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Building a SUSY Black Hole in Matrix Quantum Mechanics

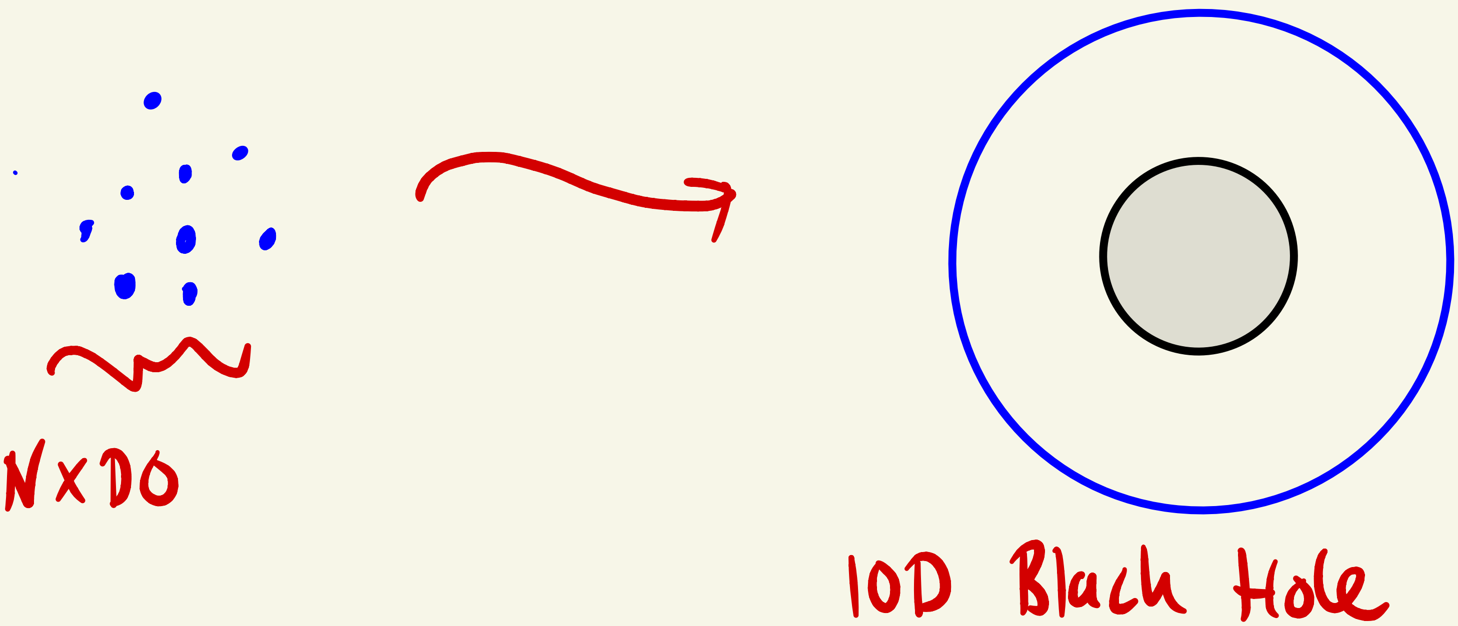
ND + Mouland + Zhao

(see also R. Mouland arXiv: 2311.13636)

Finite dimensional holography

Quantum Mechanics $\longleftrightarrow$ Gravity

eg M(atrix) model BFSS/BMN



- Non SUSY black holes

  Numerical simulation

- What about SUSY black holes?

  Analytic results?

