Finite Unified Theories:
Results and perspectives

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with
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Workshop on the Standard Model and Beyond
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What's going on?

- What happens as we approach the Planck scale? or just as we go up in energy...
- What happened in the early Universe?
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- How do we go from a fundamental theory to eW field theory as we know it?
- How do particles get their very different masses?
- What about flavour?

- **Where is the new physics??**

Search for understanding relations between parameters

addition of symmetries.

$N = 1$ SUSY GUTs.

Complementary approach: look for RGI relations among couplings at GUT scale $\longrightarrow$ Planck scale

$\Rightarrow$ **reduction of couplings**

resulting theory: less free parameters $\therefore$ more predictive

Zimmermann 1985

Remarkable: reduction of couplings provides a way to relate two previously unrelated sectors

gauge and Yukawa couplings

Kapetanakis, M.M., Zoupanos (1993), Kubo, M.M., Olechowski, Tracas, Zoupanos (1995,1996,1997); Oehme (1995); Kobayashi, Kubo, Raby, Zhang (2005); Gogoladze, Mimura, Nandi (2003,2004); Gogoladze, Li, Senoguz, Shafi, Khalid, Raza (2006,2011); M.M., Tracas, Zoupanos (2014)

Reduction of Couplings – ROC

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

$$\mu\, d\Phi/d\mu \;=\; \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i \;=\; 0.$$

g_i = **coupling,** β_i **its** β **function**

Finding the $(N-1)$ independent Φ's is equivalent to solve the reduction equations (RE)

$$\beta_g\,(dg_i/dg) = \beta_i\ ,$$

$i = 1, \cdots, N$

- Reduced theory: only one independent coupling and its β function
- complete reduction: power series solution of RE

$$g_a = \sum_{n=0} \rho_a^{(n)}\, g^{2n+1}$$

- uniqueness of the solution can be investigated at one-loop
  valid at all loops — Zimmermann, Oehme, Sibold (1984,1985)
- The complete reduction might be too restrictive, one may use fewer Φ's as RGI constraints
- SUSY is essential for finiteness

finiteness: absence of ∞ renormalizations

$$\Rightarrow \quad \beta^N = 0$$

may be achieved through RE

- SUSY no-renormalization theorems
  - $\Rightarrow$ only study one and two-loops
  - ROC guarantees that is gauge and reparameterization invariant to all loops

Reduction of couplings: the Standard Model

It is possible to make a reduced system in the Standard Model in the matter sector:
solve the REs, reduce the Yukawa and Higgs in favour of α_S gives

$$\alpha_t/\alpha_s = \frac{2}{9}\,; \quad \alpha_\lambda/\alpha_s = \frac{\sqrt{689}-25}{18} \simeq 0.0694$$

border line in RG surface, Pendleton-Ross infrared fixed line
But including the corrections due to non-vanishing gauge couplings up to two-loops, changes these relations and gives

$$M_t = 98.6 \pm 9.2 GeV$$

and

$$M_h = 64.5 \pm 1.5 GeV$$

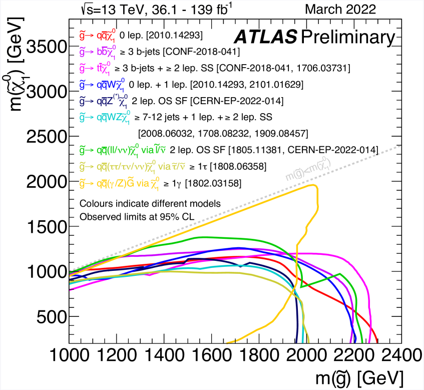
Both out of the experimental range, but pretty impressive

Kubo, Sibold and Zimmermann, 1984, 1985

SUSY in RE

Many of the reduced systems imply SUSY, even if it was not assumed a priori

Moreover: adding SUSY improves predictions $\Rightarrow$ SUSY + reduction of couplings natural



- Light SUSY in various SUSY models incompatible with LHC data
- **BUT** Different assumptions on parameters of MSSM or NMSSM lead to different predictions

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PUBNOTES/ATL-PHYS-PUB-2022-013/

Predictions in $SU(5)$ FUTs

$M_{top}^{th} \sim$ **178** GeV		large $\tan\beta$	1993
$M_{top}^{exp} =$ **176** $\pm$ **18**			1995

$M_{top}^{th} \sim$ **174**	$M_{top}^{exp} = 175.6 \pm 5.5$	heavy s-spectrum	1998
$M_{top}^{th} \sim$ **174**	$M_{top}^{exp} = 174.3 \pm 5.1$GeV	$M_{Higgs}^{th} \sim$ **115** $\sim$ **135** GeV	2003

constraints on M_h and $b \to s\gamma$ already push up the s-spectrum > 300 GeV

$M_{top}^{th} \sim$ **173**	$M_{top}^{exp} = 172.7 \pm 2.9$ GeV	$M_{Higgs}^{th} \sim$ **122** $\sim$ **126** GeV	2007
		$M_{Higgs}^{exp} =$ **126** $\pm$ **1**	2012
$M_{top}^{th} \sim$ **173**	$M_{top}^{exp} = 173.3 \pm 0.9$ GeV	$M_{Higgs}^{th} \sim$ **121** $-$ **126** GeV	2013

Constraints from Higgs and B physics$\Rightarrow$ s-spectrum > 1 TeV.

