Gauge-gravity duality of the BFSS matrix model

$\underline{\underline{X}}^I , \; \underline{\underline{\psi}}$ $\longleftrightarrow$

Toby Wiseman (Imperial)

Matrix Quantum Mechanics for M-theory Revisited. CERN Jan '24

Plan

Goal: understand quantum gravity

- Maldacena’s large N duality

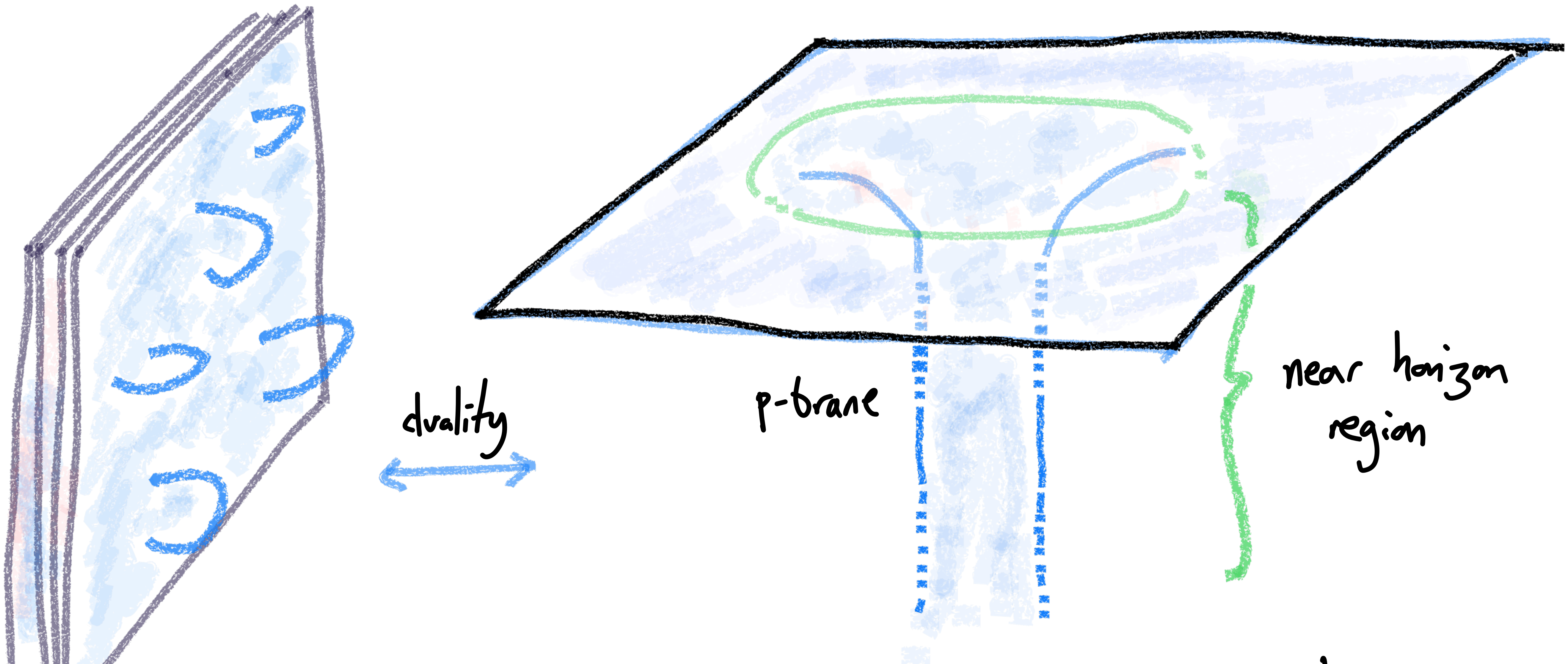
- BFSS matrix model (and SYM generalisations)

- Towards solving thermal BFSS; numerical, analytic

- Modifications of BFSS: BMN and gauging

Holography

SU(N) (p+1)-d maximal SYM <-> string theory in the decoupling limit of N Dp branes



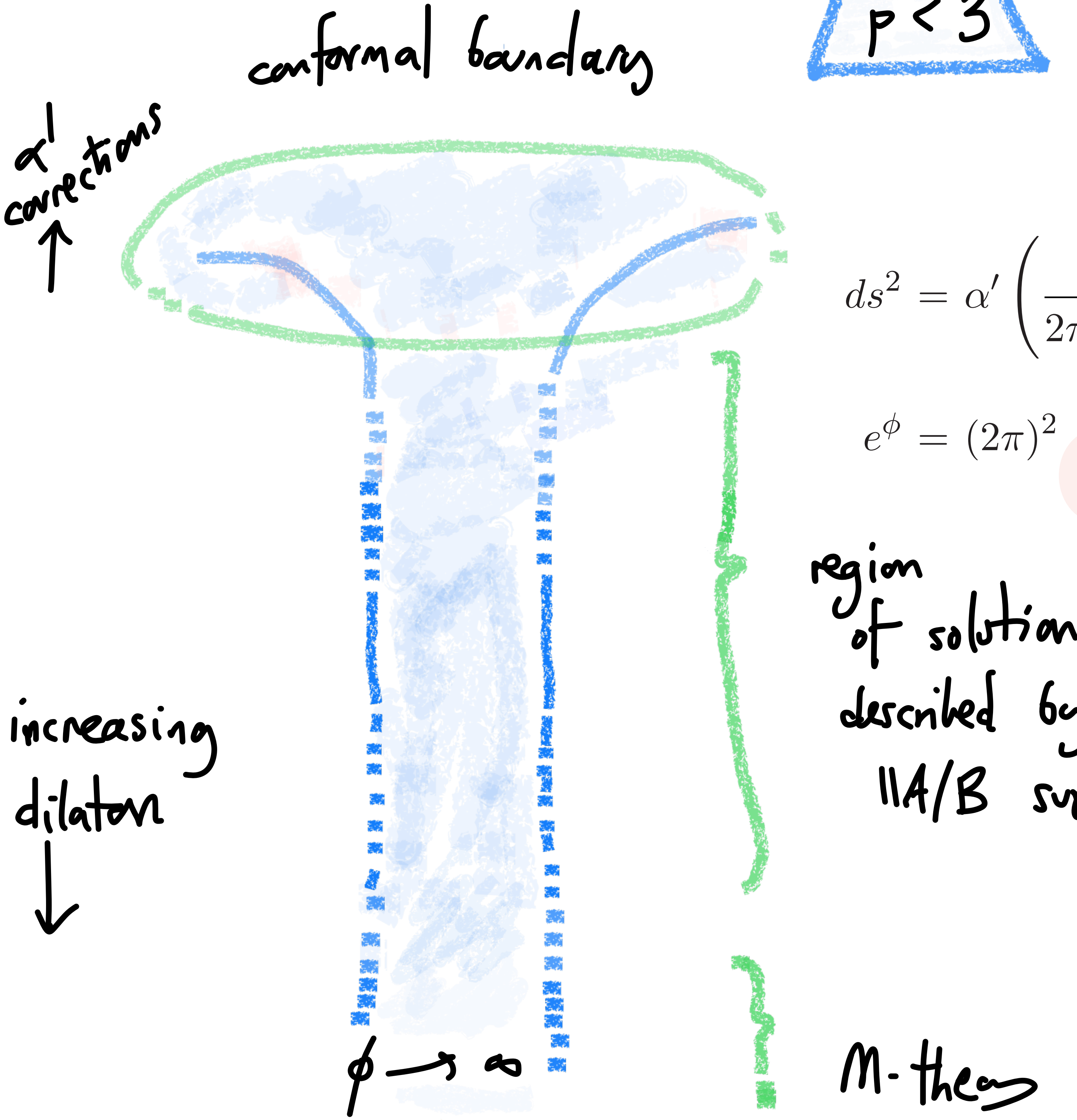
'T Hooft large N limit $\longrightarrow$ gravity

$\lambda = N g_{YM}^2$ $\qquad$ $[\lambda] = 3 - p$

Maldacena ’97; Itzhaki et al '98

Holography

SU(N) (1+p)-d maximal SYM <-> string theory in the decoupling limit of N Dp branes



$p < 3$

$U \sim$ energy scale

$$ds^2 = \alpha' \left(\frac{U^{\frac{7}{2}}}{2\pi\sqrt{b\lambda}}(-dt^2) + 2\pi\sqrt{b\lambda}\left(U^{-\frac{7}{2}}dU^2 + U^{-\frac{3}{2}}d\Omega^2\right)\right)$$

$$e^{\phi} = (2\pi)^2 \frac{\lambda}{N} \left(\frac{U^7}{4\pi^2 b\lambda}\right)^{-\frac{3}{4}} \qquad b = 240\pi^5$$

$$U / \lambda^{\frac{1}{3-p}} \ll 1$$

$p=3$ is famous AdS-CFT

Holography

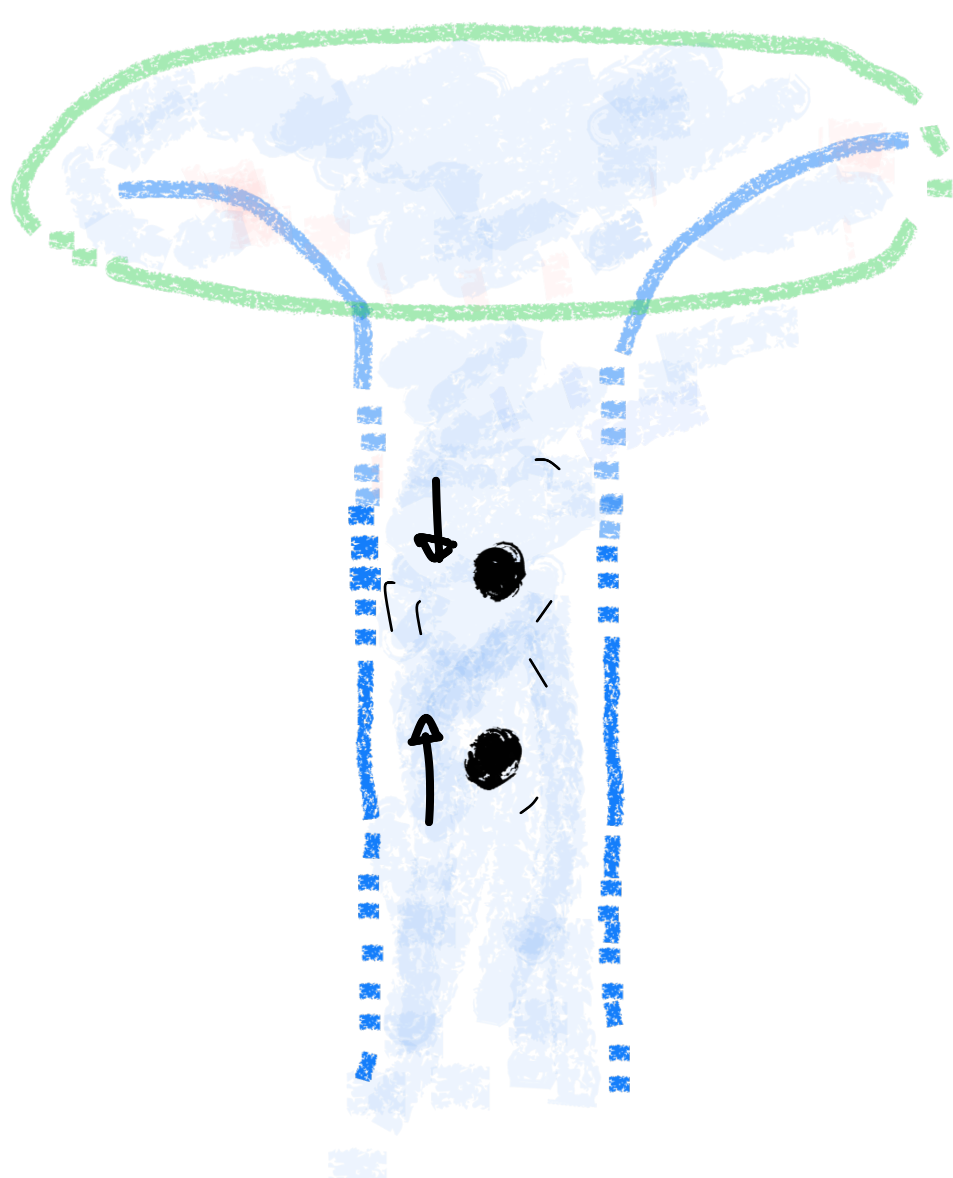
- Some basic quantities can be deduced by calculating for asymptotically flat solutions *before* taking the near horizon decoupling limit.

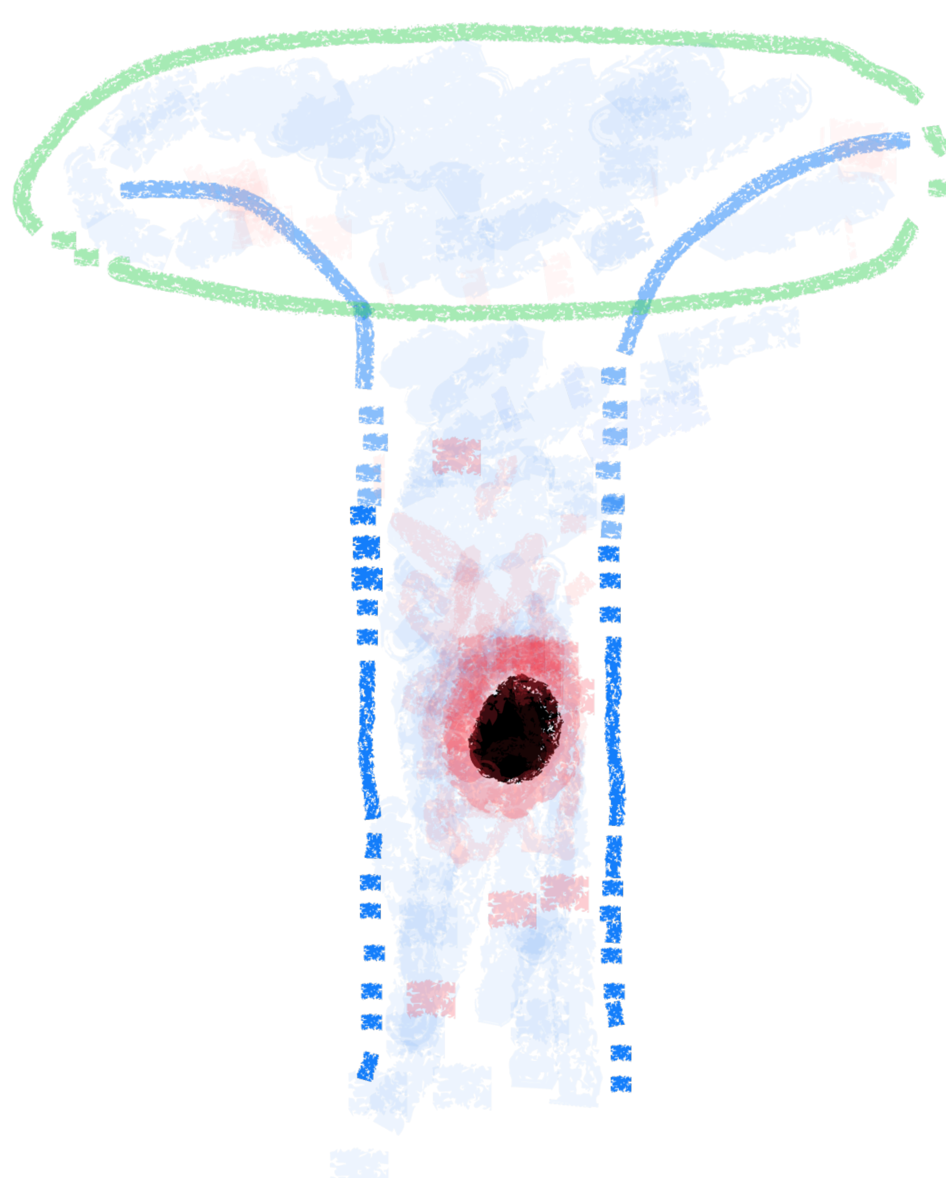
- However holographic renormalization still works in a similar manner and one can compute directly in the decoupling asymptotics.

TW, Withers ’08; Kanitscheider, Skenderis, Taylor ‘08

Holography

- Worth emphasising that ***all*** physics that occurs in bulk should be captured by the boundary theory.