This talk,

M(atrix) model of M5 branes

Berkooz, Douglas

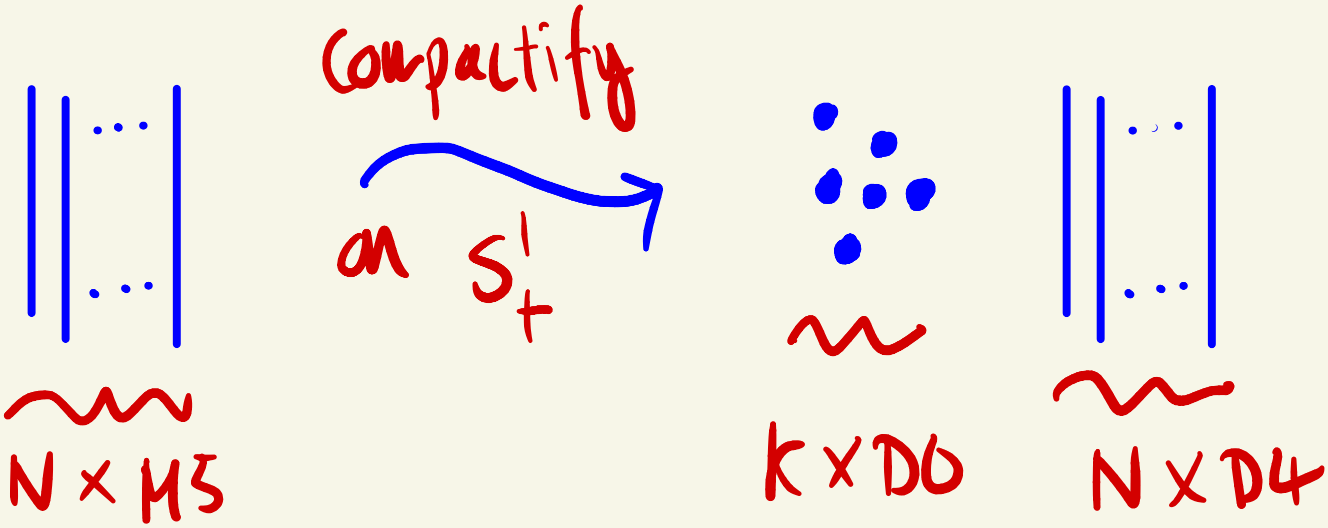
- Conformal point $Osp(4^*|4)$

- Exponential growth of BPS degeneracy

- Dual to supersymmetric ultra-spinning black hole

DLCQ of M5 branes

Berkooz + Douglas



- $U(K)$ gauged quatum mechanics
- 8 supercharges
- matter content,

		$U(K)$	$U(N)$
adjoint	$X, \tilde{X}$	$\underline{K}^2$	$\underline{1}$
fundamental	$Q, \tilde{Q}$	$\underline{K} \oplus \underline{\bar{K}}$	$\underline{N} \oplus \underline{\bar{N}}$

- parameters,

gauge coupling $\rightarrow$ e^2, ξ $\leftarrow$ FI parameter

Conformal point

$e^2 \to \infty$

$\zeta \to 0$

$$Osp(4^*|4) \subset Osp(8^*|4)$$

lightcone subgroup

(2,0) SCFT in $D=6$

Matrix model

$$\downarrow$$

IR limit: $e^2 \to \infty$

Susy QM on $\mathcal{M}^{\zeta}_{K,N}$

moduli space of K Yang-Mills instantons on $\mathbb{R}^4$

$$\mathcal{M}^{\xi}_{K,N} =$$

$$\left\{ \begin{array}{c} [X, \tilde{X}] + \sum_i Q_i \tilde{Q}_i = 0 \\ [X, X^\dagger] + [\tilde{X}, \tilde{X}^\dagger] + \sum_i |Q_i|^2 - |\tilde{Q}_i|^2 = \xi \end{array} \right\} \Big/ U(K)$$

- $\underline{\xi > 0}$ smooth hyperkähler manifold
- $\underline{\xi = 0}$ singular hyperkähler cone

Triholomorphic homothety with potential,

$$k = \mathrm{Tr}_K \left[|X|^2 + |\tilde{X}|^2 + \sum_{i=1}^{N} |Q_i|^2 + |\tilde{Q}_i|^2 \right]$$