More analyses, phenomenological and theoretical, encouraged (and done)

MM, Kapetanakis, Zoupanos 1992; MM, Heinemeyer, Kalinowski, Kotlarski, Kubo, Ma, Olechowski, Patellis, Tracas, **Zoupanos** 1993-2023

Finiteness

Finiteness = absence of divergent contributions to renormalization parameters $\Rightarrow \beta = 0$

Possible in SUSY due to improved renormalization properties

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g has a superpotential

$$W = \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \ ,$$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$ gives the following conditions:

$$\sum_i T(R_i) = 3C_2(G)\,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i)\,.$$

$C_2(G)$ quadratic Casimir invariant, $T(R_i)$ Dynkin index of R_i, C_{ijk} Yukawa coup., g gauge coup.

- **restricts the particle content of the models**
- **relates the gauge and Yukawa sectors**

- One-loop finiteness $\Rightarrow$ two-loop finiteness

Jones, Mezincescu and Yao (1984,1985)

- One-loop finiteness restricts the choice of irreps R_i, as well as the Yukawa couplings
- Cannot be applied to the susy Standard Model (SSM): $C_2[U(1)] = 0$
- The finiteness conditions allow only SSB terms

It is possible to achieve all-loop finiteness $\beta^n = 0$:

Lucchesi, Piguet, Sibold

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in g, which is solution (isolated and non-degenerate) to the reduction equations

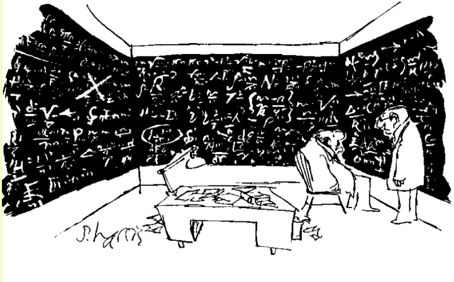
SUSY breaking soft terms

Supersymmetry is essential. It has to be broken, though…

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i \phi_j \phi_k + \frac{1}{2}\, b^{ij}\, \phi_i \phi_j + \frac{1}{2}\, (m^2)^j_i\, \phi^{*\,i} \phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

h trilinear couplings (A), b^{ij} bilinear couplings, m^2 squared scalar masses, M unified gaugino mass

Introduce over 100 new free parameters ☹



"Whatever happened to *elegant* solutions?"

RGI in the Soft Supersymmetry Breaking Sector

The RGI method has been extended to the SSB of these theories.

- One- and two-loop finiteness conditions for SSB have been known for some time
  Jack, Jones, et al.
- It is also possible to have all-loop RGI relations in the finite and non-finite cases
  Kazakov; Jack, Jones, Pickering
- SSB terms depend only on g and the unified gaugino mass M
  universality conditions

$$h = -MC, \qquad m^2 \propto M^2, \qquad b \propto M\mu$$

  but charge and colour breaking vacua
- Possible to extend the universality condition to a sum-rule for the soft scalar masses

$\Rightarrow$ better phenomenology

Kawamura, Kobayashi, Kubo; Kobayashi, Kubo, M.M., Zoupanos

Soft scalar sum-rule for the finite case

Finiteness implies

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \;\Rightarrow\; h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\, g + O(g^5)$$

If lowest order coefficients $\rho^{ijk}_{(0)}$ and $(m^2)^i_j$ satisfy diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i \,, \qquad (m^2)^i_j = m^2_j \delta^i_j$$

for all p and q.

The following soft scalar-mass sum rule is satisfied, also to all-loops

$$(\, m_i^2 + m_j^2 + m_k^2 \,)/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\Delta^{(2)} + O(g^4)$$

for $i,\ j,\ k$ with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(2)}$ is the two-loop correction =0 for universal choice

Kobayashi, Kubo, Zoupanos

based on developments by Kazakov et al; Jack, Jones et al; Hisano, Shifman; etc

Also satisfied in certain class of orbifold models, where massive states are organized into $N = 4$ supermultiples

Several aspects of Finite Models have been studied

- $SU(5)$ Finite Models studied extensively

  Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M, Kapetanakis, Zoupanos; etc
- One of the above coincides with a non-standard Calabi-Yau $SU(5) \times E_8$

