Probing the D0-brane dual gravitational background with the Berkooz-Douglas Matrix Model

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Matrix Quantum Mechanics for M-theory Revisited
CERN
January 8th 2024

Introduction

As derived by Jens Hoppe (yesterday) the membrane Hamiltonian with matrix regularization where the embedding coordinates X^a become matrices becomes

$$H_B = \mathbf{Tr}(\frac{1}{2}\sum_{a=1}^{D-1} P^a P^a - \frac{1}{4}\sum_{a,b=1}^{p} [X^a, X^b][X^a, X^b])$$

With the Gauss law constraint $[P^a, X^a] = 0$.

This appears to realize a proposed requirement of quantum gravity of Doplicher, Fredenhagen and Roberts,1995 arXiv:hep-th/0303037

The BFSS model

$$S_{SMembrane} = \int \sqrt{-G} - \int C + \quad \text{Fermionic terms}$$

The susy version only exists in 4, 5, 7 and 11 spacetime dimensions.

BFFS Model — The supersymmetric membrane à la Hoppe

$$\mathrm{H} = \mathbf{Tr}\left(\tfrac{1}{2}\sum_{a=1}^{9} P^a P^a - \frac{1}{4}\sum_{a,b=1}^{9}[X^a, X^b][X^a, X^b] + \frac{1}{2}\Theta^T \gamma^a [X^a, \Theta]\right)$$

It also describes a system of N interacting D0 branes.

Finite Temperature Model

The partition function and Energy of the model at finite temperature is

$$Z = Tr_{Phys}(\mathrm{e}^{-\beta\mathcal{H}}) \qquad \text{and} \qquad E = \frac{Tr_{Phys}(\mathcal{H}\mathrm{e}^{-\beta\mathcal{H}})}{Z} = \langle\mathcal{H}\rangle$$

The 16 fermionic matrices $\Theta_\alpha = \Theta_{\alpha A} t^A$ are quantised as

$$\{\Theta_{\alpha A}, \Theta_{\beta B}\} = 2\delta_{\alpha\beta}\delta_{AB}$$

The $\Theta_{\alpha A}$ are $2^{8(N^2-1)}$ and the Fermionic Hilbert space is

$$\mathcal{H}^F = \mathcal{H}_{256} \otimes \cdots \otimes \mathcal{H}_{256}$$

with $\mathcal{H}_{256} = \mathbf{44} \oplus \mathbf{84} \oplus \mathbf{128}$ suggestive of
the graviton (**44**), anti-symmetric tensor (**84**) and gravitino (**128**) of $11 - d$ SUGRA.

For an attempt to find the ground state see: J. Hoppe et al arXiv:0809.5270

Lagrangian formulation

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The BFSS matrix model is also the dimensional reduction of ten dimensional supersymmetric Yang-Mills theory down to one dimension:

$$
\begin{aligned}
S_M \quad &= \frac{1}{g^2}\int dt\,\mathrm{Tr}\left\{\frac{1}{2}(\mathcal{D}_0 X^i)^2 + \frac{1}{4}[X^i, X^j]^2\right.\\
&\left.\quad -\frac{i}{2}\Psi^T C_{10}\,\Gamma^0 D_0\Psi + \frac{1}{2}\Psi^T C_{10}\,\Gamma^i[X^i,\Psi]\right\}\ ,
\end{aligned}
$$

where Ψ is a thirty two component Majorana–Weyl spinor, Γ^μ are ten dimensional gamma matrices and C_{10} is the charge conjugation matrix satisfying $C_{10}\Gamma^\mu C_{10}^{-1} = -\Gamma^{\mu\,T}$.

The BMN or PWMM

The supermembrane on the maximally supersymmetric plane wave spacetime

$$ds^2 = -2dx^+dx^- + dx^a dx^a + dx^i dx^i - dx^+ dx^+ ((\frac{\mu}{6})^2 (x^i)^2 + (\frac{\mu}{3})^2 (x^a)^2)$$

with

$$dC = \mu dx^1 \wedge dx^2 \wedge dX^3 \wedge dx^+$$

so that $F_{123+} = \mu$. This leads to the additional contribution to the Hamiltonian

$$\Delta \mathrm{H}_\mu = \frac{N}{2} \mathbf{Tr} \Big((\frac{\mu}{6})^2 (X^a)^2 + (\frac{\mu}{3})^2 (X^i)^2 + \frac{2\mu}{3} i \epsilon_{ijk} X^i X^j X^k + \frac{\mu}{4} \Theta^T \gamma^{123} \Theta \Big)$$

$$
\begin{aligned}
\Delta S_\mu &= -\frac{1}{2g^2}\int_0^\beta d\tau\, \mathbf{Tr}\Big((\frac{\mu}{6})^2 (X^a)^2 + (\frac{\mu}{3})^2 (X^i)^2 \\
&\qquad + \frac{2\mu}{3} i\epsilon_{ijk} X^i X^j X^k + \frac{\mu}{4}\Psi^T \gamma^{123}\Psi \Big)
\end{aligned}
$$

The gravity dual and its geometry

Gauge/gravity duality predicts that the strong coupling regime of the theory is described by II_A supergravity, which lifts to 11-dimensional supergravity.