$\Rightarrow OSp(4^*|4)$ superconformal QM

Singleton

Representation Theory

ND + Singleton

$Osp(4^*|4) \supset \underset{M}{so(2,1)} \times \underset{J_1+J_2}{so(3)_R} \times \underset{Q_1,Q_2}{Usp(4)}$

has (semi-)short irreps containing BPS states obeying,

$$\mathcal{H} := M - J_1 - J_2 - 2(Q_1+Q_2) = 0$$

BPS states $\overset{1:1}{\longleftrightarrow}$ cohomology of

$$Q := \bar{\partial} + \bar{\partial} \mathbb{K} \wedge$$

- BPS degeneracy,

$p,q = Q_1 \mp Q_2 + d_\phi/2$

$$d(K, J_1, J_2, Q_1, Q_2) := \dim H^{[J_1, J_2]}_{p,q} \in \mathbb{N}$$

New basis for charges,

$$\mathcal{H} = M - J_1 - J_2 - 2(Q_1 + Q_2)$$

$$L_t = J_1 + J_2 + Q_1 + Q_2$$

$$L_x = J_1 - J_2 \qquad \left[\begin{matrix}\mathcal{H} \\ L_t \\ L_x \\ L_y\end{matrix}, Q\right] = 0$$

$$L_y = Q_1 - Q_2$$

$$F = 2Q_2 \qquad [F, Q] = Q$$

"Count" BPS states with <u>superconformal</u> <u>index</u> $\mathcal{I}(\mathcal{M}^{\mathcal{H}=0}_{K,N}) :=$

$$\mathrm{Tr}\left[(-1)^F e^{-\beta\mathcal{H}} t^{L_t} x^{L_x} y^{L_y}\right]$$

Superconformal index ND + Singleton

- Evaluate by localisation

$$\mathcal{I}(\mathcal{M}^{\xi=0}_{K,N}) = \chi_y(\mathcal{M}^{\xi\neq 0}_{K,N})$$

$\leftarrow$ equivariant χ_y genus

independent of $\beta, \xi, e^2 \ldots$

- Generating function,

$$G(q,t,x,y) := \sum_{K=0}^{\infty} q^K \mathcal{I}(\mathcal{M}_{K,N})$$

= Nekrasov partition function

- Coefficient of $q^K t^{L_t} x^{L_x} y^{L_y}$ is index degeneracy, $C(K, L_t, L_x, L_y)$

$$:= \sum_F (-1)^F d(K, L_t, L_x, L_y, F)$$

Warm-up $N=1$

$$\mathcal{M}_{k,1} \simeq \mathrm{Hilb}_k\, \mathbb{C}^2$$ Hilbert scheme

Exact Resummation, Awata + Kanno, Rains + de Warnaar

$$G(q,t,x,y) =$$

$$\mathrm{Pexp}\left[\frac{q}{(1-q)}\,\frac{t}{y}\cdot\frac{(1-yx)(1-y/x)}{(1-tx)(1-t/x)}\right]$$

allows evaluation of

$G(k, L_t, L_x, L_y)$ to high order.....

Sample data for $G(K, L, 0, 0)$ ......

		L		
		50	52	54
K	50	4773158006473089778	13722136087430823474	10728665632616173124
	51	6943033937905529622	14870285533157146362	-1493086283031736666
	52	9190525712121239144	13822179413239343452	-2163073510148185436 6
	53	11174425419671147488	9447220680249748082	-5074469518684269411 4
	54	12412734295210394496	5431158129565557290	-8893822944634145587 0
	57	4868162317987965318	-6300908921769692954 6	-2338973995296275169 32
	58	-4069732338637176522	-9726567340028439510 8	-2724487788437058186 14
	59	-17508654288324952938	-13621685813308401908 8	-2889900073089236969 54
	60	-35960421557696977470	-17674458392244946650 8	-2688218402207204769 58

- monotonic growth
- "Randomly" oscillating sign

Evaluate asymptotics à la Hardy - Ramanujan ......

G = contour integral

complex saddle-points

Theorem ND + Zhao

$\mathcal{M}_k := \mathrm{Hilb}_k\, \mathbb{C}^2$

$$C(k,L,0,0) = \sum_{q=1}^{k} (-1)^q \dim H^{[L,L]}_{p=k,q}(\mathcal{M}_k)$$

For $k, L \to \infty$ with k/L fixed

$$\boxed{\begin{aligned} C &= \exp(S + A)\cos(S + B) + O(1) \\ S &= \pi\left(\tfrac{8}{3}L^2 k\right)^{1/4} \\ A, B &< c\sqrt{k} \quad \text{for some constant } c \end{aligned}}$$

matches numerical data .....

