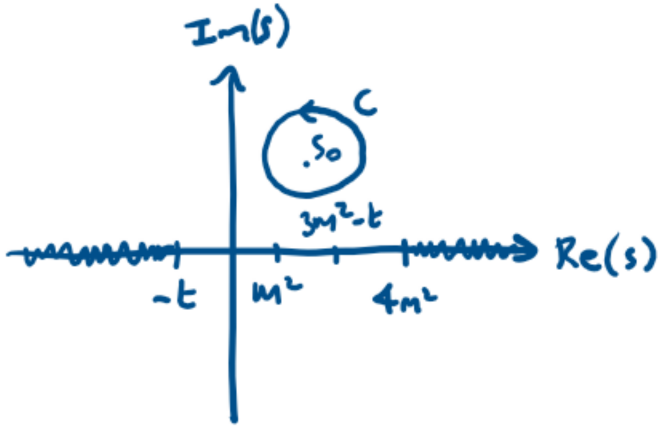


Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

EFT Positivity Bounds

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\partial_s^{(2N)} \hat{A}_{h_1h_2h_3h_4}(s_0,t) = \oint_C \frac{ds}{2\pi i} \frac{\hat{A}_{h_1h_2h_3h_4}(s,t)}{(s-s_0)^{2N+1}}$$



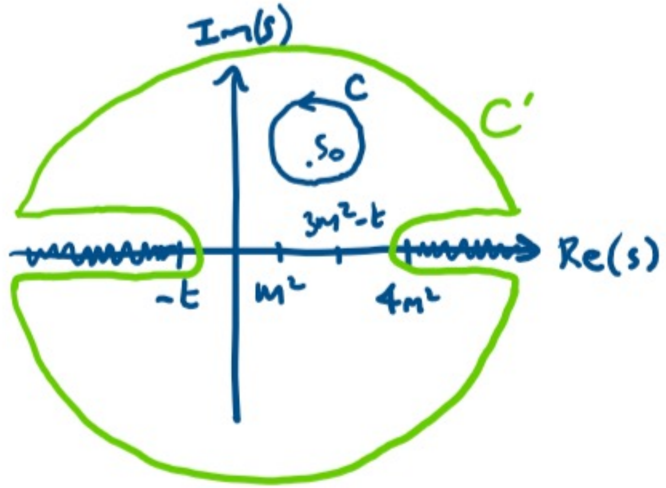
Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

EFT Positivity Bounds

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\partial_s^{(2N)} \hat{A}_{h_1h_2h_3h_4}(s_0,t) = \oint_C \frac{ds}{2\pi i} \frac{\hat{A}_{h_1h_2h_3h_4}(s,t)}{(s-s_0)^{2N+1}}$$

$$= \oint_{C'} \frac{ds}{2\pi i} \frac{\hat{A}_{h_1h_2h_3h_4}(s,t)}{(s-s_0)^{2N+1}}$$



Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

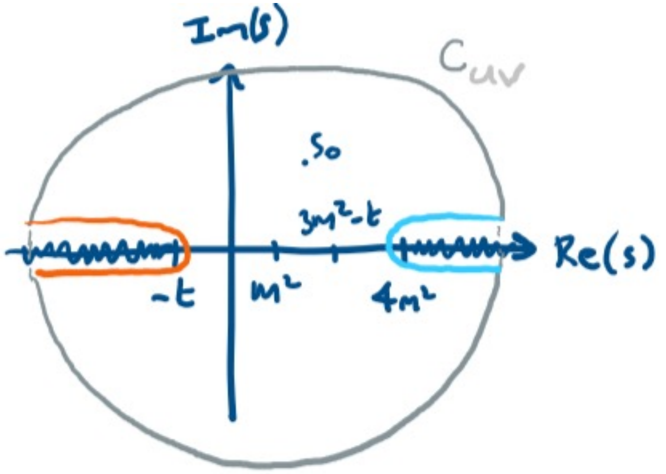
EFT Positivity Bounds

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\partial_s^{(2N)} \hat{A}_{h_1h_2h_3h_4}(s_0,t) = \oint_C \frac{ds}{2\pi i} \frac{\hat{A}_{h_1h_2h_3h_4}(s,t)}{(s-s_0)^{2N+1}}$$

$$= \oint_{C'} \frac{ds}{2\pi i} \frac{\hat{A}_{h_1h_2h_3h_4}(s,t)}{(s-s_0)^{2N+1}}$$

$$= \underbrace{I_{UV}}_{\text{locality}} + \underbrace{I_s + I_u}_{\text{unitarity}} > 0 \;?$$