The bosonic action for eleven-dimensional supergravity is given by

$$S_{11D} = \frac{1}{2\kappa_{11}^2} \int [\sqrt{-g}R - \frac{1}{2}F_4 \wedge *F_4 - \frac{1}{6}A_3 \wedge F_4 \wedge F_4]$$

where $2\kappa_{11}^2 = 16\pi G_N^{11} = \frac{(2\pi l_p)^9}{2\pi}$.

The relevant solution to eleven dimensional supergravity for the dual geometry to the BFSS model corresponds to N coincident $D0$ branes in the IIA theory. It is given by

$$ds^2 = -H^{-1}dt^2 + dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

with $A_3 = 0$

The one-form is given by $C = H^{-1} - 1$ and $H = 1 + \frac{\alpha_0 N}{r^7}$ where $\alpha_0 = (2\pi)^2 14\pi g_s l_s^7$.

A thermal bath and black hole geometry

$$ds_{11}^2 = -H^{-1}Fdt^2 + F^{-1}dr^2 + r^2 d\Omega_8^2 + H(dx_{10} - Cdt)^2$$

Set $U = r/\alpha'$ and we are interested in $\alpha' \to \infty$
$H(U) = \frac{240\pi^5\lambda}{U^7}$ and the black hole time dilation factor
$F(U) = 1 - \frac{U_0^7}{U^7}$ with $U_0 = 240\pi^5\alpha'^5\lambda$. The temperature

$$\frac{T}{\lambda^{1/3}} = \frac{1}{4\pi\lambda^{1/3}} H^{-1/2} F'(U_0) = \frac{7}{2^4 15^{1/2}\pi^{7/2}} \left(\frac{U_0}{\lambda^{1/3}}\right)^{5/2}.$$

From black hole entropy we obtain the prediction for the Energy

$$S = \frac{A}{4G_N} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{9/2} \implies \frac{E}{\lambda N^2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{14/5}$$

Checks of the predictions

We found excellent agreement with this prediction V. Filev and D.O'C. arXiv:1506.01366 and 1512.02536.
The best current results (Berkowitz, Rinaldi, Hanada, Ishiki, Shimasaki and Vranas arXiv 1606.04951) give

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{14}{5}} - (10.0 \pm 0.4)\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{23}{5}} + (5.8 \pm 0.5)\,T^{\frac{29}{5}} + \dots - \frac{5.77\,T^{\frac{2}{5}} + (3.5 \pm 2.0)\,T^{\frac{11}{5}}}{N^2} + \dots$$

Hawking Radiation?

The idea is to include a **black hole** in the gravitational system.

The Hawking termperature provides the temperature of the system.

Hawking radiation

Hanada, Hyakutake, Ishiki, and Nishimura, arXiv 1311.5607 [hep-th] argued the leakage into the flat directions was indicative of Hawking radiation.

The Berkooz Douglas Model

Adding fundamental degrees of freedom to the BFSS model yields the Berkooz–Douglas matrix model

$$\mathcal{L}=\mathcal{L}_{BFSS} + \textbf{tr}\left(D_0\bar{\Phi}^\rho D_0\Phi_\rho + i\chi^\dagger D_0\chi\right) + \mathcal{L}_{\rm int}\ ,$$

where:

$$\mathcal{L}_{\rm int}=\textbf{tr}\left(\bar{\Phi}^\alpha[\bar{X}^{\beta\dot{\alpha}},X_{\alpha\dot{\alpha}}]\Phi_\beta + \frac{1}{2}\bar{\Phi}^\alpha\Phi_\beta\bar{\Phi}^\beta\Phi_\alpha - \bar{\Phi}^\alpha\Phi_\alpha\bar{\Phi}^\beta\Phi_\beta\right)$$
$$+\textbf{tr}\left(\sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\chi}\lambda_\alpha\Phi_\beta - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\Phi}^\alpha\bar{\lambda}_\beta\chi\right)$$
$$-\sum_{i=1}^{N_f}\left((\bar{\Phi}^\rho)^i(X^a-m_i^a\mathbf{1})(X^a-m_i^a\mathbf{1})(\Phi_\rho)_i + \bar{\chi}^i\gamma^a(X^a-m_i^a\mathbf{1})\chi_i\right)$$

$a=1,\ldots,5$ are transverse to the D4-brane, m_i^a are the positions of the D4-branes, λ^ρ and $\theta^{\dot{\alpha}}$ are BFSS and χ the fundamental fermions.

