Emergent 5-branes in the BMN Matrix Model

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Matrix Quantum Mechanics for M-theory Revisited
@CERN

Introduction

Matrix Models can be
non-perturbative formulation of string/M theory

c=1 matrix model, BFSS model, IKKT model,…

BMN matrix model (a.k.a. plane-wave matrix model)
… the mass-deformation of BFSS
w/ maximal supersymmetry

[Berenstein, Maldacena, Nastase ’02]

Non-perturbative physics

D-/M-branes — how do they emerge?

Target space metric — how is it determined?

··· Emergent geometry

Introduction

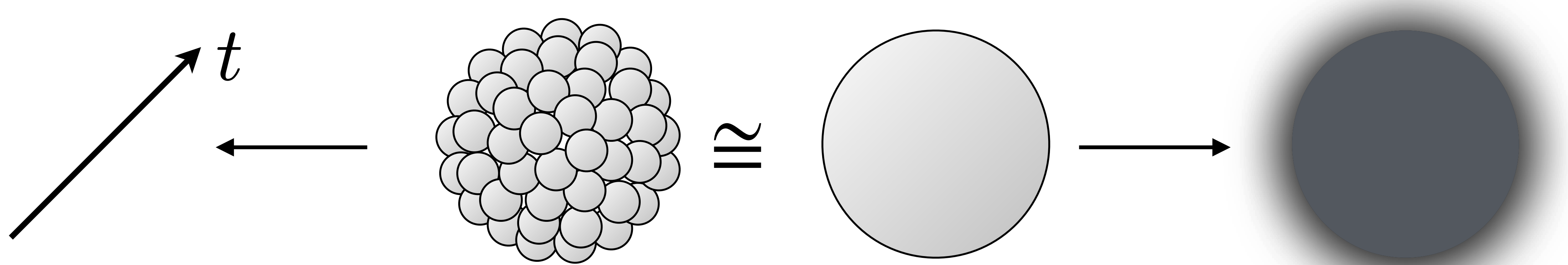
Gauge/gravity duality

[Lin, Lunin, Maldacena ’04; Lin, Maldacena ’05]

Vacuum
BMN matrix model

⟷

Gravity solution
IIA/11D SUGRA



matrix model

$$X^a = \frac{\mu}{3} L_a$$

L_a: $SU(2)$ generators
$(a = 1,2,3)$

blown-up
D0s/gravitons

p-branes

gravitational field

Bubbling geometries

Many degenerate vacua

Each matrix-model vacuum should describe the corresponding brane config. and its gravitational field.

Introduction

Gauge/gravity duality at finite T

[Costa, Greenspan, Penedones, Santos '14]