Holography

$$S_{YM} = \frac{N}{\lambda}\int dt dx^p \, \mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(D^\mu X^I)^2 - \frac{1}{4}\left[X^I, X^J\right]^2 + \Psi\left(\Gamma^\mu D_\mu - \Gamma^I\left[X^I, \cdot\,\right]\right)\Psi\right]$$

SO(9-p) flavours

- X^I are (9-p) $N \times N$ Hermitian matrices transforming in the adjoint, Ψ is a fermion also in the adjoint
- For BFSS, p=0, usually written as (by gauge fixing);

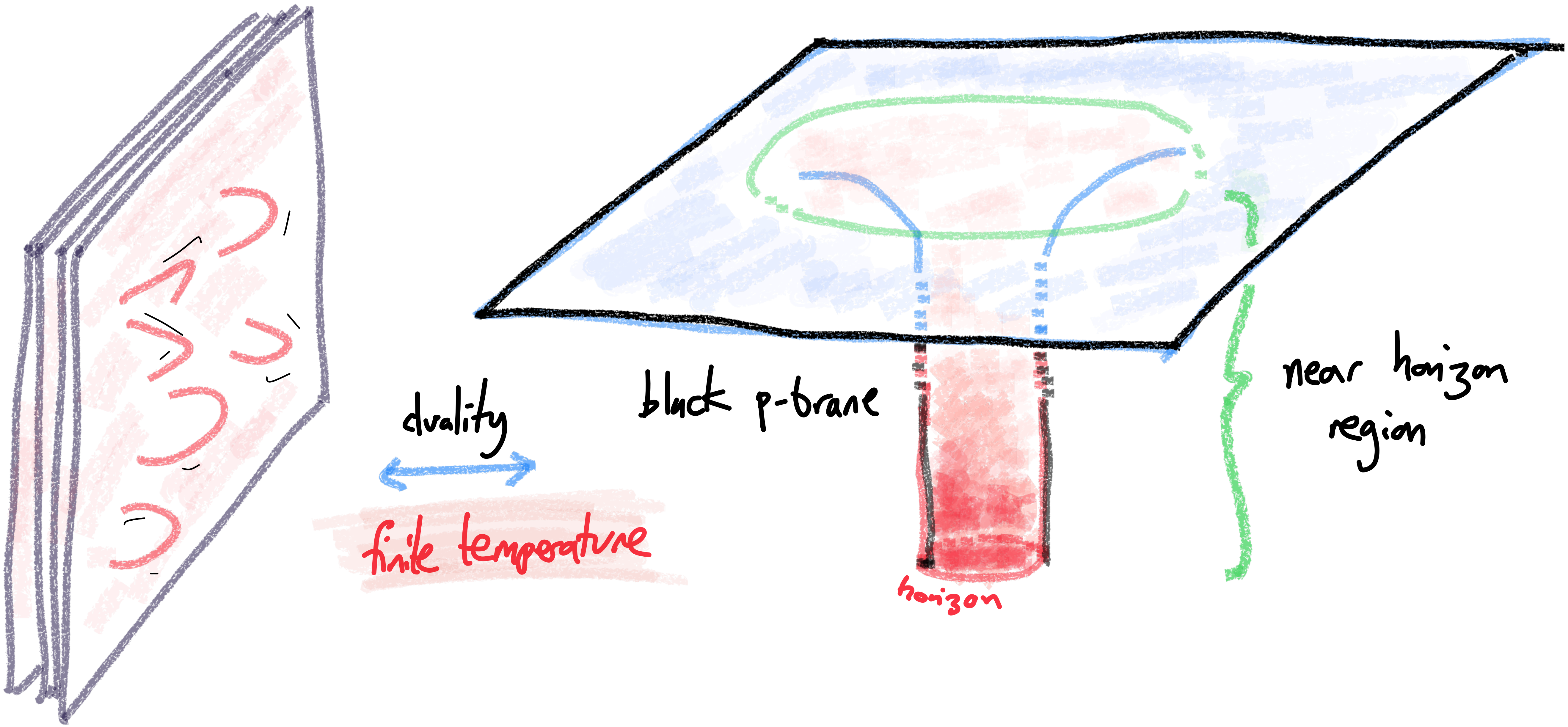
$$S_{BFSS} = \frac{N}{\lambda}\int dt \, \mathrm{Tr}\left[\frac{1}{2}(\dot{X}^I)^2 - \frac{1}{4}\left[X^I, X^J\right]^2 + \Psi\left(i\dot{\Psi} - \Gamma^I\left[X^I, \Psi\right]\right)\right]$$

- However we must remember the SU(N) singlet constraint

Holography

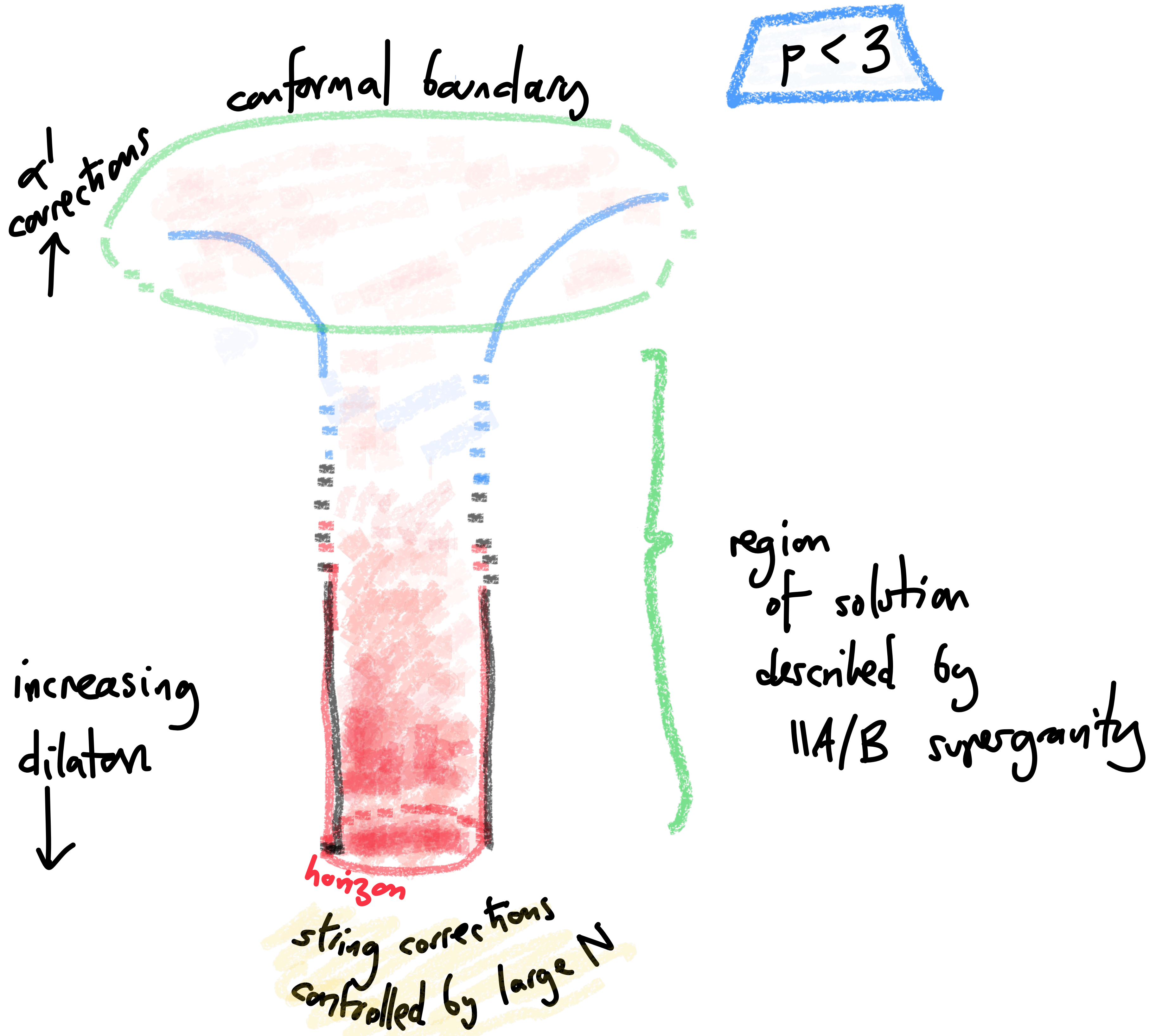
- What are the interesting questions…
  - How does locality emerge in the bulk?
  - Black holes thermodynamics — how is entropy encoded
  - Black hole evaporation and information loss?

Holography



Maldacena ’97; Itzhaki et al '98

Holography



p < 3
conformal boundary
α' corrections
region of solution described by IIA/B supergravity
increasing dilaton
horizon
string corrections controlled by large N

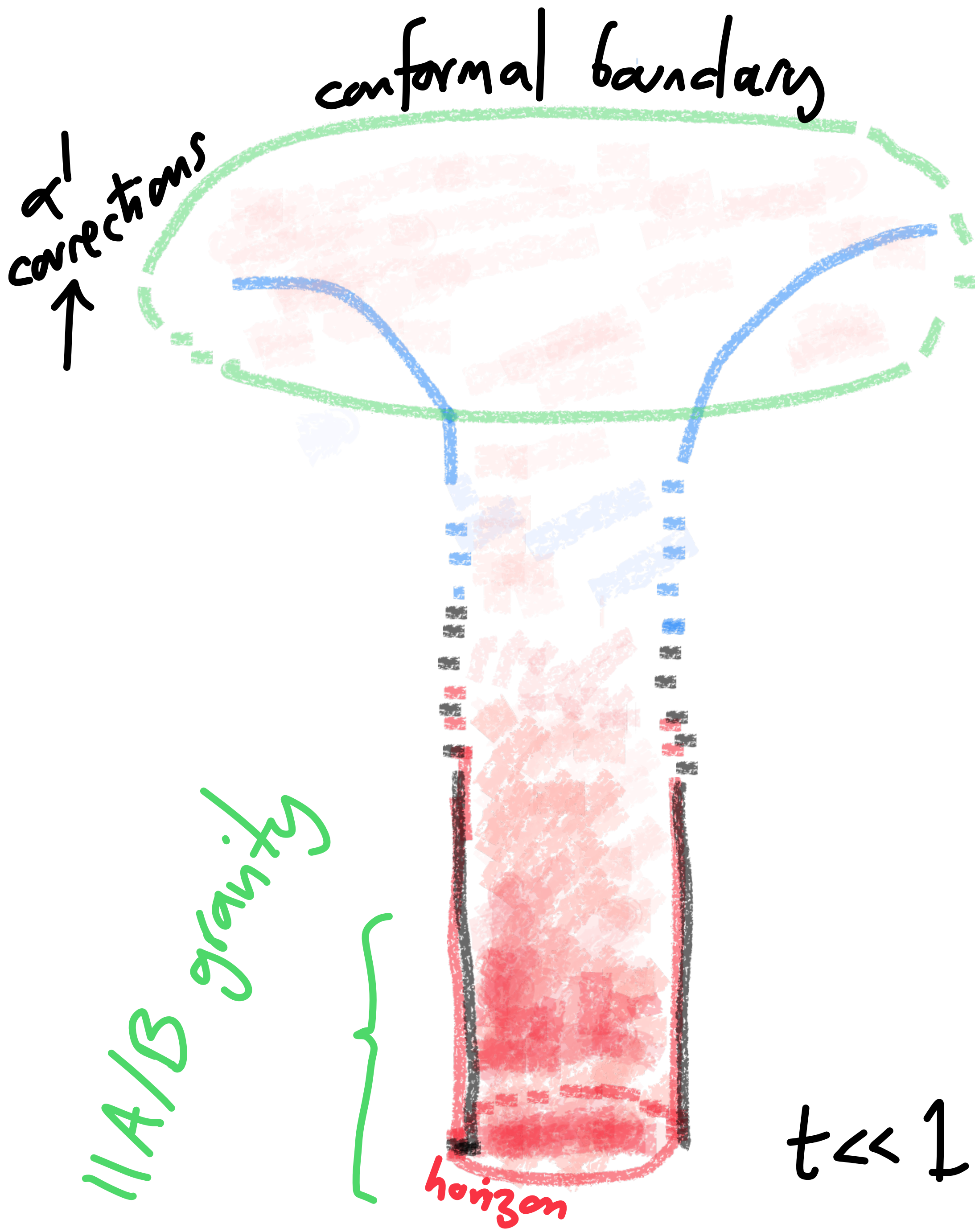
Holography

- Define dimensionless temperature $t = T\lambda^{-\frac{1}{3-p}}$
- Define dimensionless energy density $\epsilon = \rho\lambda^{-\frac{1+p}{3-p}}$
- Black hole thermodynamics: $t \ll 1$ in large N limit

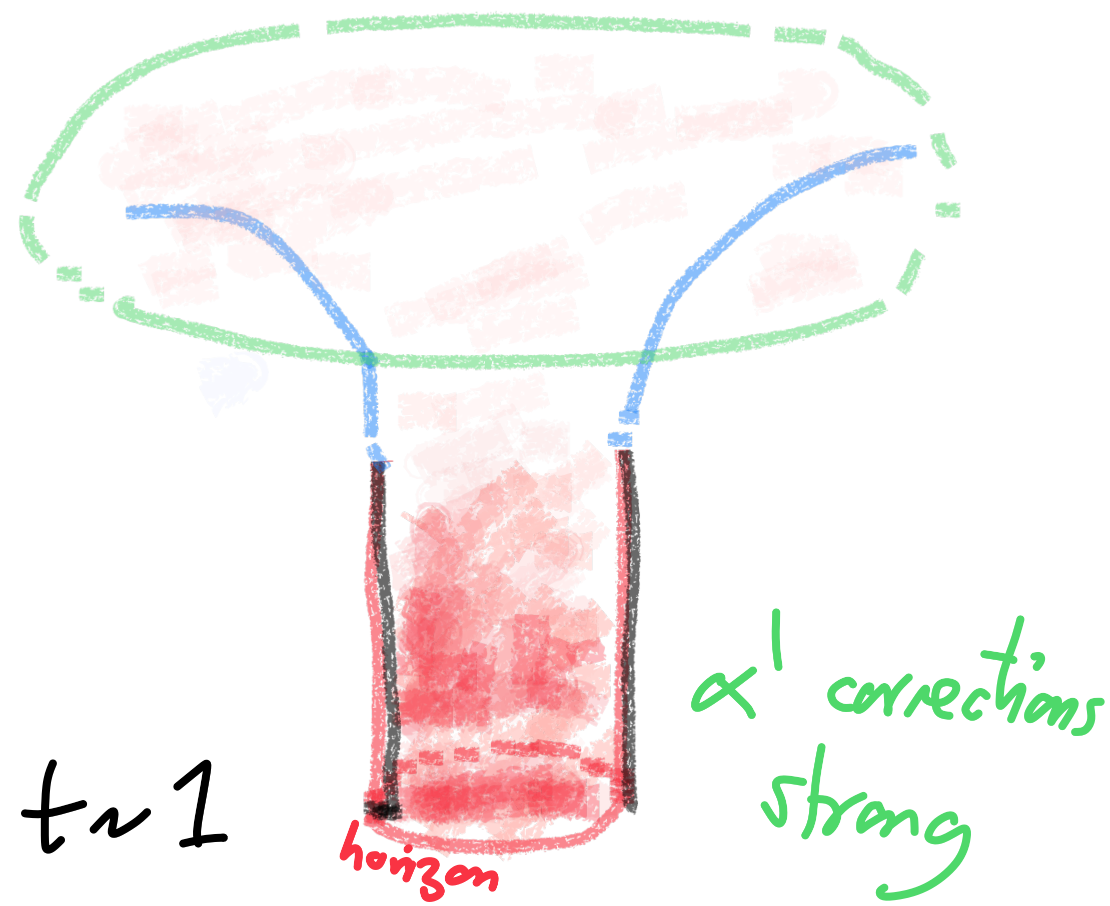
$$\epsilon = 2^{\frac{27-5p}{5-p}} (9-p) (7-p)^{-\frac{19-3p}{5-p}} N^2 \left(\pi^{\frac{13-3p}{2}} \Gamma\left(\frac{7-p}{2}\right) t^{7-p} \right)^{\frac{2}{5-p}} \sim N^2 t^{\frac{2(7-p)}{5-p}}$$

- Note; entropy $s \sim N^2 t^{\frac{9-p}{5-p}} \to 0$ as $t \to 0$; not near extremal

Holography



conformal boundary
α' corrections
IIA/B gravity
horizon
t<<1



t~1
horizon
α' corrections strong

Goal

Perform direct quantum simulation of gravity

- String theory: extremal black hole micro state counting
    - g/s degeneracy rather than thermal entropy

Strominger, Vafa '96 …… Benini, Hristov, Zaffaroni '15 ……

For QM: Dorey, Mouland, Zhao '22

- Real black holes — only have thermal entropy

- So reproducing this thermal entropy is particularly interesting.

