# Unified Matrix Theories from Nonrelativistic Strings

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Matrix Quantum Mechanics for M-Theory Revisited

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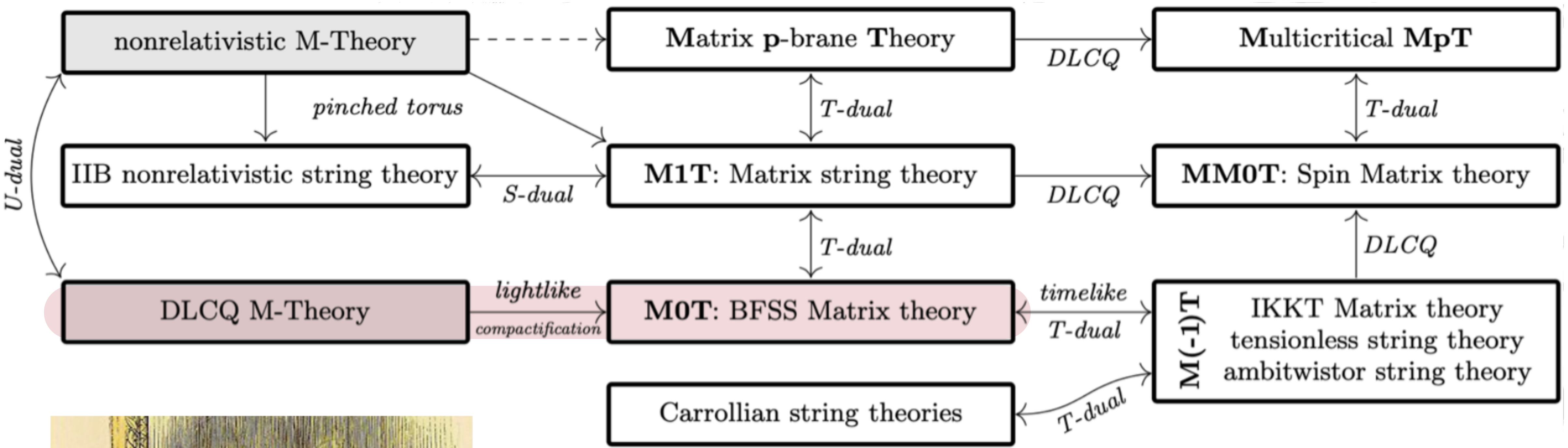
# Unification of Decoupling Limits

Unification of Decoupling Limits in String & M-Theory
Chris Blair, Johannes Lahnsteiner, Niels Obers, ZY
2311.10564



nonrelativistic M-Theory

**Matrix p-brane Theory**

**Multicritical MpT**

IIB nonrelativistic string theory

**M1T**: Matrix string theory

**MM0T**: Spin Matrix theory

DLCQ M-Theory

**M0T**: BFSS Matrix theory

**M(-1)T**

IKKT Matrix theory
tensionless string theory
ambitwistor string theory

Carrollian string theories

[Carroll 1871]

Through the Looking-Glass of Exotic String Worldsheet

and What Alice Found There ...

Worldsheet Formalism for Decoupling Limits in String Theory
Joaquim Gomis, ZY
2311.10565

# Outline

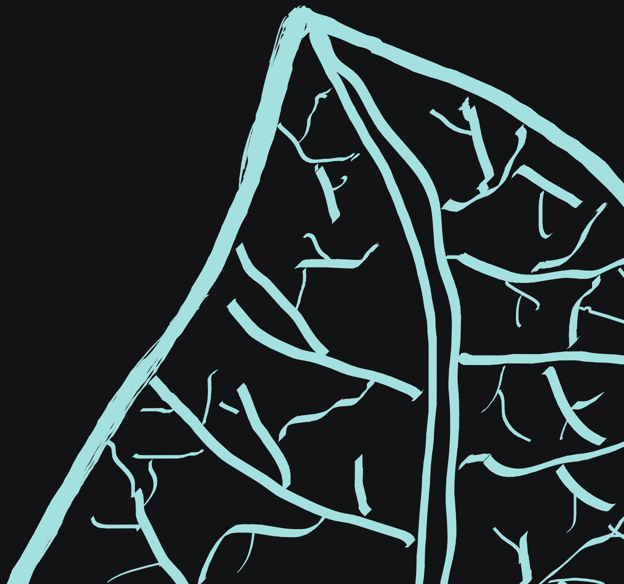
2311.10565
2311.10564

1. Fundamental strings for Matrix theory
2. Duality web from T-dualities
3. **REVIEW** of nonrelativistic string theory

2311.10564 4. Back to M-theory

see also my recent reviews

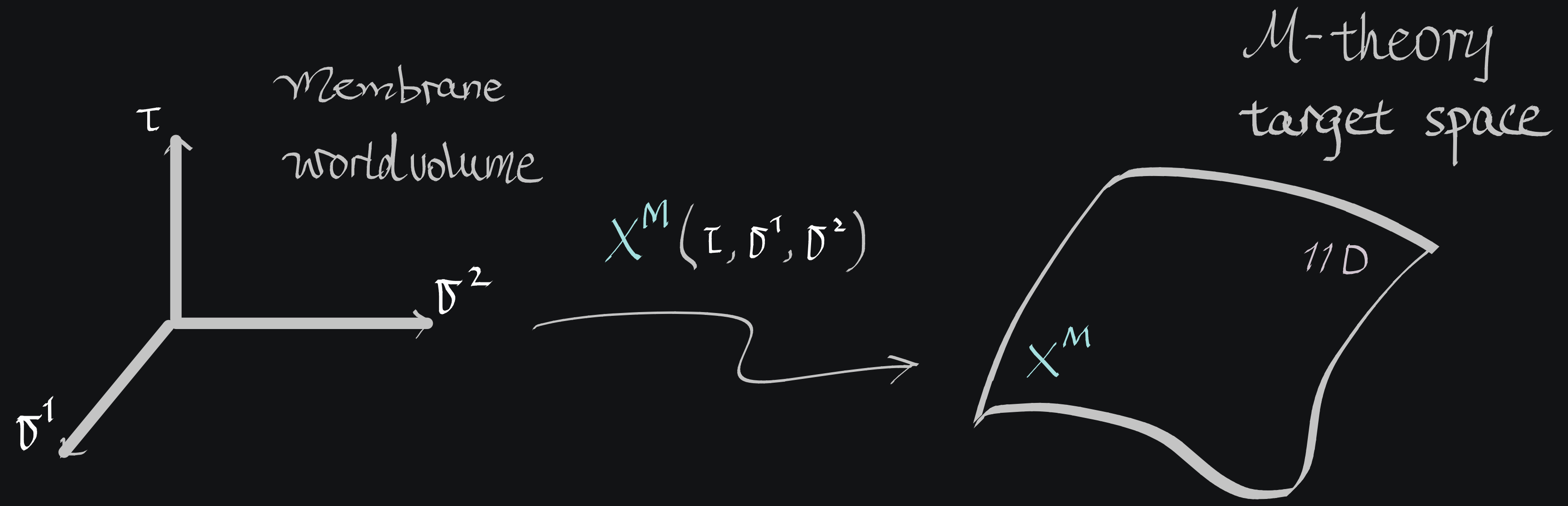
2312.12930 Section 7. Matrix theory & the string worldsheet ZY
2202.12698 Aspects of nonrelativistic strings Oling, ZY



REVIEW

# Why is M-Theory Difficult?

- membrane sigmal model: $S = -T_2 \int d^3\sigma \sqrt{-\det\left[\partial_\alpha X^M \partial_\beta X^N G_{MN}(X)\right]}$



$M = 0, 1, \cdots, 9, 10$

- Polyakov formulation: $S = -\frac{1}{2}\int d^3\sigma \left(\partial_\alpha X^M \partial^\alpha X_M + \underset{\substack{\wr \\ \text{dim.} = -\frac{1}{2}}}{g_{MNL}} X^M \partial_\alpha X^N \partial^\alpha X^L + \cdots\right)$

  not power-counting renormalizable!

  $\uparrow$ RG flow

- high-energy completion? quantum critical supermembrane: no worldvolume boost!

  renormalizable! $S = \frac{1}{2}\int d\tau\, d^2\sigma\, d^2\theta \left(D_\tau Y^M D_\tau Y^N G_{MN} - \partial_i Y^M \partial_i Y^N H_{MN}\right)$ [Hořava '08] [ZY '22]

  [Hořava, Melby-Thompson, Randall]

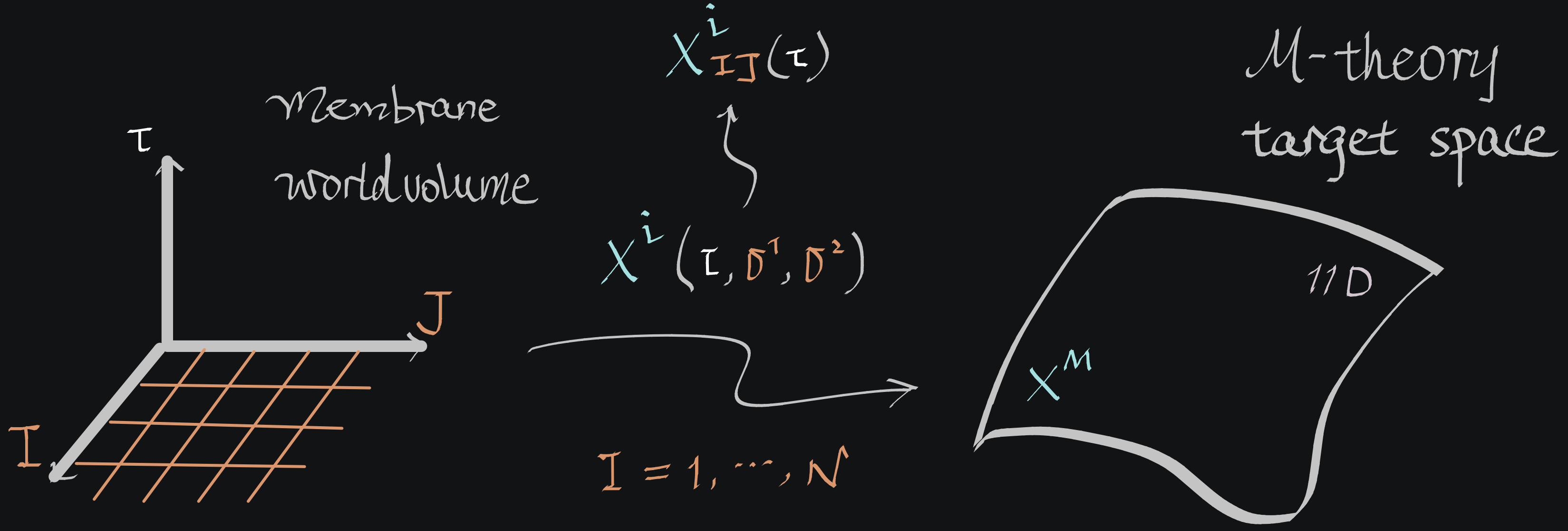
Is M-theory fundamentally nonrelativistic?

