Matrix Geometry Revisited

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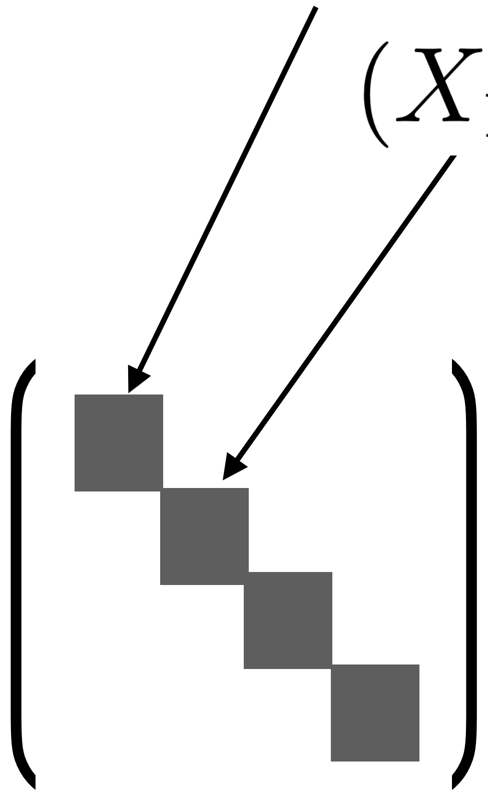
- How is the geometry encoded into matrices?
- Is matrix geometry really "noncommutative"?
- First of all, what is "matrix"?

MH, 2102.08982[hep-th] (PRD)

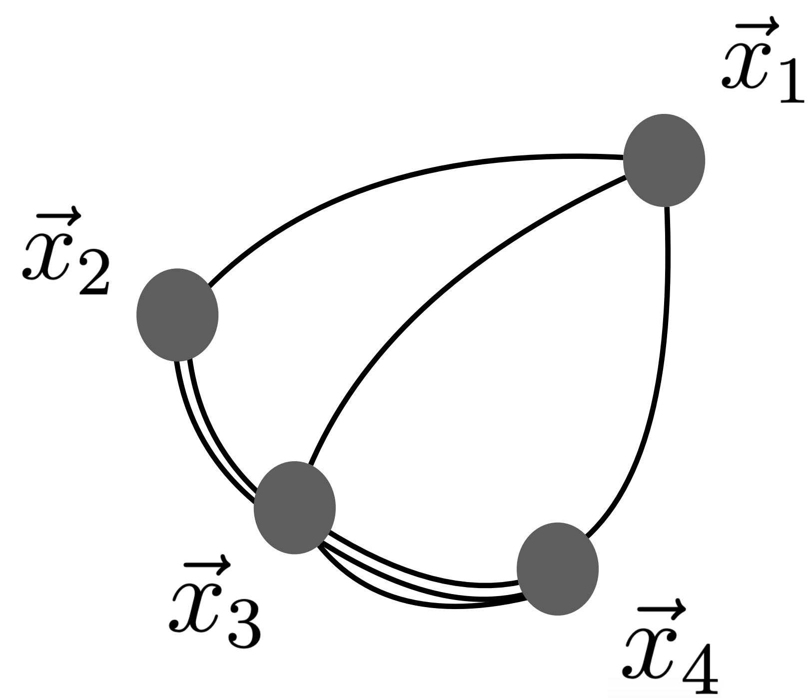
Gautam, MH, Jevicki, Peng, 2204.06472[hep-th] (JHEP)

$$(X_{1,11},\cdots,X_{9-p,11}) \equiv \vec{x}_1 \in \mathbb{R}^{9-p}$$

$$(X_{1,22},\cdots,X_{9-p,22}) \equiv \vec{x}_2 \in \mathbb{R}^{9-p}$$

$X_I =$ 

diagonal entries = location of i-th D-brane



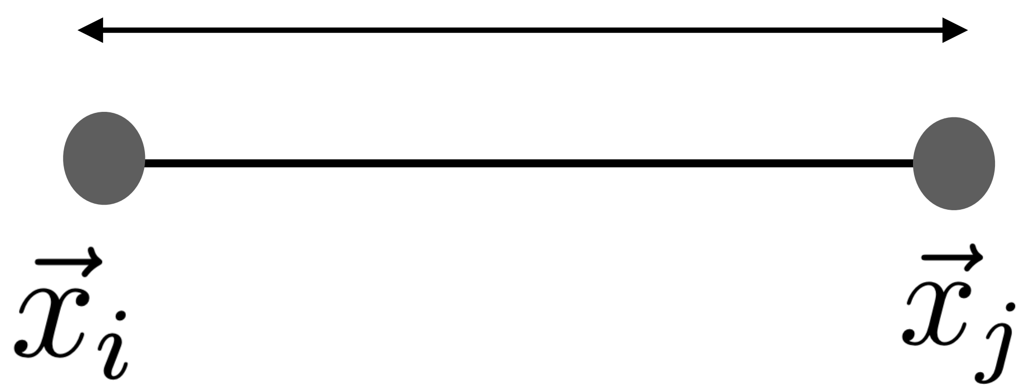
(i,j)-component
= string between
i-th & j-th D-branes

(Witten, 1995; **as low-energy effective description**)

Toby's talk this morning

String Theory

$$\text{mass} = \text{length} = |\vec{x}_i - \vec{x}_j|$$



Yang-Mills Theory

Potential term

$$\begin{aligned} V &= -\frac{g^2}{4}\mathrm{Tr}[X_I, X_J]^2 \\ &= g^2 \sum_{I \neq J} \sum_{i<j} (x_{I,i} - x_{I,j})^2 |X_{J,ij}|^2 + \cdots \end{aligned}$$

Symmetry enhancement

- $\vec{x}_1 = \cdots = \vec{x}_N$ & off-diag = 0 $\rightarrow \mathrm{U}(N)$-invariant

(All strings are massless)

- $\vec{x}_1 = \cdots = \vec{x}_{N_1},\ \vec{x}_{N_1+1} = \cdots = \vec{x}_{N_1+N_2}, \cdots$

& off-diag = 0

$\rightarrow \mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \cdots$-invariant

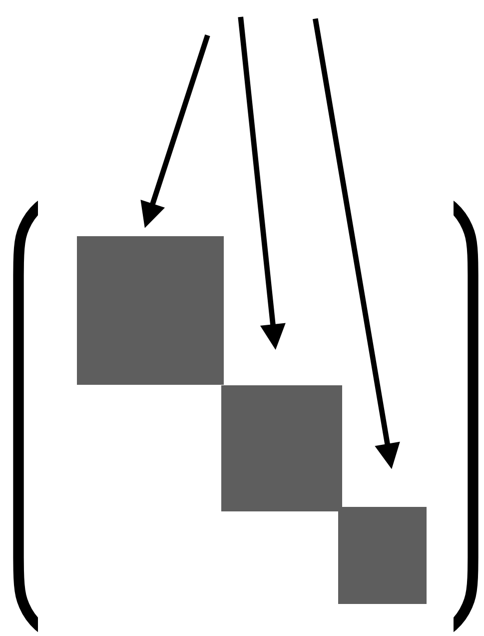
(Strings connecting D-branes in the same bunch are massless)

$$\vec{x}_1 = \cdots = \vec{x}_{N_1},\ \vec{x}_{N_1+1} = \cdots = \vec{x}_{N_1+N_2}, \ldots$$

+ off-diag excitations in each block

Non-commutative sub-matrices = various objects e.g., BH

$$X_I =$$



Large-N = (almost) 2nd Quantization

Banks, Fischler, Shenker, Susskind 1996
(in the Matrix Theory proposal)

Toby's talk this morning

A natural hope

"Diagonal elements = location of D-branes"-picture
can be used for Maldacena-type gauge/gravity duality.