BFSS model

Hoppe '82, '87; de Wit, Hoppe, Nicolai '88

Banks, Fischler, Shenker, Susskind '96

- BFSS is a gapless theory with a unique ground state

De Wit, Luscher, Nicolai '89 Fröhliche Graf, Haller, Hoppe, Yau '99 Lin, Xi '14

- Classically vacua are gauge equivalent to diagonal A^μ and X^I

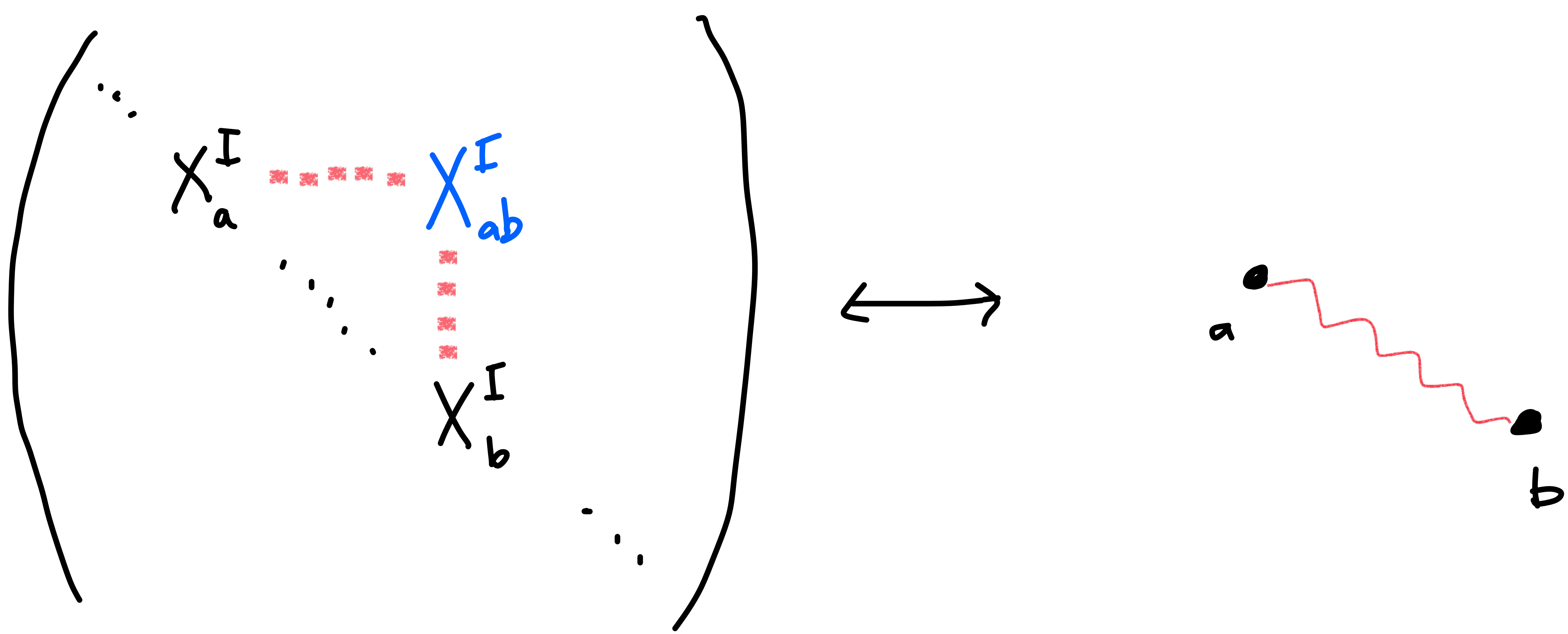
$$(\underline{\underline{A}}^\mu)_{ab} = A^\mu_a \delta_{ab} \qquad (\underline{\underline{X}}^I)_{ab} = X^I_a \delta_{ab} \qquad a = 1, \ldots, N$$

$$X^I_{ab} = \begin{pmatrix} X^I_1 & 0 & 0 & \ldots & 0 \\ 0 & X^I_2 & 0 & \ldots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \ldots & X^I_N \end{pmatrix}$$

- May view $\vec{X}_a$ as position of a'th D-brane in transverse (9-p)-dimensions

BFSS model

- We may integrate out the off-diagonal matrix components which are weakly coupled when the branes are 'well separated'
- The X^I_{ab} off-diagonal element has a mass $\sim |\vec{X}_a - \vec{X}_b|$
- For large separations these are heavy and we may integrate out.



BFSS model

- We may integrate out the off-diagonal matrix components which are weakly coupled when the branes are 'well separated'
- The X^I_{ab} off-diagonal element has a mass $\sim |\overrightarrow{X}_a - \overrightarrow{X}_b|$
- For large separations these are heavy and we may integrate out.

- These off-diagonal elements represent the open strings between the D-branes
- However due to supersymmetry there is no contribution.
- But since this is a quantum mechanics, we have to consider fluctuations… (so unique ground state)

BFSS model

- Let us promote to classical moduli (and ignore the gauge fields);

$$(\underline{\underline{\phi}}^I)_{ab} = X^I_a(t,x)\delta_{ab}$$

- The classical moduli space action is (using vector notation);

$$S^{classical} = \frac{N}{\lambda}\int dtdx^p \sum_{a=1}^{N}\left(\frac{1}{2}\partial^\mu \vec{X}_a \cdot \partial_\mu \vec{X}_a\right)$$

$$\vec{X}_{ab} = \vec{X}_a - \vec{X}_b$$

- For well separated moduli we may integrate out off-diagonal modes at 1-loop;

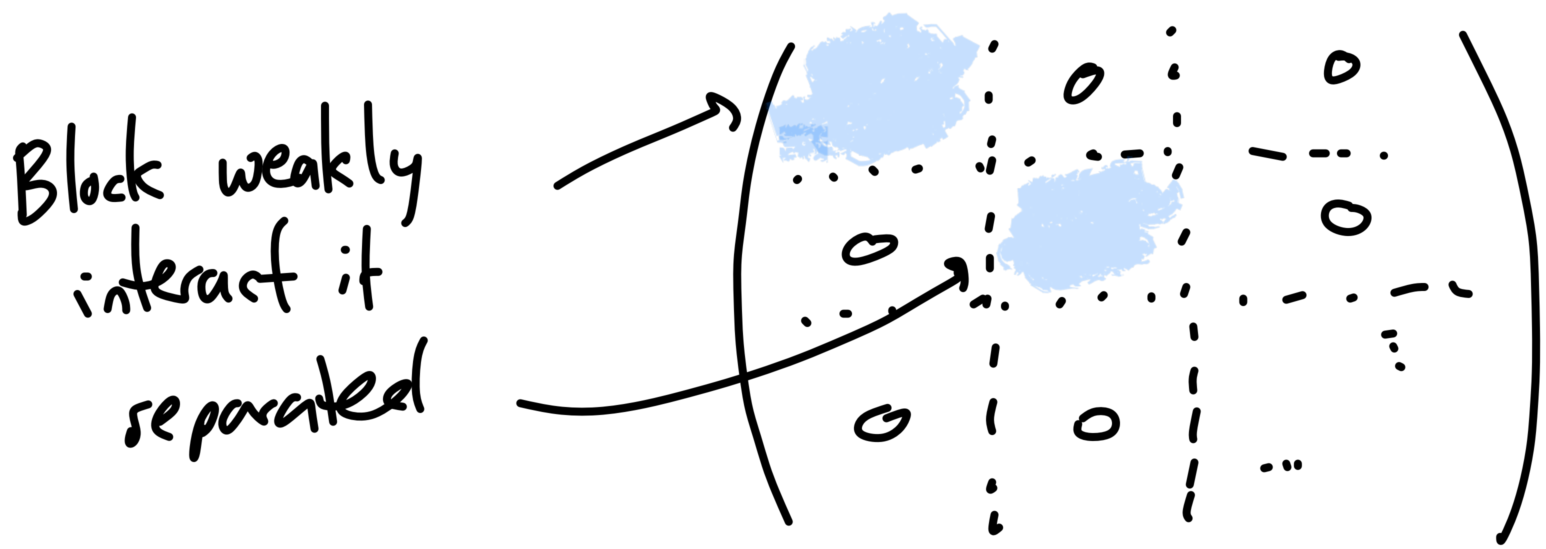
$$S^{1-loop}_{leading} = -\int d\tau dx^p \sum_{a<b} \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}}\left(2\frac{\left(\partial_\mu \vec{X}_{ab}\cdot\partial_\nu \vec{X}_{ab}\right)\left(\partial^\mu \vec{X}_{ab}\cdot\partial^\nu \vec{\phi}_{ab}\right)}{|\vec{X}_{ab}|^{7-p}} - \frac{\left(\partial_\mu \vec{X}_{ab}\cdot\partial^\mu \vec{X}_{ab}\right)^2}{|\vec{X}_{ab}|^{7-p}}\right)$$

- For BFSS this is the famous attractive;

$$S^{1-loop}_{leading} = -\frac{15}{16}\int d\tau \sum_{a<b}\frac{\left|\dot{\vec{X}}_{ab}\right|^4}{|\vec{X}_{ab}|^7}$$

BFSS model

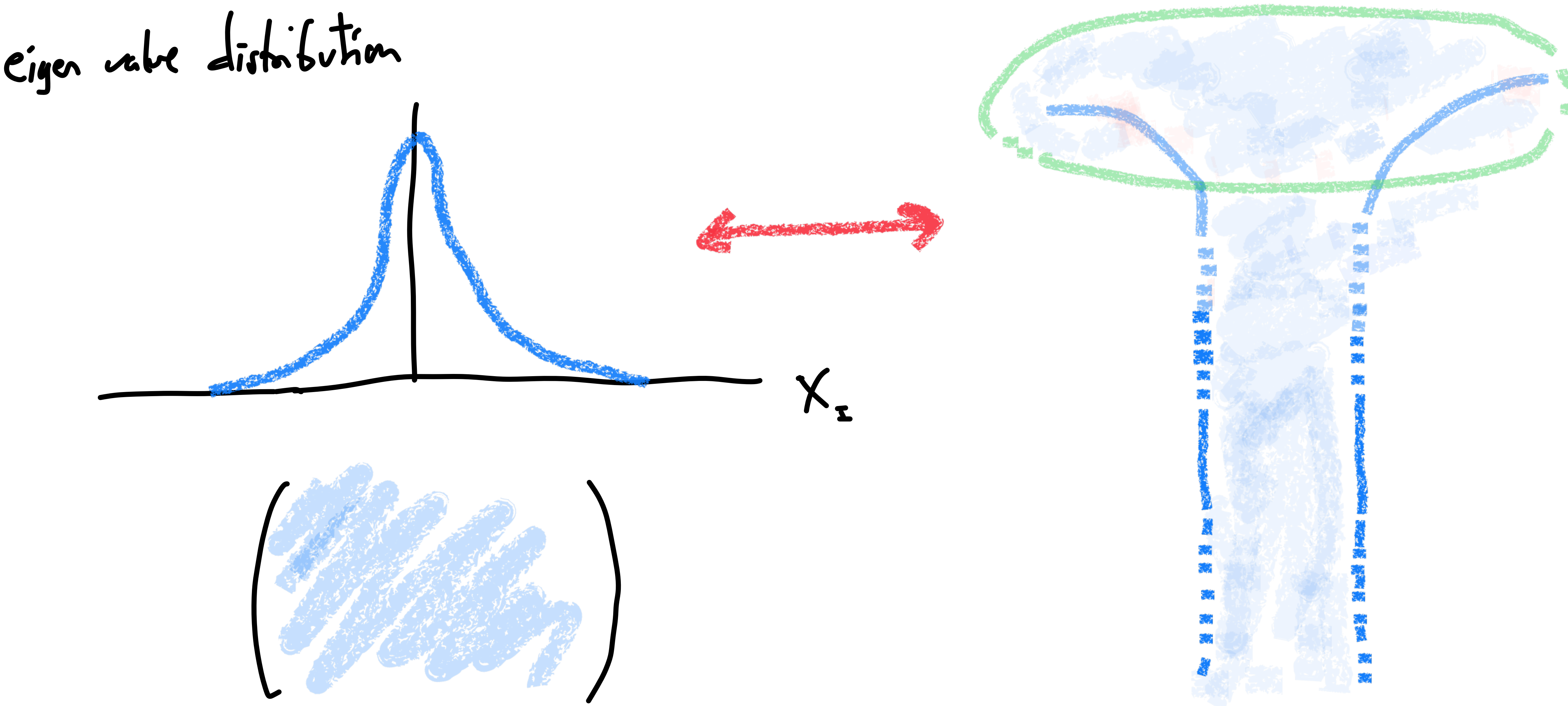
- More generally, different blocks which are well separated don't classically 'talk' to each other, and are expected to weakly interact.



- Original BFSS conjecture relates the interactions in this moduli theory to those in 11-d supergravity
- Very interesting recent progress considering scattering!
- However here we will consider the 't Hooft large N limit…

BFSS model

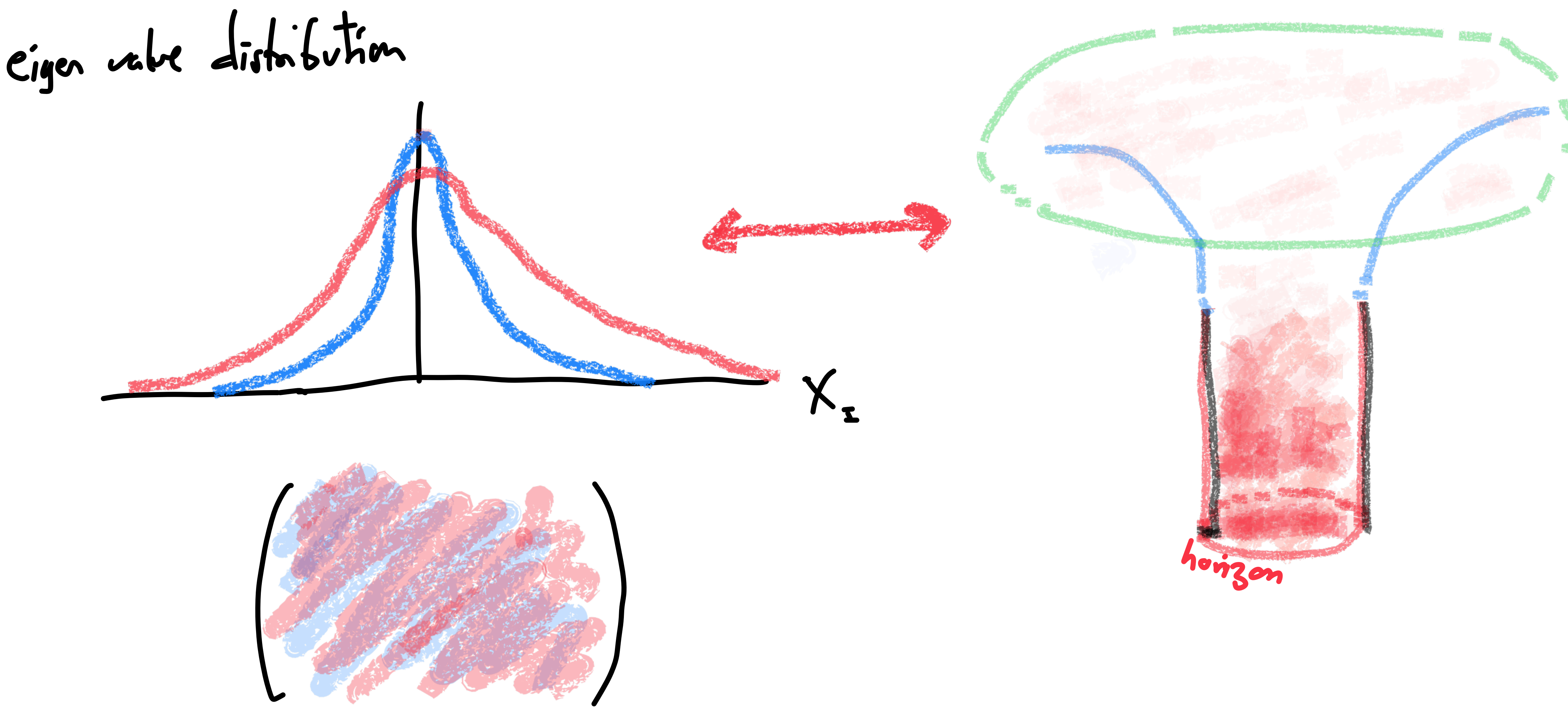
- Taking the large N limit where the brane separations are fixed implies small separations and strong coupling.
- Interpretation: fluctuations of the resulting clump of D-branes are given by the dual gravity



