Corfu 2023 **September, 06, 2023**

On non-geometric fluxes in IIB-theory

(and the induced W & $V_{\rm eff}$ of moduli fields)

George K. Leontaris

University of Ioannina

$I\omega\alpha\nu\nu\iota\nu\alpha$

GREECE

Based on hep-th/2308.15529

in collaboration with Pramod Shukla (Bose Inst., Kolkata)

Outline of the Talk

- Introductory remarks
- Anatomy of non-geometric fluxes
- Generalised Fluxes and the moduli Superpotential
- Axionic fluxes and the Scalar Potential
- Future Perspectives and Concluding Remarks

PART I

★ Introductory Remarks ★

The background

- String compactifications are characterised by moduli fields

- Viable phenomenological models must be free of massless moduli

- In the traditional approach, a key role is played by the RR and NS background (*geometric*) fluxes given in terms of the field strengths

$$F_3 = dC_2, \; H_3 = dB_2$$

- They give rise to the Gukov-Vafa-Witten (GVW) superpotential

$$W \sim \int \underbrace{(F_3 - S H_3)}_{\mathbf{G_3}} \wedge \Omega_3$$

- At least some of the complex structure (CS) moduli and the axio-dilaton $S = C_0 + ie^{-\phi}$ are stabilised by SUSY conditions.

▲ However, in contrast to CS and S moduli, Kähler moduli stabilisation is more involved, due to their **no-scale** property.

▲ A lot of work has been devoted towards a solution: combining background fluxes with perturbative and/or nonperturbative corrections to fix all closed string moduli.

▲ Origin of corrections:

• Non-perturbative terms in W arising from D3-brane instantons on D7-branes (*for refs see review 2303.04819*)

• Perturbative corrections come from KK-states propagating between D7 branes and localised EH terms emerging from 10-d action (*Antoniadis et al 1803.08941 . . . GKL and P. Shukla 2303.16689*).

▲▲ In most CY manifolds, however, CS moduli come in large numbers and geometric fluxes do not suffice to stabilise all of them.

▲ However, geometric fluxes is only a small part of a broader flux landscape.

▲ As a matter of fact, there is no compelling reason that one should restrict only in flux compactifications with geometric interpretations.

▲ The time might be ripe we try something new.

Indeed:

▲ Using T duality [a] while focusing here only on a background of NS-NS flux [b] one can go beyond the above picture and bring into the scene **non-geometric** fluxes

[a] *T-duality relates winding modes in compact space with momentum modes in its dual space through* $R \to 1/R$ *(*$R =$ *compactification radius).*

[b] *Sources of non-geometric fluxes abound; here we only consider a restricted set of specific non-geometric flux compactifications.*

Anatomy of non-geometric Fluxes

(associated with NS-NS background)

T-dualities

- **Prelude**

▲ If we start with IIB compactified on $CY_3 = \mathcal{X}$ **without** NS flux $H = dB_2$,

a dual theory in type IIA can be constructed, which is described in a mirror CY-manifold $\tilde{\mathcal{X}}$.

▲ If we add NS-NS flux $H = dB_2$ ▲

Now, (as we will demostrate) T-duality, maps H non-trivially to a deformation of the T-dual metric.

Because mirror symmetry is a generalisation of T-duality,
mirror geometry is no-longer a CY space

Indeed,

let x, y, z parametrise a 3-torus:

$$ds^2 = dx^2 + dy^2 + dz^2$$

with the identifications

$$x \to x+1, y \to y+1, z \to z+1$$

Choose B field with the only non-zero component:

$$B_{xy} = Nz$$

Integrating (*setting* $(2\pi)^2\alpha' = 1$)

$$\int H_3 = N$$

Thus, we turn on N units of NS flux

T-dualising along the x direction, the metric takes the form

$$ds^2 = (dx - Nz\,dy)^2 + dy^2 + dz^2 \tag{1}$$

This new space is called:

Twisted Torus

It is **topologically distinct** from the ordinary torus.

