The following ~~classical~~ canonical calculations refer to any dimensionality of manifold. ~~σ~~ σ stands for M coordinates σ^a ($M=2$ for membrane).

$[\ ,\ ]$ is Poisson bracket i.e

$$[F, G] = \int d\sigma \left(\frac{\delta F}{\delta x^k(\sigma)} \frac{\delta G}{\delta p^k(\sigma)} - \frac{\delta F}{\delta p^k(\sigma)} \frac{\delta G}{\delta x^k(\sigma)} \right)$$

$$- \frac{\partial F}{\partial Z} \frac{\partial G}{\partial \eta} + \frac{\partial F}{\partial \eta} \frac{\partial G}{\partial Z}$$

(see page 1 for definition of Z)

Don't try to follow details of page 6 – I have checked it

Note that the function $F_a^b(\rho, \sigma)$ on page 1 which occurs many times projects onto divergence-free functions

i.e $\frac{\partial}{\partial \rho^b} \int F_a^b(\rho\, \sigma) f^a(\sigma) d\sigma = 0$ for any f^a

One way of constructing $\zeta(\sigma)$ to satisfy

$$\frac{\partial \zeta}{\partial \sigma^a} = \frac{1}{\eta} p \cdot \frac{\partial x}{\partial \sigma^a}$$ is as follows:

Choose $G^a(\rho, \sigma)$ to satisfy $\dfrac{\partial G^a}{\partial \rho^a} = \delta(\rho, \sigma) - 1$
[and $\int G^a(\rho,\sigma) d\sigma = 0$]

Then $\displaystyle\int \frac{\partial \zeta}{\partial \rho^a} G^a(\rho, \sigma) d\rho = -\zeta(\sigma) + \int \zeta(\rho) d\rho$

$= \dfrac{1}{\eta} \displaystyle\int p \cdot \frac{\partial x}{\partial \rho^a} G^a(\rho, \sigma) d\rho$. With $\int \zeta(\rho) d\rho = Z$, we can take

$$\zeta(\sigma) = Z - \frac{1}{\eta} \int d\rho \, p \cdot \frac{\partial x}{\partial \rho^a} G^a(\rho, \sigma)$$

With this definition, $\displaystyle\frac{\partial \zeta}{\partial \sigma^a} - \frac{1}{\eta} p \cdot \frac{\partial x}{\partial \sigma^a} = -\frac{1}{\eta} \int d\rho \, p \cdot \frac{\partial x}{\partial \rho^b} F_a^b(\rho, \sigma)$

where $F_a^b(\rho, \sigma) = \dfrac{\partial G^b(\rho, \sigma)}{\partial \sigma^a} + \delta_a^b \, \delta(\rho, \sigma)$

so $\dfrac{\partial F_a^b(\rho, \sigma)}{\partial \rho^b} = \dfrac{\partial}{\partial \sigma^a} \delta(\rho, \sigma) + \dfrac{\partial}{\partial \rho^a} \delta(\rho, \sigma) = 0$

and so $\dfrac{\partial \zeta}{\partial \sigma^a} - \dfrac{1}{\eta} p \cdot \dfrac{\partial x}{\partial \sigma^a} \approx 0$

[For any dimension of manifold, our constraints are $\displaystyle\int p \cdot \frac{\partial x}{\partial \rho^b} f^b d\rho \approx 0$
when $\dfrac{\partial f^b}{\partial \rho^b} = 0$]

Lorentz generators, expressed in terms of p^i, x^i

η, Z with $[Z, \eta] = -1$, are now taken as

$$P^i = \int p^i d\sigma \; ; \; P^+ = \eta \; ; \; P^- = \frac{1}{\eta}\int \frac{p^2+h}{2} d\sigma$$

$$M^{ij} = \int (x^i p^j - x^j p^i) d\sigma \; ; \; M^{i+} = \eta X^i$$

$$(X^i = \int x^i d\sigma); \; M^{i-} = \left\{\int x^i \left(\frac{p^2+h}{2\eta}\right) d\sigma - \int \xi p^i d\sigma\right\}$$

$$= \frac{1}{\eta}\left\{\int x^i \frac{p^2+h}{2} d\sigma + \int p \cdot \frac{\partial x}{\partial p^a} G^a(\rho, \sigma) p^i(\sigma) d\rho d\sigma\right\} - ZP^i$$

$$M^{+-} = -Z\eta .$$

These should satisfy $[M^{\mu\nu}, P^\rho] = g^{\nu\rho} P^\mu - g^{\mu\rho} P^\nu$

$$[M^{\mu\nu}, M^{\rho\sigma}] = g^{\nu\rho} M^{\mu\sigma} ----$$

Easy to check except for relations with M^{i-}, ie

$[M^{i-}, P^j] = \delta_{ij} P^-$ (Easy to check since second term in M^{i-} depends only on derivatives of x)