The D4-brane as a probe of the geometry.

The dual adds N_f D4 probe branes. In the probe approximation $N_f \ll N_c$, their dynamics is governed by the Dirac-Born-Infeld action:

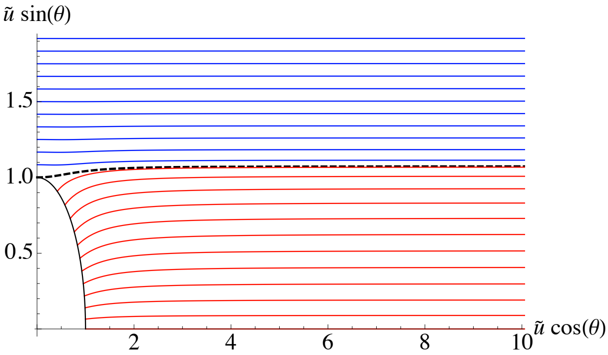
$$S_{\mathrm{DBI}} = -\frac{N_f}{(2\pi)^4\,\alpha'^{5/2}\,g_s}\int d^4\xi\,e^{-\Phi}\,\sqrt{-\det||G_{\alpha\beta}+(2\pi\alpha')F_{\alpha\beta}||}\;,$$

where $G_{\alpha\beta}$ is the induced metric and $F_{\alpha\beta}$ is the $U(1)$ gauge field of the D4-brane. For us $F_{\alpha\beta}=0$.

$$d\Omega_8^2 = d\theta^2 + \cos^2\theta\,d\Omega_3^2 + \sin^2\theta\,d\Omega_4^2$$

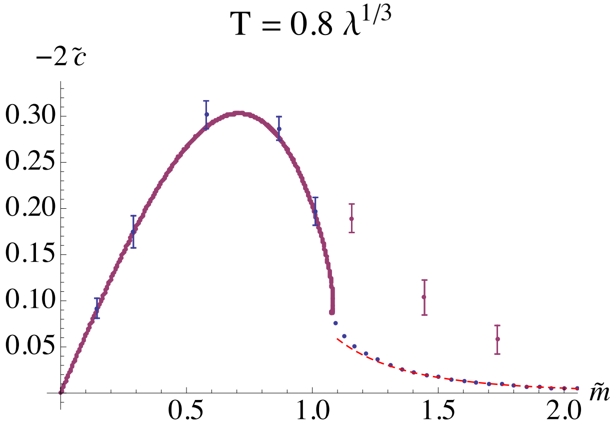
and taking a D4-brane embedding extended along: t, u, Ω_3 with a non-trivial profile $\theta(u)$.

Embeddings



$$\tilde{u} \sin \theta = m + \frac{\tilde{c}}{\tilde{u}^2} + \dots .$$

The condensate and the dual prediction



V. Filev and D. O'C. arXiv 1512.02536.

The data overlaps surprisingly well with the gravity prediction in the region where the $D4$ brane ends in the black hole.

The Backreacted Problem

For the backreacted problem we need a solution to 11-dim sugra (Filev and D. O'C. arXiv:2203.02472) in an M5-brane background of the form

$$
\begin{aligned}
ds_{11}^2 = & -K_1(u,v)\,dt^2 + K_3(u,v)(dx_{11} + A_0(u,v)\,dt)^2 \\
& + K_2(u,v)(du^2 + u^2 d\Omega_3^2) + \\
& + K_4(u,v)(dv^2 + v^2 d\Omega_4^2)\ , \qquad (1)
\end{aligned}
$$

$$
\mathcal{F}_{(4)} = F'(v)\,v^4 \sin^3\psi \sin\tilde\alpha \cos\tilde\alpha\, d\psi\wedge d\tilde\alpha \wedge d\tilde\beta \wedge d\tilde\gamma\ , \qquad (2)
$$

$$
d\Omega_3^2 = d\alpha^2 + \sin^2\alpha\, d\beta^2 + \cos^2\alpha\, d\gamma^2\ , \qquad (3)
$$