The first term in particular remains intact under the shifts

$$x \to x' = x + Ny, \;\; z \to z' = z + 1$$

and thus we have the identifications

$$(x, y, z) \cong (x+1, y, z) \cong (x, y+1, z) \cong (x+Ny, y, z+1)$$

Description through the Heisenberg Group $\mathcal{H}$:

$$h(x, y, z) = \begin{pmatrix} 1 & y & z \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix} \in \mathcal{H}$$

$\mathcal{H}$ is a simply connected 2-step nilpotent group

$$1 \to \mathbb{R} \to \mathcal{H} \to \mathbb{R}^2 \to 1$$

Matrices relevant to our discussion have the particular structure

$$h_N \equiv h(x, y, -z/N) = \begin{pmatrix} 1 & y & -\frac{z}{N} \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix}$$

(where N is associated with units of flux)

The product of two such matrices defines Translations

$h_N = h(x, y, -z/N),\ g_N = h(a, b, -c/N)$*:*

$$h_N \cdot g_N = \begin{pmatrix} 1 & b+y & -\frac{c}{N} + bx - \frac{z}{N} \\ 0 & 1 & a+x \\ 0 & 0 & 1 \end{pmatrix} \cong \begin{pmatrix} 1 & y' & -\frac{z'}{N} \\ 0 & 1 & x' \\ 0 & 0 & 1 \end{pmatrix}$$

They imply the identifications (*similar to twisted torus*)

$$x' \cong x + a,\ y' \cong y + b,\ z' \cong z - Nbx + c$$

Translations defined through $\mathcal{H}$ are non-commutative

$$h_N \cdot g_N \neq g_N \cdot h_N$$

The importance of the latter approach is that a specific group structure is revealed.

$\forall N > 0$ and ℓ, m, n integers, we define the matrices

$$\Gamma_N = \begin{pmatrix} 1 & \ell & n/N \\ 0 & 1 & m \\ 0 & 0 & 1 \end{pmatrix}$$

Now, Γ_1 defines the discrete subgroup of $\mathcal{H}$ consisting of all the integral matrices and Γ_N is the lattice containing Γ_1. It holds

$$H_1(\mathcal{H}/\Gamma_N; \mathbb{Z}) \to \mathbb{Z}^2 \oplus \mathbb{Z}_N$$

The General Case

To generalise the above case let's introduce the notation:

$$ds^2 = (dx - \omega^x_{yz} z dy)^2 + dy^2 + dz^2 \tag{2}$$

We compactify this space by identifying $x \cong x + 1, y \cong y + 1$,
However the identification $z \cong z + 1$ will induce an extra term

$$(dx - \omega^x_{yz}(z+1)dy)^2 \to (dx - \omega^x_{yz} z dy - \underbrace{\omega^x_{yz} dy}_{extra\,term})^2 \tag{3}$$

As explained, we must compactify through the chain

$$(x, y, z) \cong (x + 1, y, z) \cong (x, y + 1, z) \cong (x + \omega^x_{yz} y, y, z + 1)$$

In this way a well-defined metric is achieved globally.
Space is now T^2 along x, y and a fibered one over an S^1 in z.

Important insight can be provided through the definition of the following 1-forms

$$\begin{aligned} \eta^x &= dx - \omega^k_{ij} z dy \\ \eta^y &= dy \\ \eta^z &= dz \end{aligned}$$

Observe now that $d\eta^y = d\eta^z = 0$, while

$$d\eta^x = \omega^x_{yz} dy \wedge dz \equiv \omega^x_{yz} \eta^y \wedge \eta^z \neq 0$$

In a straightforward generalisation

$$dx^k = \omega^k_{ij} dx^j \wedge dx^k$$

where ω^k_{ij} play the role of structure constants of a Lie group associated with the isometries of the torus ($Z_{i,j,k} \rightarrow generators$)

$$[Z_i, Z_j] = \omega^k_{ij} Z_k$$

★ T-dualities along all three directions ★

▲ Assuming T-duality along x has been performed as above, we T-dualise in the y direction *(metric is independent of y)*.
▲ Locally we end up with a 'geometric torus', however, globally it cannot be described by a fixed geometry (see eg hep-th/0508133).