REVIEW

# Nonperturbative Quantization?

- membrane sigmal model: $S = -T_2 \int d^3\sigma \sqrt{-\det(\partial_\alpha X^M \partial_\beta X_M)}$

[Goldstone '82]
[Hoppe '82 '87]



$M = 0, 1, \cdots, 9, 10$

light cone gauge $X^0 - X^{10} = \tau$

$i = 1, \cdots, 9$

- SU(N) nonrel. quantum mechanics of 9 $N \times N$ matrices [de Wit, Hoppe, Nicolai '88] [Banks, Fischler, Shenker, Susskind '96] [Susskind '97] [Seiberg '97] [Sen '97]

$$S \sim \frac{1}{2g^2} \int d\tau \, \mathrm{tr}\left( \dot{X}^i \dot{X}^i + \frac{1}{4} [X^i, X^j]^2 + \text{fermions} \right)$$

nonrel. limit of a stack of $N$ D0-particles in IIA $\xleftarrow[\text{compactify}]{\text{null}}$ DLCQ M-theory

Discrete Light Cone Quantization

[Gomis, ZY '23]

# Infinite Boost v.s. Nonrelativistic Limit

Lorentz factor $\gamma = \frac{1}{\sqrt{1-v^2}}$

- DLCQ M-theory [Seiberg '97]

$$x^{10} \sim x^{10} + 2\pi R_0 \xrightarrow{\text{Lorentz boost}} \begin{cases} X^0 = \gamma\,(x^0 + v\,x^{10}) \\ X^1 = \gamma\,(x^{10} + v\,x^0) \end{cases} \qquad \text{def.} \begin{cases} X^+ = X^0 + X^1 = 2\gamma\, x^+ + \cdots \\ X^- = X^0 - v\,X^1 = \frac{1}{\gamma}\, x^0 + \cdots \end{cases}$$

$$\xrightarrow[\text{fix } R = 2\gamma R_0]{\gamma \to \infty} X^+ \sim X^+ + 2\pi R$$

- wrapped M2-brane around a compactified $X^+$? $X^+ = \sigma^2$ $\omega \propto \gamma$

String action $$S = -T \int d^2\sigma \sqrt{-\det \begin{pmatrix} 1/\omega^2 & \partial_\beta X^- \\ \partial_\alpha X^- & \partial_\alpha X^i \partial_\beta X^i \end{pmatrix}} + \cdots$$

$\alpha, \beta = 0, 1$

$i = 1, \cdots, 9$

$$\Big\downarrow \; X^- = X^0$$

$$S = -T \int d^2\sigma \sqrt{-\det\left(-\omega\, \partial_\alpha X^0 \partial_\beta X^0 + \frac{1}{\omega}\, \partial_\alpha X^i \partial_\beta X^i\right)}$$

Nambu-Goto string

$X^0 \to \sqrt{\omega}\, X^0$

$X^i \to \frac{1}{\sqrt{\omega}}\, X^i$

- infinite boost $\gamma \to \infty$ ⇒ nonrel. $\omega \to \infty$ limit of IIA effectively, infinite speed of light

[Blair, Lahnsteiner, Obers, ZY '23]

# What about D0-Branes?

- nonrelativistic limit as a BPS limit

$$S_{D0} = -\frac{1}{g_s \alpha'^{\frac{1}{2}}} \int d\tau \, \mathrm{tr} \left\{ \sqrt{- \dot{X}^{\mu} \dot{X}_{\mu} \det\left(1 - \tfrac{1}{4}[X^i, X^j]^2\right)} \right\} + \frac{1}{g_s \alpha'^{\frac{1}{2}}} \int C_1$$

$$\xrightarrow{\omega \to \infty} \frac{1}{2} \frac{1}{g_s \alpha'^{\frac{1}{2}}} \int d\tau \, \mathrm{tr} \left( \dot{X}^i \dot{X}^i + \frac{1}{4} [X^i, X^j]^2 \right)$$

$\mu = 0, \cdots, 9$

$X^0 \longrightarrow \sqrt{\omega} X^0$

$X^i \longrightarrow \frac{1}{\sqrt{\omega}} X^i$

$g_s \longrightarrow \omega^{-\frac{3}{2}} g_s$

$C_1 \longrightarrow \omega^2 g_s^{-1} dX^0$

Static gauge

$X^0 = \tau$

see also

[Gopakumar, Minwalla, Seiberg, Strominger '00]

[Harmark '00] [Gomis, Ooguri '00]

[Danielsson, Guijosa, Kruczenski '00]

- BFSS Matrix theory from a full-fledged String theory

DLCQ M-theory

$\big\downarrow$ compactify over a lightlike circle

Matrix 0-brane theory

a decoupling limit of IIA

D0-branes: BFSS Matrix theory

D2-branes: noncommutative Yang-Mills

fundamental string

# Polyakov String in Matrix 0-Brane Theory [Gomis, ZY '23]

- Nambu-Goto string

$$S = - T \int d^2\sigma \sqrt{-\det \begin{pmatrix} 0 & \partial_\beta X^0 \\ \partial_\alpha X^0 & \partial_\alpha X^i \partial_\beta X^i \end{pmatrix}}$$

see also [Batlle, Gomis, Not '16]

- Polyakov formulation flat gauge

$$S_P = \frac{T}{2} \int d^2\sigma \left( \partial_\sigma X^0 \partial_\sigma X^0 + \partial_\tau X^i \partial_\tau X^i + \lambda\, \partial_\tau X^0 \right)$$

tropological sigma models
for Gromov-Witten invariants;
nonequilibrium string theory?

[Albrychiewicz, Ellers, Valiente, Hořava '23]

target space Galilei boost $\delta X^0 = 0$ $\delta X^i = v^i X^0$ $\delta \lambda = -2 v^i \partial_\tau X^i$

Worldsheet Carroll boost $\delta \tau = v^i \sigma$ $\delta \sigma = 0$ $\delta \lambda = 2 v^i \partial_\sigma X^0$

Worldsheet Galilean Conformal Algebra $\approx$ $BMS_3$ [Bagchi '10]

[Gomis, ZY '23]

# Worldsheet Topology: Nodal Spheres

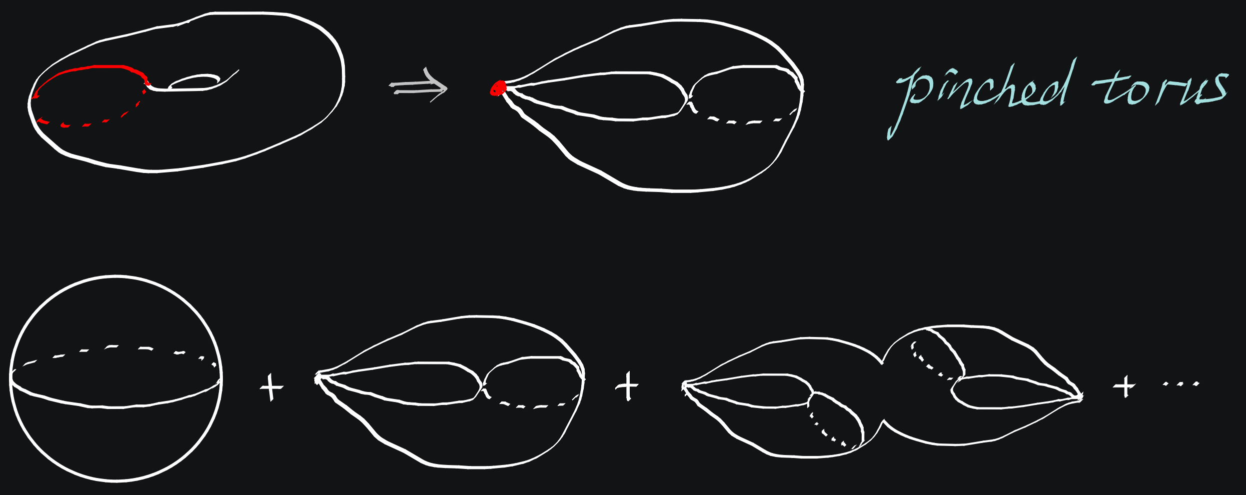
- MOT string from a nonrel. limit $\begin{cases} X^0 \to \sqrt{\varpi}\, X^0 \\ X^i \to \frac{1}{\sqrt{\varpi}} X^i \end{cases}$ $\quad \begin{matrix} \tau \to \varpi^{-1}\tau \\ \sigma \to \sigma \end{matrix}$

$$S = -\frac{T}{2}\int d^2\sigma\, \partial_\alpha X^\mu \partial^\alpha X_\mu$$

$$= -\frac{T}{2}\int d^2\sigma \left[ \underbrace{\varpi^2 \partial_\tau X^0 \partial_\tau X^0}_{-\lambda\, \partial_\tau X^0 - \frac{\lambda^2}{4\omega^2}} - \left(\partial_\sigma X^0 \partial_\sigma X^0 + \partial_\tau X^i \partial_\tau X^i\right) + \mathcal{O}(\omega^{-2}) \right]$$

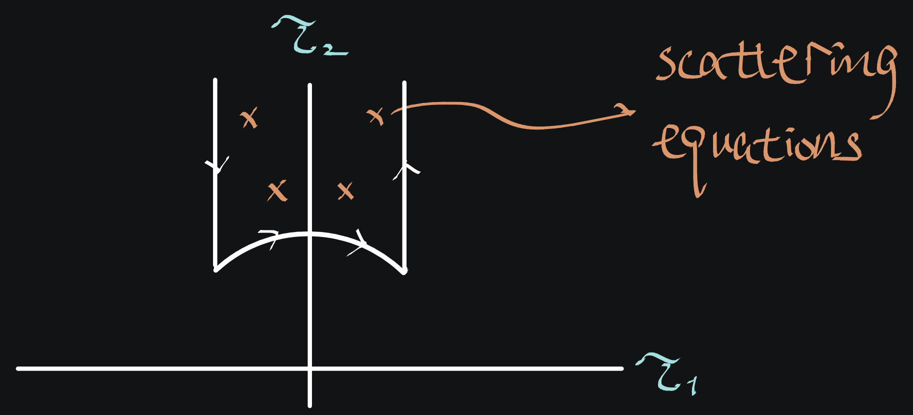
- Singular topology from $\varpi \to \infty$

modulus $\tau \longrightarrow i\infty$



ambitwistor string amplitudes

$\tau \to i\infty$



[Geyer, Mason, Monteiro, Tourkine '15]

[Gomis, ZY '23]

# Duality Web from T-Dualities

MOT string $i=1, \cdots, 9$

$$S_{MOT} = \frac{T}{2}\int d^2\sigma\left(\partial_\sigma X^0 \partial_\sigma X^0 + \partial_\tau X^i \partial_\tau X^i + \lambda\, \partial_\tau X^0\right)$$