Analyticity (causality) of 2-to-2 scattering amplitude allows contour deformation sensitive to UV at large s

EFT Positivity Bounds

Contour integral isolates higher-dimensional operator contributions for choice of N

$$\partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) > 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

EFT Positivity Bounds

e.g. Contour integral isolates dimension-8 operator contributions for N = 1

$$\mathcal{L}_{\rm EFT}[v] = \bar{c}_8 \frac{\mathcal{O}_8}{\Lambda^4}$$

$$\bar{c}_8 \sim \partial_s^{(2N)} \hat{A}_{h_1 h_2 h_3 h_4}(s_0, t) > 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

Potential Positivity Bounds

Scalar potentials with a stable vev can contribute to positivity bounds

$$\mathcal{L}_{\mathrm{EFT}}[H] = c_8 \frac{\mathcal{O}_8}{\Lambda^4} + c_{10} \frac{|H|^2 \mathcal{O}_8}{\Lambda^6}$$

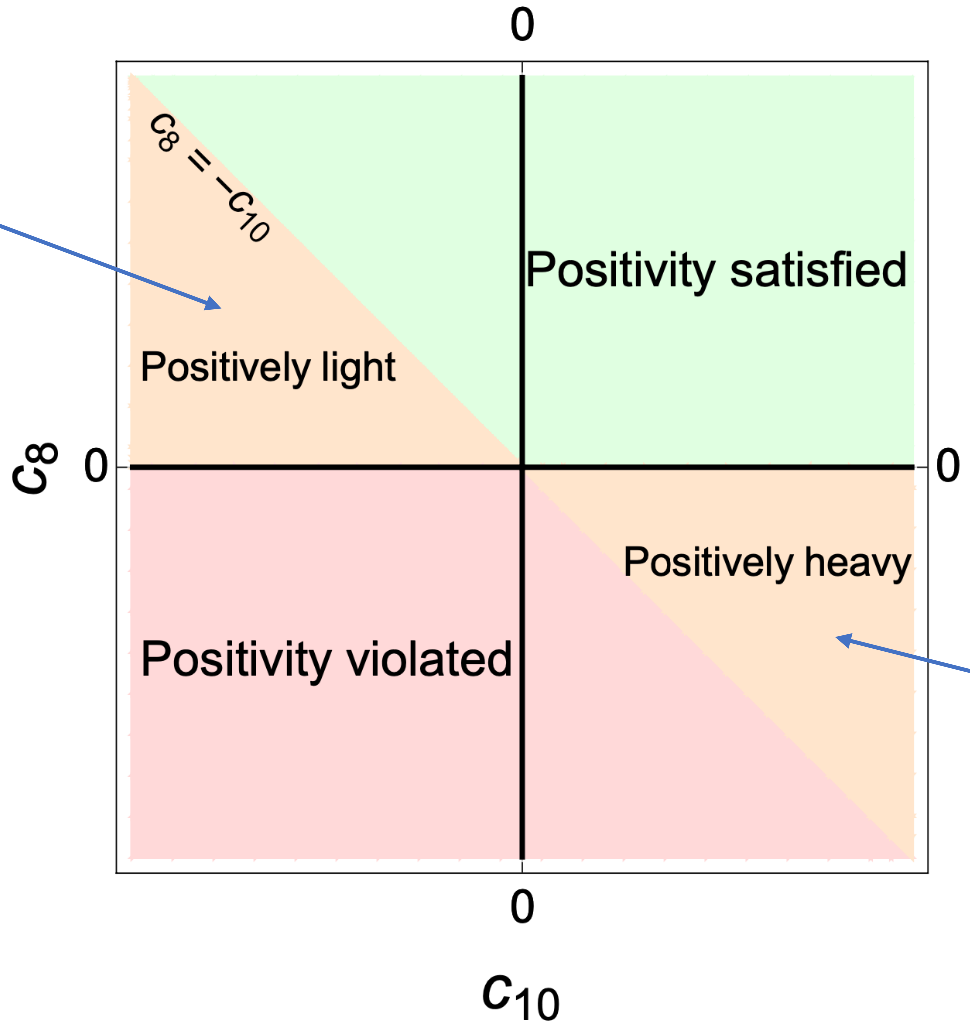
$$c_8 + \frac{v^2}{\Lambda^2} c_{10} \geq 0$$