Matrix Model $X_1, \cdots, X_9 \to \mathbb{R}^9$

4d Super Yang-Mills $X_1, \cdots, X_6 \to \mathbb{R}^6 = \mathbb{R}_{>0} \times S^5$

$\mathbb{R}^{1,3} \longrightarrow \mathrm{AdS}_5$

- Polchinski (1998) and Susskind (1999) pointed out a subtlety.

Toby's talk this morning

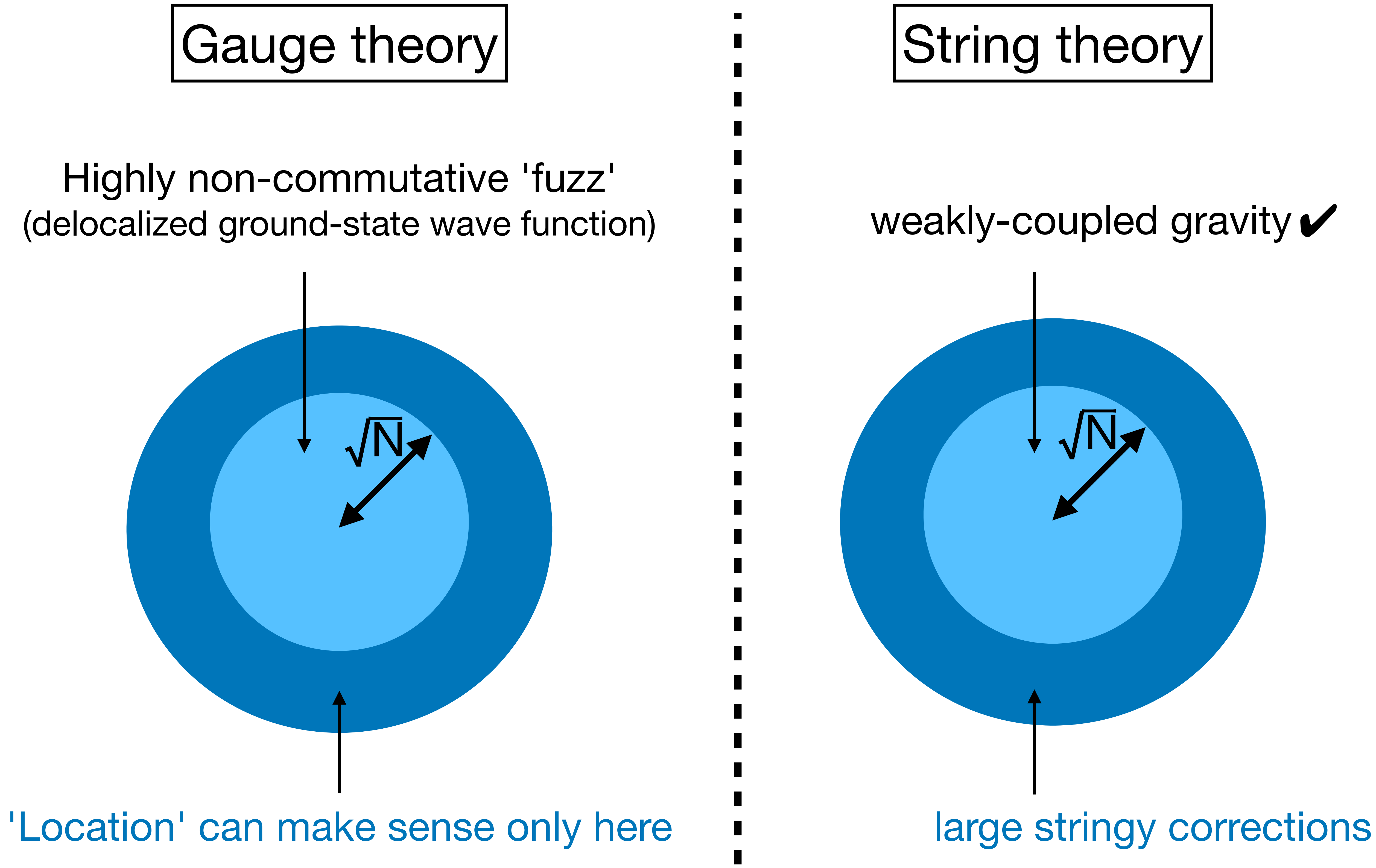
- There is a simple resolution. (MH, 2021)

A subtlety

(Polchinski, hep-th/9903165)

Toby's talk this morning

[This is for p<3. Similar argument applies to p=3.]



(Polchinski, hep-th/9903165)

$$\langle \mathrm{Tr} X_I^2 \rangle \sim N^2 \text{ } (\leftarrow \text{'t Hooft counting})$$

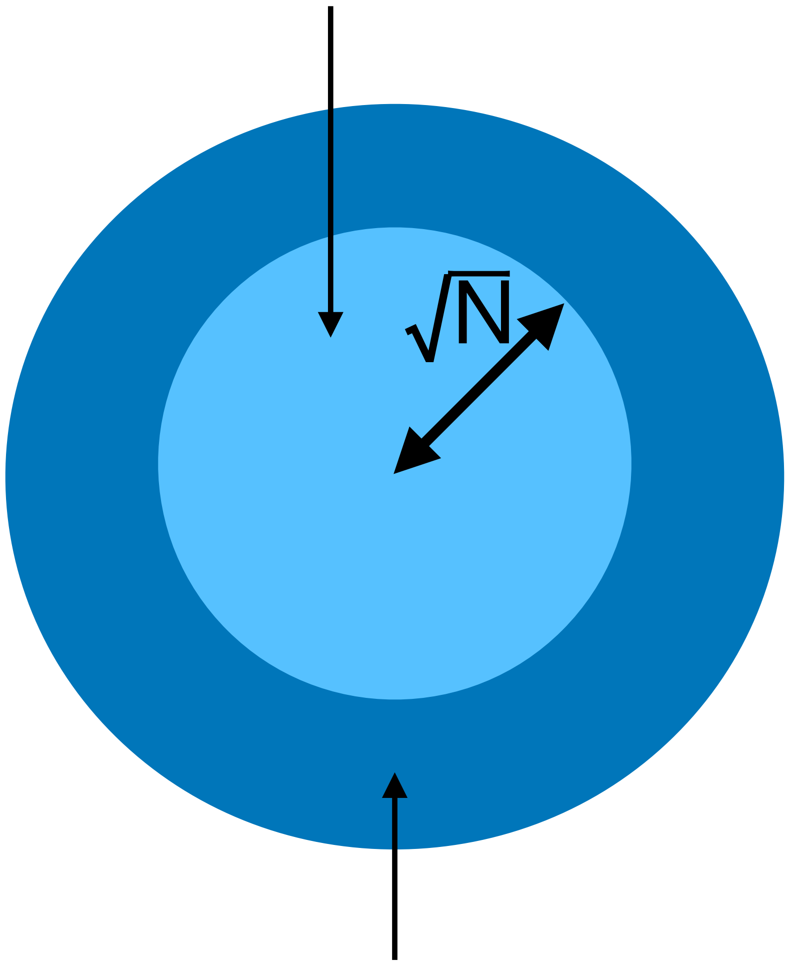
$$\text{Eigenvalues of } X_I^2 \sim N$$

$$\text{Eigenvalues of } X_I \sim \sqrt{N}$$

Highly non-commutative 'fuzz'
(delocalized ground-state wave function)