▲ In general for compactifications on $T^6 \sim T_1^2 \times T_2^2 \times T_3^3$, under three successive T-dualities, the three-form flux $H_3 = dB_2$ implies the following '**geometric**' and **non-geometric** fluxes:

$$\underbrace{H_{mnp} \xrightarrow{T_m} \omega^m_{np}}_{\text{geometric}} \xrightarrow{T_n} \underbrace{Q^{mn}_p \xrightarrow{T_p} R^{mnp}}_{\text{non}-\text{geometric}} . \tag{4}$$

Furthermore, (*as we will see*) S-duality invariance of the type IIB superstring compactification requires the inclusion of additional fluxes, which are S-dual to the (non)-geometric fluxes (*see eg, Font et al, hep-th/0602089, Gao & Shukla 1501.07248*)

PART II

★ Generalised Fluxes and the Superpotential ★

$\star$ Framework $\star$

Type IIB compactification on $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ **orbifold** with

Compexified coordinates

$$z^1 = x^1 + U_1 x^2, z^2 = x^3 + U_2 x^4, z^3 = x^5 + U_3 x^6$$

and $\mathbb{Z}_2$ actions:

$$\theta \quad : \quad (z^1, z^2, z^3) \to (-z^1, -z^2, z^3) \qquad (5)$$

$$\bar{\theta} \quad : \quad (z^1, z^2, z^3) \to (z^1, -z^2, -z^3) \qquad (6)$$

Orientifold action:

$\mathcal{O} = \Omega_p I_6 (-)^{F_L}$,

$I_6 : (z^1, z^2, z^3) \to (-z^1, -z^2, -z^3)$

Some Definitions

We introduce the prepotantial $\mathcal{F}$ in terms of projective coordinates and the symplectic period vectors $(\mathcal{X}^K, \mathcal{F}_K)$ we also define:

$$\mathcal{F} = \frac{\mathcal{X}^1\mathcal{X}^2\mathcal{X}^3}{\mathcal{X}^0} = U^1U^2U^3$$

$$\Omega_3 = \mathcal{X}^K\alpha_K - \mathcal{F}_K\beta^K, \; (\mathcal{F}_i = U_jU_k, \ldots)$$

$$S = C_0 + ie^{-\phi} \equiv C_0 + is, \;\; s = \frac{1}{g_s}$$

$$J = t^a\mu_a = t^1 dx^1 \wedge dx^2 + \cdots$$

$$\mathcal{J} = C_4 - \frac{i}{2} J \wedge J$$

▲ S-duality ▲

All pairs of fluxes transform under $SL(2,Z)$ according to:

$$\begin{pmatrix} \mathcal{A} \\ \mathcal{B} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \mathcal{A} \\ \mathcal{B} \end{pmatrix}, \; a,b,c,d \in \mathbb{Z}, \; ad-bc=1$$

In particular, the following $SL(2,\mathbb{Z})$ transformations

$$\mathbf{S} \to \mathbf{S}+\mathbf{1}, \; \mathbf{S} \to -\mathbf{1}/\mathbf{S}$$

interchange the fluxes according to:

$$F \to -H, \quad H \to F$$

and for $S = C_0 + ie^{-\phi} \equiv C_0 + is$, $(s = 1/g_s)$:

$$C_0 \to -\frac{C_0}{(C_0^2+s^2)}, \; s \to \frac{s}{(C_0^2+s^2)}, \; \frac{s}{C_0} \to -\frac{s}{C_0}$$