T-dualize $X^1$

T-dualize $X^0$

M1T string $a=0,1$ $i=2,\cdots,9$

$$S_{M1T} = \frac{T}{2}\int d^2\sigma\left(-\partial_\sigma X^a \partial_\sigma X_a + \partial_\tau X^i \partial_\tau X^i + \lambda_a \partial_\tau X^a\right)$$

D1-strings: **Matrix string theory**

[Motl '97] [Dijkgraaf, Verlinde, Verlinde '97]

tensionless string [Linström, Sundborg, Theodoridis '91]

$$S_{M(-1)T} = \frac{T}{2}\int d^2\sigma\, \partial_\tau X^\mu \partial_\tau X_\mu \qquad \mu = 0, \cdots, 9$$

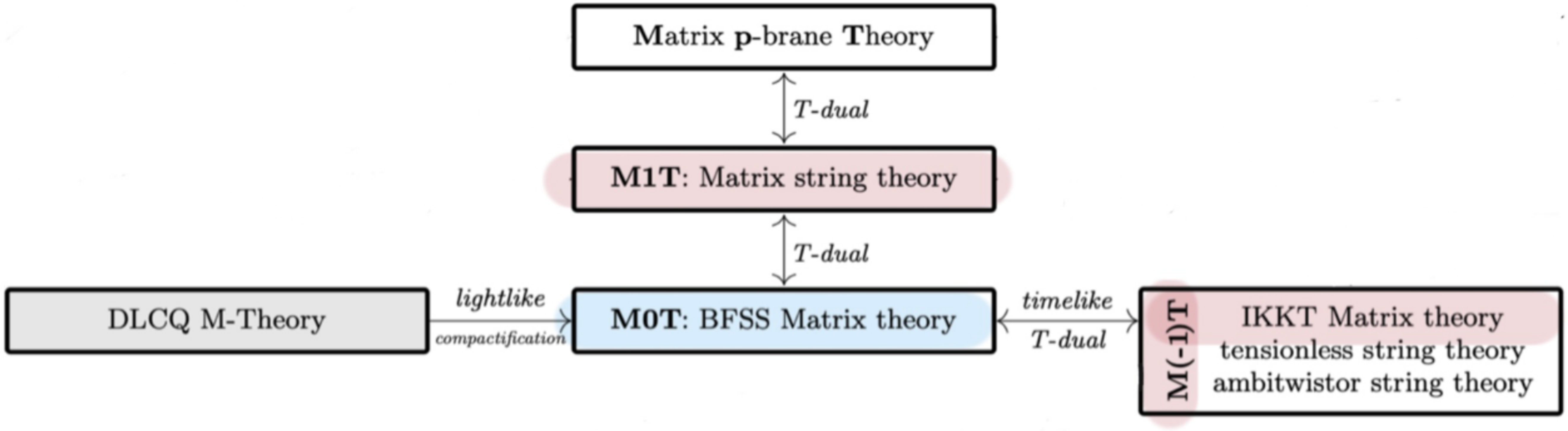
D(-1)-instantons: **IKKT Matrix theory**

[Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

[Blair, Lahnsteiner, Obers, ZY '23]

$p=3$ $\mathcal{N}=4$ SYM (coupled to non-Lorentzian geometry)

AdS$_5$/CFT$_4$?



**Matrix p-brane Theory**

*T-dual*

**M1T**: Matrix string theory

*T-dual*

DLCQ M-Theory

*lightlike*

*compactification*

**M0T**: BFSS Matrix theory

*timelike*

*T-dual*

**M(-1)T**

IKKT Matrix theory

tensionless string theory

ambitwistor string theory

[Gomis, ZY '23]

# Strings in Carroll-Like Spacetimes

M(-1)T/tensionless string

$$S_{M(-1)T} = \frac{T}{2}\int d^2\sigma\, \partial_\tau X^\mu \partial_\tau X_\mu \qquad \mu = 0, \cdots, 9$$

D(-1)-instantons: IKKT Matrix theory

[Ishibashi, Kawai, Kitazawa, Tsuchiya '96]

T-dualize $X^1$

M(-2)T string $\quad a = 0, 2, \cdots, 9$

$$S_{M(-2)T} = \frac{T}{2}\int d^2\sigma \left( \partial_\sigma X^1 \partial_\sigma X^1 + \partial_\tau X^a \partial_\tau X_a + \lambda_1 \partial_\tau X^1 \right)$$

target-space Carroll-like boost

$$\delta X^a = v^a X^1 \qquad \delta X^1 = 0 \qquad \delta \lambda_1 = -2\, v_a \partial_\tau X^a$$

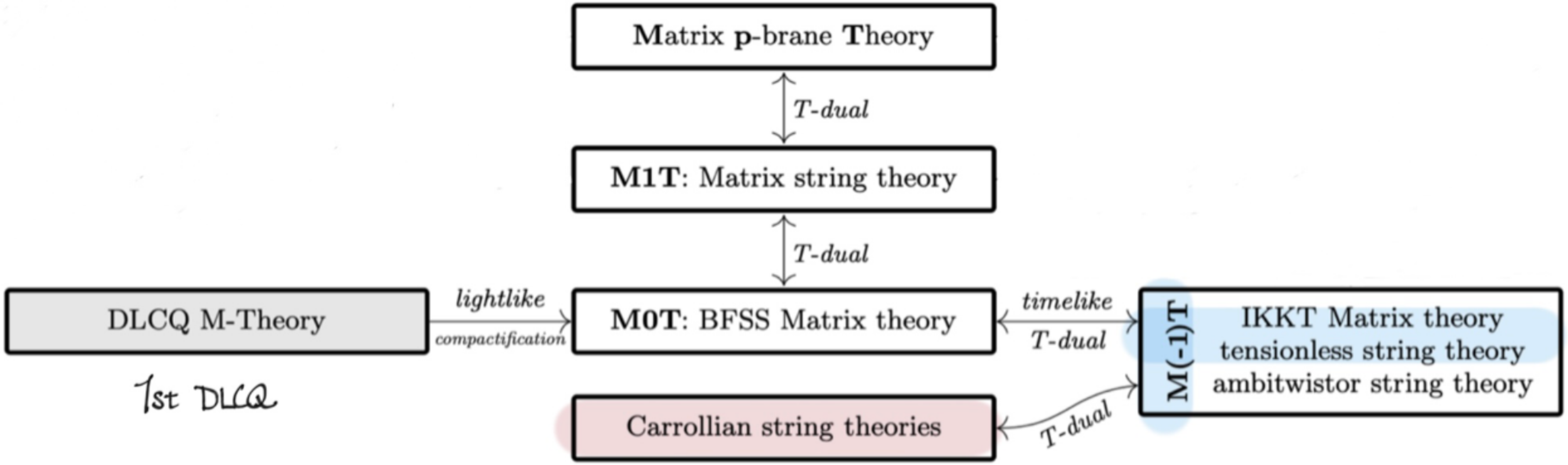
light excitations:

1-dim. spacelike branes [Hull '98] [Gutperle, Strominger '02]

flat space holography? ⇒ Carrollian/Celestial?

IKKT & instanton?

[Blair, Lahnsteiner, Obers, ZY '23]



**Matrix p-brane Theory**

*T-dual*

**M1T**: Matrix string theory

*T-dual*

DLCQ M-Theory

*lightlike* *compactification*

**M0T**: BFSS Matrix theory

*timelike* *T-dual*

**M(-1)T**

IKKT Matrix theory
tensionless string theory
ambitwistor string theory

Carrollian string theories

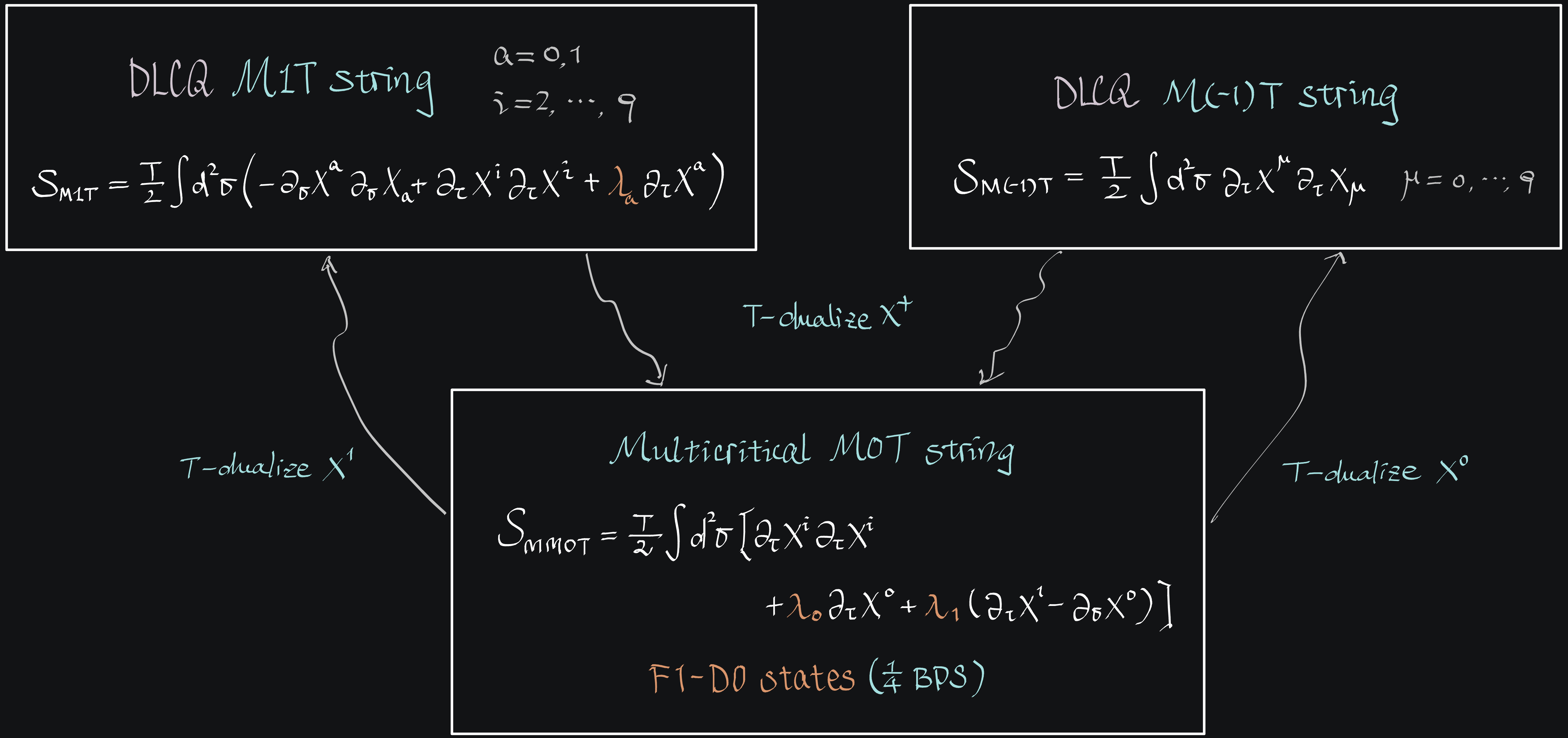
*T-dual*

1st DLCQ

MpT, $p<-1$

[Gomis, ZY '23]