<u>Eigenvalues are large...</u>

$$\langle \mathrm{Tr}[X_I, X_J]^2 \rangle \sim N^3 \text{ } (\leftarrow \text{'t Hooft counting})$$



Diagonalize X_1
$\to \sum_{i,j=1}^{N} (X_1^{ii} - X_1^{jj})^2 |X_{J,ij}|^2 \sim N^3$
$\to |X_{J,ij}| \sim 1$

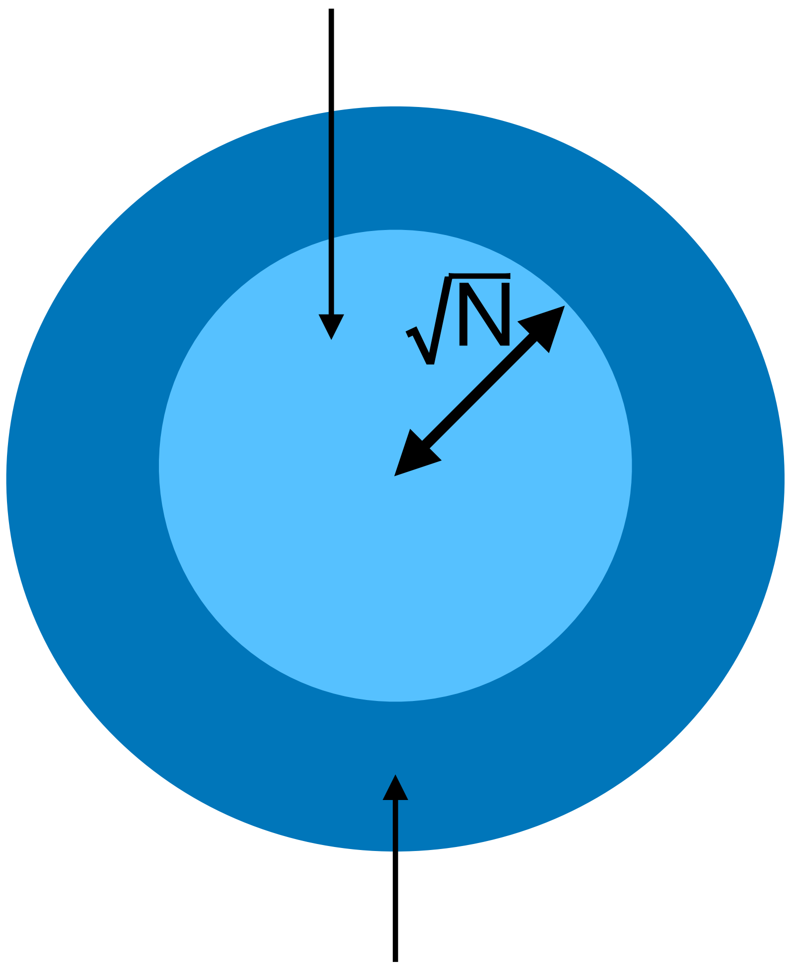
<u>Not close to diagonal at all...</u>

'Location' can make sense only here

We show this is wrong.

Highly non-commutative 'fuzz'

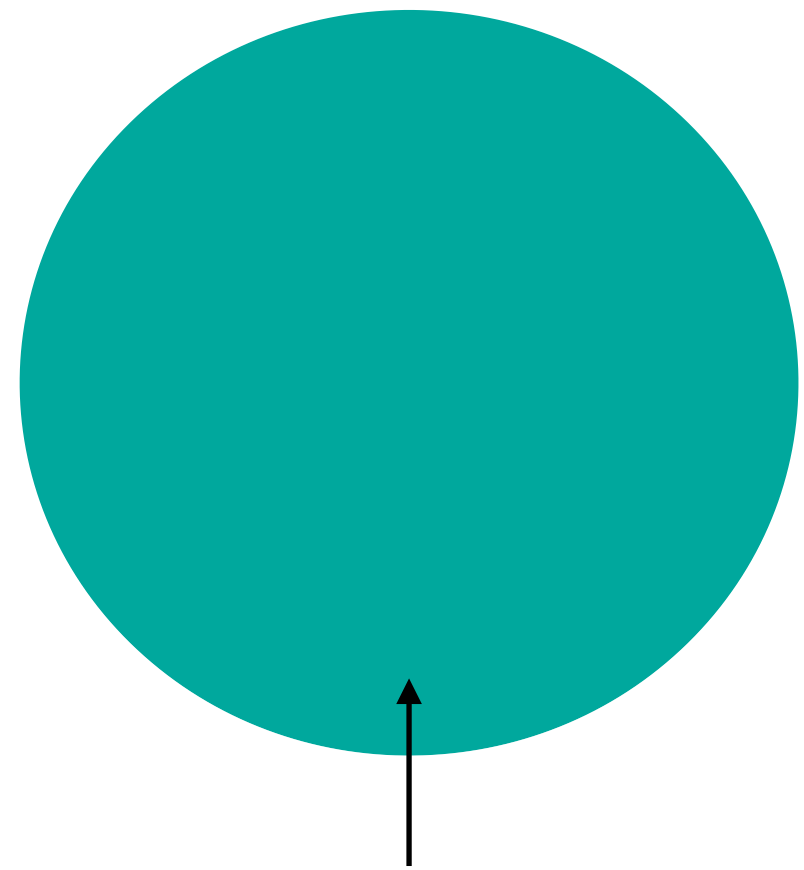
(delocalized ground-state wave function)



'Location' can make sense only here

(Polchinski, hep-th/9903165)

This is the actual physics.



'Location' can make sense everywhere

(M.H., 2102.08982[hep-th])

$$\langle \mathrm{Tr} X_I^2 \rangle \sim N^2 \ (\leftarrow \text{'t Hooft counting})$$ Correct

We must be careful here

$$\text{Eigenvalues of } X_I^2 \sim N$$

Correct

$$\text{Eigenvalues of } X_I \sim \sqrt{N}$$

We consider matrix model for simplicity.

(Generalization to QFT is straightforward.)

$$\hat{H} = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 + \frac{1}{2}\hat{X}_I^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2\right)$$

(+fermion)

I=1,2,...,9

(this mass term is not essential)

Low-energy states
= (superpositions of) **wave packets**.

$$|\Phi\rangle = \int_{\mathbb{R}^{9N^2}} dX|X\rangle\langle X|\Phi\rangle \equiv \int_{\mathbb{R}^{9N^2}} dX\,\underline{\Phi(X)}|X\rangle$$

Extended smoothly in $\mathbb{R}^{9N^2}$

$X_{I,ij} \in \mathbb{R}^{9N^2}$ is not uniquely determined.