★ The scalar potential V with F, and H fluxes ★

To appreciate the impact of the non-geometric fluxes on the final theory, let's recall again the well known Gukov-Vafa-Witten superpotential

$$W_{IIB} = \int (F_3 - S H_3) \wedge \Omega_3$$

The only invariant components of the F_3 and H_3 fluxes surviving under the orientifold action are,

$$H_3 : \quad H_{135}\,,\, H_{146}\,,\, H_{236}\,,\, H_{245}\,,\, H_{246}\,,\, H_{235}\,,\, H_{145}\,,\, H_{136}\,,$$

$$F_3 : \quad F_{135}\,,\, F_{146}\,,\, F_{236}\,,\, F_{245}\,,\, F_{246}\,,\, F_{235}\,,\, F_{145}\,,\, F_{136}\ .$$

★ The Scalar Potential ★

In the absence of odd-moduli G^a, the Kähler metric acquires a block diagonal form corresponding to each of the S, U^i and T_α classes of moduli.

The resulting scalar potential V, derived from the formula:

$$e^{-K}\, V \quad = \quad K^{\mathcal{A}\overline{\mathcal{B}}}\,(D_{\mathcal{A}}W)\,(\overline{D}_{\overline{\mathcal{B}}}\overline{W}) - 3|W|^2$$

is expressed in terms of the components of F_3, H_3, and contains 361 distinct terms.

However, defining the following "Axionic" fluxes:

$$\mathbb{F}_{ijk} = F_{ijk} - C_0\; H_{ijk}, \qquad \mathbb{H}_{ijk} = H_{ijk},$$

V can be grouped into three types only with 160 terms:

$$V = V_1 + V_2 + V_3$$

$$V_1 = \frac{1}{4\,s\,\mathcal{V}}\left[\frac{1}{3!}\;\mathbb{F}_{ijk}\,\mathbb{F}_{i'j'k'}\,g^{ii'}\,g^{jj'}g^{kk'}\right],$$

$$V_2 = \frac{1}{4\,s\,\mathcal{V}}\left[\frac{1}{3!}\,(s^2)\,\mathbb{H}_{ijk}\,\mathbb{H}_{i'j'k'}\,g^{ii'}\,g^{jj'}g^{kk'}\right],$$

$$V_3 = \frac{1}{4\,s\,\mathcal{V}}\left[(+2\,s)\times\left(\frac{1}{3!}\times\frac{1}{3!}\;\mathbb{H}_{ijk}\;\mathcal{E}^{ijklmn}\;\mathbb{F}_{lmn}\right)\right].$$

$g^{ii'}$ are elements of the torus metric, and $\mathcal{E}^{ijklmn} = \epsilon^{ijklmn}/\mathcal{V}$.

The origin of V is attributed to the kinetic pieces of IIB action which also includes a Chern-Simons (CS) term:

$$\mathcal{S} \equiv \frac{1}{2}\int d^{10}x\,\sqrt{-g}\,\left(\mathcal{L}_{\mathbb{F}\mathbb{F}} + \mathcal{L}_{\mathbb{H}\mathbb{H}}\right) + S_{CS}$$

$$S_{CS} = -\int d^{10}x\,C^{(4)}\wedge F\wedge H$$

Implementation of T- and S-dualities

The most generic (tree-level) flux induced superpotential will be derived in a series of iterative steps by the T/S dual completions

To start with we first present the IIB/ IIA duality disctionary for the **Moduli fields.** The first line of the table shows type IIB axio-dilaton S, odd G_a, T_α Kähler, and U_i (CS) moduli. Their T-dual IIA moduli appear in the second line.