$[M^{i-}, P^+] = P^i$ (easy)

$[M^{i-}, P^-] = 0$ (see page 3)

$[M^{i+}, M^{j-}] = \delta_{ij} M^{+-} - M^{ij}$ (see page 3)

$[M^{i-}, M^{j-}] = 0$ (see page 5)

Relations with M^{ij}, M^{+-} are manifest from the form of M^{i-}

③

$$[M^{i+}, M^{j-}] = \left[X^i, \int x^j \frac{p^2+h}{2}\, d\sigma + \int p\cdot\frac{\partial x}{\partial \rho^a} G^a(\rho\,\sigma)\, p^j(\sigma)\, d\rho\, d\sigma\right]$$
$$- [\eta X^i, Z P^j]$$

$$= \int x^j p^i\, d\sigma + \int \frac{\partial x^i}{\partial \rho^a} G^a(\rho\,\sigma)\, p^j(\sigma)\, d\rho\, d\sigma + \delta_{ij} \int\!\!\int p\cdot\frac{\partial x}{\partial \rho^a} G^a(\rho\,\sigma)\, d\rho\, d\sigma$$
$$- X^i P^j - \delta_{ij}\, \eta Z$$

Third term is zero since $\int G^a(\rho,\sigma)\, d\sigma = 0$

Second term $= -\int x^i(\rho) \{\delta(\rho\,\sigma) - 1\} p^j(\sigma)\, d\rho\, d\sigma$

$= -\int x^i p^j\, d\sigma + X^i P^j$ so $[M^{i+}, M^{j-}] = \delta_{ij} M^{+-} - M^{ij}$ (Correct)

$$[M^{i-}, P^-] = -\left[Z, \frac{1}{\eta}\right]\left(\int \frac{p^2+h}{2}\, d\sigma\right) P^i$$
$$+ \frac{1}{\eta^2}\left[\int x^i \frac{p^2+h}{2}\, d\sigma, \int \frac{p^2+h}{2}\, d\sigma\right]$$
$$+ \frac{1}{\eta^2}\left[\int\!\!\int p\cdot\frac{\partial x}{\partial \rho^a} G^a(\rho\,\sigma)\, p^i(\sigma)\, d\rho\, d\sigma, \int \frac{p^2+h}{2}\, d\sigma\right]$$

$$\eta^2 [M^{i-}, P^-] = -P^i \int \frac{p^2+h}{2}\, d\sigma + \int \frac{p^2+h}{2}\, d\sigma\, p^i$$
$$- \int \frac{\partial}{\partial \sigma^a}\left(x^i h h^{ab} \frac{\partial x^k}{\partial \sigma^b}\right) p^k\, d\sigma + \int x^i p^k \frac{\partial}{\partial \sigma^a}\left(h h^{ab} \frac{\partial x^k}{\partial \sigma^b}\right) d\sigma$$
$$- \int d\rho\, d\sigma \frac{\partial}{\partial \rho^a}\left\{p^k(\rho) G^a(\rho\,\sigma)\, p^i(\sigma)\right\} p^k(\rho) + \int d\rho\, d\sigma \frac{\partial x^k}{\partial \rho^a} G^a(\rho\,\sigma)\, p^i(\sigma) \frac{\partial}{\partial \rho^b}\left(h h^{bc} \frac{\partial x^k}{\partial \rho^c}\right)$$
$$+ \int d\rho\, d\sigma\, p\cdot\frac{\partial x}{\partial \rho^a} G^a(\rho\,\sigma) \frac{\partial}{\partial \sigma^b}\left(h h^{bc} \frac{\partial x^i}{\partial \sigma^c}\right)$$

$$= -P^i \int \frac{p^2+h}{2}\, d\sigma + \int p^i\, \frac{p^2+h}{2}\, d\sigma$$

$$- \int \frac{\partial x^i}{\partial \sigma^a}\, h\, h^{ab}\, p \cdot \frac{\partial x}{\partial \sigma^b}\, d\sigma - \int \frac{p^2(\rho)}{2} \left\{ \delta(\rho,\sigma) - 1 \right\} p^i(\sigma)\, d\rho\, d\sigma$$

$$- \int d\rho\, d\sigma\, \frac{1}{2} h(\rho) \left\{ \delta(\rho\,\sigma) - 1 \right\} p^i(\sigma)$$

$$- \int d\rho\, d\sigma\, p \cdot \frac{\partial x}{\partial \rho^a}\, \frac{\partial G^a(\rho,\sigma)}{\partial \sigma^b}\, h\, h^{bc}\, \frac{\partial x^i}{\partial \sigma^c}$$