Thermal state
BMN matrix model

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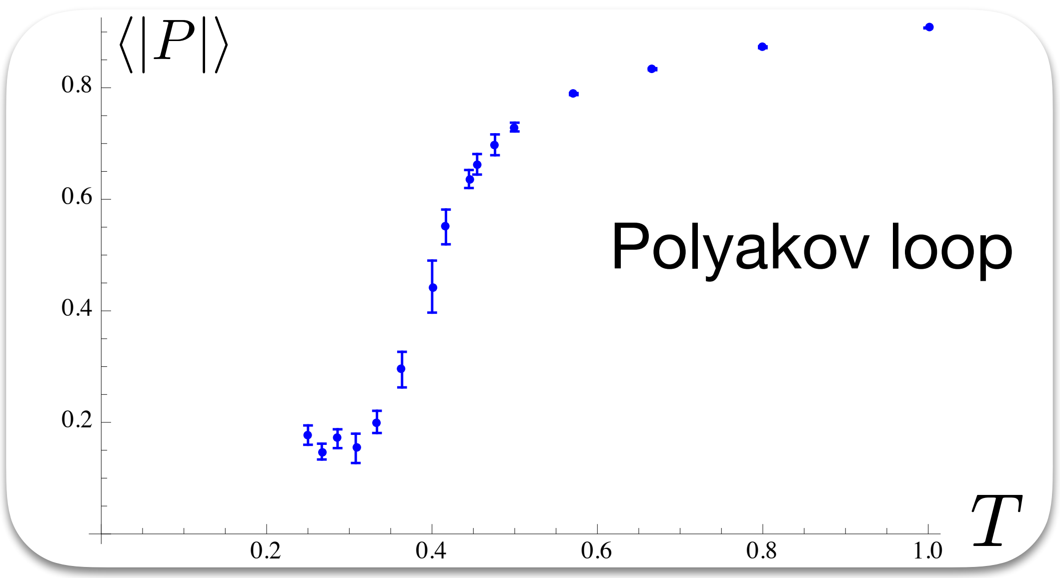
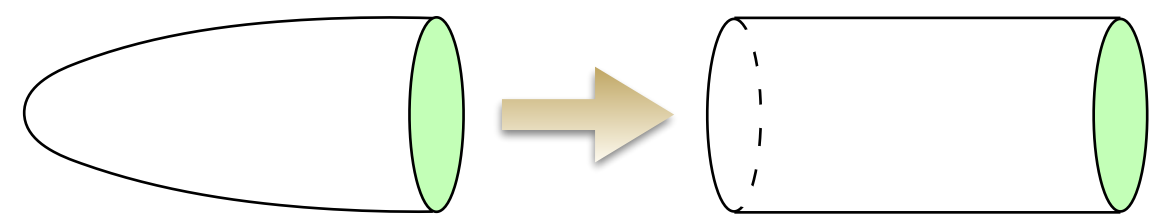
Thermal geometry
IIA/11D SUGRA

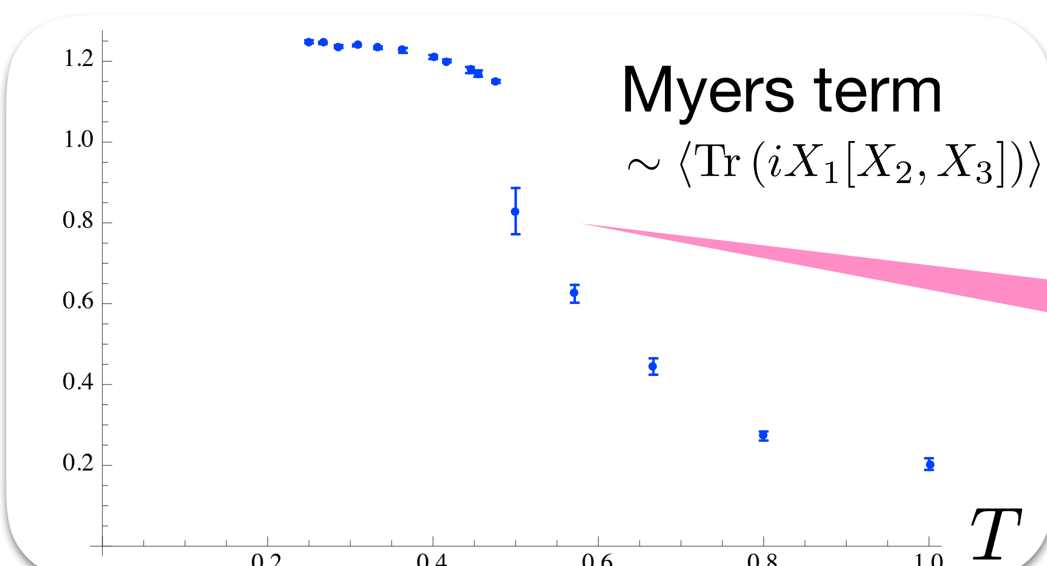


Hawking-Page-like transition



The agreement is numerically checked.

[Y.A., Filev, Kovacik, O'Connor '18;
Bergner, Bodendorfer, Hanada, Pateloudis,
Rinaldi, Schäfer, Vranas, Watanabe '21;…]



Transitions between different brane configs have been observed.