BFSS model

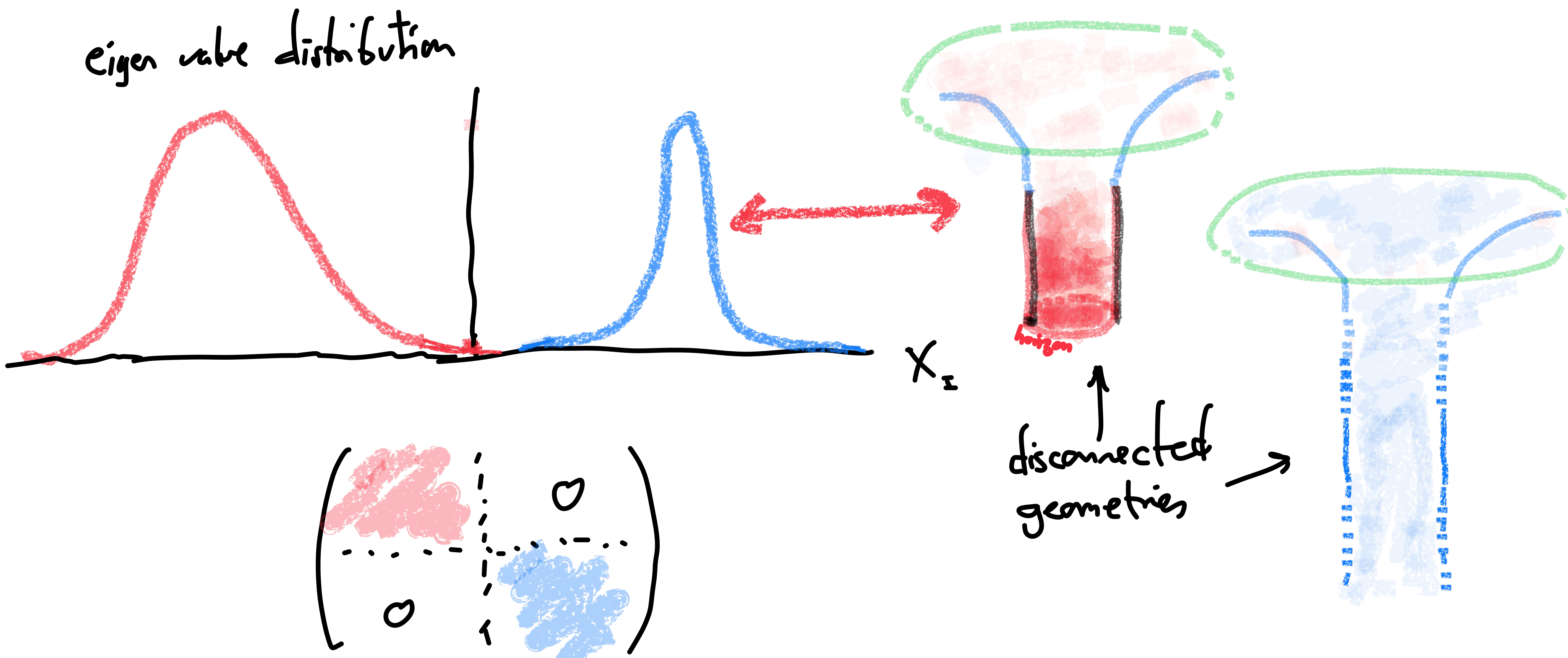
- Turning on finite temperature, the dual geometry will contain a black hole.

- However this clump of branes isn't the black hole — it is the entire dual geometry.



BFSS model

- Turning on finite temperature, the dual geometry will contain a black hole.
- However this clump of branes isn't the black hole — it is the entire dual geometry.



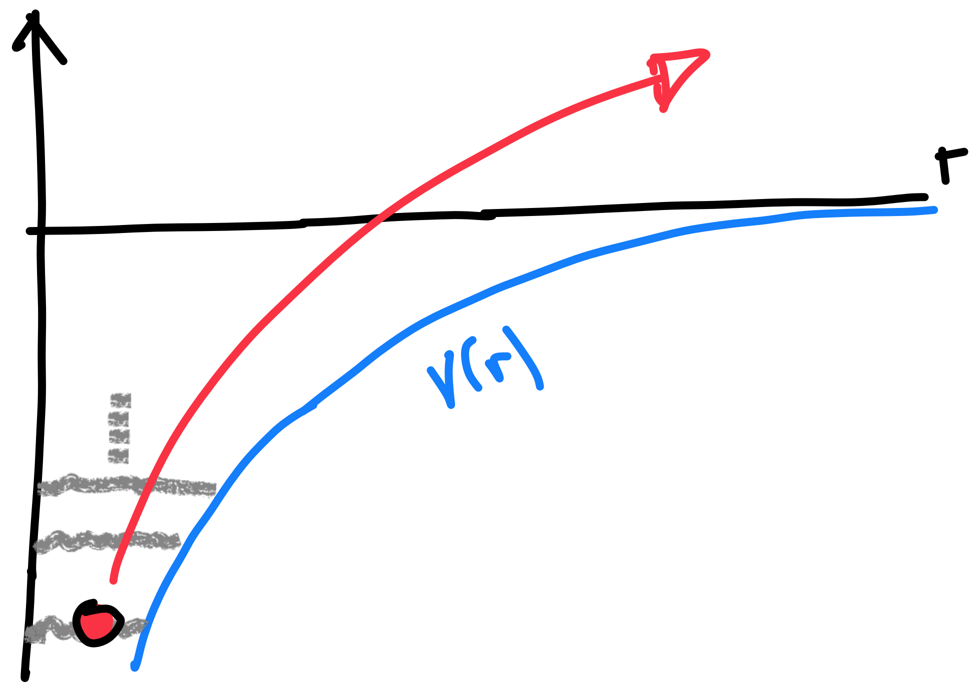
BFSS model

Thermal behaviour

- When the diagonal components of X^I are well separated they behave as free QM particles, and lead to divergence in **thermal** partition function *cf*. H atom

$$X^I = \begin{pmatrix} x_1^I & 0 & 0 & \dots & 0 \\ 0 & x_2^I & 0 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & x_N^I \end{pmatrix} + \delta X^I$$

Catterall, TW '09



- Interpretation: Hawking radiation of D0-branes from decoupling region

- Thermal behaviour is meta-stable at large N

BFSS model

Thermal behaviour

$$S_{BFSS} = \frac{N}{\lambda} \oint^{\beta} d\tau \, \mathrm{Tr} \left[\frac{1}{2} (D_\tau X^I)^2 - \frac{1}{4} \left[X^I, X^J \right]^2 + \Psi \left(\Gamma^\tau D_\tau - \Gamma^I \left[X^I, \cdot \, \right] \right) \Psi \right]$$

Banks, Fischler, Schenker, Susskind '96

- Introduce finite temperature using Euclidean time, $\tau \sim \tau + \beta$
- Consider 't Hooft limit; $N \to \infty$ with $t \sim O(1)$
- High temp/energy - usual QM ~ hot strings $\frac{\epsilon}{N^2} \sim t$

$t >> 1$

$t << 1$

- Low temp/energy - Dual to IIA sugra/M theory $\frac{\epsilon}{N^2} = 7.41 \, t^{14/5}$

BFSS model

Thermal behaviour

$$S_{BFSS} = \frac{N}{\lambda} \oint^{\beta} d\tau \, \mathrm{Tr} \left[\frac{1}{2}(D_\tau X^I)^2 - \frac{1}{4} \left[X^I, X^J\right]^2 + \Psi \left(\Gamma^\tau D_\tau - \Gamma^I \left[X^I, \cdot\,\right] \right) \Psi \right]$$

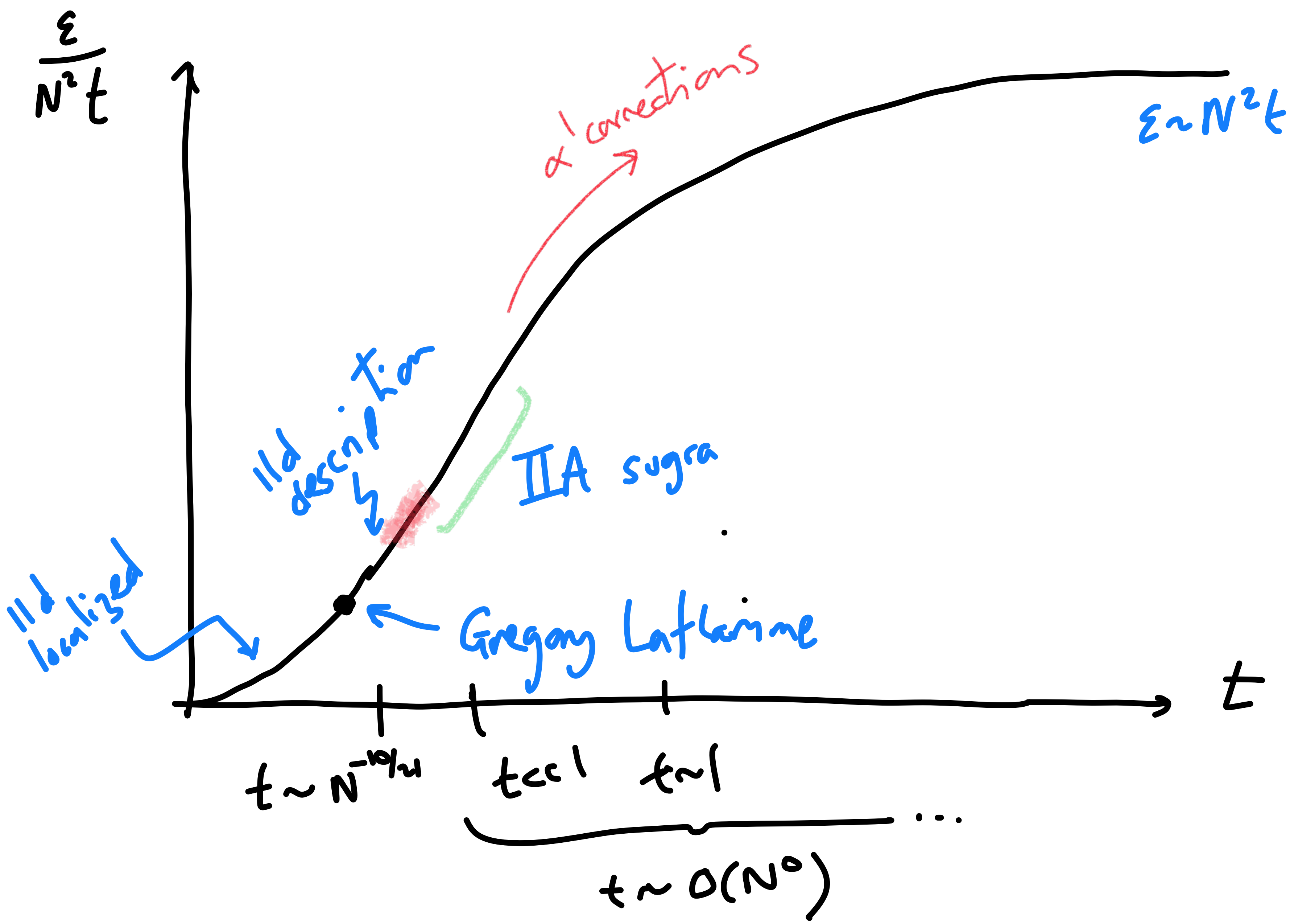
- At very low temperature $t \sim N^{-10/21}$ the IIA dual becomes strongly coupled — the dilation is large near the horizon
- Then one may pass to 11-d sugra to describe the solution — a black string boosted on the 11-circle.
- At a yet *smaller* temperature $t \sim N^{-5/9}$ (when the entropy $S \sim N$) we expect a Gregory-Laflamme instability to a localized black hole, still boosted on the 11-circle

BFSS model

Thermal behaviour

$$S_{BFSS} = \frac{N}{\lambda} \oint^{\beta} d\tau \, \mathrm{Tr} \left[\frac{1}{2} (D_\tau X^I)^2 - \frac{1}{4} \left[X^I, X^J \right]^2 + \Psi \left(\Gamma^\tau D_\tau - \Gamma^I \left[X^I, \cdot \, \right] \right) \Psi \right]$$

- Sketch of expected large N behaviour;

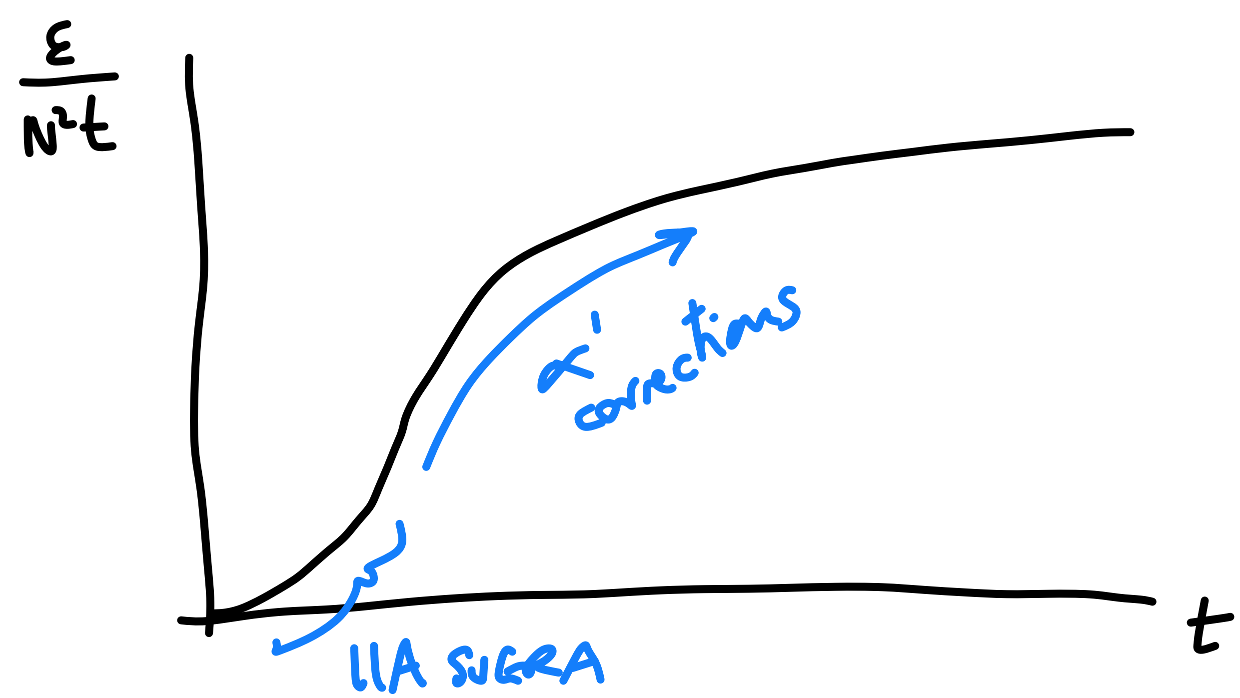


BFSS model

Thermal behaviour

$$S_{BFSS} = \frac{N}{\lambda}\oint^{\beta} d\tau \, \mathrm{Tr}\left[\frac{1}{2}(D_\tau X^I)^2 - \frac{1}{4}\left[X^I, X^J\right]^2 + \Psi\left(\Gamma^\tau D_\tau - \Gamma^I\left[X^I, \cdot\,\right]\right)\Psi\right]$$