IIB	S	G^a	T_α	U^i	g_s
IIA	$\mathrm{N}^{\mathbf{0}}$	N^k	U_λ	T^a	z^0

Orientifold actions imply:

$$h^{1,1}_{-} = 0, \; h^{2,1}_{+} = 0$$

The superpotential with non-geometric fluxes

Using type T-duality the IIB superpotential coming from IIA (hep-th/0602089) takes the form:

$$\mathcal{W}_{\rm IIB} = \int_X \Big[\underbrace{F - S\,H}_{G_3} + Q^\alpha\,T_\alpha\Big]_3 \wedge \Omega_3\,, \qquad (7)$$

Due to the T-dual emerging term $Q^\alpha\,T_\alpha$, the underlying S-duality of the type IIB supergravity is no longer a symmetry of the effective scalar potential

S-duality is preserved if the superpotential $\mathcal{W}_{\rm IIB}$ is completed with a new flux P^α:

$$Q^\alpha\,T_\alpha \longrightarrow (Q^\alpha - S P^\alpha)\,T_\alpha$$

Incorporating the new term $\mathcal{W}_{\rm IIB}$ *takes the form:*

$$\mathcal{W}_{\rm IIB} = \int_X \Big[F - S\,H + (Q^\alpha - \underbrace{S P^\alpha)\,T_\alpha}_{\text{new term}} \Big]_3 \wedge \Omega_3 \,, \tag{8}$$

Observations:

▲ *The inclusion of* Q^α, P^α *fluxes introduces Kähler* T_a *in* $\mathcal{W}_{\rm IIB}$.

▲ The new term ST_α in $\mathcal{W}_{\rm IIB}$, implies a term $\mathbf{N^0 U}_\lambda \in \mathcal{W}_{\rm IIA}$

▲ Applying successively T-dualities between $IIA \leftrightarrow IIB$ and implementing S duality completions, we end up with new T_α terms,

$$\propto T_\alpha T_\beta, \& \propto T_\alpha T_\beta T_\gamma$$

which require two new sets of fluxes

$$(Q', P') \;\&\; (H', F')$$

These non-geometric contributions generate a **huge number of terms** in the scalar potential V.
A way to handle them and provide a compact form for V is to write it in terms of axionic flux combinations
(*as we have already done for GWV case*).
The superpotential can be written schematically as follows:

$$W_{\text{IIB}}^{\text{gen}} = \int_{X_3} \Big[(F - S\,H) + (Q - S\,P) \triangleright \mathcal{J} + + (P' - S\,Q') \diamond \mathcal{J}^2 + (H' - S\,F') \odot \mathcal{J}^3 \Big] \wedge \Omega_3 \,, \quad (9)$$

(P', Q'): their indices have non-geometric structure similar to Q_i^{jk}
(H', F'): indices have non-geometric structure similar to R^{ijk}

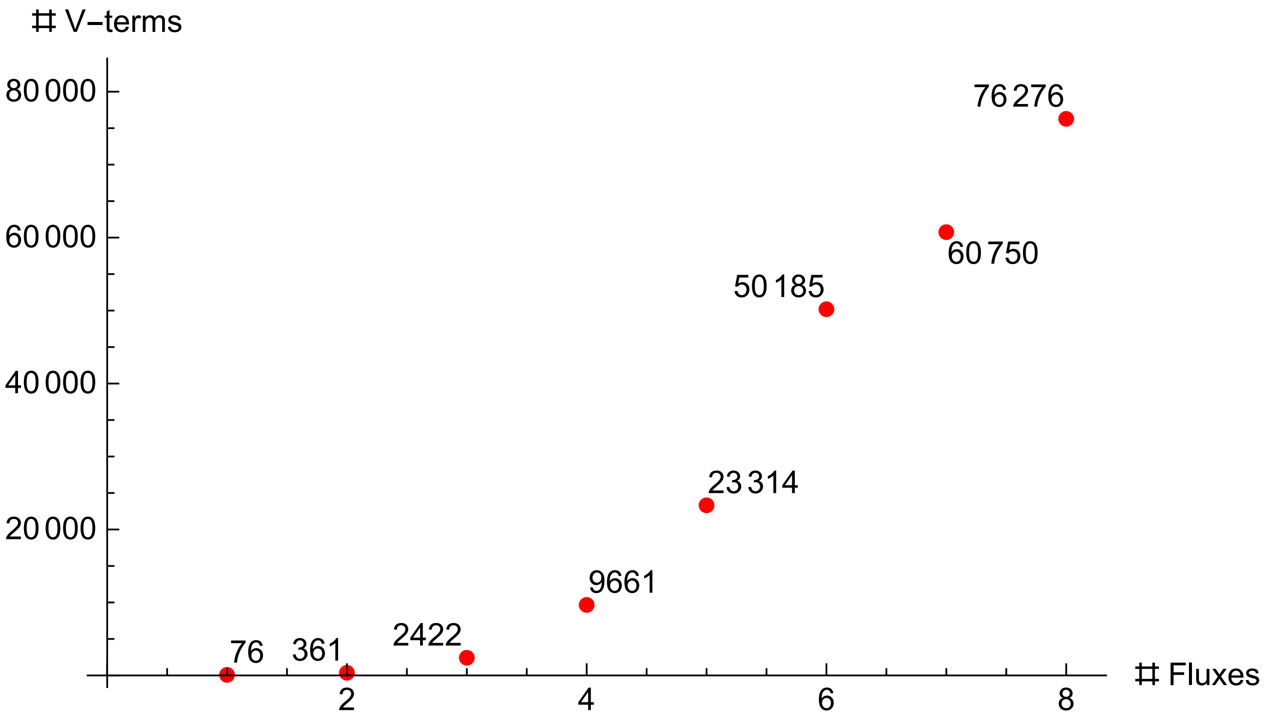
The induced scalar potential V can be written in terms of S-dual pairs and S-self dual terms as follows