(μ=5, L=24, N=11) [Y.A., Filev, Kovacik, O'Connor '18]

Introduction

The vacua of the BMN model

The BMN model has many degenerate vacua, protected by susy.

[Dasgupta, Sheikh-Jabbari, Raamsdonk '02]

$$X^a = \frac{\mu}{3}\begin{pmatrix} \mathbf{1}_{N_2^1} \otimes L_a^{[N_5^1]} & & \\ & \mathbf{1}_{N_2^2} \otimes L_a^{[N_5^2]} & \\ & & \ddots \end{pmatrix}$$

$(a = 1,2,3)$

$L_a^{[N_5]}$: N_5 dimensional irrep. matrix

N_2: multiplicity of this irrep.

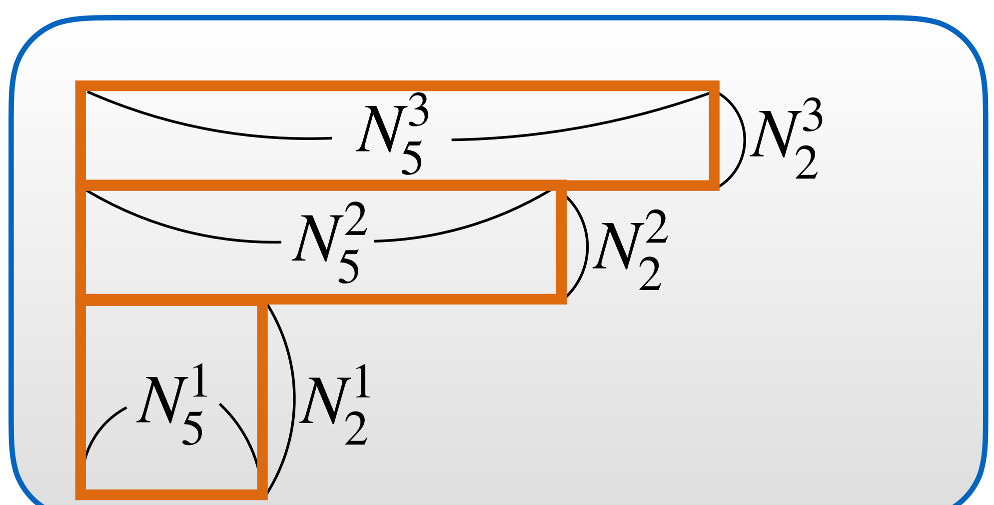
$$X^m = 0$$

$(m = 4,\dots,9)$

The vacuum is parameterised by $\{N_2^s, N_5^s\}_{s=1,2,\dots}$

Partition of N

$$N = \sum_s N_2^s N_5^s$$



[Maldacena, Sheikh-Jabbari, Raamsdonk '02]