# The 2nd DLCQ



DLCQ M1T string

$a = 0, 1$

$i = 2, \cdots, 9$

$$S_{M1T} = \frac{T}{2}\int d^2\sigma \left(-\partial_\sigma X^a \partial_\sigma X_a + \partial_\tau X^i \partial_\tau X^i + \lambda_a \partial_\tau X^a\right)$$

DLCQ M(-1)T string

$$S_{M(-1)T} = \frac{T}{2}\int d^2\sigma \, \partial_\tau X^\mu \partial_\tau X_\mu \qquad \mu = 0, \cdots, 9$$

T-dualize $X^+$

T-dualize $X^1$

T-dualize $X^0$

Multicritical M0T string

$$S_{MM0T} = \frac{T}{2}\int d^2\sigma \Big[\partial_\tau X^i \partial_\tau X^i + \lambda_0 \partial_\tau X^0 + \lambda_1 \left(\partial_\tau X^1 - \partial_\sigma X^0\right)\Big]$$

F1-D0 states ($\frac{1}{4}$ BPS)

REVIEW

# Spin Matrix Limits of AdS/CFT

- Spin Matrix theory: near BPS limits of $\mathcal{N}=4$ SYM on $\mathbb{R}\times S^3$ [Harmark, Orselli '06]

$$\lambda_t \to 0 \qquad E-Q \to 0 \qquad \frac{E-Q}{\lambda_t} \to \text{fixed}$$

$J_1, J_2$: angular momenta on $S^3$

$S_1, S_2, S_3$: R-charges

- integrable
- finite N generalizations of spin chains

- bulk perspective: from $AdS_5\times S^5$ to non-Lorentzian geometry

[Harmark, Hartong, Obers '17]
[Harmark, Hartong, Menculini, Obers, ZY '18]
[Harmark, Hartong, Obers, Oling '21]

$$ds^2 = R^2\left(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2 + d\Omega_5^2\right)$$

$$\lambda_t = \frac{R^4}{l_s^4} \qquad E = i\partial_t \qquad \underbrace{J_1, J_2 \quad S_1, S_2, S_3}_{Q}$$

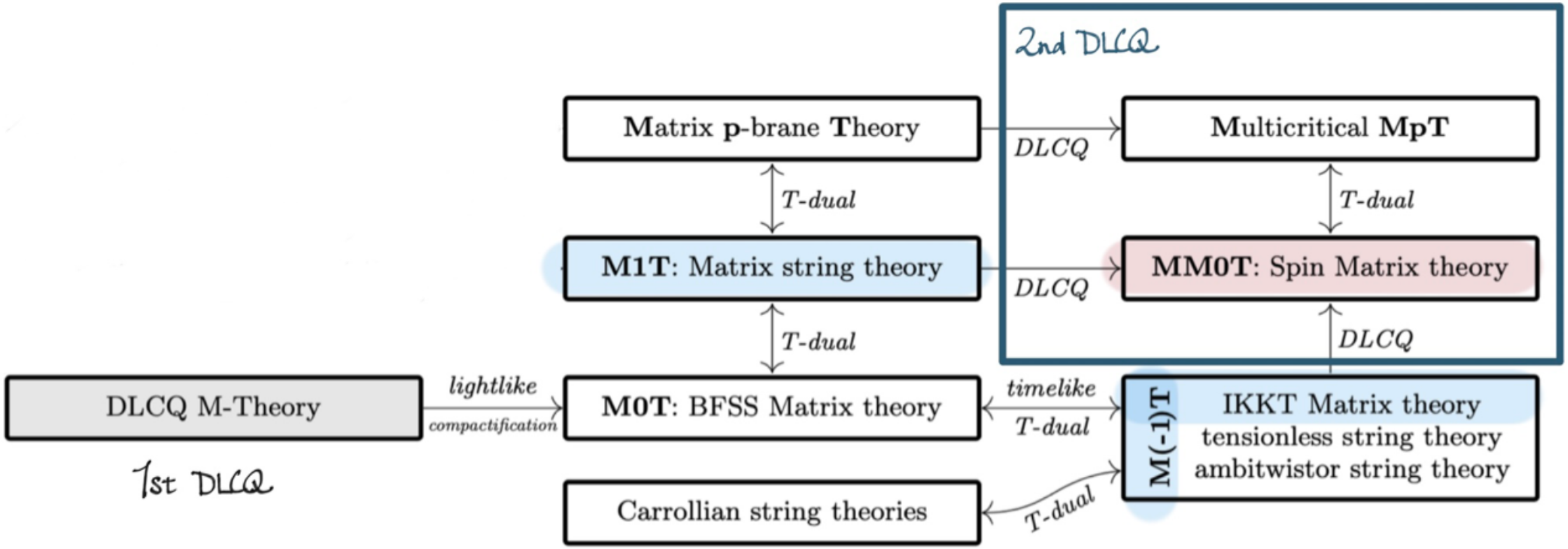
$$PSU(1,2|3):\ Q = J_1 + J_2 + S_1 + S_2 + S_3$$

Spin Matrix limit $\longrightarrow$ same worldsheet theory as in MMOT

$$S_{MMOT} = \frac{T}{2}\int d^2\sigma\left(\partial_\tau X^\mu \partial_\tau X^\nu E_{\mu\nu} + \lambda_a \partial_\tau X^\mu \tau_\mu{}^a - \lambda_1 \partial_\sigma X^\mu \tau_\mu{}^0\right) - T\int B$$

[Blair, Lahnsteiner, Obers, ZY '23]

3rd DLCQ: Kaluza-klein monopole?



2nd DLCQ

**Matrix p-brane Theory**

**Multicritical MpT**

**M1T**: Matrix string theory

**MM0T**: Spin Matrix theory

DLCQ M-Theory

**M0T**: BFSS Matrix theory

IKKT Matrix theory
tensionless string theory
ambitwistor string theory

**M(-1)T**

Carrollian string theories

*DLCQ*

*T-dual*

*lightlike compactification*

*timelike T-dual*

1st DLCQ

CHY formalism of Galilean QFT in MM0T?

[Gomis, ZY '23]

[Blair, Lahnsteiner, Obers, ZY '23]

# Decoupling Limits of Type II Superstring Theory

Matrix p-brane Theory $\xrightarrow{\text{lightlike T-dual}}$ Multicritical Matrix (p-1)-brane Theory

$$X^a \to \sqrt{\omega}\, X^a \qquad a = 0, \cdots, p$$

$$X^i \to \frac{1}{\sqrt{\omega}} X^i \qquad i = p+1, \cdots, 9$$

$$g_s \to \omega^{\frac{p-3}{2}} g_s$$

$$C_{p+1} \to \omega^2 g_s^{-1} dX^0 \wedge \cdots \wedge dX^p$$

- critical background configurations

|  | 0 1 ⋯ p | p+1 ⋯ 9 |
|---|---|---|
| Dp | × × ⋯ × |  |

light excitations: $\frac{1}{2}$ BPS

$$X^0 \to \omega X^0 \qquad X^1 \to X^1$$

$$X^a \to \frac{1}{\sqrt{\omega}} X^a \qquad a = 2, \cdots, p$$

$$X^i \to \frac{1}{\sqrt{\omega}} X^i \qquad i = p+1, \cdots, 9$$

$$g_s \to \omega^{\frac{p-3}{2}} g_s$$

$$C_p \to \omega^2 g_s^{-1} dX^0 \wedge dX^2 \wedge \cdots \wedge dX^p$$

$$B \to -\omega\, dX^0 \wedge dX^1$$

|  | 0 | 1 | 2 ⋯ p | p+1 ⋯ 9 |
|---|---|---|---|---|
| D(p-1) | X |  | X ⋯ X |  |
| F1 | X | X |  |  |

light excitations: $\frac{1}{4}$ BPS

U-duality & BPS states
[Dijkgraaf, de Boer, Harmark, Obers]

[Blair, Lahnsteiner, Obers, ZY '23]

# Non-Lorentzian Geometries

Matrix p-brane Theory $\xrightarrow{\text{lightlike T-dual}}$ Multicritical Matrix (p-1)-brane Theory

$$G_{\mu\nu} = \omega\, \tau_{\mu\nu} + \frac{1}{\omega} E_{\mu\nu}$$

$$e^{\Phi} = \omega^{\frac{p-3}{2}} e^{\phi}$$

$$C_{p+1} = \omega^2 e^{-\Phi}\, \tau^0 \wedge \cdots \wedge \tau^p + c_{p+1}$$

$$G_{\mu\nu} = -\omega^2 \tau_\mu{}^0 \tau_\nu{}^0 + \tau_\mu{}^1 \tau_\nu{}^1 + \omega\, \tau_\mu{}^a \tau_\nu{}^a + \frac{1}{\omega} E_{\mu\nu}$$

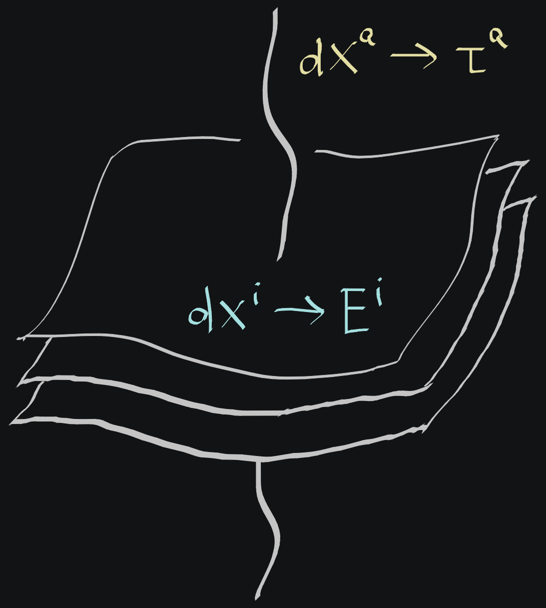
$$e^{\Phi} = \omega^{\frac{p-3}{2}} e^{\phi} \qquad a = 2, \cdots, p$$

$$C_p = \omega^2 e^{-\Phi}\, \tau^0 \wedge \tau^2 \wedge \cdots \wedge \tau^p + \omega\, \tau^0 \wedge \tau^1 \wedge c_{p-2} + c_p$$

$$B = -\omega\, \tau^0 \wedge \tau^1 + b \qquad C_q = \omega\, \tau^0 \wedge \tau^1 \wedge c_{q-2} + c_q$$