General Case $\quad N \gg 1$ $\quad$ ND, Mouland, Zhao

$$G(K, L_t, L_x, L_y) \overset{K, L_t \to \infty}{\sim} \cos(\pi F^*)\, e^{S^*}$$

where $\tau = S^* + i\pi F^*$ is determined by quartic eqn,

$$\boxed{\tau^2 (\tau - 2\pi i L_y)^2 = \frac{8\pi^2}{3} N^3 K \times \big(\tau + 2\pi i (L_t + L_x + L_y)\big)\big(\tau + 2\pi i (L_t - L_x + L_y)\big)}$$

Holographic Dual

ND, Moulland, Zhao
Moulland

- General story, Maldacena, Martelli, Tachikawa

$$
\begin{tikzcd}
\text{CFT}_D \arrow[rr, leftrightarrow, "\text{AdS/CFT}"'] \arrow[d, "\text{Penrose limit/orbifold}"] & & \text{AdS}_{D+1} \arrow[d] \\
\text{Conformal QM} \arrow[rr, leftrightarrow] & & \text{AdS pp-wave}
\end{tikzcd}
$$

Thermal state (under CFT_D)

Black hole (under AdS_{D+1})

Thermal state (under Conformal QM)

ultra-spinning black hole (under AdS pp-wave)

Kerr-Newman black holes in AdS_7

$U(1)\times U(1)$ gauged SUGRA

Conserved charges,

$$\underset{\text{mass}}{\tilde{M}}\ ,\quad \underbrace{J_1, J_2, J_3}_{\text{angular momenta}} \quad \underbrace{Q_1, Q_2}_{\text{electric charges}}$$

BPS bound $OSp(8^*|4)$

$$\tilde{M} \geq J_1 + J_2 + J_3 + 2(Q_1 + Q_2)$$

Black hole solutions:

$Q_1 = Q_2$ Chow 2008

Generic Bobev, David, Hong, Mouland 2023

Ultra-spinning Limit

$J_3 \to \infty$, $\mathbb{Z}_L$ orbifold
$K = J_3/L$ fixed

Ultraspinning black hole,

- AdS pp-wave asymptotics
  Osp(4*|4) isometry
- Space-like S^1 in bulk becomes null on boundary
- Non-compact horizon
  finite area
- Conserved charges

$$\underset{\text{mass}}{M}, \underset{\text{null boundary momentum}}{K}, \underbrace{J_1, J_2}_{\text{transverse angular momenta}}, \overbrace{Q_1, Q_2}^{\text{electric charges}}$$

obeys $Osp(4^*|4)$ BPS bound,

$$M \geq J_1 + J_2 + 2(Q_1 + Q_2) \qquad -\circledast$$

- SUGRA discription valid for

$$K \gg N^{7/3} \gg 1 \qquad N^3 = \frac{3\pi^2}{16} \frac{R_{AdS}^5}{G_N}$$

- Ultra-spinning SUSY black holes

ND, Mouland, Zhao
Mouland

  - saturate $\circledast$
  - exist only on codimension one subspace in

$$(\underset{\substack{\uparrow \\ \text{Boundary null momentum}}}{K}, \underbrace{J_1, J_2}_{\text{transverse angular momenta}}, \overbrace{Q_1, Q_2}^{\text{electric charges}})$$

Constraint $\Rightarrow$ $\exists$ exactly one* black hole for each value of the "global charges"

$$K, L_t, L_x, L_y$$

*known

with entropy,

$$S = S^*(K, L_t, L_x, L_y)$$

and "fermion number"

$$F = 2Q_2 = F^*(K, L_t, L_x, L_y)$$

- Consistent with QM assumption,

$$G(K, L_t, L_x, L_y) \underset{K, L_t \to \infty}{\overset{N \gg 1}{\sim}} \cos(\pi F^*)\, e^{S^*}$$

Conclusion / outlook

- **conformal** $e^2 = \infty$, $\xi = 0$

BPS ensemble in QM $\xleftrightarrow{\text{AdS/CFT}}$ ultra-spinning SUSY black hole

- **non-conformal** $e^2, \xi > 0$

- large BPS degeneracy
- dual black holes?
  M5 branes in pp-wave
- simulation?