$$
\begin{aligned}
V &= \underbrace{V_{\mathbb{F}\mathbb{F}} + V_{\mathbb{H}\mathbb{H}}}_{\mathrm{S-dual\,pair}} + \underbrace{V_{\mathbb{Q}\mathbb{Q}} + V_{\mathbb{P}\mathbb{P}}}_{\mathrm{S-dual\,pair}} + \cdots \\
&+ \underbrace{V_{\mathbb{F}\mathbb{H}} + V_{\mathbb{Q}\mathbb{P}} + V_{\mathbb{P}'\mathbb{Q}'} + V_{\mathbb{H}'\mathbb{F}'}}_{\mathrm{S-self\,dual\,terms}}
\end{aligned} \tag{10}
$$

We demonstrate its S-duality property by examining the terms

$$
\begin{aligned}
V_{\mathbb{F}\mathbb{F}} + V_{\mathbb{H}\mathbb{H}} &\propto \frac{1}{s}\,\mathbb{F}_{ijk}\,\mathbb{F}_{i'jk'} + s\,\mathbb{H}_{ijk}\,\mathbb{H}_{i'jk'} \\
&\propto \underbrace{\frac{1}{s}\,F_{ijk}\,F_{i'jk'} + \frac{s^2 + C_0^2}{s}\,H_{ijk}\,H_{i'jk'}}_{\mathrm{S-dual\,terms}} - 2\underbrace{\frac{C_0}{s}\,F_{ijk}\,H_{i'jk'}}_{self\,\mathrm{S}-dual\,term},
\end{aligned}
$$

Here is the list-plot of the number of V terms vs the number of standard fluxes $\{F, H, Q, P, P', Q', H', F'\}$