$\tau_{\mu\nu} = \tau_\mu{}^a \tau_\nu{}^b \eta_{ab}$

$E_{\mu\nu} = E_\mu{}^i E_\nu{}^i$



generalized Newton-Cartan / Carroll geometry

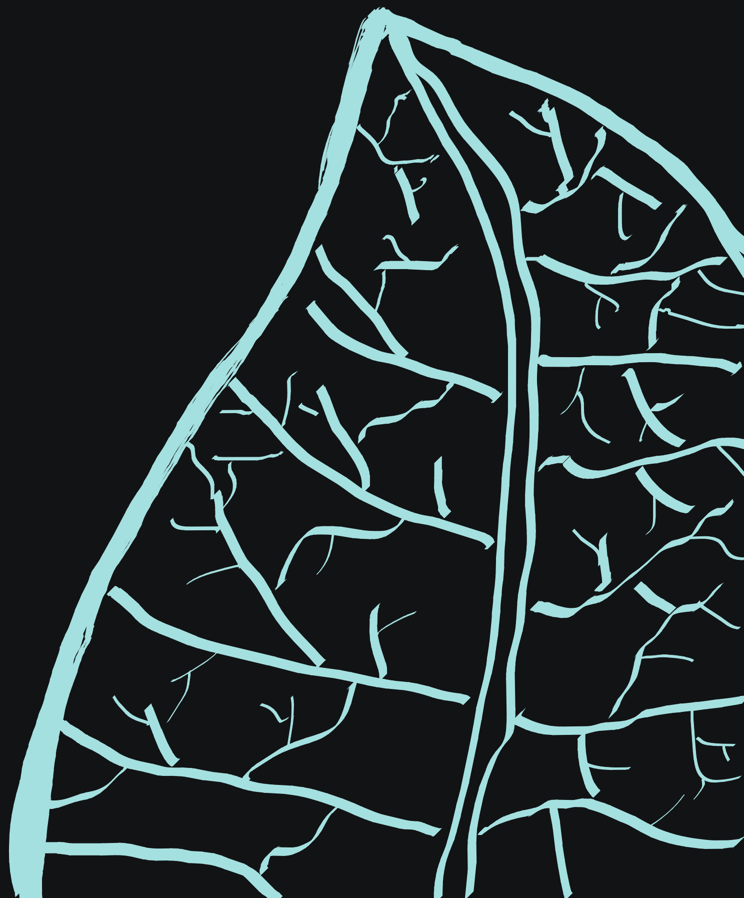
w/ torsional constraints $d\tau \sim 0$

due to quantum consistency / supersymmetry

implications for Matrix theory / SUGRA correspondence?

REVIEW

# S-Duality & Nonrelativistic String Theory



# S-Dual of Matrix 1-Brane Theory

REVIEW

D1-string $F = dA$

F1-string

$$\begin{tikzcd}
S_{D1} = -\frac{1}{g_s\alpha'}\int d^2\sigma\sqrt{-\det\left(\partial_\alpha X^\mu\partial_\beta X_\mu + F_{\alpha\beta}\right)} + \frac{1}{g_s\alpha'}\int C_2 \arrow[rr, leftrightarrow, "S\text{-dual}"] \arrow[d, "\omega\to\infty"] & & S = -T\int d^2\sigma\sqrt{-\det\left(\partial_\alpha X^\mu\partial^\alpha X_\mu\right)} - T\int B \arrow[d, "\omega\to\infty"'] \\
S^{M1T}_{D1} = -\frac{1}{2g_s\alpha'}\int d^2\sigma\sqrt{-\tau}\left(\tau^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^i - \frac{1}{2}\tau^{\alpha\gamma}\tau^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}\right) \arrow[u, bend left, "T\bar{T}\text{-deformation}"] & & S_{NR} = -\frac{T}{2}\int d^2\sigma\sqrt{-\tau}\,\tau^{\alpha\beta}\partial_\alpha X^i\partial_\beta X^i \arrow[u, bend right, "T\bar{T}\text{-deformation}"']
\end{tikzcd}$$

$$X^a \longrightarrow \sqrt{\omega}\,X^a \qquad a = 0, 1$$
$$X^i \longrightarrow \frac{1}{\sqrt{\omega}}X^i \qquad i = 2, \cdots, 9$$
$$g_s \longrightarrow \omega^{-1} g_s$$
$$C_2 \longrightarrow \omega^2 g_s^{-1}\, dX^0 \wedge dX^1$$

$$X^a \longrightarrow \omega\, X^a \qquad a = 0, 1$$
$$X^i \longrightarrow X^i \qquad i = 2, \cdots, 9$$
$$g_s \longrightarrow \omega\, g_s$$
$$B \longrightarrow -\omega^2\, dX^0 \wedge dX^1$$

[Blair '20]

$$\tau_{\alpha\beta} = \partial_\alpha X^a\partial_\beta X_a$$

Matrix string theory in M1T

[Dijkgraaf, Verlinde, Verlinde '97]
[Ebert, Sun, ZY '21]
[Blair, Lahnsteiner, Obers, ZY '23]

nonrelativistic string theory

[Klebanov, Maldacena '00]
[Gomis, Ooguri '00]
[Danielsson, Guijosa, Kruzcenski '00]
[Andringa, Bergshoeff, Gomis, de Roo '12]

# S-Dual of Matrix 1-Brane Theory

REVIEW

- Branched $SL(2,\mathbb{Z})$ & polynomial realization

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad ad-bc=1$$

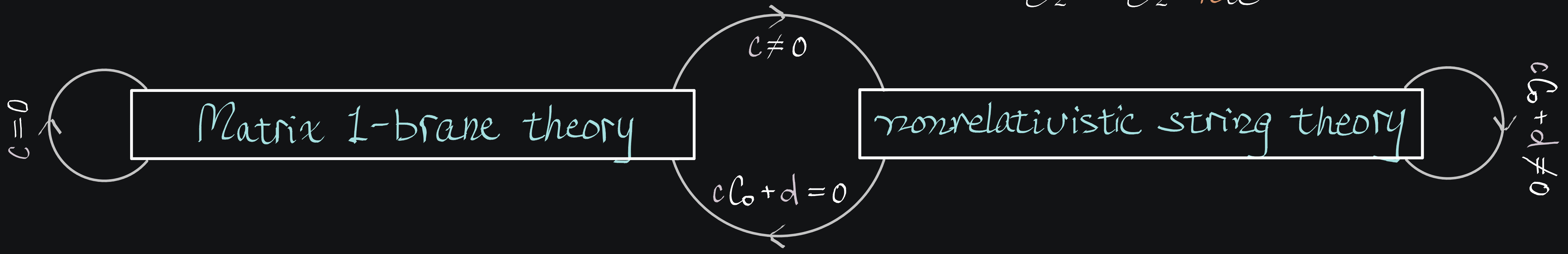
$$\mathcal{B} = e^{-\frac{\Phi}{2}} B$$

$$\mathcal{C}_2 = e^{\frac{\Phi}{2}} (C_2 + C_0 B)$$

$$\mathcal{B} \to \mathcal{B} - \kappa\, \mathcal{C}_2 + \tfrac{1}{2} \kappa^2\, \tau^0 \wedge \tau^1$$

$$\mathcal{C}_2 \to \mathcal{C}_2 - \kappa\, \mathcal{B}$$

$$\kappa = \frac{c\, e^{-\Phi}}{c C_0 + d}$$



Matrix 1-brane theory

nonrelativistic string theory

$c \neq 0$

$c C_0 + d = 0$

$c = 0$

$c C_0 + d \neq 0$

non-Lorentzian IIB SUGRA:

[Bergshoeff, Grosvenor, Lahnsteiner, ZY, Zorba '22 '23]

[Ebert, ZY '23]

classical invariant theory & non-Lorentzian bootstrap?

[Blair, Lahnsteiner, Obers, ZY to appear]

see also nonrel. string expansion [Hare, Hartong '21]

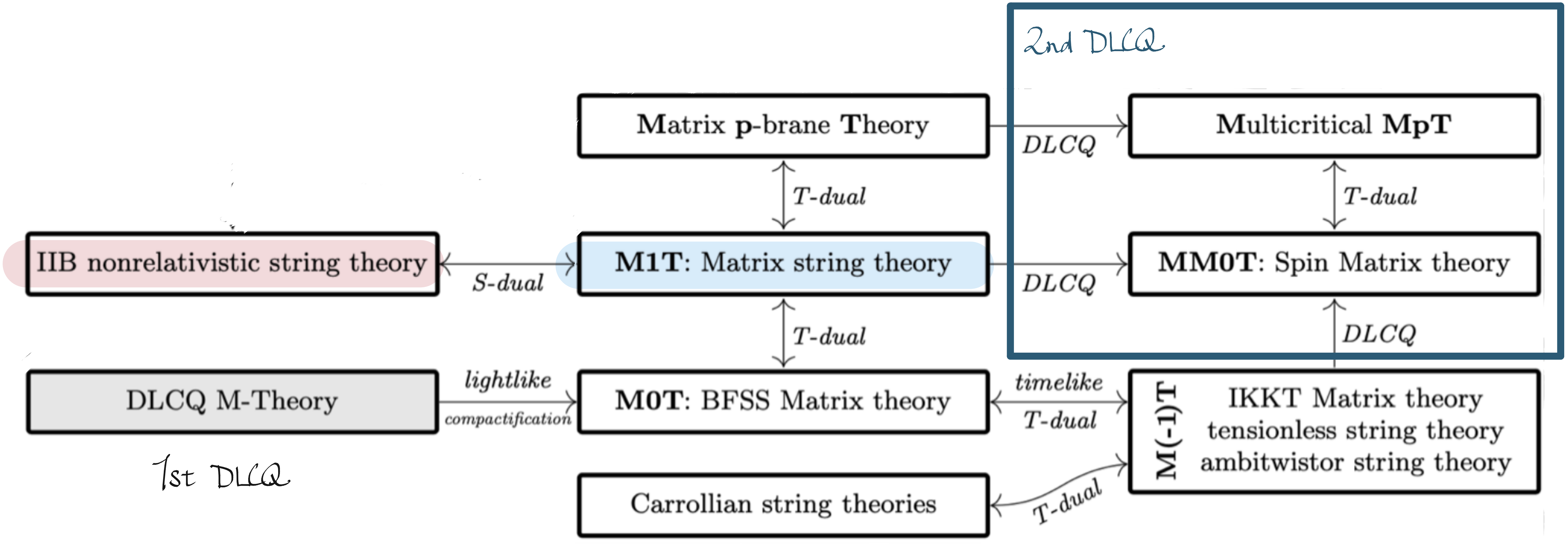
- D3-branes

$$\text{noncommutative Yang-Mills} \xleftrightarrow{\text{S-dual}} \text{noncommutative open string}$$

[Gopakumar, Maldacena, Minwalla, Strominger '00]

AdS/CFT: e.g. [Hashimoto, Itzhaki '99] [Maldacena, Russo '00] [Guijosa, Rosas-López '23] ...