$$
d\Omega_4^2 = d\psi^2 + \sin^2\psi\, d\tilde\Omega_3^2\ , \quad d\tilde\Omega_3^2 = d\tilde\alpha^2 + \sin^2\tilde\alpha\, d\tilde\beta^2 + \cos^2\tilde\alpha\, d\tilde\gamma^2\ .
$$

$$
\int \mathcal{F}_{(4)} = \frac{8}{3}\pi^2\, v^4\, F'(v) = -Q_5 \qquad \text{the M5-brane charge.}
$$

gives

$$
F(v) = 1 + \frac{Q_5}{8\pi^2 v^3} \equiv 1 + \frac{v_5^3}{v^3} = 1 + \frac{N_f}{N_c}\frac{4\pi^3\alpha'^3\lambda}{v^3}\ ,
$$

The solution preserving supersymmetry is given by:

$$ds_{11}^2 = \left(1 + \frac{v_5^3}{v^3}\right)^{-1/3} \left(-H(u,v)^{-1}\, dt^2 + \right.$$
$$+H(u,v)\left(dx_{11} + (H(u,v)^{-1} - 1)\, dt\right)^2 +$$
$$\left. du^2 + u^2\, d\Omega_3^2\right) + \left(1 + \frac{v_5^3}{v^3}\right)^{2/3} \left(dv^2 + v^2\, d\Omega_4^2\right) \ .$$

Note: Supersymmetry does not restrict the shape of the function $H(u,v)$. The equation of motion for H can be obtained either by using the Einstein equations or by requiring that the angular momentum along x_{11} is conserved.

Equation for $H(u, v)$

The non-trivial equation requiring a solution is:

$$\partial_v^2 H(u,v) + \frac{4}{v}\partial_v H(u,v) + \left(1 + \frac{v_5^3}{v^3}\right)\left(\partial_u^2 H(u,v) + \frac{3}{u}\partial_u H(u,v)\right) = 0$$

Perturbation in v_5 recovers the probe limit.

In the $v \to \infty$ limit of equation (equivalent to the $v_5 \to 0$ limit) $SO(9)$ symmetry is recovered and

$$H_0(u, v) = 1 + \frac{r_0^7}{(u^2 + v^2)^{7/2}} \ ,$$

The parameter r_0^7 is proportional to the number of D0–branes, N_c:

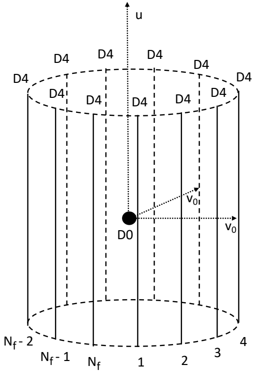
$$r_0^7 = N_c \, 60 \, \pi^3 \, g_s \, \alpha'^{7/2} \ .$$

$$1 + \frac{v_5^3}{v^3} = 1 + \frac{N_f}{N_c} \frac{4\pi^3 \lambda}{(v/\alpha'^3)} \ .$$

Perturbation in v_5 recovers the probe approximation.

Instability of overlap intersection

There is an instability in the system when the D0-branes lie in the D4-branes. We move them off into a shell



D0-branes at the origin surrounded by uniform density of D4-branes separated in the $\mathbb{R}^5$ transverse to the D4–branes and a distance $v = v_0$ from the D0-branes.

Just as in electrostatics, the solution interior to the shell is the same as that in the absence of the D4s, however the interior expression is modified from

$$H(u,v) = 1 + \frac{r_0^7}{r^7} \qquad \text{to} \qquad H(u,v) = 1 + \frac{\gamma^3 r_0^7}{r^7}$$

where $\gamma^2 = 1 + \frac{N_f}{N_c}\frac{\lambda}{2m_q^3}$ and $r^2 = u^2 + \gamma^2 v^2$ is the interior radial coordinate. The dependence on N_f/N_c is because the parameter r_0 is measured at infinity in u at fixed v.

The backreacted exterior solution takes the fom

$$H(u,v) \;\; = \;\; 1 + \frac{r_0^7}{(u^2+v^2)^{\frac{7}{2}}}\left[1 + \frac{v_5^3}{v_0^3} H_c\left(\frac{u}{v_0}, \frac{v}{v_0}, \frac{v_5}{v_0}\right)\right] \; .$$

It is similar to the leading perturbative solution $H_c \sim 1$. The principal effect of increasing v_5/v_0 is that the geometry outside of the shell approaches that of the D4-branes geometry in the absence of D0-branes.

We need a black hole solution

Work in progress

For useful comparisons with numerical simulations we need an 11-dim gravitational M5-brane solution in the presence of a black hole. This seems accessible only via numerics.

Thanks for Your Attention!