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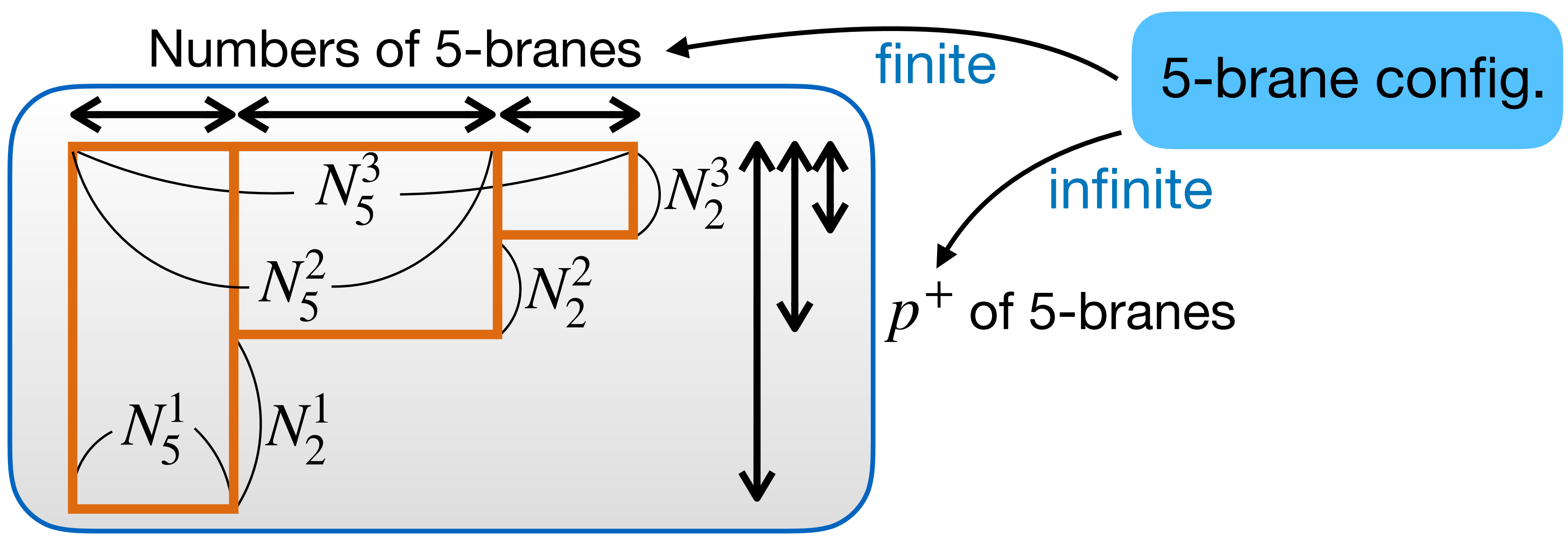


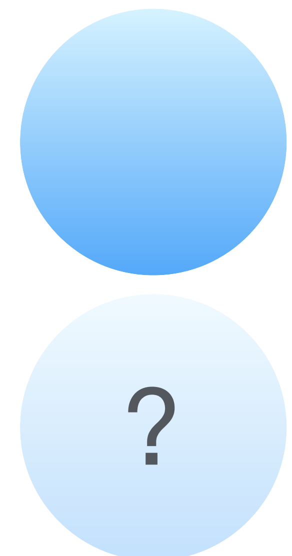
[Maldacena, Sheikh-Jabbari, Raamsdonk '02]

Introduction

<u>5-brane geometry in the Matrix Model</u>

Vacuum: $X^a = \frac{\mu}{3} L_a$ (a=1,2,3) → fuzzy 2-sphere ··· 2-brane!

$X^m = 0$ (m=4,...,9) → should be (fuzzy) 5-sphere…



5-branes have been a long-standing problem

To solve this problem, we ought to notice that geometrical features should be seen at strong coupling

Localisation

Localisation

Supersymmetric localisation computation

[Pestun '07;…]

Symmetry tends to make difficult computation possible.

The (new) SUSY localisation technique is a very powerful tool!

- applicable to a large class of supersymmetric theories

  not only topological field theories

  but also gauge theories on curved space, supergravity theories, …

- can make an infinite-dim. path integral to a finite-dim. integral

$$\int [dX(\tau)] \to \int \left[\prod_{i=1}^{N} dm_i \right]$$

- its result is exact and valid in the strong-coupling regime

Localisation

How the localisation works

[Pestun '07;...]

Let us consider a system with a fermionic trf. δ

$$Z_{\mathcal{O}} = \int [dX]\, \mathcal{O} e^{-S[X]} \qquad \delta S = 0 \qquad \delta \mathcal{O} = 0$$

Introduce a deformation δV whose bosonic part is positive-definite.

$$Z_{\mathcal{O}}(t) := \int [dX]\, \mathcal{O} e^{-S[X] - t\delta V[X]} \qquad \delta^2 V = 0$$

Then $$\frac{dZ_{\mathcal{O}}(t)}{dt} := -\int [dX]\, \delta(\mathcal{O} V e^{-S[X] - t\delta V[X]}) = 0$$

if the surface term vanishes or $[dX]$ is SUSY inv.