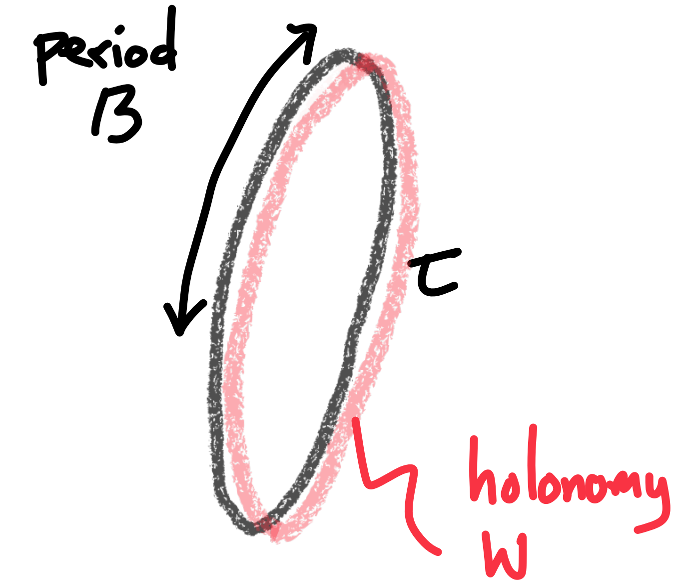
- Sketch of expected large N behaviour;



- Two types of correction;
  - 1/N corrections are `quantum gravity' corrections — BUT the whole low energy curve is quantum gravity
  - Corrections in t are classical α'

BFSS model

Thermal behaviour



$$S_{BFSS} = \frac{N}{\lambda} \oint^{\beta} d\tau \,\mathrm{Tr} \left[\frac{1}{2} (D_\tau X^I)^2 - \frac{1}{4} \left[X^I, X^J \right]^2 + \Psi \left(\Gamma^\tau D_\tau - \Gamma^I \left[X^I, \cdot \, \right] \right) \Psi \right]$$

Banks, Fischler, Schenker, Susskind '96

- Simple observables; energy, Polyakov loop - diagnoses horizon

Witten '98

$$W = P e^{\oint dx A} \quad P = \frac{1}{N} \left\langle \left| \mathrm{Tr}(W) \right| \right\rangle$$

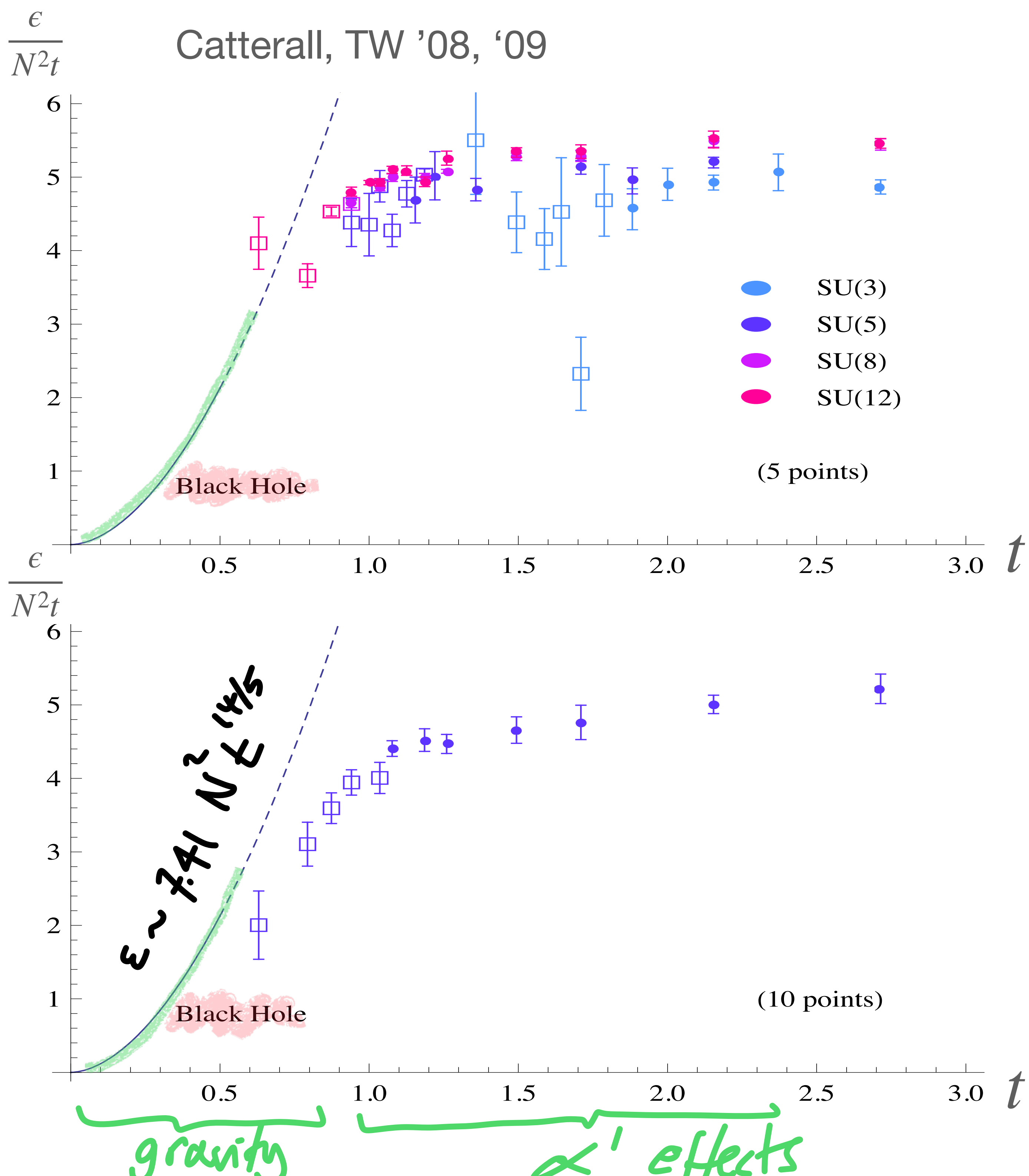
- More subtle ones; Maldacena loop, entanglement