Positivity mandated by unitarity, locality, causality (and Lorentz invariance) of UV

Positively light Higgs

A unitary, local, and causal UV theory that lives in $|c_8| \ll |c_{10}|$ EFT parameter space *necessarily* has restricted vev v

$$\frac{v^2}{\Lambda^2} < \frac{|c_8|}{|c_{10}|}$$

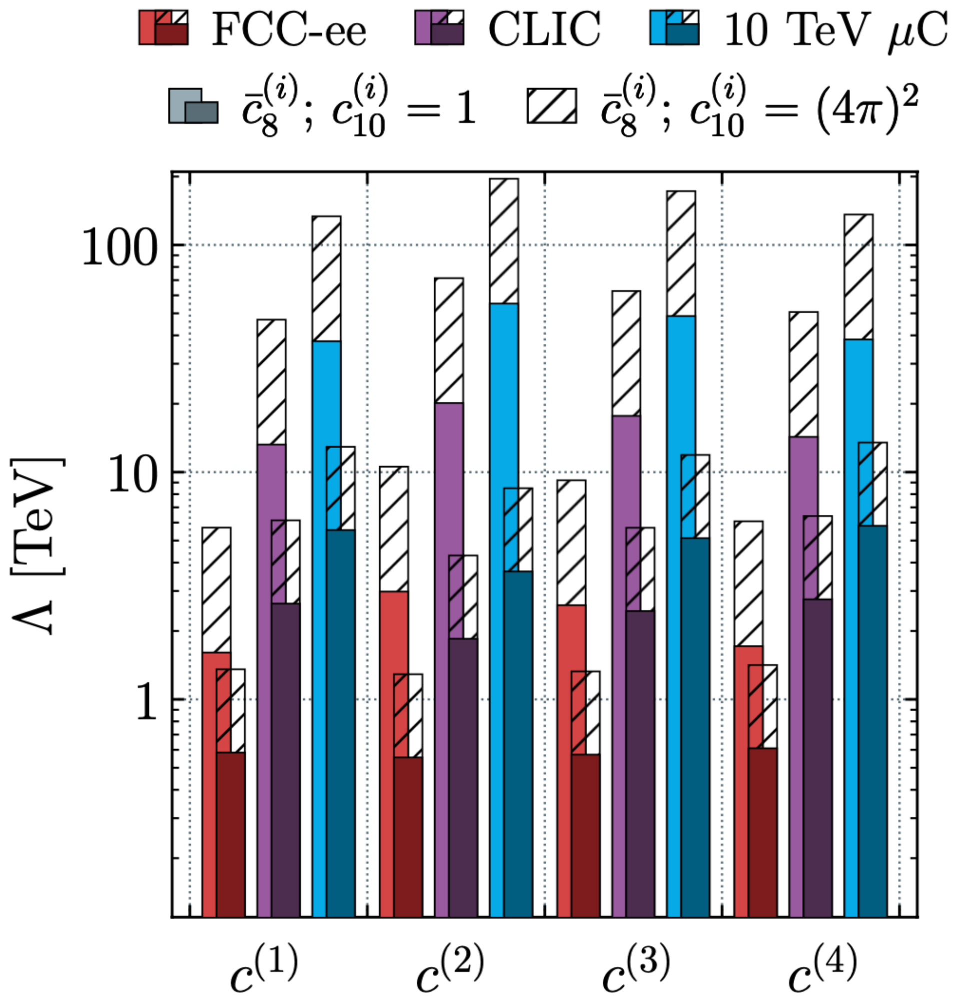


$$c_8 + \frac{v^2}{\Lambda^2} c_{10} \geq 0$$

$$\frac{v^2}{\Lambda^2} > \frac{|c_8|}{|c_{10}|}$$

Positively light Higgs

This scenario could *in principle* be established experimentally for a little hierarchy up to O(10) TeV



$$\mathcal{L}_{\rm EFT}[H] = c_8 \frac{\mathcal{O}_8}{\Lambda^4} + c_{10} \frac{|H|^2 \mathcal{O}_8}{\Lambda^6}$$

$$\mathcal{O}_8^{(1)} = \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{e}_i \gamma_\mu e_i\right) ,$$