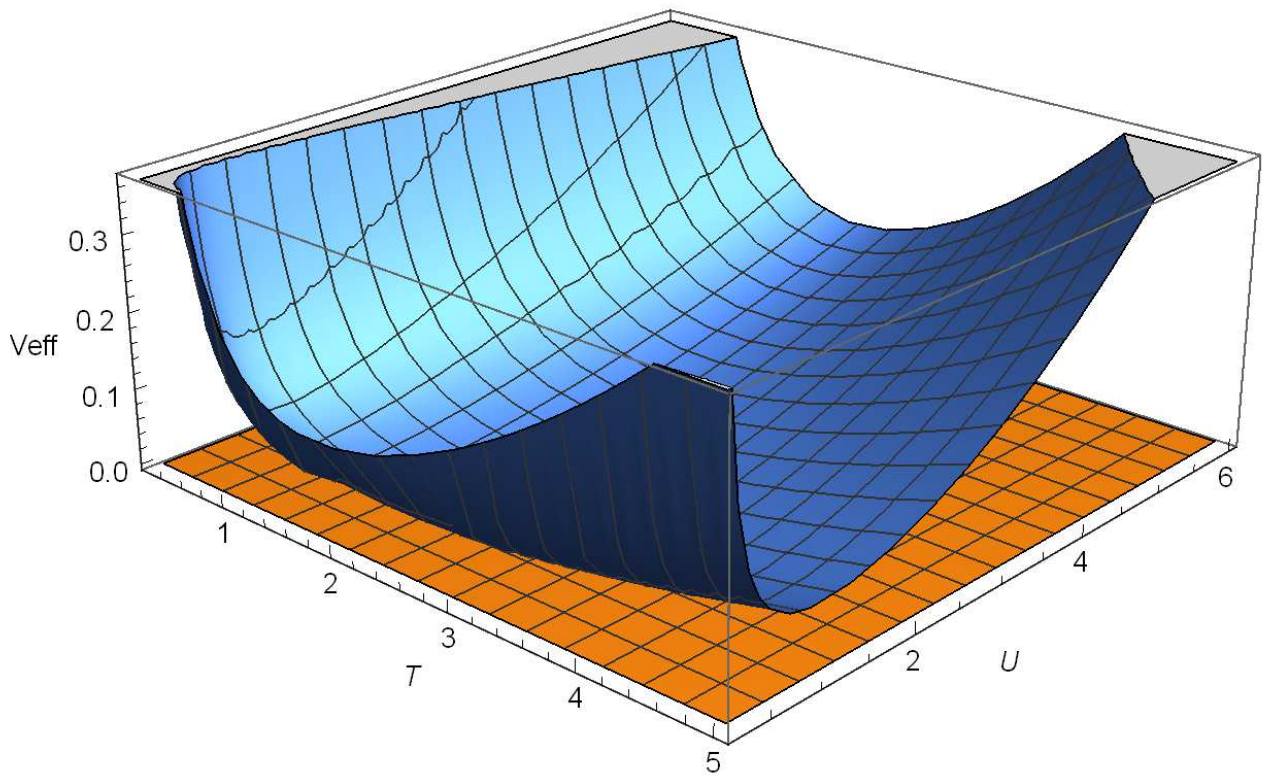


...it's a huge number of terms, however, V *simplifies dramatically if it is expressed in terms of* **axionic fluxes**

The axionic flux structure can be appreciated by observing:

	Standard fluxes	# V-terms	Axionic fluxes	# V-terms
1)	F	76	$\mathbb{F}$	76
2)	F, H	361	$\mathbb{F}, \mathbb{H}$	160
3)	F, H, Q	2422	$\mathbb{F}, \mathbb{H}, \mathbb{Q}$	772
4)	F, H, Q, P	9661	$\mathbb{F}, \mathbb{H}, \mathbb{Q}, \mathbb{P}$	2356
5)	F, H, Q, P, P'	23314	$\mathbb{F}, \mathbb{H}, \mathbb{Q}, \mathbb{P}, \mathbb{P}'$	4855
6)	F, H, Q, P P', Q'	50185	$\mathbb{F}, \mathbb{H}, \mathbb{Q}, \mathbb{P},$ $\mathbb{P}', \mathbb{Q}'$	8326
7)	$F, H, Q, P,$ P', Q', H'	60750	$\mathbb{F}, \mathbb{H}, \mathbb{Q}, \mathbb{P},$ $\mathbb{P}', \mathbb{Q}', \mathbb{H}'$	9603
8)	$F, H, Q, P,$ P', Q', H', F'	76276	$\mathbb{F}, \mathbb{H}, \mathbb{Q}, \mathbb{P},$ $\mathbb{P}', \mathbb{Q}', \mathbb{H}', \mathbb{F}'$	10888

The investigation of V in its full generality is a tremendous task. Here is the plot of a 'naive' case obtained by 'random' flux choices



A minimum of V_{eff} for a simple isotropic case, where all U_i, T_i moduli are assumed to be the same: $U_i = U$, $T_i = T$.