Frenkel, Hartnoll '23

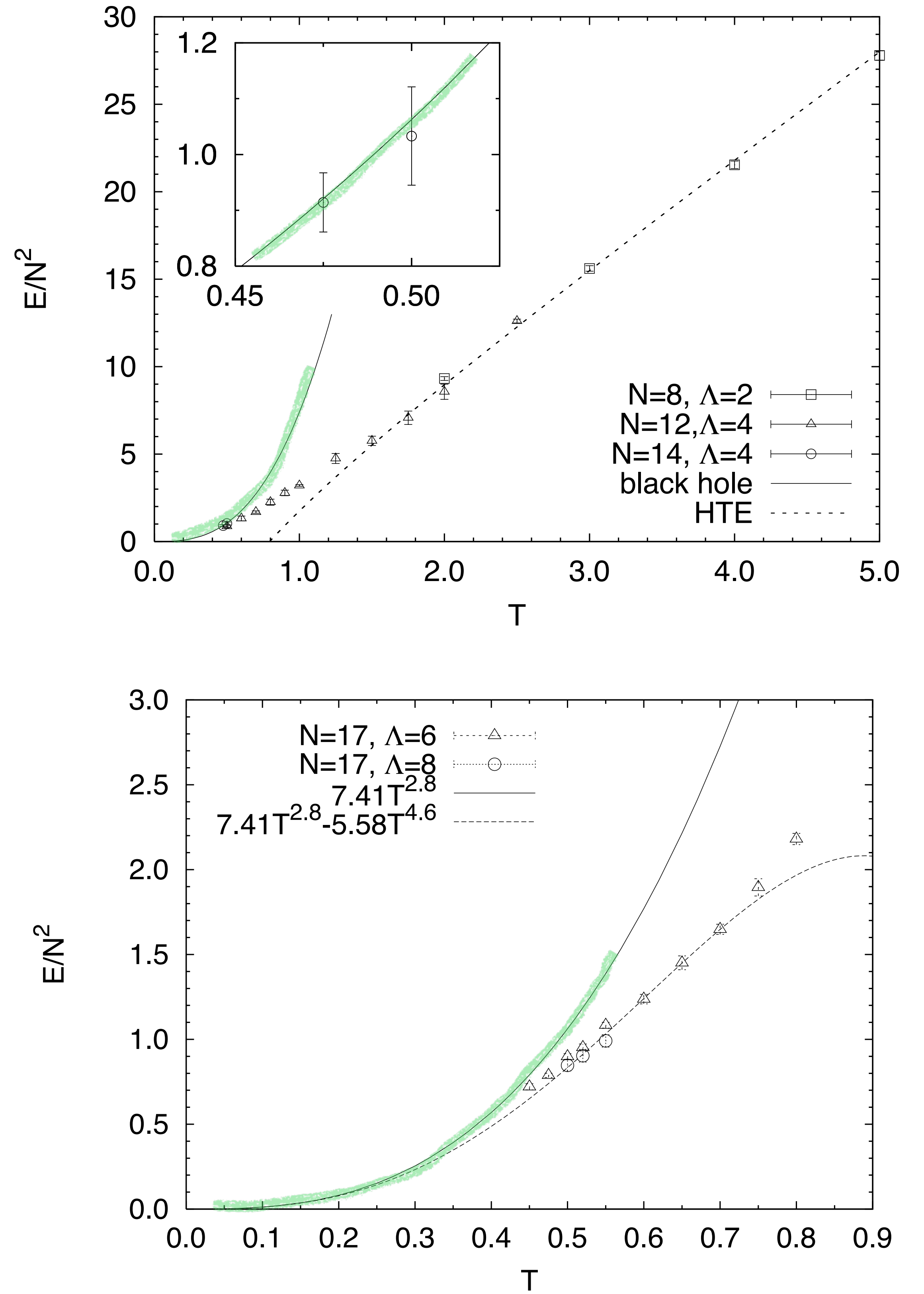
BFSS model

Lattice results

Catterall, TW '08, '09

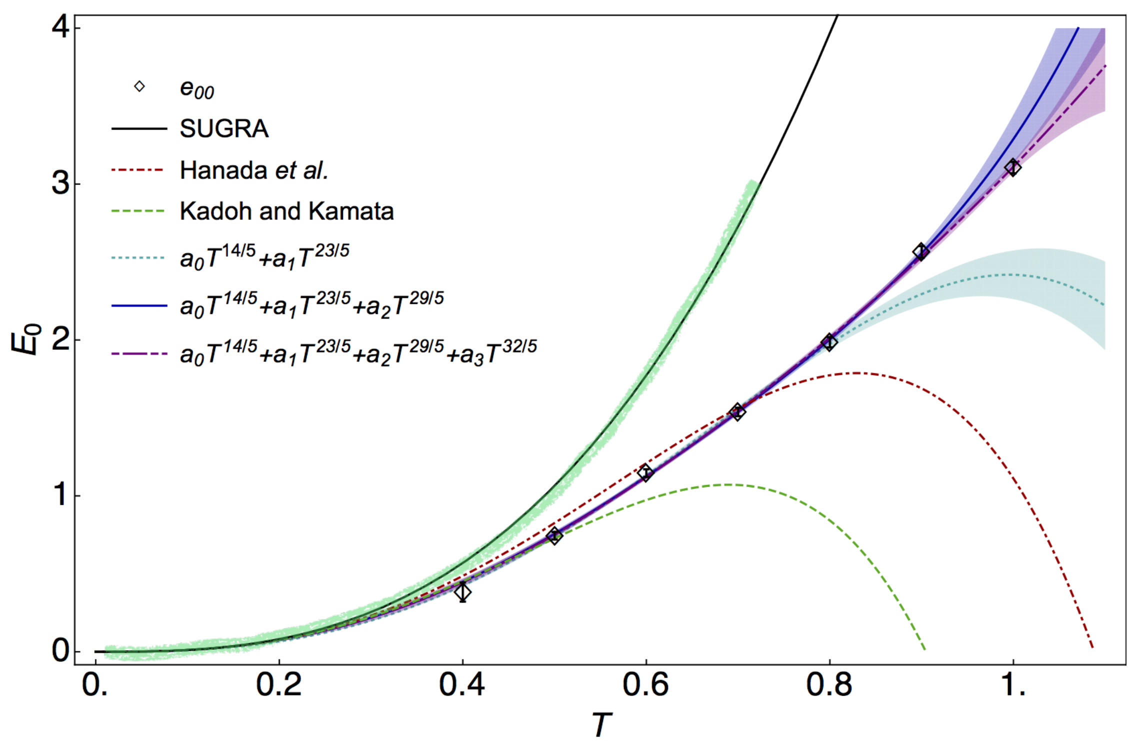


Anagnostopoulos, Hanada, Nishimura, Takeuchi '07

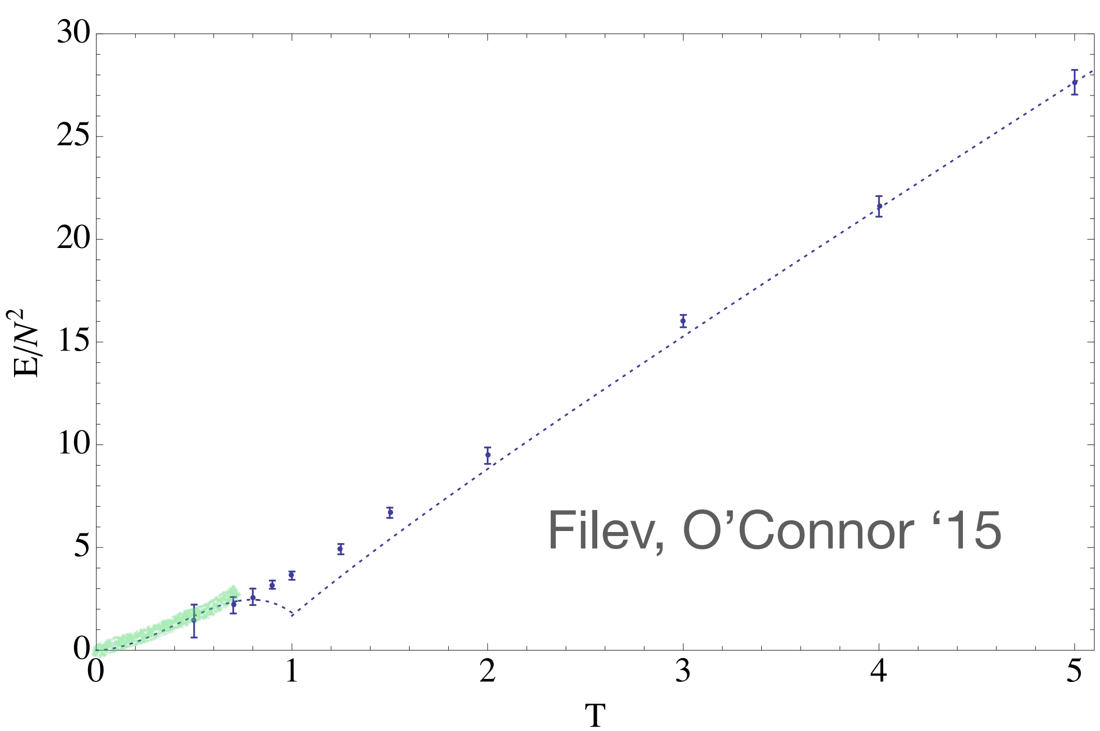


BFSS model

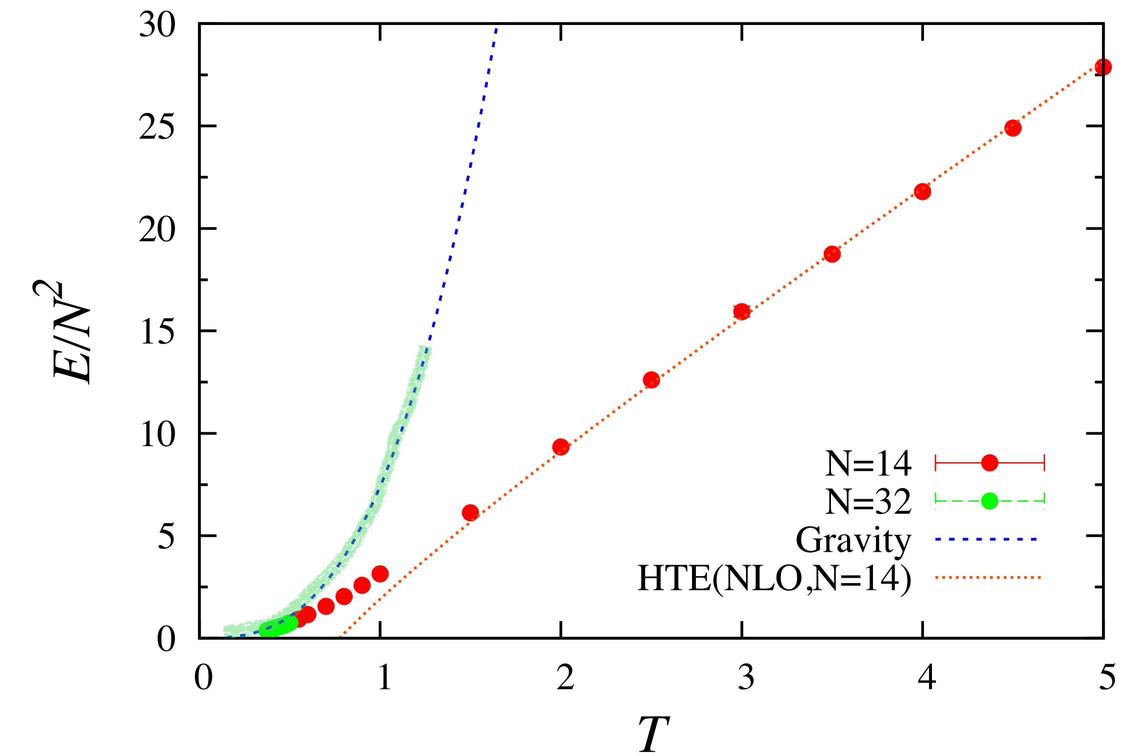
Lattice results

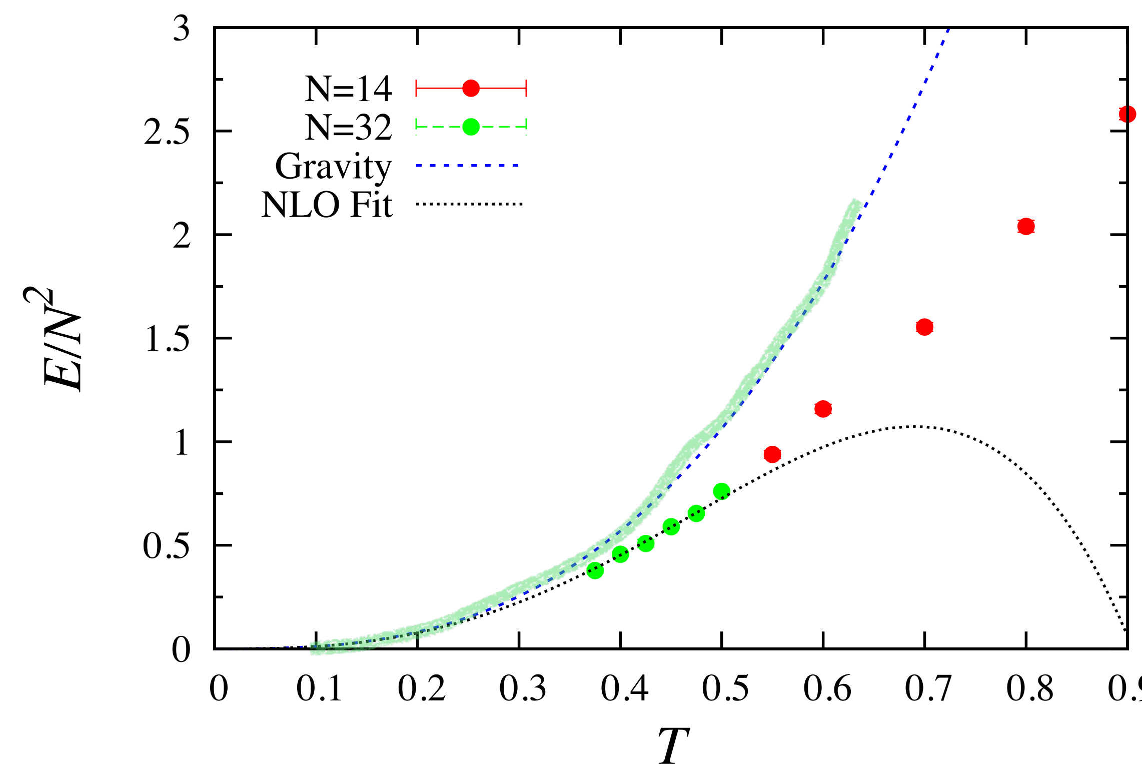


Berkowitz, Rinaldi, Hanada, Ishiki, Shimasaki, Vranas ‘16



Filev, O’Connor ‘15



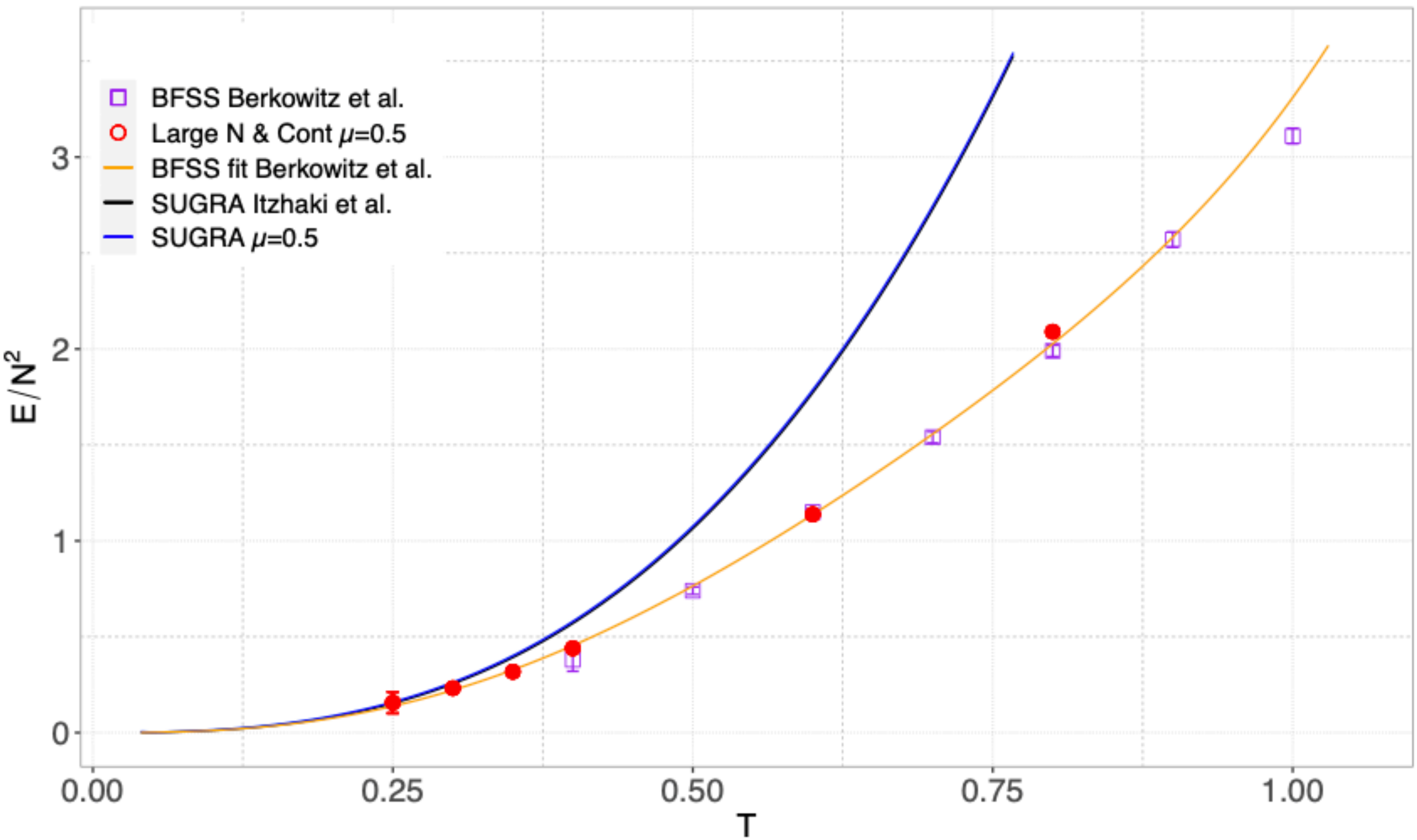


Kadoh, Kamata ‘15

BFSS model

Lattice results

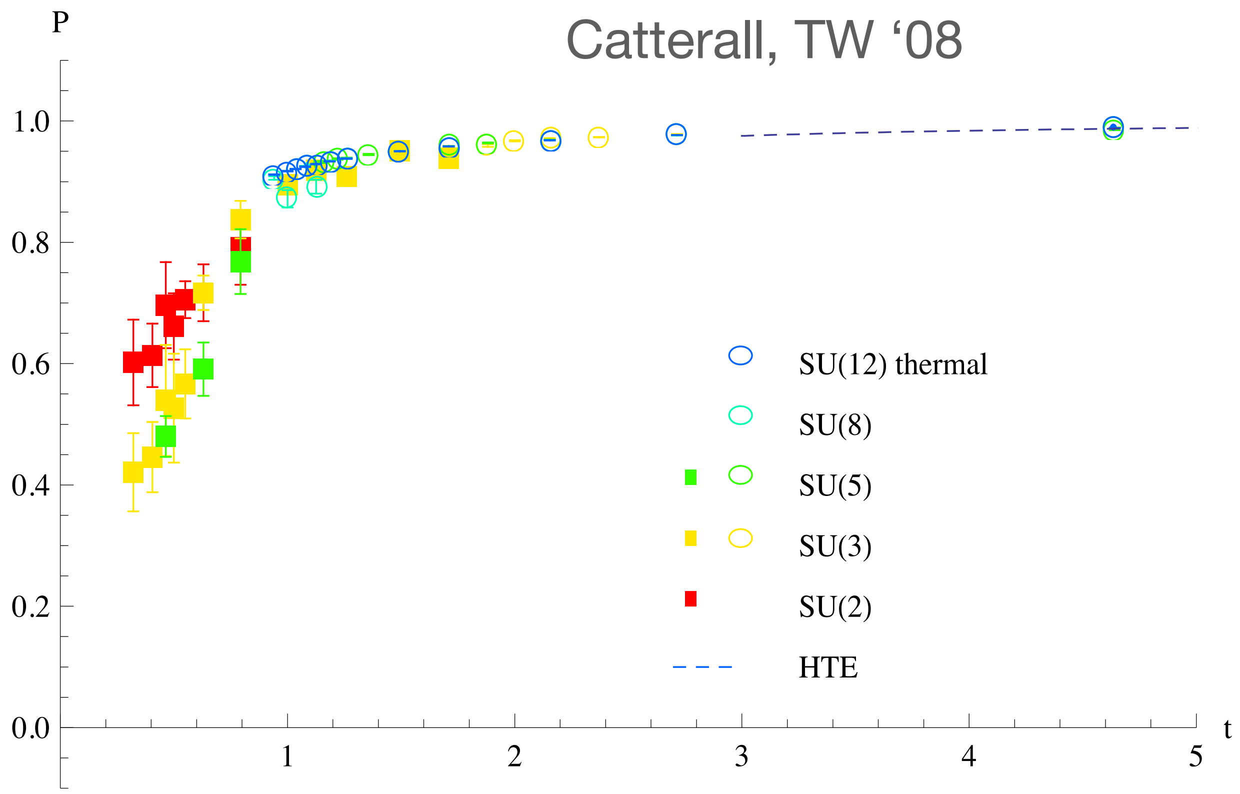
MCSMC: Pateloudis, Bergner, Hanada, Rinaldi, Schafer, Vranas, Watanabe, Bodendorfer '23



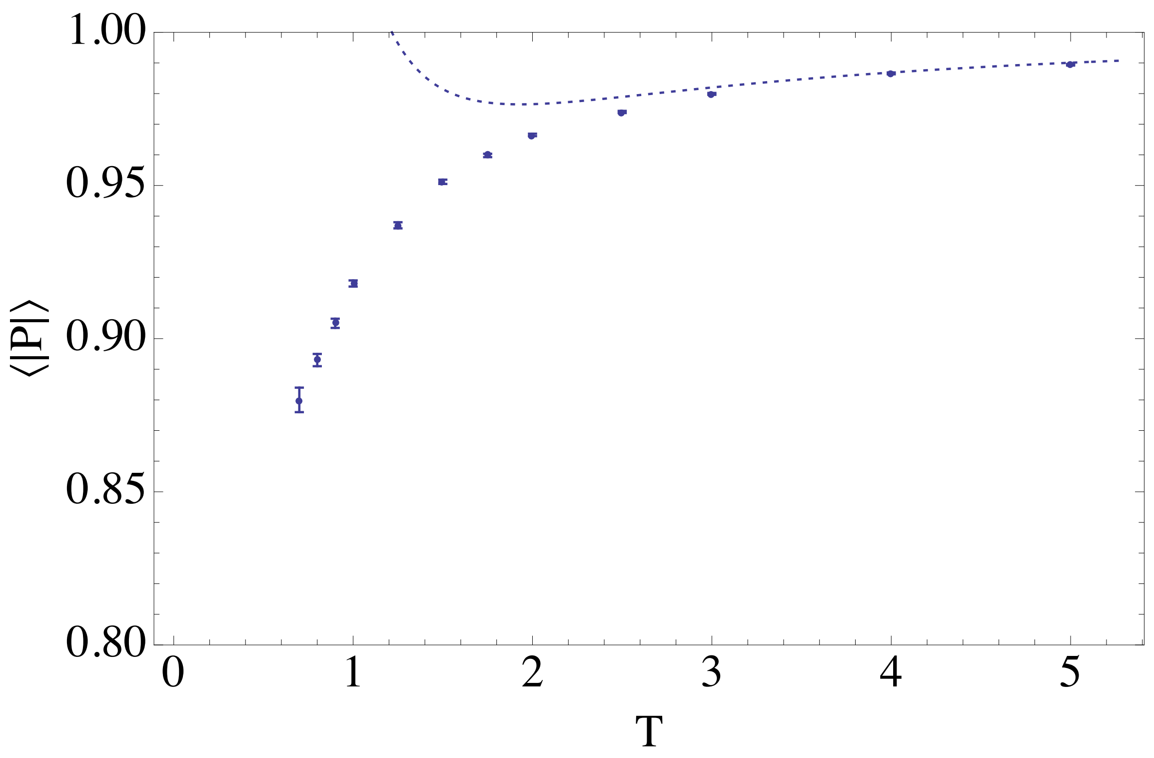
BFSS model

Lattice results

Catterall, TW ‘08



Filev, O’Connor ‘15



BFSS model

Analytic directions

- Gaussian approximation and gap equations

Kabat, Lifschytz ‘99

Kabat, Lifschytz and Lowe ‘01

Lin, Shan, Wang, Yin ‘13

- Exciting progress QM bootstrap

Lin ’20

Han, Hartnoll, Krustoff ’20

Berenstein, Hulsey ‘21

….