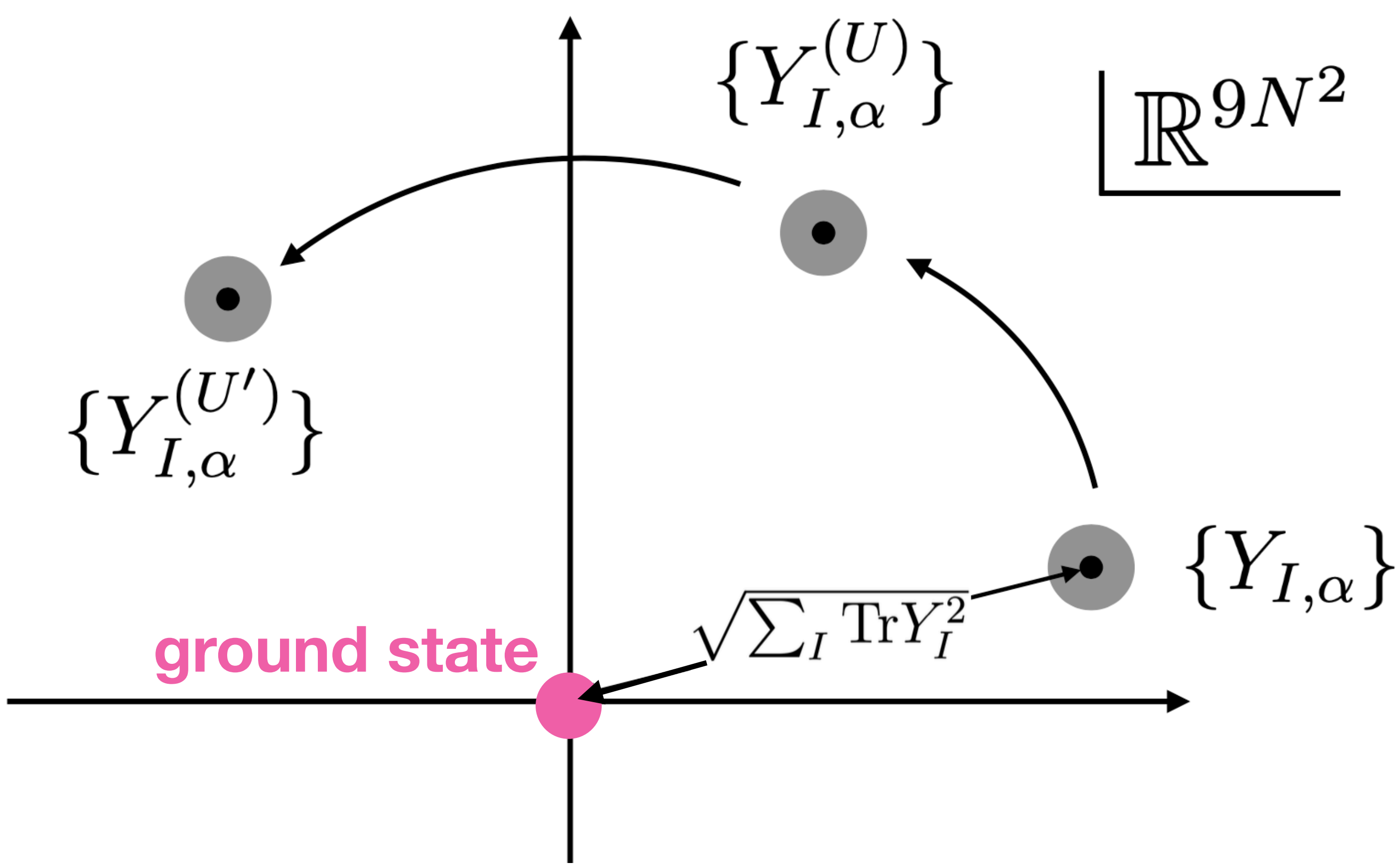
Physically meaningful 'matrices'
= center of a wave packet in $\mathbb{R}^{9N^2}$

$\{Y_{I,\alpha}\}$

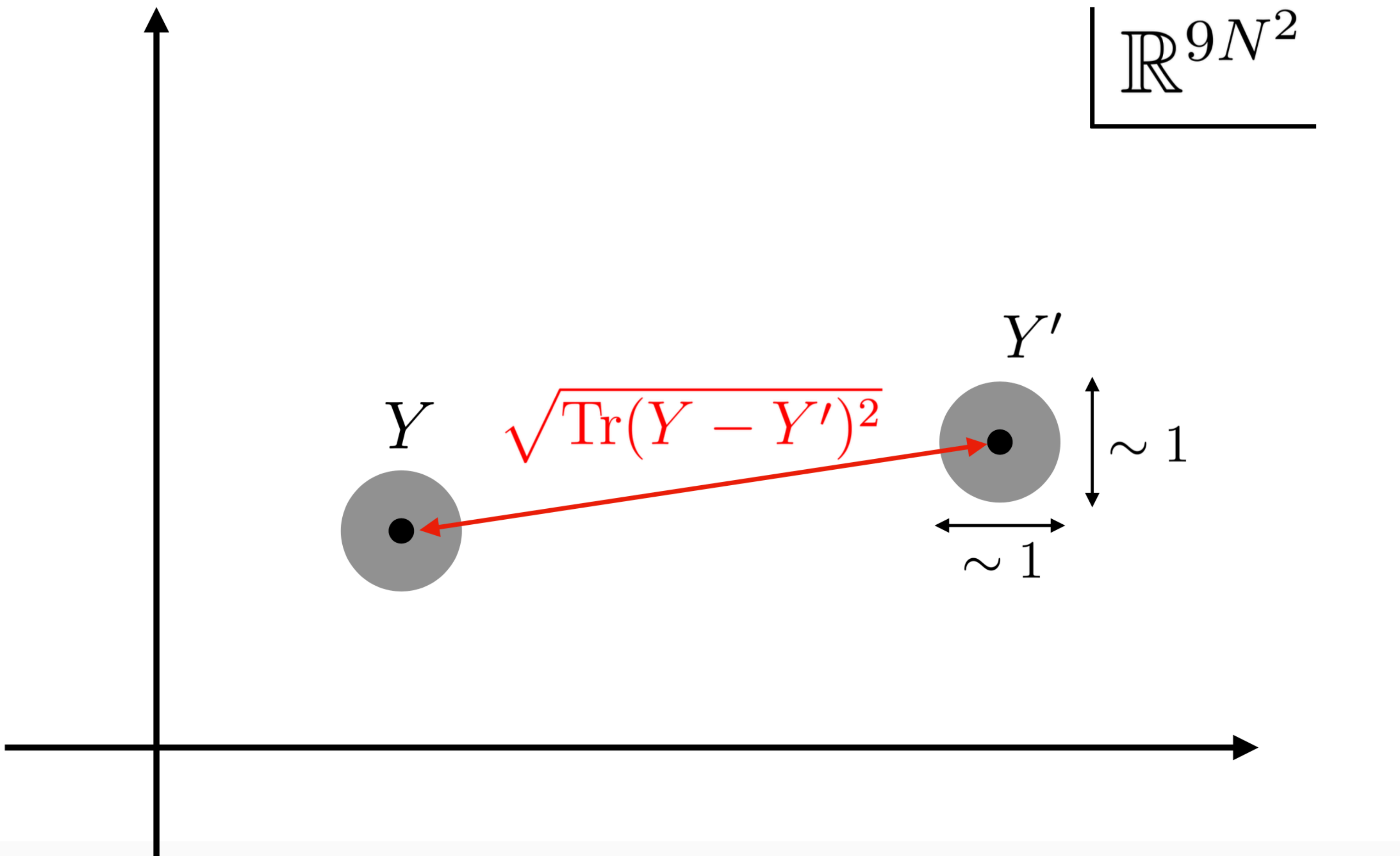
SU(N) gauge transformation:

$$\hat{X}_{I,ij} \to \hat{U}\hat{X}_{I,ij}\hat{U}^{-1} = (U\hat{X}_I U^{-1})_{ij}$$

$$|X\rangle \to \hat{U}|X\rangle = |U^{-1}XU\rangle$$

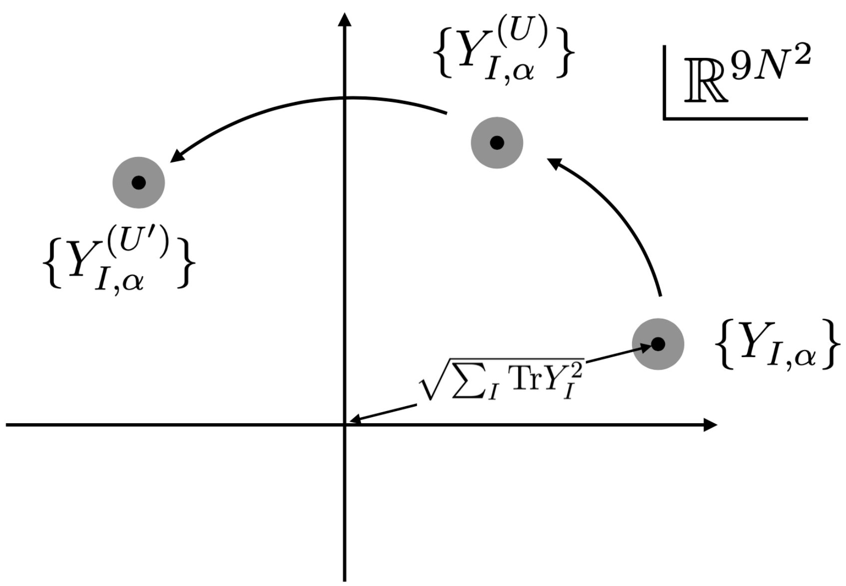


- The location of the wave packet moves → diagonalizable
- The size of the wave packet does not change.
- Y=0 for the ground state.

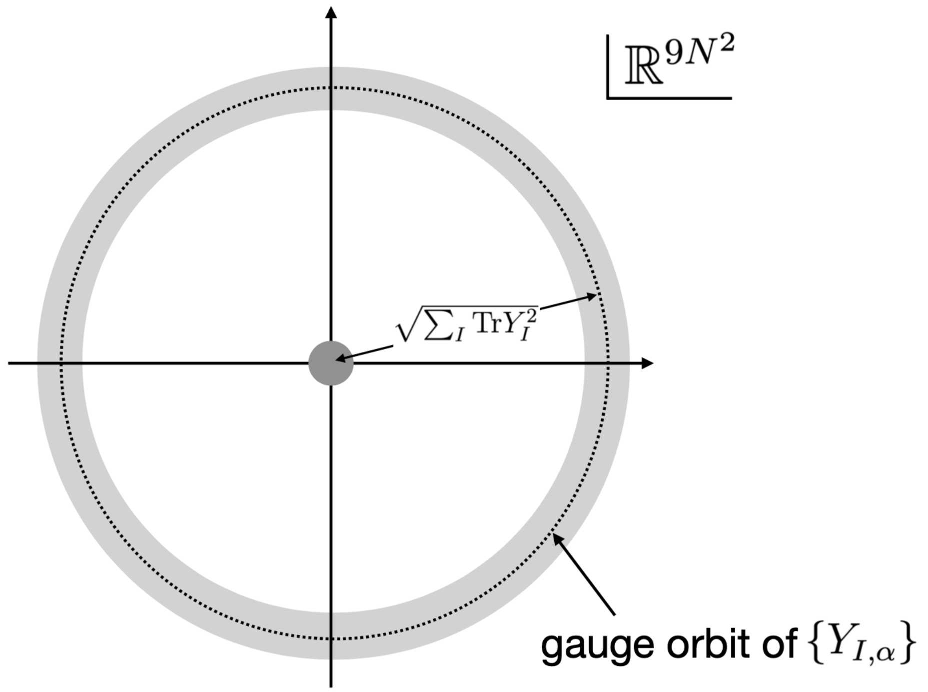


Almost **no** overlap, if the distance between centers is larger than 1.

If you are a singlet-Hilbert-space lover:



SU(N)-symmetrization



$$Z(T) = \frac{1}{\mathrm{vol}(G)} \int_G dg \mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}} (\hat{g} e^{-\hat{H}/T})$$

projection to singlet

(~Polyakov loop; gauge field A_0)

The linear combination of all SU(N)-equivalent states

$$Z(T) = \mathrm{Tr}_{\mathcal{H}_{\mathrm{inv}}} (e^{-\hat{H}/T})$$

Gauged Gaussian Matrix Model

$$\hat{H}_{\mathrm{Gaussian}} = \sum_{\alpha=1}^{N^2} \left(\frac{1}{2} \hat{P}_{I,\alpha}^2 + \frac{1}{2} \hat{X}_{I,\alpha}^2 \right)$$

I=1,2,...,9

Ground state = Fock vacuum

$$|\text{ground state}\rangle = |\{0\}\rangle = \otimes_{I,\alpha}|0\rangle_{I,\alpha}$$

$$\langle X|\text{ground state}\rangle = \frac{1}{\pi^{9N^2/4}} \exp\left(-\frac{1}{2}\sum_I \mathrm{Tr}X_I^2\right)$$

Gauge-invariant wave packet localized around the origin.

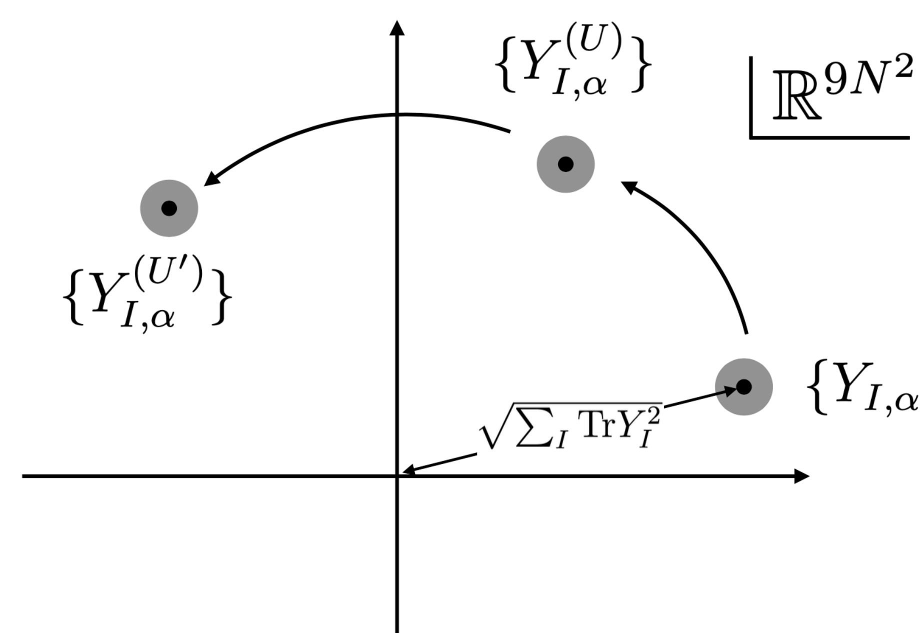
Generic wave packets = coherent states

(in Gaussian matrix model)

$$
\begin{aligned}
|\text{wave packet at } \{Y_{I,\alpha}\}\rangle &= e^{-i\sum_{I=1}^{9}\sum_{\alpha=1}^{N^2} Y_{I,\alpha}\hat{P}_{I,\alpha}}|\text{ground state}\rangle \\
&= e^{-i\sum_{I=1}^{9}\mathrm{Tr}(Y_I\hat{P}_I)}|\text{ground state}\rangle.
\end{aligned}
$$