  Greene et al; Kapetanakis, M.M., Zoupanos
- Finite theory from compactified string model also exists (albeit not good phenomenology) Ibáñez
- Criteria for getting finite theories from branes Hanany, Strassler, Uranga
- $N = 2$ finiteness Frere, Mezincescu and Yao
- Models involving three generations Babu, Enkhbat, Gogoladze
- Some models with $SU(N)^k$ **finite** $\Longleftrightarrow$ **3 generations,** good phenomenology with $SU(3)^3$ Ma, M.M, Zoupanos
- Relation between commutative field theories and finiteness studied

  Jack and Jones
- Proof of conformal invariance in finite theories Kazakov
- Inflation from effects of curvature that break finiteness

  Elizalde, Odintsov, Pozdeeva, Vernov

$SU(5)$ Finite Models

Example: two models with $SU(5)$ gauge group. The matter content is

$$3\,\overline{\mathbf{5}} + 3\,\mathbf{10} + 4\,\{\mathbf{5} + \overline{\mathbf{5}}\} + \mathbf{24}$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- The soft scalar masses obey a sum rule
- At the M_{GUT} scale the gauge symmetry is broken $\Rightarrow$ MSSM
- At the same time finiteness is broken
- Assume two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{\mathbf{5} + \overline{\mathbf{5}}\}$ coupled mainly to the third generation

The difference between the two models is the way the Higgses couple to the **24**

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.

The superpotential which describes the two models takes the form

$$W = \sum_{i=1}^{3} [\, \frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \,] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4$$

$$+ g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + \sum_{a=1}^{4} g_a^f \, H_a \, \mathbf{24} \, \overline{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3$$

find isolated and non-degenerate solution to the finiteness conditions

The unique solution implies discrete symmetries, $Z_n \times Z_m \times \ldots$
We will do a partial reduction, only third generation

The finiteness relations give at the M_{GUT} scale

Model A

- $g_t^2 = \frac{8}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{6}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 + m_{\mathbf{5}}^2 + m_{\mathbf{10}}^2 = M^2$

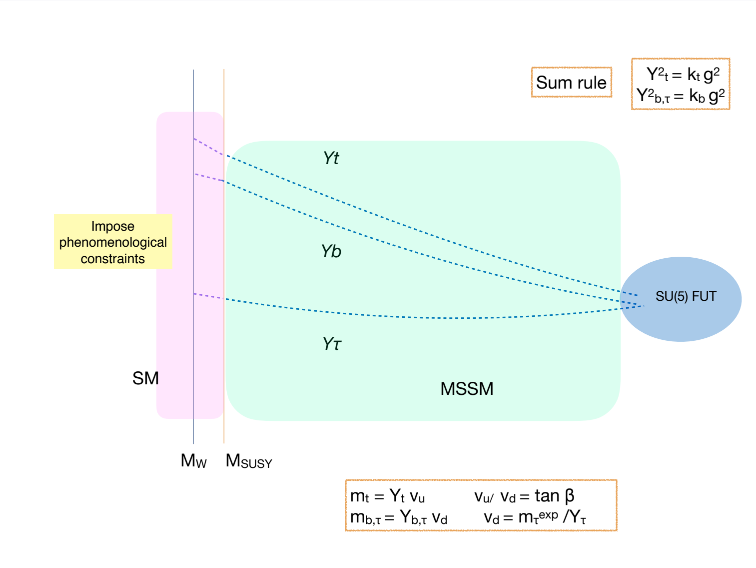
- **3 free parameters:**
  M, $m_{\mathbf{5}}^2$ and $m_{\mathbf{10}}^2$

Model B

- $g_t^2 = \frac{4}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{3}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}$
- $m_{\mathbf{5}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$

- **2 free parameters:**
  M, $m_{\mathbf{5}}^2$

FUTs at work



Sum rule
$Y^2_t = k_t g^2$
$Y^2_{b,\tau} = k_b g^2$
Yt
Impose phenomenological constraints
Yb
SU(5) FUT
Yτ
SM
MSSM
M_W
M_{SUSY}
$m_t = Y_t v_u$ $v_u / v_d = \tan\beta$
$m_{b,\tau} = Y_{b,\tau} v_d$ $v_d = m_\tau^{exp} / Y_\tau$

Phenomenology

The gauge symmetry is broken below M_{GUT} $\Rightarrow$
Boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at M_{GUT}

$\Rightarrow$ MSSM.

- Fix the value of $m_\tau \Rightarrow \tan\beta \Rightarrow M_{top}$ and m_{bot}
- Assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

We also

- Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- Include radiative corrections to bottom and tau, plus resummation **(very important!)**
- Estimate theoretical uncertainties

Top and bottom masses...

First top and bottom masses (depends on SSB) were predicted, now constraints:

Predictions:

- **FUTB:**
  $M_{top} \sim 172 \sim 174\ GeV$
  Theoretical uncertainties $\sim 4\%$
- large $\tan\beta$

- Δb and $\Delta\tau$ included resummation done.
  Depend mainly on $\tan\beta$ and unified gaugino mass M.
- **FUTB** $\mu < 0$ **favoured**

Now include the rest...