The original path integral can be computed by config. localised around $\delta V = 0$:

$$Z_{\mathcal{O}} = Z_{\mathcal{O}}(0) = \lim_{t \to +\infty} Z_{\mathcal{O}}(t)$$

➡ 1-loop computation becomes exact

Localisation

BMN-model action

Let us consider a double-Wick-rotated theory $(\tau = it,\ X^0 = -iX^9)$

(to construct susy)

$$S = \frac{1}{g^2}\int d\tau\, \mathrm{tr}\left(\frac{1}{2}(D_\tau X^i)^2 + \frac{1}{4}\left(-\frac{\mu}{3}\varepsilon^{abc}X_c - i[X^a, X^b]\right)^2 - \frac{1}{2}[X^a, X^m]^2 - \frac{1}{4}[X^m, X^n]^2 + \frac{1}{2}\left(\frac{\mu}{6}\right)^2 X_m X^m + \sum_{I=1}^{7} K_I K_I + \text{fermions}\right)$$

$(i=1,\ldots,8,0,\ \ a=1,2,3,\ \ m=4,\ldots,8,0)$

Off-shell SUSY (9 supercharges)

$$\delta_s X^i = i\epsilon\gamma^i\Psi, \qquad \delta_s A = i\epsilon\gamma_\tau\Psi,$$

$$\delta_s\Psi = \left(D_\tau X_i\gamma^{\tau i} - \frac{i}{2}[X_i, X_j]\gamma^{ij} - \frac{\mu}{3}X_a\gamma^a\gamma^{123} + \frac{\mu}{6}X_m\gamma^m\gamma^{123}\right)\epsilon + K_I\nu^I$$

$$\delta_s K_I = i\nu_I\left(\gamma^\tau D_\tau\Psi - i\gamma^i[X_i, \Psi] - \frac{\mu}{24}\varepsilon_{abc}\gamma^{abc}\Psi\right)$$

[Berkovits '93]

Killing spinor eq.: $\partial_\tau\epsilon = -\frac{\mu}{12}\gamma_\tau\gamma^{123}\epsilon$

ϵ & ν_I's satisfy conditions that make the SUSY closes off-shell

Localisation

Quarter-BPS sector (4 supercharges)

[Y.A., Ishiki, Okada, Shimasaki '12]

- $SO(5,1) \to SO(4)\times SO(1,1)$: $\quad \gamma^{4567}\epsilon = \epsilon$

  ($\mathcal{N}=4$ vector multiplet
  $\to \mathcal{N}=2$ vector + $\mathcal{N}=2$ hyper)

- BPS Wilson loop : $\quad \gamma^{03}\epsilon(\tau) = \epsilon(-\tau)$

  (dim. reduction of
  the half-BPS Wilson loop
  in $\mathcal{N}=4$ SYM on $R\times S^3$)

$$\partial_\tau \epsilon = -\frac{\mu}{12}\gamma_\tau \gamma^{123}\epsilon$$

$\Rightarrow$ $$\phi = X^3 + \sinh\frac{\mu\tau}{6}X^8 + \cosh\frac{\mu\tau}{6}X^0$$ is invariant