(More generally, $e^{-i\sum_{I=1}^{9}\mathrm{Tr}(Y_I\hat{P}_I - Q_I\hat{X}_I)}|\text{ground state}\rangle$)

$$
\langle X|Y, Q=0\rangle = \frac{1}{\pi^{9N^2/4}}\exp\left(-\frac{1}{2}\sum_I \mathrm{Tr}(X_I - Y_I)^2\right)
$$



Correct symmetry enhancement pattern

- i-th D-brane at $\vec{y}_i \in \mathbb{R}^9$

$$e^{-i\sum_{k=1}^{N} \vec{y}_k \cdot \hat{\vec{P}}_{kk}} |\text{ground state}\rangle$$

$$\vec{y}_1 = \cdots = \vec{y}_{N_1},\ \vec{y}_{N_1+1} = \cdots = \vec{y}_{N_1+N_2}, \cdots$$

$$\to \mathrm{U}(N_1) \times \mathrm{U}(N_2) \times \cdots\text{-invariant}$$

Correct 'Higgsing' effect

$$\hat{H} = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 + \frac{1}{2}\hat{X}_I^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2\right)$$

$$e^{-i\sum_{I=1}^{9}\mathrm{Tr}(Y_I\hat{P}_I)}|\mathrm{ground\ state}\rangle$$

$$e^{i\sum_I \mathrm{Tr}(Y_I\hat{P}_I)}H(\hat{P},\hat{X})e^{-i\sum_I \mathrm{Tr}(Y_I\hat{P}_I)} = H(\hat{P},\hat{X}+Y)$$

Mass term due to Higgsing:

$$\sum_{I\neq J}\sum_{i,j}(y_{I,i} - y_{I,j})^2|\hat{X}_{J,ij}|^2$$

Wave packet in interacting theories

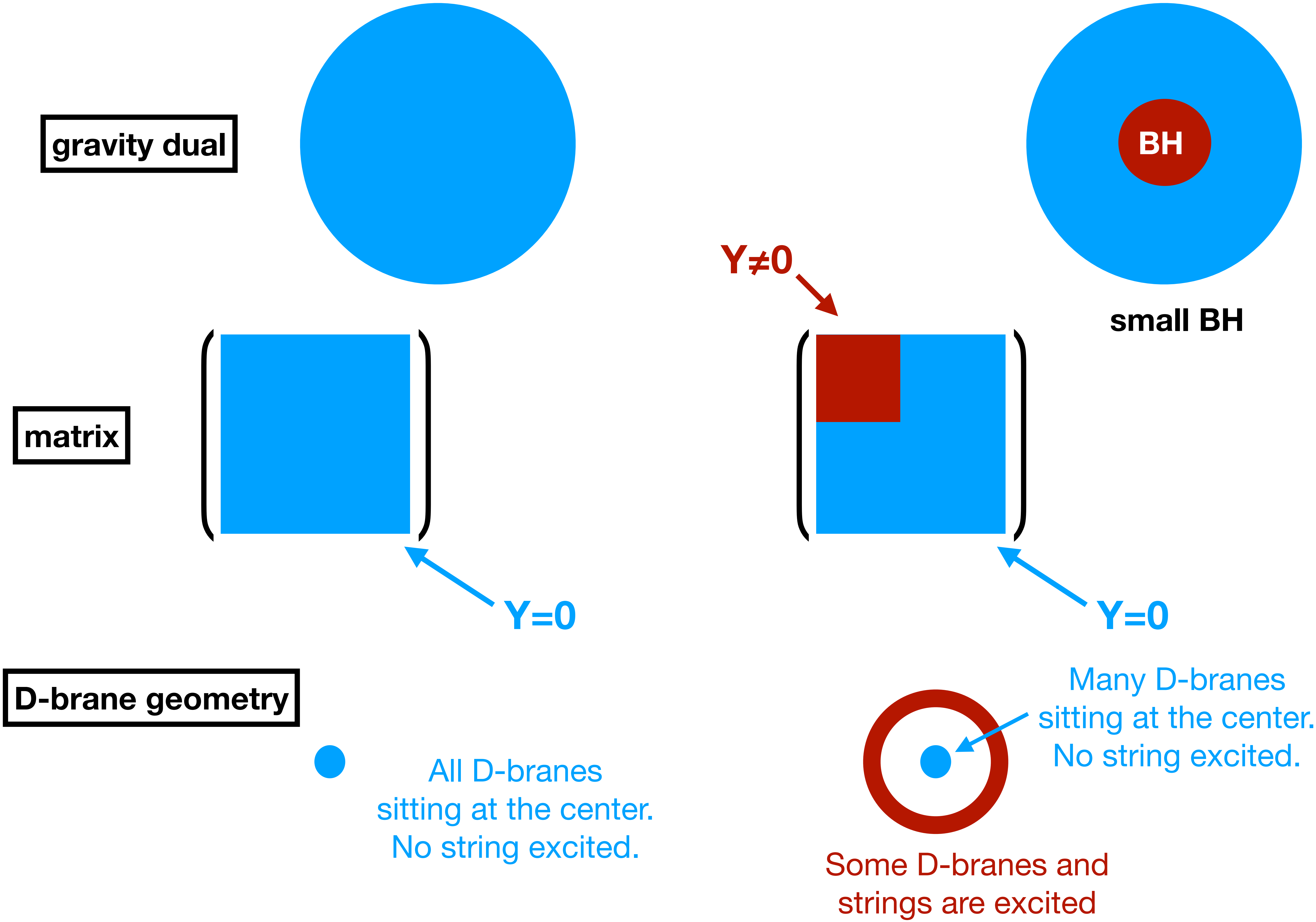
Some corrections to the coherent state are needed.

A natural construction

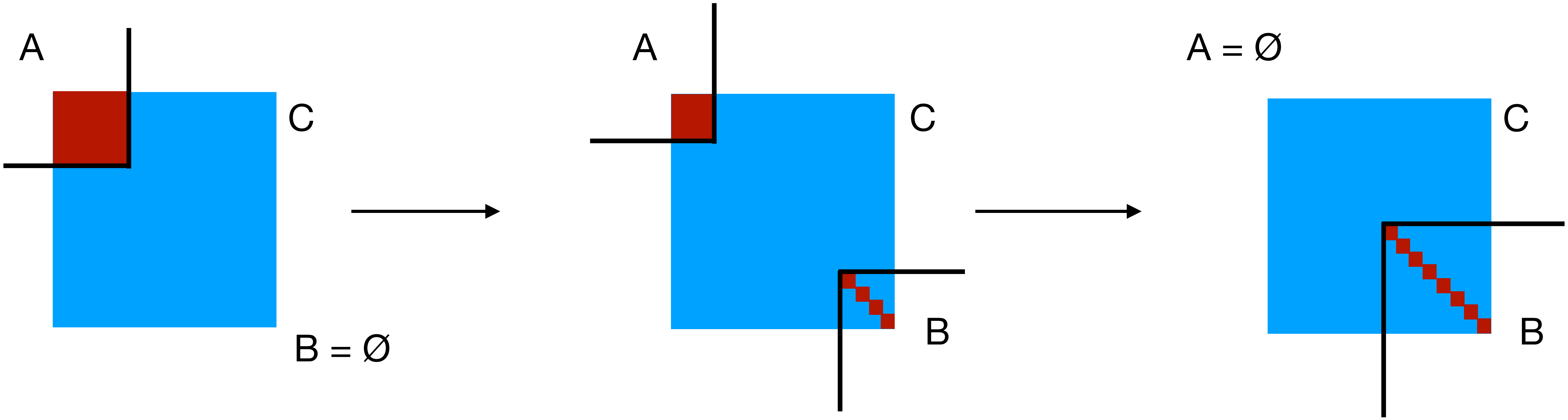
$$\text{Minimize } \langle\Phi|\hat{H}|\Phi\rangle \text{ satisfying}$$
$$\langle\Phi|\hat{X}_I|\Phi\rangle = Y_I \text{ and } \langle\Phi|\hat{P}_I|\Phi\rangle = Q_I$$