- Scaling symmetry

TW ’13

Biggs, Maldacena ‘23

BFSS model

- Can we understand the temperature behaviour?

$$\epsilon = 2^{\frac{27-5p}{5-p}} (9-p) (7-p)^{-\frac{19-3p}{5-p}} N^2 \left(\pi^{\frac{13-3p}{2}} \Gamma\left(\frac{7-p}{2}\right) t^{7-p} \right)^{\frac{2}{5-p}}$$

N dependence

transcendental part

temperature

BFSS model

Smilga ’07; TW ’13

Morita, Shiba, TW, Withers ’13 and ‘14

Biggs, Maldacena ‘23

- Can we understand the temperature behaviour?

$$\epsilon = 2^{\frac{27-5p}{5-p}} (9-p)(7-p)^{-\frac{19-3p}{5-p}} N^2 \left(\pi^{\frac{13-3p}{2}} \Gamma\left(\frac{7-p}{2}\right) t^{7-p} \right)^{\frac{2}{5-p}}$$

- Recall the moduli space theory;

$$S \sim \int d\tau dx^p \left(L_{classical} + L_{1-loop} + \dots \right)$$

$$L_{classical} = \frac{N}{\lambda} \sum_{a=1}^{N} \left(\partial \vec{X}_a \right)^2 \qquad L_{1-loop} \sim \sum_{a<b} \frac{\left(\partial_\mu \vec{X}_{ab} \right)^4}{|\vec{X}_{ab}|^{7-p}}$$

$$L_{n-loop} \sim \sum_{a_1<a_2<\ldots<a_n} \frac{\left(\partial \vec{X} \right)^{2+2n}}{|\vec{X}|^{(7-p)n}}$$

Morita, Shiba, TW, Withers ’13
Biggs, Maldacena ‘23

- Then there is a scaling symmetry;

$$\tau \to \Lambda^{-1}\tau \qquad \vec{x} \to \Lambda^{-1}\vec{x} \qquad \vec{X}_a \to \Lambda^{\frac{2}{5-p}} \vec{X}_a \qquad S \to \Lambda^{\frac{(3-p)^2}{5-p}} S$$

- This constrains energy density to scale as; $\epsilon \sim t^{\frac{2(7-p)}{5-p}}$

BFSS model

- While the SYM doesn't have conformal symmetry for $p \neq 3$ leading to AdS in the dual, there is a scaling symmetry in the dual gravity

- Recall the moduli space theory; $S \sim \int d\tau dx^p \left(L_{classical} + L_{1-loop} + \dots \right)$

$$L_{classical} = \frac{N}{\lambda} \sum_{a=1}^{N} \left(\partial \overrightarrow{X}_a \right)^2$$

$$L_{1-loop} = - \sum_{a<b} \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}} \left(2 \frac{\left(\partial_\mu \overrightarrow{X}_{ab} \cdot \partial_\nu \overrightarrow{X}_{ab}\right)^2}{|\overrightarrow{X}_{ab}|^{7-p}} - \frac{\left(\partial_\mu \overrightarrow{X}_{ab} \cdot \partial^\mu \overrightarrow{X}_{ab}\right)^2}{|\overrightarrow{X}_{ab}|^{7-p}} \right)$$

- When does this become strong coupled? When,

$$\langle L_{classical} \rangle \sim \langle L_{1-loop} \rangle$$

BFSS model

- Estimate thermal vevs as;

$$\vec{X}_a \sim \vec{X}_{ab} \sim \chi \qquad \partial_\mu \sim \pi T$$

$$\sum_a \sim N \qquad \sum_{a<b} \sim N^2$$

- Proceed *keeping transcendental numbers!*

$$\langle L_{classical} \rangle = \langle \frac{N}{\lambda} \sum_{a=1}^{N} \left(\partial \vec{X}_a \right)^2 \rangle \sim \frac{N^2}{\lambda} \pi^2 T^2 \chi^2$$

$$\langle L_{1-loop} \rangle = \langle -\sum_{a<b} \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}} \left(2 \frac{\left(\partial_\mu \vec{X}_{ab} \cdot \partial_\nu \vec{X}_{ab}\right)^2}{|\vec{X}_{ab}|^{7-p}} - \frac{\left(\partial_\mu \vec{X}_{ab} \cdot \partial^\mu \vec{X}_{ab}\right)^2}{|\vec{X}_{ab}|^{7-p}} \right) \rangle \sim N^2 \frac{\Gamma\left(\frac{7-p}{2}\right)}{(\pi)^{\frac{1+p}{2}}} \frac{\pi^4 T^4 \chi^4}{\chi^{7-p}}$$

- So strong coupling implies;

$$\frac{N^2}{\lambda} \pi^2 T^2 \chi^2 \sim N^2 \frac{\Gamma\left(\frac{7-p}{2}\right)}{(4\pi)^{\frac{1+p}{2}}} \pi^4 T^4 \chi^{p-3} \qquad \chi = \left(\Gamma\left(\frac{7-p}{2}\right) \pi^{\frac{3-p}{2}} \lambda T^2 \right)^{\frac{1}{5-p}}$$

BFSS model

- Estimate energy density as;

$$\rho \sim \langle L_{classical} \rangle = \langle \frac{N}{\lambda} \sum_{a=1}^{N} \left(\partial \overrightarrow{X}_a \right)^2 \rangle \sim \frac{N^2}{\lambda} \pi^2 T^2 \chi^2 \sim \frac{N^2}{\lambda} \pi^2 T^2 \left(\Gamma \left(\frac{7-p}{2} \right) \pi^{\frac{3-p}{2}} \lambda T^2 \right)^{\frac{2}{5-p}}$$

- So,

$$t = T \lambda^{-\frac{1}{3-p}}$$

$$\epsilon = \rho \lambda^{-\frac{1+p}{3-p}} \sim N^2 \pi^2 \left(\Gamma \left(\frac{7-p}{2} \right) \pi^{\frac{3-p}{2}} t^{7-p} \right)^{\frac{2}{5-p}}$$

correct N and transcendental dependence!

- Compare to earlier;

$$\epsilon = 2^{\frac{27-5p}{5-p}} (9-p) (7-p)^{-\frac{19-3p}{5-p}} N^2 \left(\pi^{\frac{13-3p}{2}} \Gamma \left(\frac{7-p}{2} \right) t^{7-p} \right)^{\frac{2}{5-p}}$$

Maldacena, Milekhin '18

To gauge or not to gauge…

To gauge or not to gauge…

- Do we have to gauge? We previously gauge fixed to write;

$$S_{BFSS} = \frac{N}{\lambda}\int dt\,\mathrm{Tr}\left[\frac{1}{2}(\dot{X}^I)^2 - \frac{1}{4}\left[X^I, X^J\right]^2 + \Psi\left(i\dot{\Psi} - \Gamma^I\left[X^I, \Psi\right]\right)\right]$$

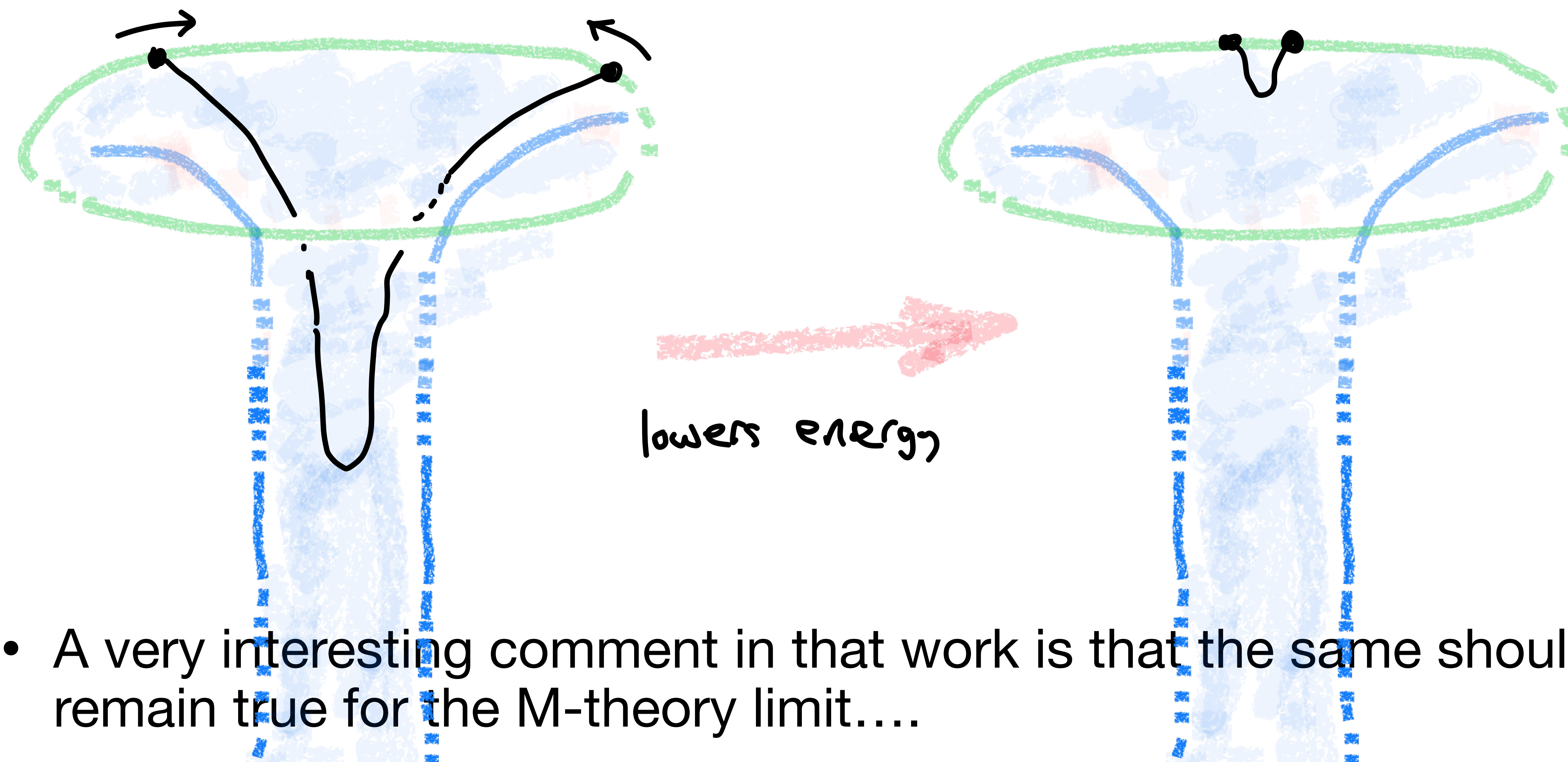
- Recall there is the singlet constraint
- Maldacena & Milekhin '18 argue that after dropping the singlet constraint, the gravity dual is still valid — the non-singlet sector states all live at high energy.
- Non-singlet states can be thought of as adding Wilson line insertion, $Tr_R P e^{i\int A_t dt}$ — gravity dual is a string hanging from the 'boundary'
- Note, this is not a susy `Maldacena' loop

To gauge or not to gauge…

- Gravity dual is a string with *Neumann* boundary conditions

Alday, Maldacena '07

- Then it is intuitive that the lowest energy state will be where the string end points come together; argue that $E \sim \lambda^{1/3}$

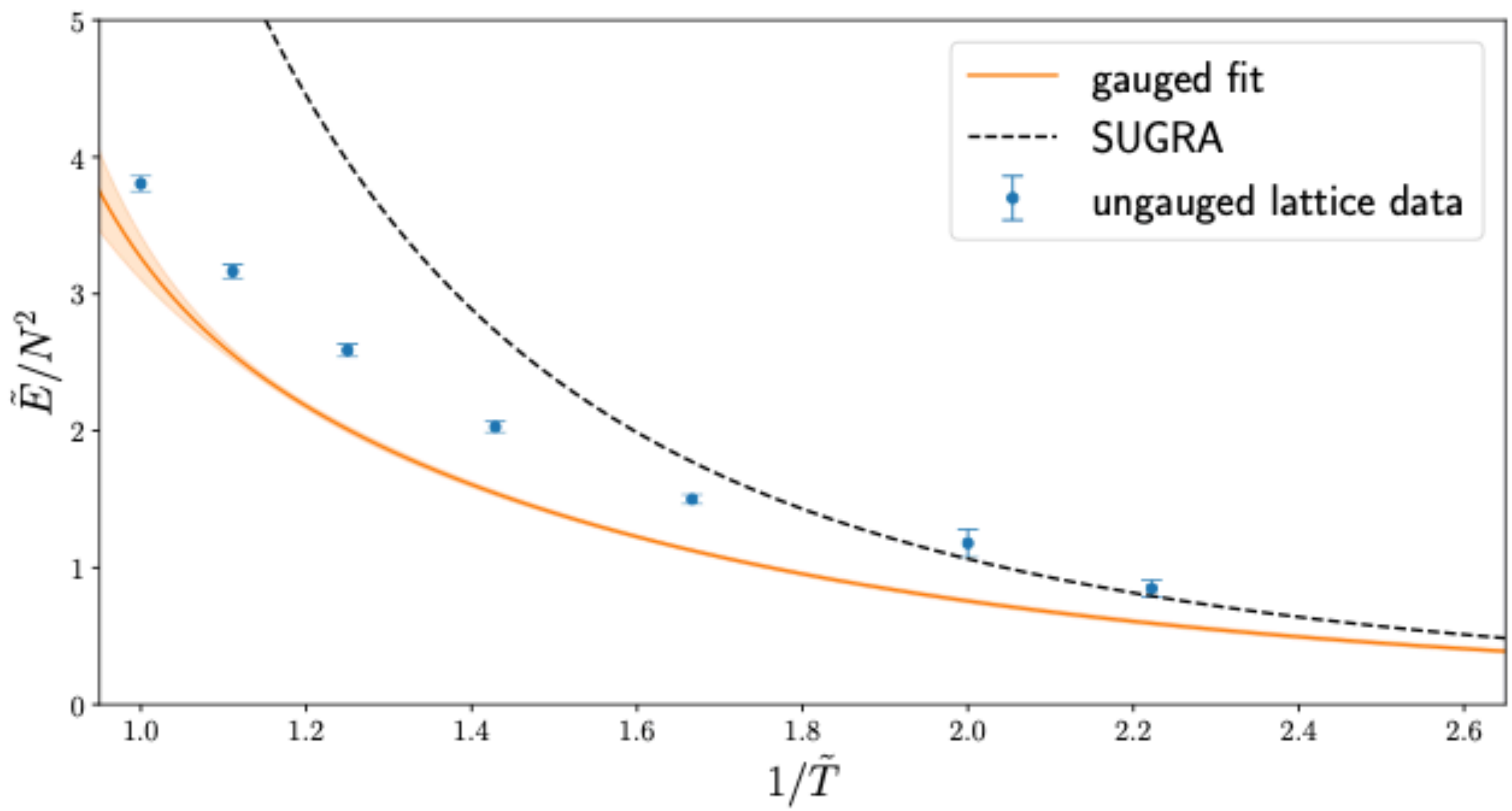


- A very interesting comment in that work is that the same should remain true for the M-theory limit….

To gauge or not to gauge…

Lattice results

- Do we have to gauge?



Berkowitz, Hanada, Rinaldi, Vranas ‘18

Patekoudis, Bergner, Bodendorfer, Hanada, Rinaldi, Schafer ‘22

BMN deformation

Berenstein, Maldacena, Nastase '02

- In Catterall & TW '09 it was argued that an IR regulator mass should be added to control the divergence in the partition function.

- Further it was argued that the natural regulator is the BMN model, since this still has a gravity dual

$$S_{BFSS} = \frac{N}{\lambda} \int dt \, \mathrm{Tr} \left[\frac{1}{2} (\dot{X}^I)^2 - \frac{1}{4} \left[X^I, X^J \right]^2 + \Psi \left(i \dot{\Psi} - \Gamma^I \left[X^I, \Psi \right] \right) \right]$$

$$\Delta S_{BMN} = \frac{N}{\lambda} \int dt \, \mathrm{Tr} \left[\frac{\mu^2}{2} \sum_{I=1,2,3} (\dot{X}^I)^2 + \frac{\mu^2}{8} \sum_{I=4\ldots9} (\dot{X}^I)^2 + i\mu \sum_{I,J,K=1,2,3} \epsilon^{IJK} X_I X_J X_K \right]$$

- There is a mass deformation preserving gravity dual and max susy that breaks the flavour $SO(9) \rightarrow SO(3) \times SO(6)$

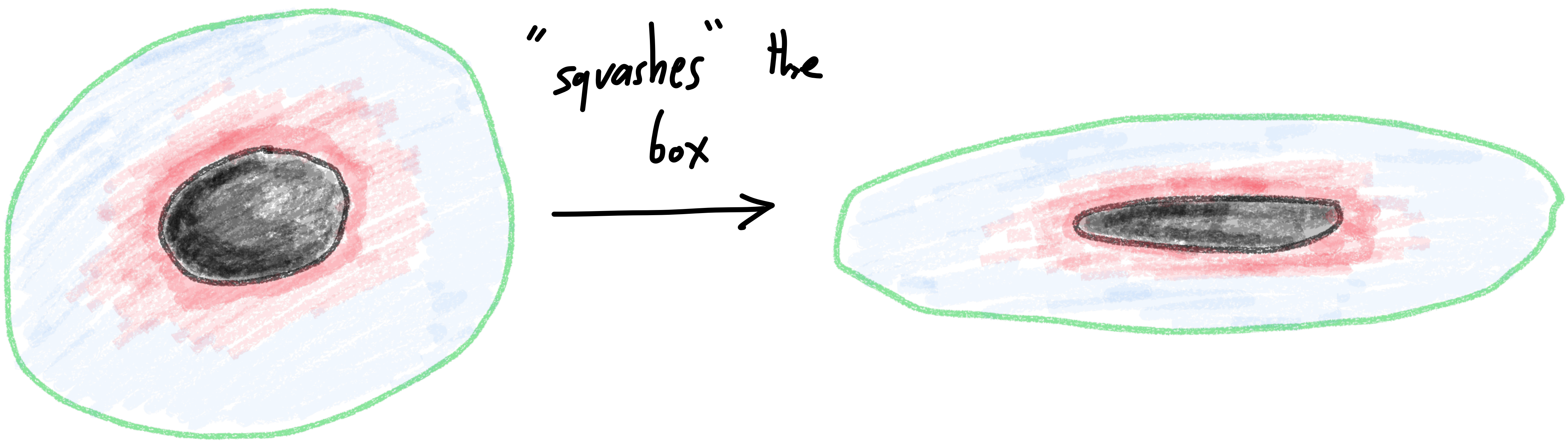
- Now also discrete 'fuzzy sphere' vacua at finite mass μ