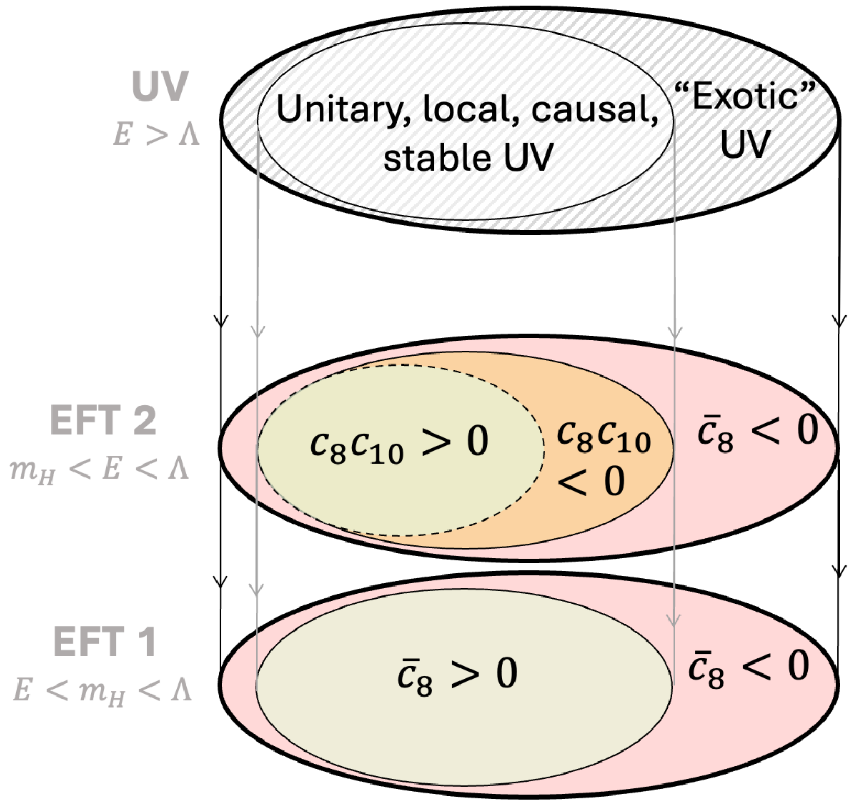
$$\mathcal{O}_8^{(2)} = \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{L}_i \gamma_\mu L_i\right) ,$$

$$\mathcal{O}_8^{(3)} = D^\nu \left(\bar{e}_i L_i\right) D_\nu \left(\bar{L}_i e_i\right) ,$$

$$\mathcal{O}_8^{(4)} = \partial^\nu \left(\bar{L}_i \gamma^\mu L_i\right) \partial_\nu \left(\bar{L}_i \gamma_\mu L_i\right) ,$$

Conclusion

There exists a region of EFT parameter space where positivity is conditional upon a scalar vev hierarchy



Relates an *a priori* unrelated IR observable to a restricted Higgs vev through reasonable UV assumptions
c.f. Fifth force and Weak Gravity Conjecture = light Higgs [1407.7865 Cheung & Remmen]

Conclusion

Everything about the SM Higgs potential is highly non-generic:

Dimension-0 operator (cosmological constant) balanced between implosion and explosion

Dimension-4 operator (Higgs quartic) on boundary of vacuum stability and instability

$$\mathcal{L} = \Lambda^4 + \Lambda^2\mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \ldots$$

Dimension-2 operator (Higgs mass) tuned between unbroken and phase phases

Higher-dimensional operators may also place on the edge of positive and non-positive theory space!