Some speculations

Gautam, MH, Jevicki, Peng, 2204.06472[hep-th] (JHEP)



The confined sector can be used to probe the geometry outside the BH.

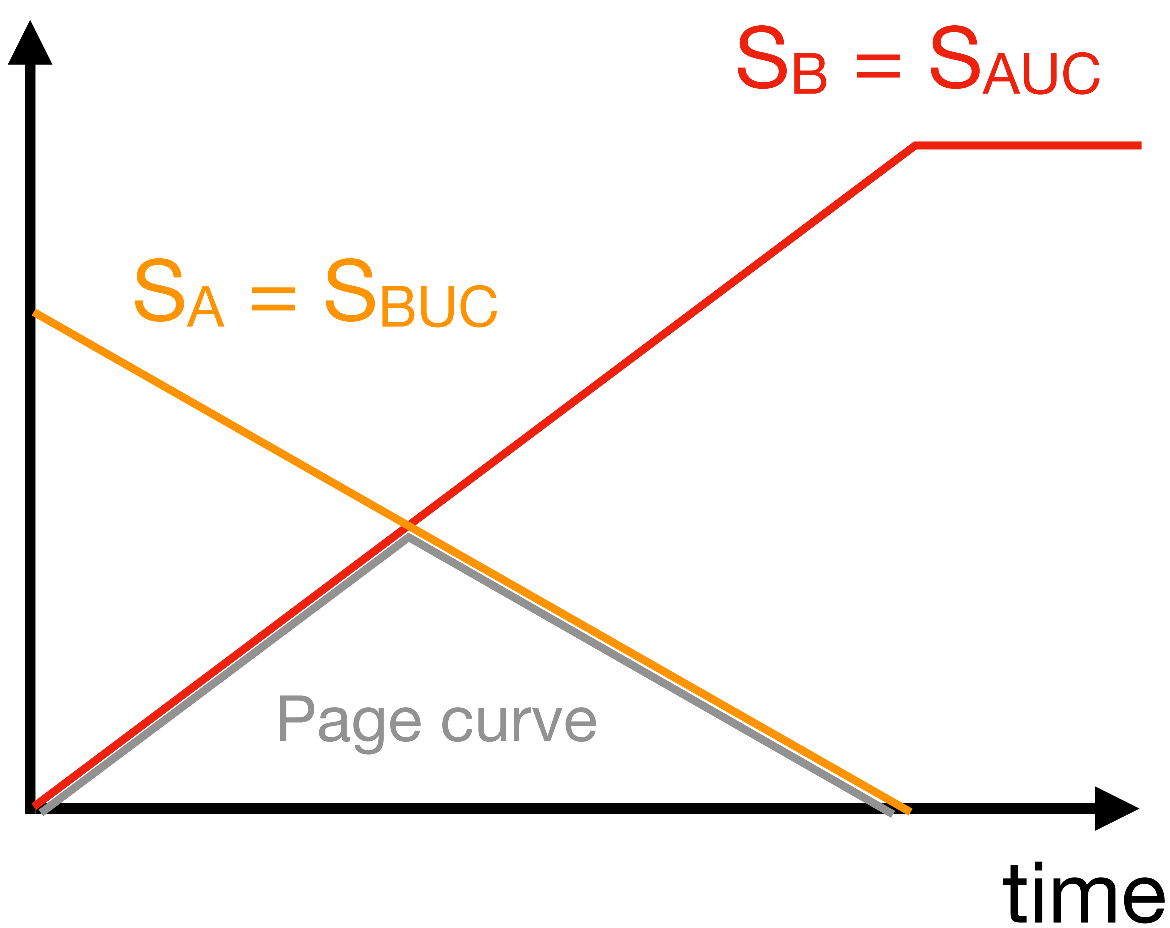


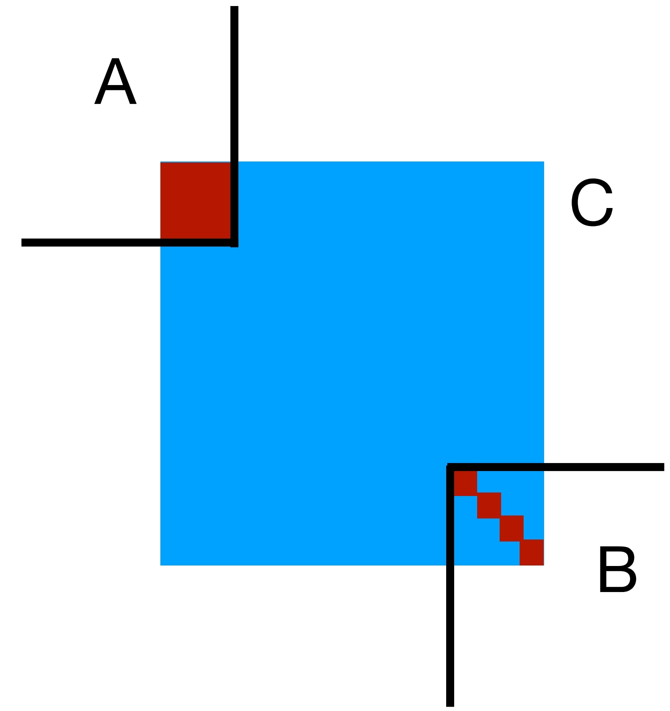
- Small BH in AdS can evaporate.
- There is a natural partitioning of matrix degrees of freedom for some states.
- Extended Hilbert space factorizes. → Entanglement can be defined.