[I have used $\frac{\partial x^k}{\partial \rho^a} \frac{\partial}{\partial \rho^b} h\, h^{bc} \frac{\partial x^k}{\partial \rho^c} = \frac{1}{2} \frac{\partial h}{\partial \rho^a}$ and several integrations by parts]

$$\cancel{= -P^i \int \frac{p^2+h}{2}\, d\sigma +}$$

$$= - \int d\sigma\, h\, h^{bc} \frac{\partial x^i}{\partial \sigma^c} \left\{ \int d\rho\, p \cdot \frac{\partial x}{\partial \rho^a} \left(\frac{\partial G^a}{\partial \sigma^b}(\rho,\sigma) + \delta^a_b\, \delta(\rho,\sigma) \right) \right\}$$

and as on page 1, term in $\{\ \}$ ≈ 0 by constraints.

so $[M^{i-}, P^-] \approx 0$ (correct)

$$[M^{i-}, M^{j-}]\,\eta^2$$

$$= \left[\int x^i\left(\frac{p^2+h}{2}\right)d\sigma,\ \int x^j\left(\frac{p^2+h}{2}\right)d\sigma\right]$$

$$+ \left[\int x^i\left(\frac{p^2+h}{2}\right)d\sigma,\ \int p\cdot\frac{\partial x}{\partial\rho^a}\,G^a(\rho,\sigma)\,p^j(\sigma)\,d\rho\, d\sigma\right] - i\leftrightarrow j$$

$$+ \left[\int p\cdot\frac{\partial x}{\partial\rho^a}\,G^a(\rho\,\sigma)\,p^i(\sigma)\,d\rho\, d\sigma,\ \int p\cdot\frac{\partial x}{\partial\lambda^b}\,G^b(\lambda,\mu)\,p^j(\mu)\,d\lambda\, d\mu\right]$$

$$+ P^j\left\{\int x^i\left(\frac{p^2+h}{2}\right)d\sigma + \int p\cdot\frac{\partial x}{\partial\rho^a}\,G^a(\rho\,\sigma)\,p^i(\sigma)\,d\rho\, d\sigma\right\} - i\leftrightarrow j$$

$$= \left\{\int d\sigma\ \frac{p^2+h}{2}\ x^j p^i - \int d\sigma\ \frac{\partial}{\partial\sigma^a}\left(x^i h h^{ab}\frac{\partial x^k}{\partial\sigma^b}\right)x^j p^k\right\} \ \cancel{-\ i\leftrightarrow j}$$

$$+ \int d\sigma\, d\rho\ \frac{p^2(\sigma)+h(\sigma)}{2}\left\{\frac{\partial x^i}{\partial\sigma^a}\,G^a(\sigma,\rho)\,p^j(\rho) + \cancel{p\cdot\frac{\partial x}{\partial\rho^a}\,G^a(\rho,\sigma)\,\delta_{ij}}\right\}$$

$$- \int d\sigma\, d\rho\ \frac{\partial}{\partial\sigma^a}\left(x^i h h^{ab}\frac{\partial x^k}{\partial\sigma^b}\right)\left\{\frac{\partial x^k}{\partial\sigma^c}\,G^c(\sigma,\rho)\,p^j(\rho)\right\}$$

$$- \int d\sigma\, d\rho\ \frac{\partial}{\partial\sigma^a}\left(x^i h h^{ab}\frac{\partial x^j}{\partial\sigma^b}\right)\ p\cdot\frac{\partial x}{\partial\rho^c}\,G^c(\rho,\sigma)$$

$$+ \int d\sigma\, d\rho\ x^i(\sigma)\,p^k(\sigma)\,\frac{\partial}{\partial\sigma^a}\left\{p^k(\sigma)\,G^a(\sigma,\rho)\right\}p^j(\rho)$$

$$- \int d\rho\, d\sigma\, d\lambda\ \frac{\partial}{\partial\rho^a}\left\{p^k(\rho)\,G^a(\rho\,\sigma)\right\}p^i(\sigma)\left\{\frac{\partial x^k}{\partial\rho^b}\,G^b(\rho,\lambda)\,p^j(\lambda) \cancel{+ p\cdot\frac{\partial x}{\partial\lambda^b}\,G^b(\lambda,\rho)}\right\}$$

$$- \int d\rho\, d\sigma\, d\lambda\ \frac{\partial}{\partial\rho^a}\left\{p^j(\rho)\,G^a(\rho\,\sigma)\right\}p^i(\sigma)\ p\cdot\frac{\partial x}{\partial\lambda^b}\,G^b(\lambda,\rho)$$

$$+ P^j\left\{\int x^i\,\frac{p^2+h}{2}\,d\sigma + \int p\cdot\frac{\partial x}{\partial\rho^a}\,G^a(\rho\,\sigma)\,p^i(\sigma)\,d\rho\, d\sigma\right\}$$