$(\propto \epsilon_1\gamma^\mu\epsilon_2 X_\mu)$

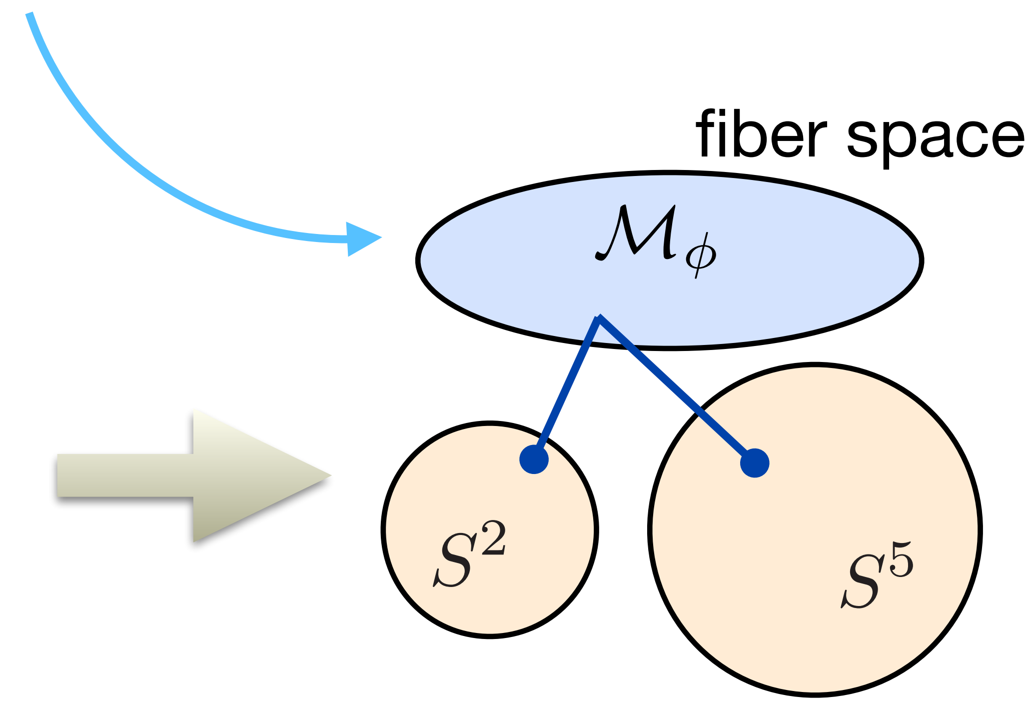
Localisation

<u>What will this 1/4-BPS mean?</u> [Y.A., Ishiki, Okada, Shimasaki '12]

In the original BMN model,

$$\phi = X^3 + i\left(\sin\frac{\mu t}{6}X^8 - \cos\frac{\mu t}{6}X^9\right)$$

This would describe the radial directions of $S^2 \times S^5$



insertion of ϕ
$\langle f(\phi(\tau))\rangle$

$R\times SO(3)\times SO(6)$
$\rightarrow SO(2)\times SO(5)$
a point on $S^2 \times S^5$

We expect it describes low-energy d.o.f. In fact, ϕ turns out to be time-independent, meaning that it only has a "0-energy mode."

Localisation

Localisation for the BMN model

[Y.A., Ishiki, Okada, Shimasaki '12]

- Wick rotation to the Euclidean theory (but not compactified)
- B.C.: All fields approach the same vac. config. at $\tau \to \pm\infty$.
- Fermionic sym.: off-shell SUSY + BRST sym. $\delta = \delta_s + \delta_B$
  (the 1/4-BPS)
- Deformation δV: $\delta V = \delta\left(\int d\tau\, \mathrm{tr}[\Psi\overline{\delta\Psi}] + V_{\mathrm{gh}}\right)$

  Hermitian conjugate after the Wick rot.

Localising locus: (basically, a solution to $\delta_s\Psi = 0$)

$$\hat{X}^a(\tau) = \frac{\mu}{3}L_a \qquad \hat{X}^9(\tau) = -\frac{\mu M}{6\cosh\frac{\mu\tau}{6}} \qquad ([L_a, M] = 0)$$

same as the vacuum

"moduli"

other X's are zero

$$\hat{\phi} = \frac{\mu}{6}(2L_3 + iM)$$

($\mu = 6$ from now on, set by rescaling)

Localisation

<u>Partition fn. in the 1/4-BPS sector</u>

[Y.A., Ishiki, Okada, Shimasaki '12]

$$Z_1 = \int [dX]\, e^{-S[X]-t\delta V[X]}$$

$$= \lim_{t\to\infty} \int [dM][d\tilde{X}]\, e^{-S[\hat{X}]-t\delta V[\hat{X}+\frac{1}{\sqrt{t}}\tilde{X}]} = \int [dM]\, Z_{1-\text{loop}}\, e^{-S[\hat{X}]}$$