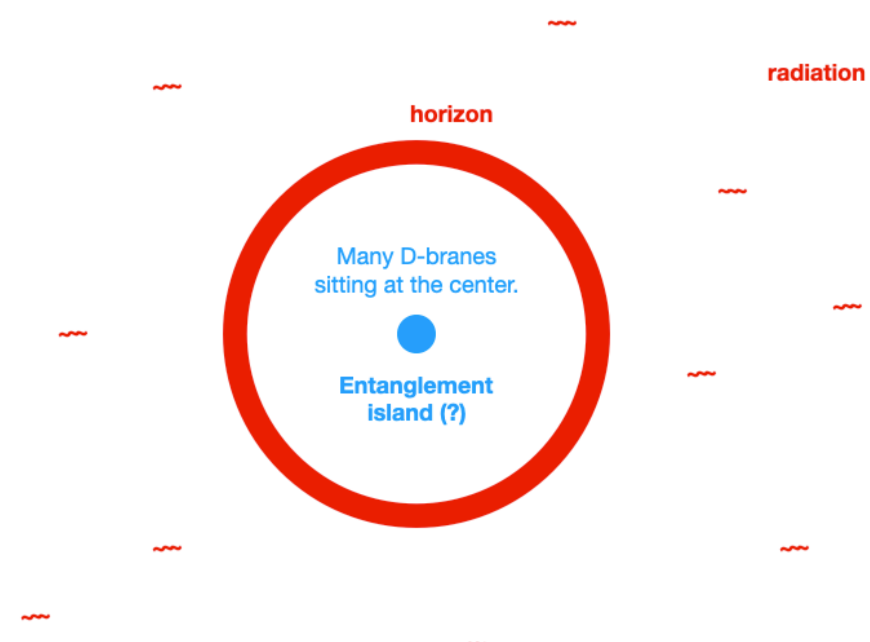
$$\mathcal{H}_{\rm ext} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

Partitioning colors,
instead of space

Partitioning colors, instead of space



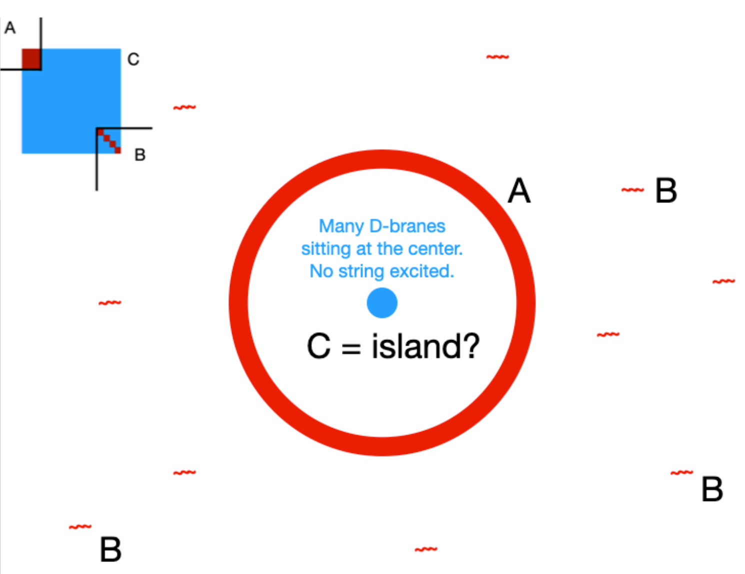




Meaning of region C?

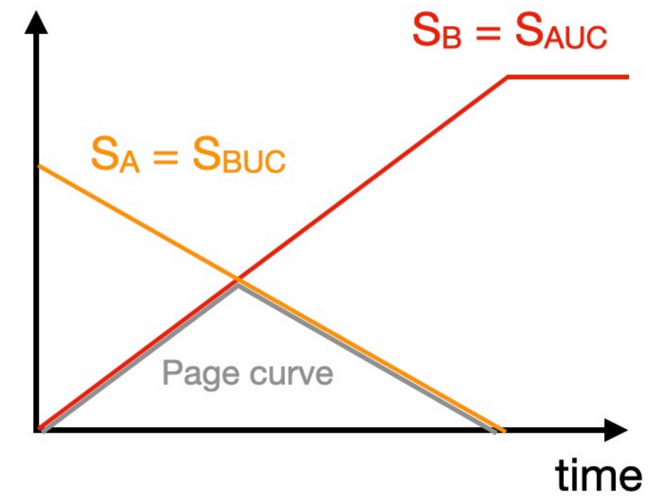
- D-branes in region C ~ BPS black hole
- Asymptotically AdS geometry is created by D-branes in region C
- BH described by A can sit away from the center for some time

  → Region C is not always "behind the horizon"
- Region C ~ AdS entire geometry?



- Suppose Bob has access to BUC, i.e., he has $\hat{\rho}_{B\cup C}$

- Suppose Bob has access to BUC, i.e., he has $\hat{\rho}_{B\cup C}$
- Smaller von Neumann entropy → more information
- Page curve = the best of Bob's knowledge

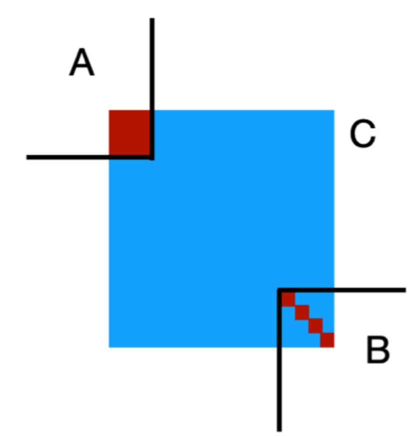


<u>Before Page time</u> $S_B < S_{B\cup C}$

At best, the coarse-grained entropy of region B.
Bob cannot see region C.

$$\mathcal{H}_{\rm ext} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

<u>After Page time</u> $S_B > S_{B\cup C}$



Bob can get information from region C.
Bob can see region C.

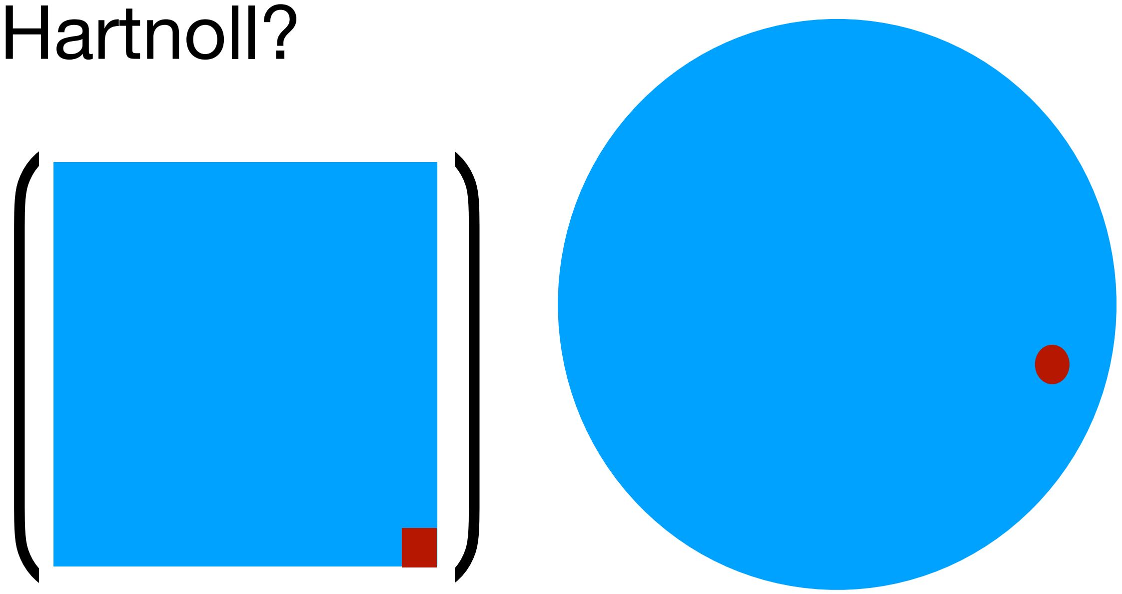
Future directions

- Shut up and *get a number*

    - Probe BPS p-brane geometry by a D-brane

        numerical method like Han-Hartnoll?

    - Quantum simulation



- Dual M-theory geometry of D0-brane matrix model?