★ Conclusions ★

▲ Fundamental string dualities have been used to construct a genaralised superpotential $\mathcal{W}_{IIB}$ of type IIB string theory on $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold.

▲ Completion arguments of S/T dualities between IIB-IIA superpotentials required the incorporation of four sets of 3-form fluxes resulting to $\mathcal{W}_{IIB}$ with 128 terms

▲ Computations give a huge 4-d scalar potential which is greatly simplified when expressed in terms of "Axionic Fluxes"

▲ Generalised constraints from Bianchi Identities and Tadpole conditions eliminate a considerable amount of terms.

▲ The higher dimensional origin of such terms is an open issue which should be addressed.

★ *Thank You* ★

★ *APPENDIX* ★

Definitions of bases:

$$\alpha_{0,1,2,3} \rightarrow dx^1 \wedge dx^3 \wedge dx^5,\ dx^2 \wedge dx^3 \wedge dx^5,\ dx^1 \wedge dx^4 \wedge dx^5, \cdots$$

$$\beta_{0,1,2,3} \rightarrow dx^1 \wedge dx^3 \wedge dx^5,\ dx^1 \wedge dx^4 \wedge dx^5, dx^1 \wedge dx^3 \wedge dx^6, \cdots$$

$$\mu_{1,2,3} \rightarrow dx^1 \wedge dx^2,\ dx^3 \wedge dx^4,\ dx^5 \wedge dx^6$$

$$\tilde{\mu}^{1,2,3} \rightarrow dx^3 \wedge dx^4 \wedge dx^5 \wedge dx^6, \cdots$$

In a general (geometric) setup, $\mathcal{W}_{IIB}$ can be expressed in terms of generalised axionic fluxes $\mathbb{F}, \mathbb{H}, \ldots$ where the various "products" involved are as follows:

$$
\begin{aligned}
(Q \triangleright \mathcal{J})_{a_1 a_2 a_3} &= \frac{3}{2}\, Q^{b_1 b_2}_{[\underline{a}_1}\, \mathcal{J}_{\underline{a}_2\, \underline{a}_3] b_1 b_2}\,, \\
\left(P' \diamond \mathcal{J}^2\right)_{a_1 a_2 a_3} &= \frac{1}{4}\, P'^{c, b_1 b_2 b_3 b_4}\, \mathcal{J}_{[\underline{a}_1 \underline{a}_2 | c b_1|}\, \mathcal{J}_{\underline{a}_3] b_2 b_3 b_4}\,, \\
\left(H' \odot \mathcal{J}^3\right)_{a_1 a_2 a_3} &= \frac{1}{192}\, H'^{c_1 c_2 c_3, b_1 b_2 b_3 b_4 b_5 b_6}\, \mathcal{J}_{[\underline{a}_1 \underline{a}_2 | c_1 c_2|} \\
&\quad \times \mathcal{J}_{\underline{a}_3] c_3 b_1 b_2}\, \mathcal{J}_{b_3 b_4 b_5 b_6}\,, \\
&\vdots \qquad\qquad \vdots
\end{aligned}
$$

$$
\mathbb{H}_\Lambda = H_\Lambda + \rho_\alpha\, P^\alpha{}_\Lambda + \frac{1}{2}\rho_\alpha\, \rho_\beta\, Q'^{\alpha\beta}{}_\Lambda - \frac{1}{6}\, \rho_\alpha\, \rho_\beta\, \rho_\gamma\, F'^{\alpha\beta\gamma}{}_\Lambda
$$