[Blair, Lahnsteiner, Obers, ZY '23]



2nd DLCQ

Matrix p-brane Theory

Multicritical MpT

IIB nonrelativistic string theory

**M1T**: Matrix string theory

**MM0T**: Spin Matrix theory

DLCQ M-Theory

**M0T**: BFSS Matrix theory

**M(-1)T**

IKKT Matrix theory
tensionless string theory
ambitwistor string theory

Carrollian string theories

1st DLCQ

REVIEW

# Nonrelativistic String Theory as a Pivot Theory

Nambu-Goto: $S_{NG} = -\frac{T}{2}\int d^2\sigma \sqrt{-\tau}\, \tau^{\alpha\beta}\partial_\alpha X^i \partial_\beta X^i$

$\tau_{\alpha\beta} = \partial_\alpha X^a \partial_\beta X_a \qquad i = 2, \cdots, 9$

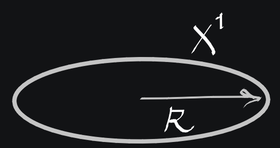
noncritical target space dim.

$S \sim \int d^2\sigma \left(\partial\Phi\bar\partial\Phi + \lambda\bar\partial X^+ + \bar\lambda\partial X^- - e^{-\Phi}\lambda\bar\lambda - \frac{1}{2}R\Phi\right)$

AdS$_3$/CFT$_2$ [Eberhardt, Gaberdiel, Gopakumar '19]

chiral 2D Yang-Mills [Komatsu, Maity]

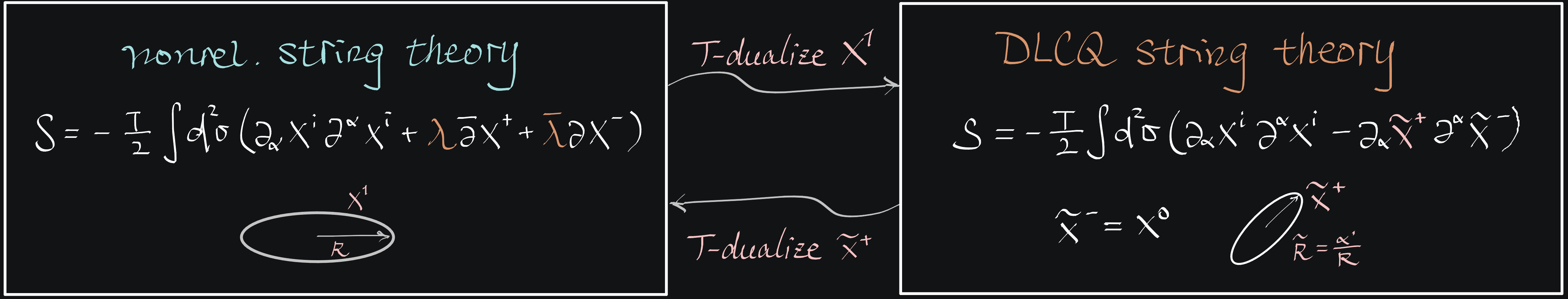
Polyakov: $S_p = -\frac{T}{2}\int d^2\sigma \left(\partial_\alpha X^i \partial^\alpha X^i + \underbrace{\lambda\bar\partial X^+ + \bar\lambda\partial X^-}_{\beta\gamma \text{ system}}\right)$ [Gomis, Ooguri '00]



1. string spectrum: $\mathcal{E} = \frac{\alpha'}{2wR}\left[k_i k^i + \frac{2}{\alpha'}(N + \bar N - 2)\right]$ Galilei invariant
2. target space string Galilei boost: $\delta X^a = 0$, $\delta X^i = \Lambda^i{}_a X^a$
3. relativistic worldsheet! Standard CFT techniques
4. unitary and UV-complete nonrel. quantum gravity

REVIEW

# First Principles Definition of DLCQ String Theory



nonrel. string theory

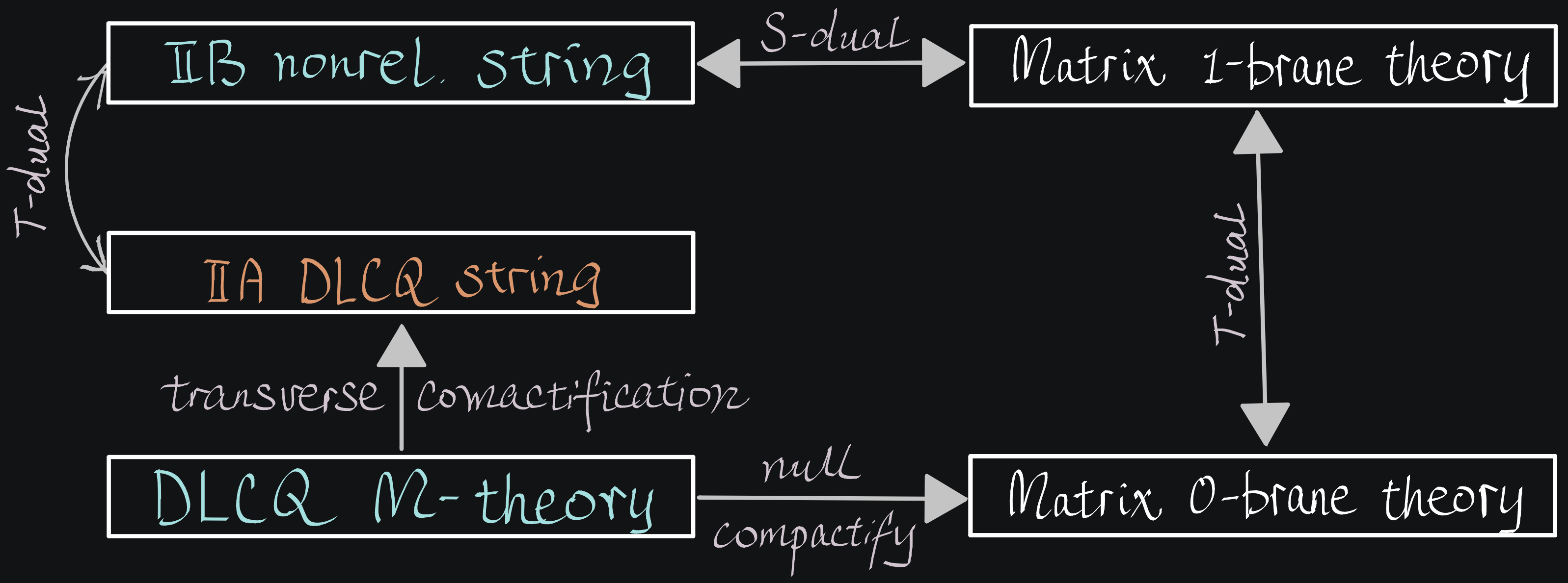
$$S = -\frac{T}{2}\int d^2\sigma\left(\partial_\alpha X^i \partial^\alpha X^i + \lambda \bar{\partial} X^+ + \bar{\lambda}\partial X^-\right)$$

DLCQ string theory

$$S = -\frac{T}{2}\int d^2\sigma\left(\partial_\alpha X^i \partial^\alpha X^i - \partial_\alpha \tilde{X}^+ \partial^\alpha \tilde{X}^-\right)$$

$\tilde{X}^- = X^0$

[Bergshoeff, Gomis, ZY '18]



IIB nonrel. string

Matrix 1-brane theory

IIA DLCQ string

DLCQ M-theory

Matrix 0-brane theory

null-reduction perspective

[Harmark, Hartong, Obers '17] [Klusoň '18]

[Harmark, Hartong, Menculini, Obers, ZY '18]

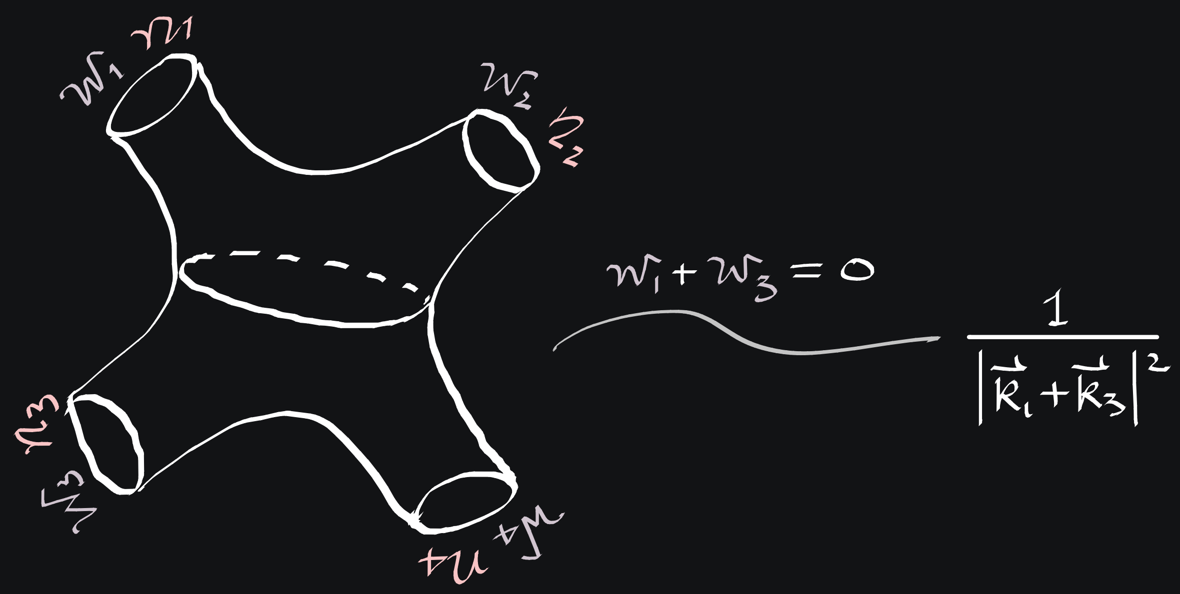
[Harmark, Hartong, Menculini, Obers, Oling '19]

REVIEW

# String Amplitudes & Winding KLTs

- no massless graviton
- instantaneous gravitational interaction
  between winding strings
- localized moduli space

[Gomis, Ooguri '00] [Danielsson, Guijosa, Kruczenski '00 '00] [Bilal '98]