Once top was found, we look for the solutions that satisfy the following constraints:

Facts of life:

- Right masses for top and bottom
- B physics observables

$\mathrm{BR}(b \to s\gamma) SM/MSSM$:
$|BRbsg - 1.089| < 0.27$

$\mathrm{BR}(B_u \to \tau\nu) SM/MSSM$:
$|BRbtn - 1.39| < 0.69$

$\Delta M_{B_s} SM/MSSM$: 0.97 ± 20

$\mathrm{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9}$

Results:
$M_H =\sim 121 - 126$ GeV

Heavy s-spectrum

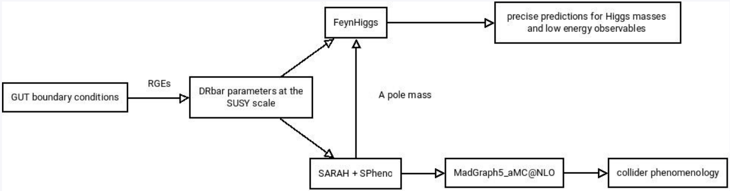
Heinemeyer, MM, Zoupanos, JHEP 2008

Once the Higgs was found, we can use the experimental value as constraint $\Rightarrow$ restrict more M and s-spectrum

Experimental challenge

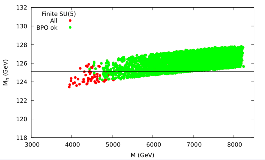
- Can they be tested at HL-LHC or FCC?
- Constraints: Top, bottom, and Higgs masses, B physics
- tan β always large, heavy s-spectrum common to all, but details differ
- Test models, calculate expected cross sections at 14 Tev (HL-LHC) and 100 TeV (FCC)

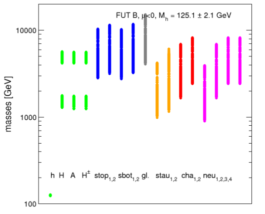
Heinemeyer, Kalinowski, Klotarski, MM, Patellis, Tracas, Zoupanos, Eur. Phys. J. C (2021) 81:185



Results

With latest FeynHiggs and experimental constraints ⇒ collider phenomenology:





- Top and bottom quark masses within 2σ
- Heavy SUSY spectrum
  ⇒ consistent with non-observation
- From collider searches
  ⇒ challenging even for the FCC
- Lightest neutralino 100% of DM
  ⇒ Over abundance of DM

BUT take into account:

- Only third generation included
- R parity breaking ⇒ neutrino masses and gravitino as DM
- Possible to extend to 3 generations

FUTs

- Finiteness provides us with an UV completion of our QFT
- Boundary conditions for RGE of the MSSM
- RGI takes the flow in the right direction for the third generation and Higgs masses
  Taking into account experimental constraints
  $\Rightarrow$ susy spectrum high
- Experimentally challenging

- Are there other finite models?
- Can it give us insight into the flavour structure?
- Can we have successful reduction of couplings in a SM-like theory?

$SU(N)^k$

3 generations $\leftrightarrow$ finite Consider the gauge group

$$\mathrm{SU}(N)_1 \times \mathrm{SU}(N)_2 \times \cdots \times \mathrm{SU}(N)_k$$

with n_f copies of $(N, \bar{N}, 1, \ldots, 1) + (1, N, \bar{N}, \ldots, 1) + \cdots + (\bar{N}, 1, 1, \ldots, N)$.

The one-loop β-function coefficient

$$\beta = \left(-\frac{11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N \,.$$

$\Rightarrow n_f = 3$ is a solution of $\beta = 0$, independently of the values of N and k.

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3),$$

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*),$$

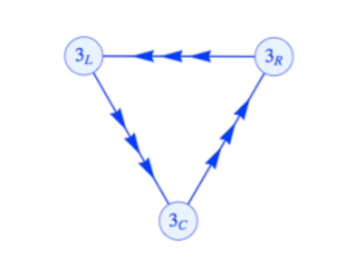
$SU(3)^3$ singled out as the only possible phenomenological model

2-loop $SU(3)^3$ out of several possibilities

$SU(3)^3$ 2-loop finite trinification model, parametric solution of reduction equations

$$f^2 = r\left(\frac{16}{9}\right)g^2\,, \quad f'^2 = (1-r)\left(\frac{8}{3}\right)g^2\,,$$

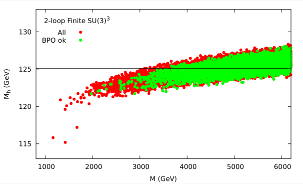
r parameterizes different solutions to boundary conditions, f, f' Yukawa for quarks and leptons respectively



- Finiteness implies 3 generations
- Good top
  and bottom masses, depend on a parameter
- Large tan β
- Heavy SUSY spectrum
- Possibility of having neutrino masses
- Consistent with seesaw mechanism
- At high energies vector-like down type quarks
- Also needs extra symmetries