- However gravity only emerges in $t, \mu \ll 1$ limit

BMN deformation

Berenstein, Maldacena, Nastase ‘02

- Now gravity computation is **much** more subtle



- Vacuum geometry is known

Headrick, Kitchen, TW ‘09

- Required numerical GR methods to find black hole solutions in technical `Tour de Force’ by Costa, Greenspan, Penedones, Santos ’14

- Conjecture confining phase transition — dual to gravity gas

BMN deformation

Lattice results

Catterall, Van Anders ‘10

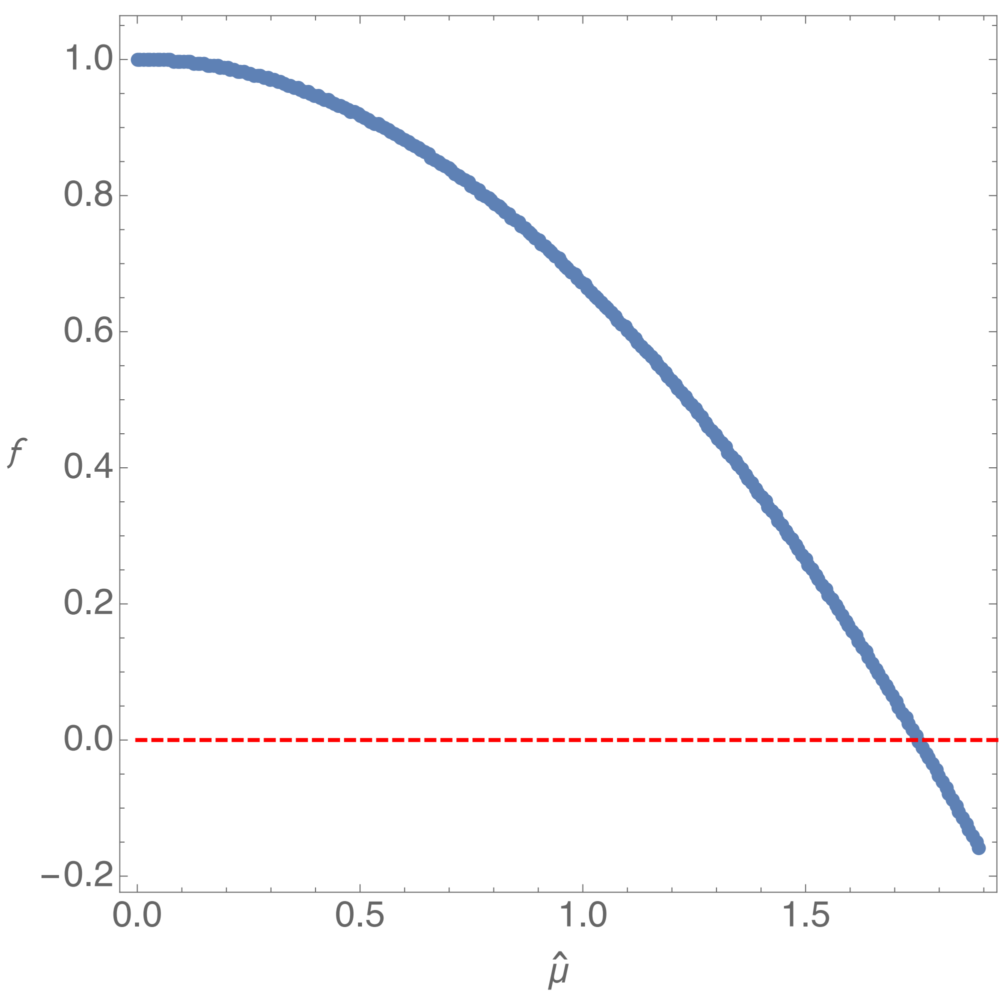
Asano, Filev, Kovacik, O’Connor ‘18

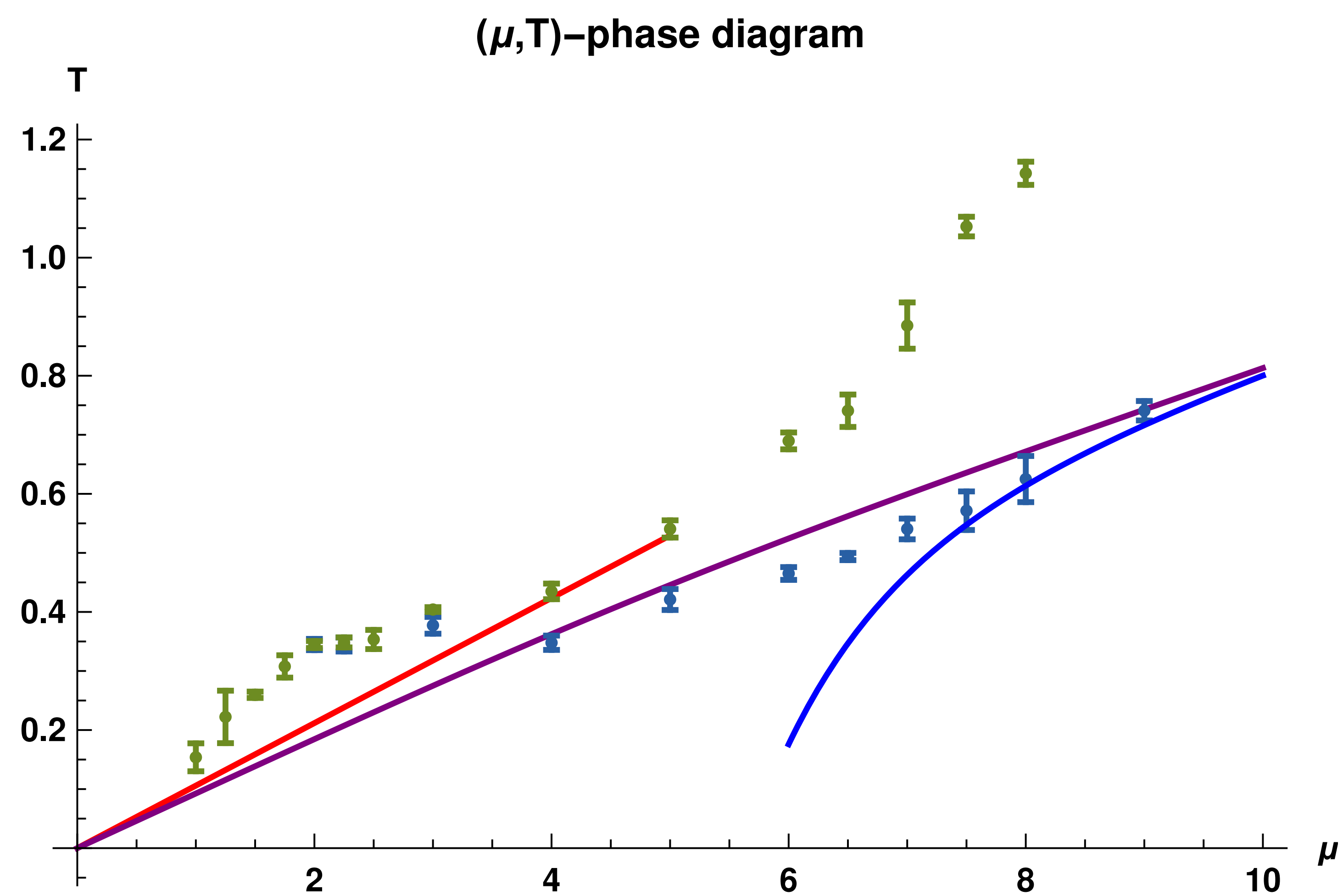
Schaich, Jha, Joseph ‘20

Bergner, Hanada et al ‘22

Costa, Greenspan, Penedones, Santos ‘14

$$\frac{T_c}{\mu} = \frac{7}{12\pi\widehat{\mu}_c} = 0.105905(57)\,.$$





BMN deformation

Lattice results

Catterall, Van Anders '10

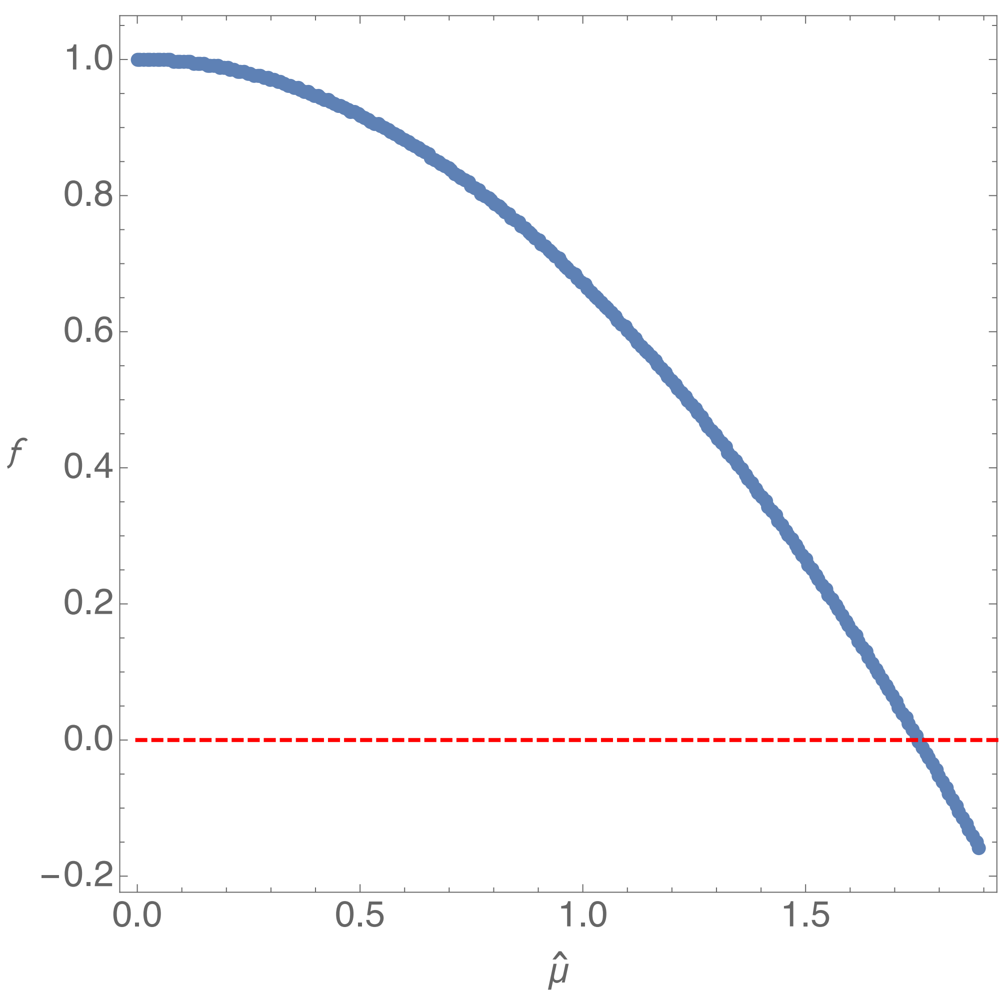
Asano, Filev, Kovacik, O'Connor '18

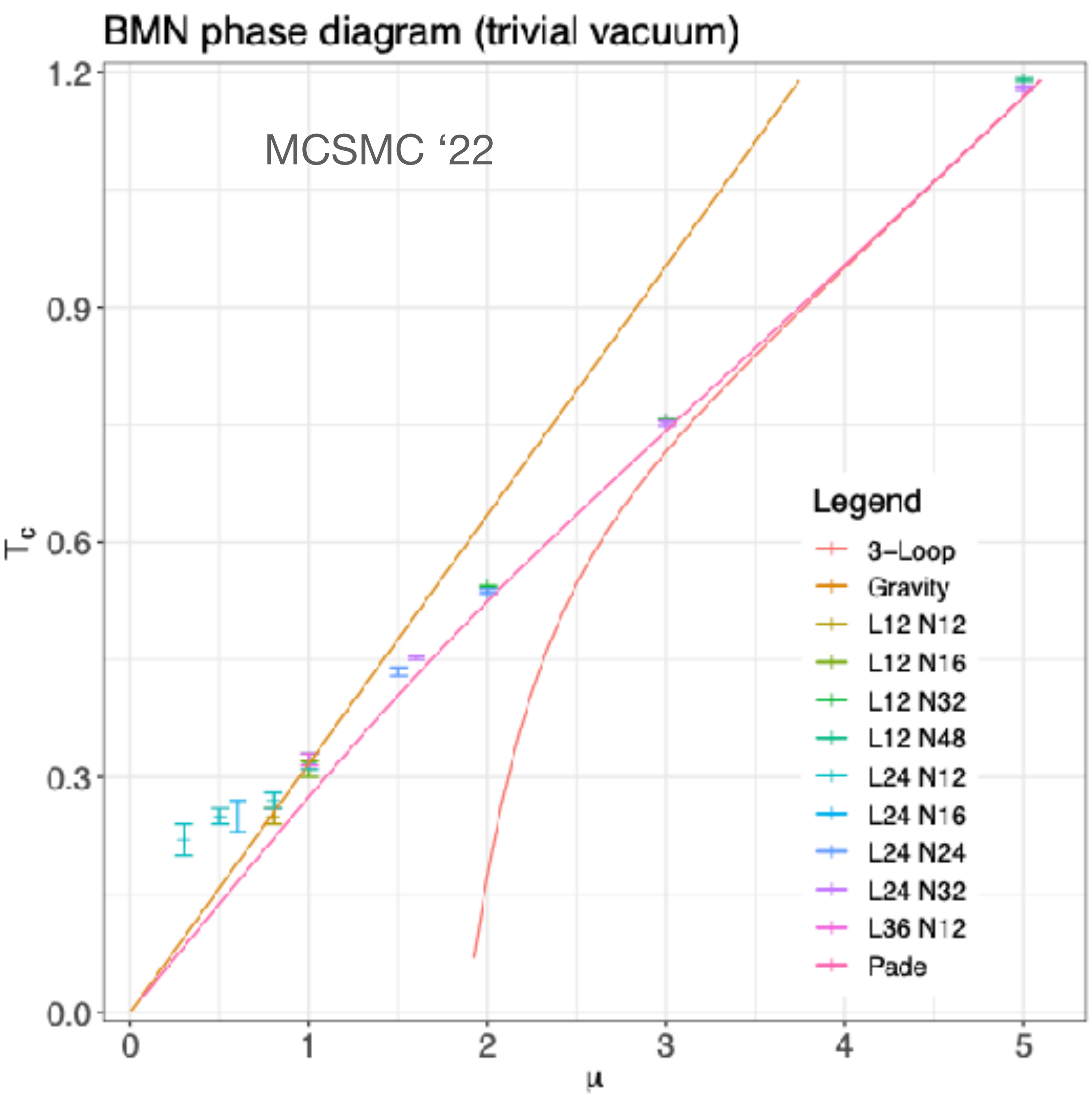
Schaich, Jha, Joseph '20

Bergner, Hanada et al '22

Costa, Greenspan, Penedones, Santos '14

$$\frac{T_c}{\mu} = \frac{7}{12\pi\widehat{\mu}_c} = 0.105905(57)\,.$$





Summary

- Significant progress in BFSS matrix theory at 'large N'.
- Where to go now?
  - Improve further off lattice simulations
  - New analytic approaches
  - New observables - entanglement entropy?
  - Develop better understanding for why gravity behaviour seen
- Dynamics and quantum computing? Maldacena '23

  Rinaldi, Han, Hassan, Feng, Nori, McGuigan, Hanada '21

- M(atrix) theory limit and finite N?

  Miller, Strominger, Tropper, Wang '23

  Tropper, Wang '23

  Herderschee, Maldacena '23

The End

Thanks for listening!