$w_1 + w_3 = 0$

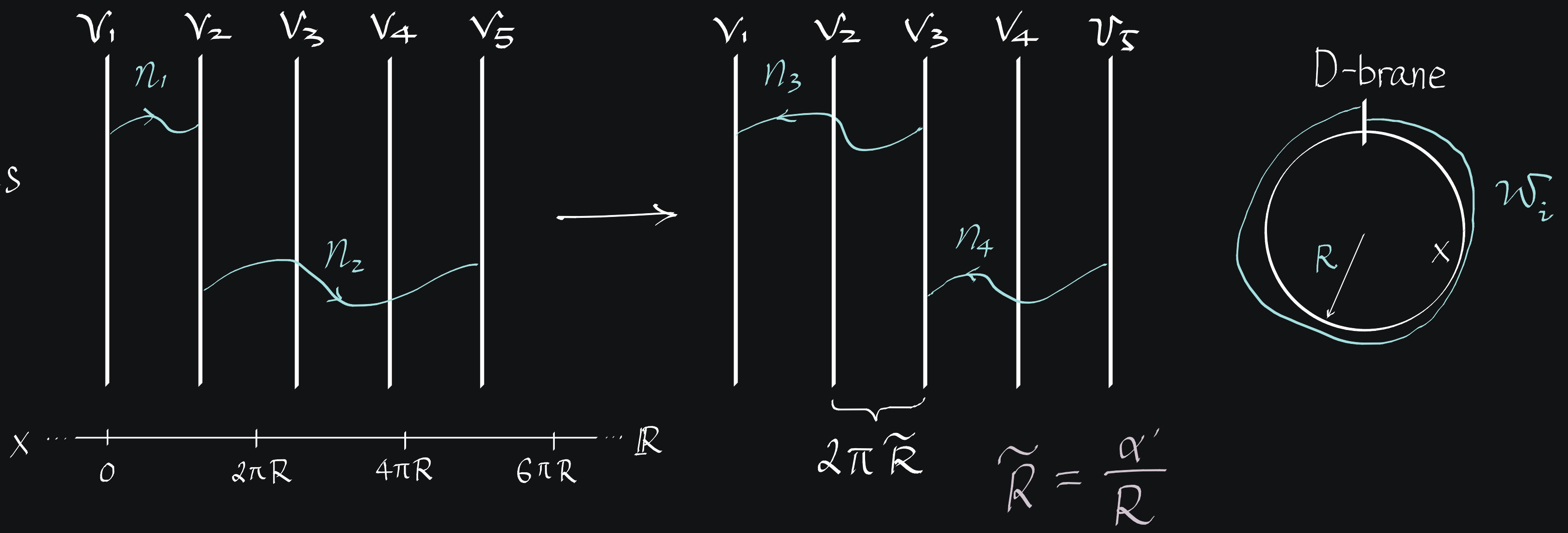
$$\frac{1}{|\vec{k}_1 + \vec{k}_3|^2}$$

- KLTs with D-branes [Gomis, ZY, Yu '21] [ZY, Yu '21]

open string scattering w/ DD $\tilde{R} \to 0$

array of $n_+ + 1$ D-branes

$n_+ = n_1 + n_2$



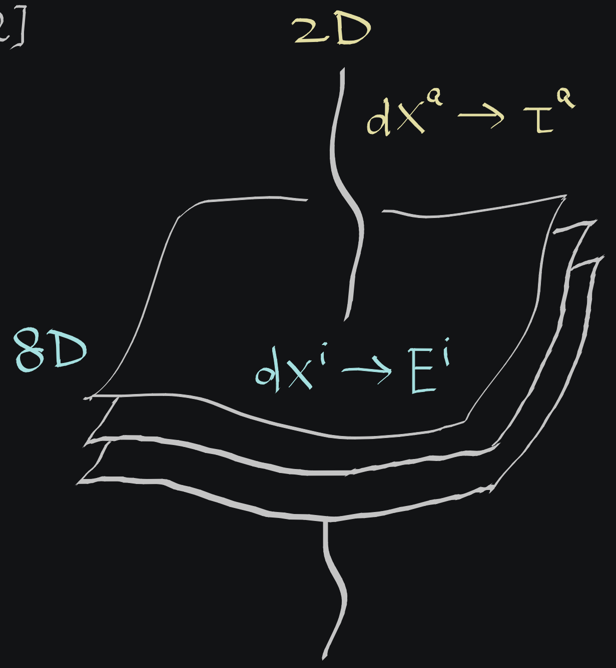
$\tilde{R} = \frac{\alpha'}{R}$

REVIEW

# Interacting Sigma Models

- Strings in string Newton-Cartan geometry [Andringa, Bergshoeff, Gomis, de Roo '12]

$$S = -\frac{T}{2}\int d^2\sigma \left[\partial X^\mu \bar\partial X^\nu (E_{\mu\nu} + b_{\mu\nu}) + \lambda \bar\partial X^\mu \tau_\mu^{+} + \bar\lambda \partial X^\mu \tau_\mu^{-}\right]$$



$\beta$-functions [Gomis, Oh, ZY '19][Gallegos, Gürsoy, Zinnato 19]
[Bergshoeff, Gomis, Rosseel, Şimşek, ZY '19] [ZY, Yu '19] [ZY '21]

($T\bar{T}$ deformation [Blair '20])

- torsional deformation : $\lambda\bar\lambda$ is generated quantum mechanically
  → back to relativistic string theory

- noncentral extensions of string Galilei algebra $\Rightarrow$ torsional constraints on $\tau^a$

trans. momentum
$\uparrow$
$[G_{ai}, P_j] = \delta_{ij} Z_a$
$\downarrow$ string Galilei boost $\Downarrow$ $D\tau^a = 0$
$\mathcal{N} = 2$ SUSY? (type II)

trans. momentum
$\uparrow$
$[G_{+i}, P_j] = \delta_{ij} Z_+$
$\downarrow$ string Galilei boost $\Downarrow$ $D\tau^- = 0$
$\mathcal{N} = 1$ SUSY (heterotic, type I)

no $\lambda\bar\lambda$ generated at all loops!

[ZY '21]

similar constraints from SUSY
[Bergshoeff, Lahnsteiner, Romano, Rosseel, Şimşek '21]
[Bergshoeff, Blair, Lahnsteiner, Rosseel]

# Open Strings and D-Branes

see also [Danielsson, Guijosa, Kruzcenski '00]

REVIEW

- open string vertex operators [Gomis, ZY, Yu '20]

$$S_{\text{boundary}} = \frac{1}{2\pi\alpha'}\int d\tau \left[ \nu(\lambda - \bar{\lambda}) + i A_i \partial_\tau X^i \right]$$

$$G_{\mu\nu} = \omega^2 \tau_{\mu\nu} + E_{\mu\nu}$$

$$B = -\omega^2 \tau^0 \wedge \tau^1 + b$$

$$\Phi = \phi + \ln \omega$$

$$C_q = \omega^2 \tau^0 \wedge \tau^1 \wedge c_{q-2} + c_q$$

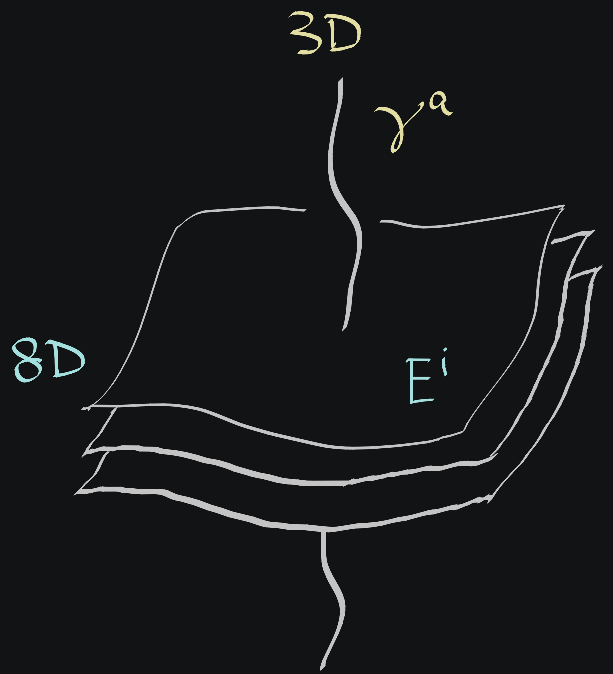
- D-brane actions from β-functions [Gomis, ZY, Yu '20] [Ebert, Sun, ZY '21]

$$S_{Dp} = -\int d^{p+1}\sigma\, e^{-\phi} \sqrt{-\det \begin{pmatrix} 0 & \tau_\beta^+ \\ \tau_\alpha^- & E_{\alpha\beta} + b_{\alpha\beta} + F_{\alpha\beta} \end{pmatrix}} - \int \sum_q c_q\, e^{b+F} \Big|_{p+1}$$

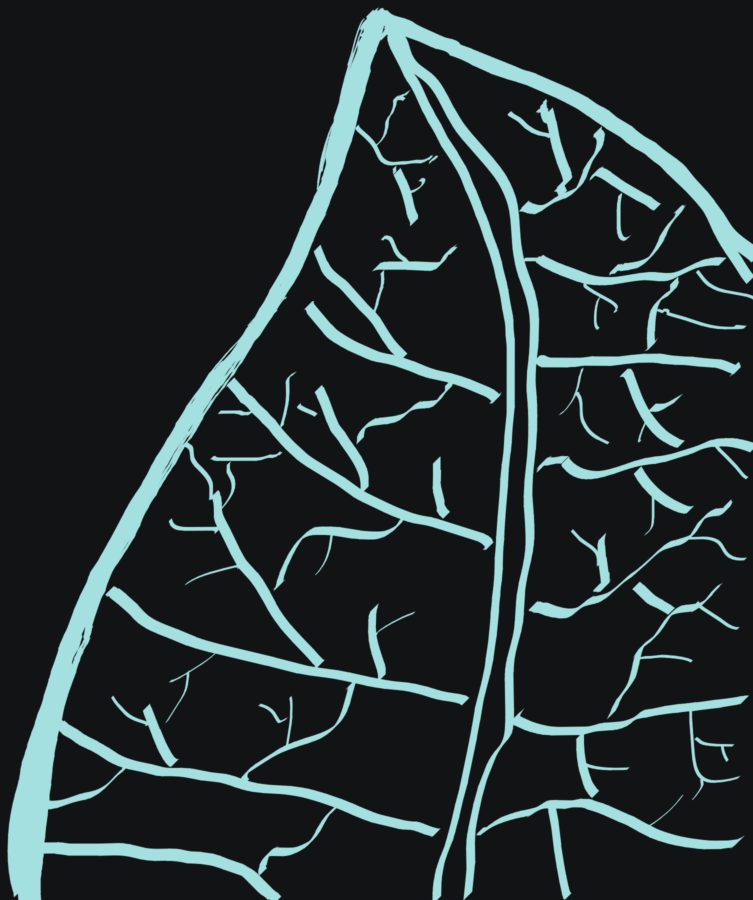
- nonrelativistic M-theory [Gomis, Ooguri '00] [Gopakumar, Minwalla, Seiberg, Strominger '00] [Bergshoeff, Berman, van der Schaar, Sundell '00] [Ebert, ZY '23] [García, Guijosa, Vergara '02] [Blair, Gallegos, Zinnato '21]

electromagnetic dual of D2-brane [Ebert, Sun, ZY '21]

$$S_{M2} = -\int d^3\sigma \sqrt{-\gamma}\, \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu E_{\mu\nu} - \int A_3$$



# Back to M-Theory



[Blair, Lahnsteiner, Obers, ZY '23]