Results for $SU(3)^3$

- Requiring that top and bottom lie within experimental bounds gives a lower bound on M
- Not trivial to find r that fits both top and bottom quark masses
- Incorporate sum rule, follow procedure $\Rightarrow$ Higgs mass



- Too much CDM, if 100% is neutralino, other mechanisms can be incorporated.
- Neutrinos can naturally be incorporated (along with a lot of exotics)
- Very heavy spectrum, but heavy Higgs sector testable at FCC-hh

Can we have successful reduction of couplings in a SM-like theory?
YES, with SUSY
We assume a covering GUT, reduced top-bottom system

Y_τ not reduced, its reduction gives imaginary values

$$\frac{Y_t^2}{4\pi} = G_t^2 \frac{g_3^2}{4\pi} + c_2 \left(\frac{g_3^2}{4\pi}\right)^2 ; \quad \frac{Y_b^2}{4\pi} = G_b^2 \frac{g_3^2}{4\pi} + p_2 \left(\frac{g_3^2}{4\pi}\right)^2$$

where

$$G_t^2 = \frac{1}{3} + \frac{71}{525}\rho_1 + \frac{3}{7}\rho_2 + \frac{1}{35}\rho_\tau , \qquad G_b^2 = \frac{1}{3} + \frac{29}{525}\rho_1 + \frac{3}{7}\rho_2 - \frac{6}{35}\rho_\tau$$

$$\rho_{1,2} = \frac{g_{1,2}^2}{g_3^2} = \frac{\alpha_{1,2}}{\alpha_3} , \qquad \rho_\tau = \frac{g_\tau^2}{g_3^2} = \frac{\frac{Y_\tau^2}{4\pi}}{\alpha_3}$$

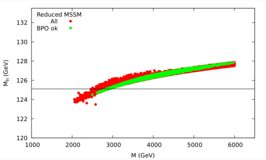
$\rho_{1,2}, \rho_\tau$ corrections from the non-reduced part, assumed smaller as energy increases

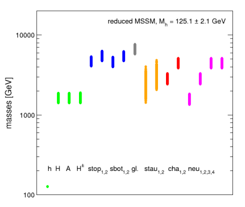
c_2 and p_2 can also be found (long expressions not shown)

Higgs mass and s-spectrum

RMSSM has lightest s-spectrum!

- Possible
  to have reduction of couplings
  in MSSM, third family of quarks
- Up to now
  only attempted in SM or in GUTs
- Reduced system further
  constrained by phenomenology:
- Large tan β
- SUSY spectrum $M_{LSP} \geq 1 \; TeV$
- DM abundance OK (below limit),
  possible to add a SUSY axion?





Prospects for FCC

Model	top/bottom masses	Higgs mass	SUSY spectra	heavy Higgs spectra	CDM
$\sim$ FUT $SU(5)$	OK/OK	OK	$\gtrsim$ 2.0 TeV	$\gtrsim$ 5.5 TeV	too much
✓ FUT $SU(3)^3$	OK/OK	OK	$\gtrsim$ 1.5 TeV	$\gtrsim$ 6.4 TeV	feasible
$\sim$ RMin $SU(5)$	OK/bot 4σ	OK	$\gtrsim$ 1.2 TeV	$\sim$ 2.5 TeV	too much
✗ RMSSM	OK/OK	OK	$\sim$ 1.0 TeV	$\sim$ 1.3 TeV	OK

- RMSSM already excluded by LHC searches
- The rest testable only at FCC-hh at 2 σ, only part at 5 σ
- Exception: $SU(3)^3$ heavy Higgs sector testable at FCC-hh
- In $SU(5)$ models you can have neutrino masses and gravitino as DM $\Rightarrow$ $\not{R}$

So, now what?
Perspectives for the models

$SU(5)$ with three generations

Can we include 3 generations?

- First obvious step: include all generations
- Not easy, 2 ways:
  Rotate to MSSM
  Keep all Higgses
- First very simple approach: get diagonal solution for quark masses, no SUSY breaking

- Rotation of Higgs sector $\Rightarrow$ impacts proton decay and doublet-triplet splitting
- Now include off-diagonal terms $\Rightarrow$
  again need discrete symmetries, but possible to get interesting "textures"

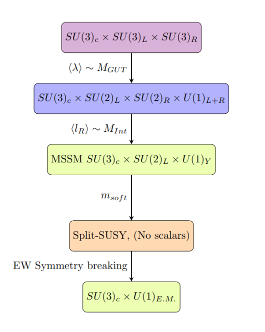
$m_u(M_Z)$	$m_c(M_Z)$	$m_t(M_Z)$	$m_d(M_Z)$	$m_s(M_Z)$	$m_b(M_Z)$	$m_\tau(M_Z)$	$\tan\beta$	$\chi^2_{r_{min}}$
$0.0012GeV$	$0.626GeV$	$171.8GeV$	$0.00278GeV$	$0.0595GeV$	$2.86GeV$	$1.74623GeV$	57.4	0.152

Work in progress

MM, L.O. Estrada-Ramos, G. Patellis, G. Zoupanos

Split SUSY in FUT $SU(3)^3$

We can implement a split SUSY scenario in finite $SU(3)^3$



- Similar to coset space dimensional reduction (see Patellis talk) but not identical...
- We have to implement the sum rule
- More than one candidate to dark matter

Work in progress

MM, L.E. Reyes, G. Patellis, G. Zoupanos + ...