$-\ i\leftrightarrow j$ throughout

$$\int d\sigma\, \frac{p^2+h}{2}\, x^j p^i + \int d\sigma\, h h^{ab}\, p\cdot\frac{\partial x}{\partial\sigma^b}\, x^i \frac{\partial x^j}{\partial\sigma^a}$$

$$+ \int d\sigma\, d\rho\, \frac{p^2+h}{2}\, \frac{\partial x^i}{\partial\sigma^a} \int G^a(\sigma,\rho)\, p^j(\rho)\, d\rho \;-\; \int d\sigma d\rho\, \frac{\partial x^i}{\partial\sigma^a}\, h(\sigma)\, G^a(\sigma\rho)\, p^j(\rho)$$

$$\cancel{- \int d\sigma d\rho\, x^i(\sigma)\, \frac{1}{2}\frac{\partial h}{\partial\sigma^a}\, G^a(\sigma\rho)\, p^j(\rho)} + \int d\sigma\, d\rho\, x^i(\sigma)\, h h^{ab}\, \frac{\partial x^j}{\partial\sigma^b}\, p\cdot\frac{\partial x}{\partial\rho^c}\, \frac{\partial G^c(\rho\sigma)}{\partial\sigma^a}$$

$$+ \int d\sigma d\rho\, \frac{\partial x^i}{\partial\sigma^a}\, \frac{1}{2} h\, G^a(\sigma,\rho)\, p^j(\rho) + \int d\sigma d\rho\, x^i(\sigma)\, \frac{1}{2} h\, (\delta(\sigma,\rho)-1)\, p^j(\rho)$$

$$\pm \int d\sigma d\rho\, \frac{\partial x^i(\sigma)}{\partial\sigma^a}\, \frac{p^2(\sigma)}{2}\, G^a(\sigma\rho)\, p^j(\rho) + \int d\sigma d\rho\, x^i(\sigma)\, \frac{1}{2} p^2(\sigma) \{\delta(\sigma,\rho)-1\}\, p^j(\rho)$$

$$- \int d\rho\, d\sigma\, d\lambda\, \frac{\partial p}{\partial\rho^a}\cdot\frac{\partial x}{\partial\rho^b}\, G^a(\rho\sigma)\, G^b(\rho\lambda)\, p^i(\sigma)\, p^j(\lambda)$$

$$- \int d\rho\, d\sigma\, d\lambda\, p\cdot\frac{\partial x}{\partial\rho^b} \{\delta(\rho\sigma)-1\}\, p^i(\sigma)\, G^b(\rho\lambda)\, p^j(\lambda)$$

$$+ \int d\rho\, d\sigma\, d\lambda\, p^j(\rho)\, p^i(\sigma)\, p\cdot\frac{\partial x}{\partial\lambda^b}\, G^a(\rho\sigma)\, \frac{\partial G^b(\lambda\rho)}{\partial\rho^a}$$

$$+ P^j \int x^i\, \frac{p^2+h}{2}\, d\sigma + P^j \int p\cdot\frac{\partial x}{\partial\rho^a}\, G^a(\rho\sigma)\, p^i(\sigma)\, d\rho\, d\sigma$$

$$- \; i \leftrightarrow j$$

$$= \int d\sigma \left(x^i \frac{\partial x^j}{\partial\sigma^a} - x^j \frac{\partial x^i}{\partial\sigma^a}\right) h h^{ab} \int d\rho\, p\cdot\frac{\partial x}{\partial\rho^c} \left\{\frac{\partial G^c(\rho\sigma)}{\partial\sigma^b} + \delta(\rho\sigma)\, \delta^c_b\right\}$$

$$- \int d\rho\, d\sigma \left(p^i(\rho)\, p^j(\sigma) - p^j(\rho)\, p^i(\sigma)\right) G^a(\rho,\sigma) \int d\lambda\, p\cdot\frac{\partial x}{\partial\lambda^b} \left\{\frac{\partial G^b(\lambda\rho)}{\partial\rho^a} + \delta(\lambda\rho)\, \delta^b_a\right\}$$

$$+ \int d\rho\, d\sigma\, p^i(\rho)\, p^j(\sigma) \int_{d\lambda} G^a(\lambda,\rho)\, G^b(\lambda,\sigma) \left(\frac{\partial x}{\partial\lambda^a}\cdot\frac{\partial p}{\partial\lambda^b} - \frac{\partial x}{\partial\lambda^b}\cdot\frac{\partial x}{\partial\lambda^a}\right)$$

$$\approx 0 \quad \text{(correct)}$$