# Nonrelativistic M-Theory

- back to DLCQ M-theory

$$X^+ \to \omega X^+$$
$$X^- \to \frac{1}{\omega} X^-$$

infinite boost: $\omega \to \infty$

- Compactify over $T^3$

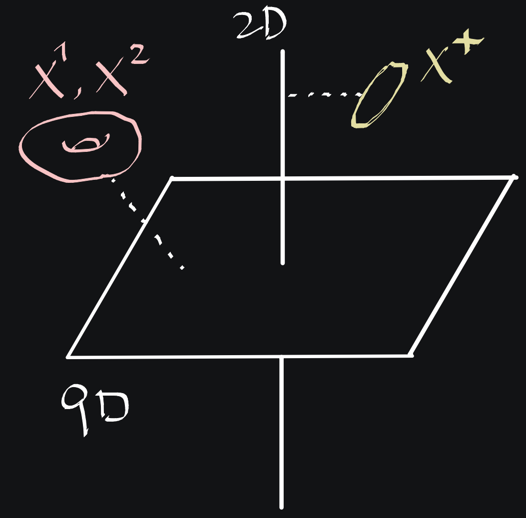
|  | $X^+$ | $X^1$ | $X^2$ |
|---|---|---|---|
| radius | $R_+$ | $R_1$ | $R_2$ |

- U-dual frame $\xrightarrow{\omega \to \infty}$ nonrelativistic M-theory

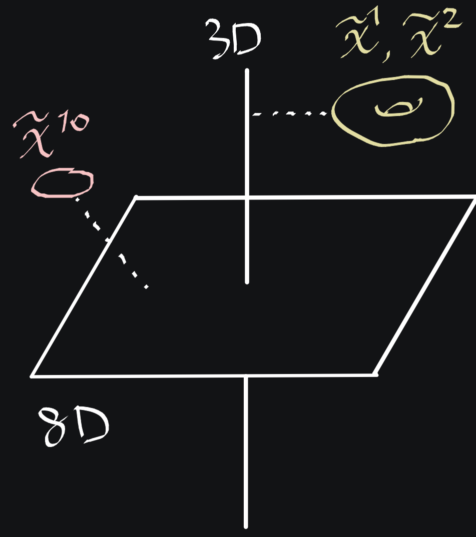
$$ds^2 = \omega^{\frac{4}{3}} \left[ -(dx^0)^2 + (d\tilde{x}^1)^2 + (d\tilde{x}^2)^2 \right] + \omega^{-\frac{2}{3}} \left[ \sum_{i=3}^{9} dx^i dx^i + (d\tilde{x}^{10})^2 \right]$$

$$A_3 = -\omega^2 dx^0 \wedge d\tilde{x}^1 \wedge d\tilde{x}^2$$

DLCQ M-theory



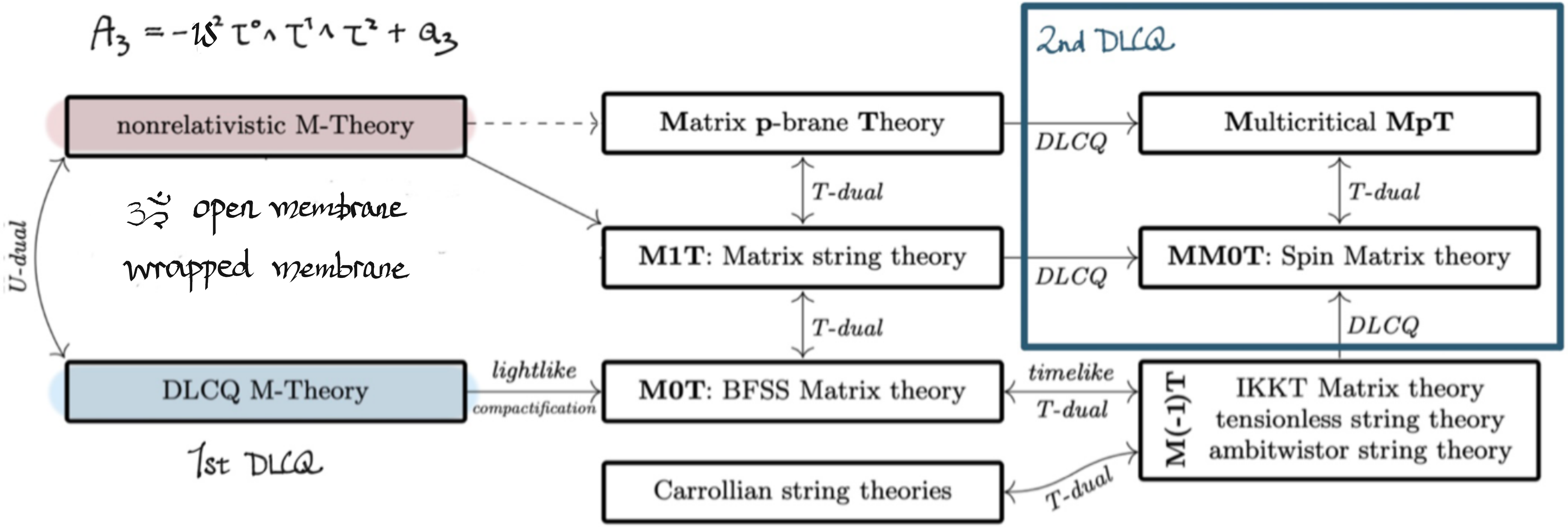
nonrelativistic M-theory



[Blair, Lahnsteiner, Obers, ZY '23]

$$G_{\mu\nu} = \omega^{\frac{4}{3}} \overset{3D}{\tau_{\mu\nu}} + \omega^{-\frac{2}{3}} \overset{8D}{E_{\mu\nu}}$$

$$A_3 = -\omega^2 \tau^0 \wedge \tau^1 \wedge \tau^2 + a_3$$



ॐ open membrane

wrapped membrane

1st DLCQ

[Blair, Lahnsteiner, Obers, ZY '23]

# Anisotropic Compactification of Nonrelativistic M-Theory

DLCQ M-theory

$T^3$ ↕ U-dual

nonrelativistic M-theory

+

2nd DLCQ

$T^4$ ↕ U-dual

→ IIB nonrel. string theory

$$\tau = C_0 + i\omega^{-1} e^{-\Phi} \xrightarrow{\omega \to \infty} \text{real}$$

[Ebert, ZY '23]

[Gomis, ZY '23]

→ Matrix 1-brane theory

$$\tau = C_0 + i\omega e^{-\Phi} \xrightarrow[+SL(2,\mathbb{Z})]{\omega \to \infty} i\infty$$

polynomial realization of $SL(2,\mathbb{Z})$: $\kappa = \dfrac{c\,e^{-\Phi}}{c\,C_0 + d}$

↓

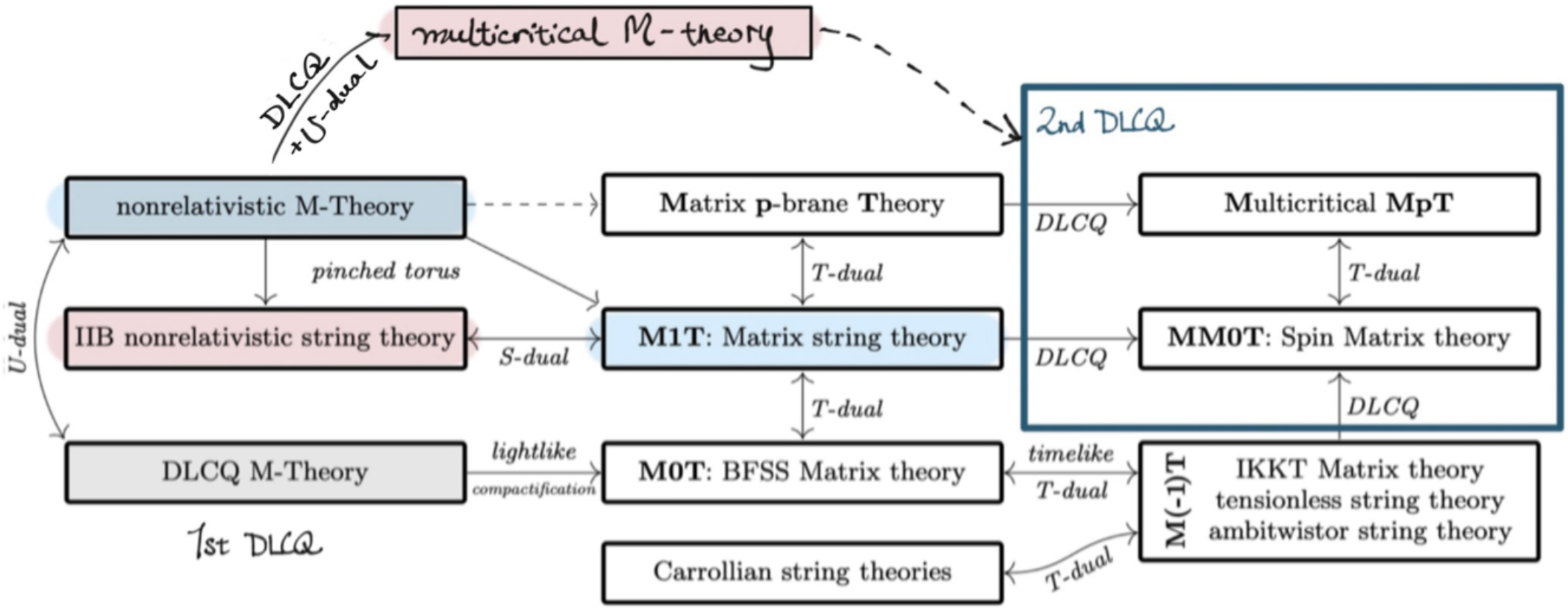
Galilei boost "velocity"

multicritical M-theory $\xrightarrow[\text{over } x^1]{\text{compactify}}$ multicritical Matrix 2-brane theory

$$G_{\mu\nu} = -\varpi^2 \tau_\mu{}^0 \tau_\nu{}^0 + \tau_\mu{}^1 \tau_\nu{}^1 + \varpi\, \tau_\mu{}^a \tau_\nu{}^a + \frac{1}{\varpi} E_{\mu\nu}$$ critical background

$$A_3 = -\varpi\, \tau^0 \wedge \tau^1 \wedge \tau^2 - \varpi^2\, \tau^0 \wedge \tau^3 \wedge \tau^4 + a_3$$ orthogonal M2-branes

[Blair, Lahnsteiner, Obers, ZY '23]



multicritical M-theory

DLCQ + U-dual

2nd DLCQ

nonrelativistic M-Theory

Matrix p-brane Theory

DLCQ

Multicritical MpT

pinched torus

T-dual

T-dual

IIB nonrelativistic string theory

S-dual

**M1T**: Matrix string theory

DLCQ

**MM0T**: Spin Matrix theory

U-dual

T-dual

DLCQ

DLCQ M-Theory

lightlike compactification

**M0T**: BFSS Matrix theory

timelike T-dual

**M(-1)T**

IKKT Matrix theory
tensionless string theory
ambitwistor string theory

Carrollian string theories

T-dual

1st DLCQ

Many new adventures ahead...

Thank You!