Reduction of couplings in 2HDM

- First attempt at 2HDM by Denner $\Rightarrow$ too low top and Higgs masses, not known then.
  You can reduce top, Higgs, bottom with $\alpha_s \Rightarrow$ other couplings zero

  Denner, NPB 347 (1990)

- Re-did Denner analysis, in type I, II, X and flipped 2HDM, similar (not identical) results:

	Tipo I/X (GeV)	Tipo II/Y (GeV)
m_t	≤ 99.9	94.7
m_H	≤ 55.8	50.6
m_h	0	1.7
$m_{H^\pm}$	0	36.0
m_A	0	0

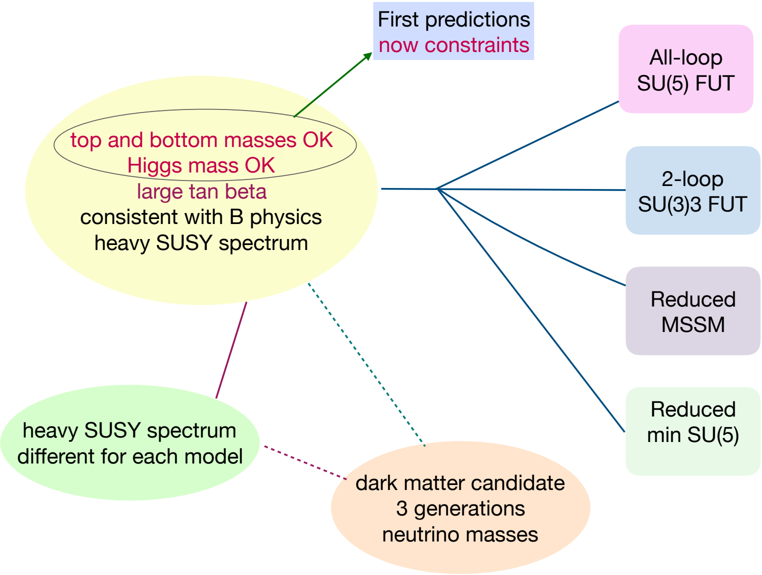
This could provide a limit or some guide to multi-Higgs models...

Miguel Angel May M.Sc. Thesis (2023)

- Ongoing effort: 2HDM with corrections from first two generations

  MM, May Pech, Patellis, Zoupanos + Branco, Rebelo + ...

GYU from reduction of couplings at work



Conclusions

- Reduction of couplings: powerful principle implies Gauge Yukawa Unification
  $\Rightarrow$ predictive models
- Possible SSB terms $\Rightarrow$ satisfy a sum rule among soft scalars
- Finiteness $\Rightarrow$ reduces greatly the number of free parameters
  - completely finite theories $SU(5)$
  - 2-loop finite theories $SU(3)^3$
- Reduced MSSM

- Successful prediction for top quark and Higgs boson mass
- Large tan β
- Satisfy BPO constraints (not trivial)
- Heavy SUSY spectrum
- Most of the spectra too heavy to be tested at FCC:
  - RMSSM excluded
  - $SU(3)^3$ heavy Higgs sector could be tested

Outlook

Some open questions and future work in reduction of couplings

- Are there more finite and reduced models? Yes...
- Do all fermions acquire masses the same way? ??
- Is it possible to include the three generations in a reduced or finite model? Yes...
- How to incorporate flavour? possible, aided by symmetries
- How to include neutrino masses? Yes... $\not R$ for ($SU5$), natural for $SU(3)^3$
- Is it indispensible to have SUSY for successful reduced theories? So far it looks like that, but non-SUSY multi-Higgs might be possible
- How to make better use symmetries $\Leftrightarrow$ reduction of couplings? ?

Results for FUT SU(5): CDM, Higgs and s-spectra

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde g}$	$M_{\tilde\chi^0_1}$	$M_{\tilde\chi^0_2}$	$M_{\tilde\chi^0_3}$	$M_{\tilde\chi^0_4}$	$M_{\tilde\chi^\pm_1}$	$M_{\tilde\chi^\pm_2}$
FUTSU5-1	5.688	5.688	5.688	8.966	2.103	3.917	4.829	4.832	3.917	4.833
FUTSU5-2	7.039	7.039	7.086	10.380	2.476	4.592	5.515	5.518	4.592	5.519
FUTSU5-3	16.382	16.382	16.401	12.210	2.972	5.484	6.688	6.691	5.484	6.691
	$M_{\tilde e_{1,2}}$	$M_{\tilde\nu_{1,2}}$	$M_{\tilde\tau}$	$M_{\tilde\nu_\tau}$	$M_{\tilde d_{1,2}}$	$M_{\tilde u_{1,2}}$	$M_{\tilde b_1}$	$M_{\tilde b_2}$	$M_{\tilde t_1}$	$M_{\tilde t_2}$
FUTSU5-1	3.102	3.907	2.205	3.137	7.839	7.888	6.102	6.817	6.099	6.821
FUTSU5-2	3.623	4.566	2.517	3.768	9.059	9.119	7.113	7.877	7.032	7.881
FUTSU5-3	4.334	5.418	3.426	3.834	10.635	10.699	8.000	9.387	8.401	9.390

Table 5: Masses for each benchmark of the Finite $N = 1$ $SU(5)$ (in TeV).

scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV	scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV
$\tilde\chi^0_2\tilde\chi^0_3$	0.01	0.01		$\tilde\nu_i\tilde\nu^*_j$	0.02	0.01	0.01
$\tilde\chi^0_3\tilde\chi^0_4$	0.03	0.01		$\tilde u_i\tilde\chi^-_1, \tilde d_i\tilde\chi^+_1 + h.c.$	0.15	0.06	0.02
$\tilde\chi^0_2\tilde\chi^+_1$	0.17	0.08	0.03	$\tilde q_i\tilde\chi^0_1, \tilde q^*_i\tilde\chi^0_1$	0.08	0.03	0.01
$\tilde\chi^0_3\tilde\chi^+_2$	0.05	0.03	0.01	$\tilde q_i\tilde\chi^0_2, \tilde q^*_i\tilde\chi^0_2$	0.08	0.03	0.01
$\tilde\chi^0_4\tilde\chi^+_2$	0.05	0.03	0.01	$\tilde\nu_i\tilde e^*_j, \tilde\nu^*_i\tilde e_j$	0.09	0.04	0.01
$\tilde g\tilde g$	0.20	0.05	0.01	$Hb\bar b$	2.76	0.85	
$\tilde g\tilde\chi^0_1$	0.03	0.01		$Ab\bar b$	2.73	0.84	
$\tilde g\tilde\chi^0_2$	0.03	0.01		$H^+b\bar t + h.c.$	1.32	0.42	
$\tilde g\tilde\chi^+_1$	0.07	0.03	0.01	H^+W^-	0.38	0.12	
$\tilde q_i\tilde q_j, \tilde q_i\tilde q^*_j$	3.70	1.51	0.53	HZ	0.09	0.03	
$\tilde\chi^+_1\tilde\chi^-_1$	0.10	0.05	0.02	AZ	0.09	0.03	
$\tilde\chi^+_2\tilde\chi^-_2$	0.03	0.02	0.01				
$\tilde e_i\tilde e^*_j$	0.23	0.13	0.05				
$\tilde q_i\tilde g, \tilde q^*_i\tilde g$	2.26	0.75	0.20				

Table 6: Expected production cross sections (in fb) for SUSY particles in the FUTSU5 scenarios.

Results for $SU(3)^3$: CDM, Higgs and s-spectra

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde g}$	$M_{\tilde\chi^0_1}$	$M_{\tilde\chi^0_2}$	$M_{\tilde\chi^0_3}$	$M_{\tilde\chi^0_4}$	$M_{\tilde\chi^\pm_1}$	$M_{\tilde\chi^\pm_2}$
FSU33-1	7.029	7.029	7.028	6.526	1.506	2.840	6.108	6.109	2.839	6.109
FSU33-2	6.484	6.484	6.431	8.561	2.041	3.817	7.092	7.093	3.817	7.093
FSU33-3	6.539	6.539	6.590	10.159	2.473	4.598	6.780	6.781	4.598	6.781
	$M_{\tilde e_{1,2}}$	$M_{\tilde\nu_{1,2}}$	$M_{\tilde\tau}$	$M_{\tilde\nu_\tau}$	$M_{\tilde d_{1,2}}$	$M_{\tilde u_{1,2}}$	$M_{\tilde b_1}$	$M_{\tilde b_2}$	$M_{\tilde t_1}$	$M_{\tilde t_2}$
FSU33-1	2.416	2.415	1.578	2.414	5.375	5.411	4.913	5.375	4.912	5.411
FSU33-2	3.188	3.187	2.269	3.186	7.026	7.029	6.006	7.026	6.005	7.029
FSU33-3	3.883	3.882	2.540	3.882	8.334	8.397	7.227	8.334	7.214	7.409

Table 8: Masses for each benchmark of the Finite $N = 1$ $SU(3)^3$ (in TeV).

scenarios $\sqrt{s}$	FSU33-1 100 TeV	FSU33-2 100 TeV	FSU33-3 100 TeV	scenarios $\sqrt{s}$	FSU33-1 100 TeV	FSU33-2 100 TeV	FSU33-3 100 TeV
$\tilde\chi^0_1\tilde\chi^0_1$	0.04	0.01	0.01	$\tilde q_i\tilde g, \tilde q^*_i\tilde g$	22.12	3.71	1.05
$\tilde\chi^0_2\tilde\chi^0_2$	0.04	0.01		$\tilde\nu_i\tilde\nu^*_j$	0.10	0.03	0.01
$\tilde\chi^0_2\tilde\chi^+_1$	0.58	0.16	0.07	$\tilde u_i\tilde\chi^-_1, \tilde d_i\tilde\chi^+_1 + h.c.$	1.22	0.25	0.08
$\tilde\chi^0_3\tilde\chi^+_2$	0.02	0.01	0.01	$\tilde q_i\tilde\chi^0_1, \tilde q^*_i\tilde\chi^0_1$	0.55	0.13	0.05
$\tilde\chi^0_4\tilde\chi^+_2$	0.02	0.01	0.01	$\tilde q_i\tilde\chi^0_2, \tilde q^*_i\tilde\chi^0_2$	0.60	0.13	0.04
$\tilde g\tilde g$	2.61	0.30	0.07	$\tilde\nu_i\tilde e^*_j, \tilde\nu^*_i\tilde e_j$	0.36	0.12	0.04
$\tilde g\tilde\chi^0_1$	0.20	0.05	0.02	$Hb\bar b$	0.71	1.23	1.19
$\tilde g\tilde\chi^0_2$	0.20	0.04	0.01	$Ab\bar b$	0.72	1.23	1.18
$\tilde g\tilde\chi^+_1$	0.42	0.09	0.03	$H^+b\bar t + h.c.$	0.37	0.75	0.58
$\tilde q_i\tilde q_j, \tilde q_i\tilde q^*_j$	25.09	6.09	2.25	H^+W^-	0.10	0.25	0.19
$\tilde\chi^+_1\tilde\chi^-_1$	0.37	0.10	0.04	HZ	0.02	0.04	0.04
$\tilde e_i\tilde e^*_j$	0.39	0.12	0.06	AZ	0.02	0.04	0.04

Table 9: Expected production cross sections (in fb) for SUSY particles in the FSU33 scenarios.

Results for RMSSM: CDM, Higgs and s-spectra

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}^0_1}$	$M_{\tilde{\chi}^0_2}$	$M_{\tilde{\chi}^0_3}$	$M_{\tilde{\chi}^0_4}$	$M_{\tilde{\chi}^\pm_1}$	$M_{\tilde{\chi}^\pm_2}$
RMSSM-1	1.393	1.393	1.387	7.253	1.075	3.662	4.889	4.891	1.075	4.890
RMSSM-2	1.417	1.417	1.414	7.394	1.098	3.741	4.975	4.976	1.098	4.976
RMSSM-3	1.491	1.491	1.492	7.459	1.109	3.776	5.003	5.004	1.108	5.004
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
RMSSM-1	2.124	2.123	2.078	2.079	6.189	6.202	5.307	5.715	5.509	5.731
RMSSM-2	2.297	2.139	2.140	2.139	6.314	6.324	5.414	5.828	5.602	5.842
RMSSM-3	2.280	2.123	2.125	2.123	6.376	6.382	5.465	5.881	5.635	5.894

Table 11: Masses for each benchmark of the Reduced MSSM (in TeV).

Since $M_A \lesssim 1.5$ TeV and large $\tan\beta$, RMSSM is excluded by searches $H/A \to \tau\tau$ at ATLAS.

Reduction of dimensionless parameters

Any RGI relation among couplings $g_1, ..., g_A$ of a renormalizable theory can be written as

$$\Phi(g_1, \cdots, g_A) \ = \ \text{const.},$$

which has to satisfy the partial differential equation

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla}\Phi \cdot \vec{\beta} \ = \ \sum_{a=1}^{A} \beta_a \frac{\partial \Phi}{\partial g_a} \ = \ 0 \ ,$$

where β_a is the β-function of g_a.

Equivalent to solving a set of ordinary differential equations $\rightarrow$ reduction equations:

$$\beta_g \frac{dg_a}{dg} = \beta_a \ , \ a = 1, \cdots, A \ ,$$

where g and β_g are the primary coupling and its β-function, respectively, the counting on a does not include g.

Zimmermann 1985; Oehme and Zimmermann 1985; Oehme 1986

Solutions of RE's

- The Φ_a's can impose a maximum of $(A-1)$ independent RGI "constraints" in the A-dimensional space of parameters, which could be expressed in terms of a single coupling g.
- However, the general solutions of RE's contain as many integration constants as the number of equations.
- Solution: power series solutions to the RE's, which preserve perturbative renormalizability

$$g_a = \sum_n \rho_a^{(n)} g^{2n+1} \ ,$$

- Reduced theory: only one independent coupling and its β function