Exact result
(up to instanton effect)

$$S[\hat{X}] = \frac{2}{g^2}\,\mathrm{tr}\, M^2 = \frac{2}{g^2} \sum_{s=1}^{\Lambda} \sum_{i=1}^{N_2^s} N_5^s {m_{si}}^2$$

$$Z_{1-\text{loop}} = \prod_{s,t=1}^{\Lambda} \prod_{J=\frac{|N_5^s - N_5^t|}{2}}^{\frac{N_5^s+N_5^t}{2}-1} \prod_{i=1}^{N_2^s} \prod_{j=1}^{N_2^t}{}' \left[\frac{\{(2J+2)^2 + (m_{si}-m_{tj})^2\}\{(2J)^2 + (m_{si}-m_{tj})^2\}}{\{(2J+1)^2 + (m_{si}-m_{tj})^2\}^2} \right]^{\frac{1}{2}}$$

contribution from superpartners of $X^\tau, X^1, X^2, X^3, X^8$

contribution from $X^4, \cdots, X^7$

$$\hat{X}^a = 2 \begin{pmatrix} \mathbf{1}_{N_2^1} \otimes L_a^{[N_5^1]} & & \\ & \ddots & \\ & & \mathbf{1}_{N_2^\Lambda} \otimes L_a^{[N_5^\Lambda]} \end{pmatrix}$$

(Λ blocks)

$$M = \begin{pmatrix} M_1 \otimes \mathbf{1}_{N_5^1} & & \\ & \ddots & \\ & & M_\Lambda \otimes \mathbf{1}_{N_5^\Lambda} \end{pmatrix}$$

(m_{si} are eigenvalues of M_s)

Localisation

The VEV of any 1/4-BPS operators can be computed by a simple matrix integral

τ-independent

$$\langle f_1(\phi(\tau_1)) f_2(\phi(\tau_2)) \cdots \rangle = \Big\langle f_1((2L_3 + iM)) f_2((2L_3 + iM)) \cdots \Big\rangle_{MM}$$

$$\left(\langle \mathcal{O} \rangle_{MM} = \frac{1}{Z_1} \int [dM]\, \mathcal{O}\ Z_{1-\text{loop}}\, e^{-S[\hat{X}]} \right)$$

The eigenvalue distribution for the sth block:

$$\rho^{(s)}(q) = \left\langle \sum_{i=1}^{N_2^s} \delta(q - m_{si}) \right\rangle_{MM}$$

(support: $[-q_m^{(s)}, q_m^{(s)}]$)

Localisation

Eigenvalue distribution

[Y.A., Ishiki, Okada, Shimasaki '14]

The saddle point eq. for $S_{\text{eff}}[M] = S[\hat{X}] - \ln Z_{1-\text{loop}}$

at large $q_m^{(s)}$ for the sth block is a Fredholm-type integral eq.

$$\rho^{(s)}(q) - \frac{1}{\pi}\sum_{t=1}^{\Lambda}\int_{-q_m^{(t)}}^{q_m^{(t)}} dq' \left[\frac{N_5^s + N_5^t}{(N_5^s + N_5^t)^2 + (q-q')^2} - \frac{|N_5^s - N_5^t|}{(N_5^s - N_5^t)^2 + (q-q')^2}\right]\rho^{(t)}(q') = \frac{\beta_s}{\pi} - \frac{2N_5^s}{\pi g^2}q^2$$

β_s: Lagrange multiplier

In the $\Lambda = 1$ case,

$$\rho(q) - \frac{1}{\pi}\int_{-q_m}^{q_m} dq' \frac{2N_5}{(2N_5)^2 + (q-q')^2}\rho(q') = \frac{\beta}{\pi} - \frac{2N_5}{\pi g^2}q^2$$

This can be solved in special limits.

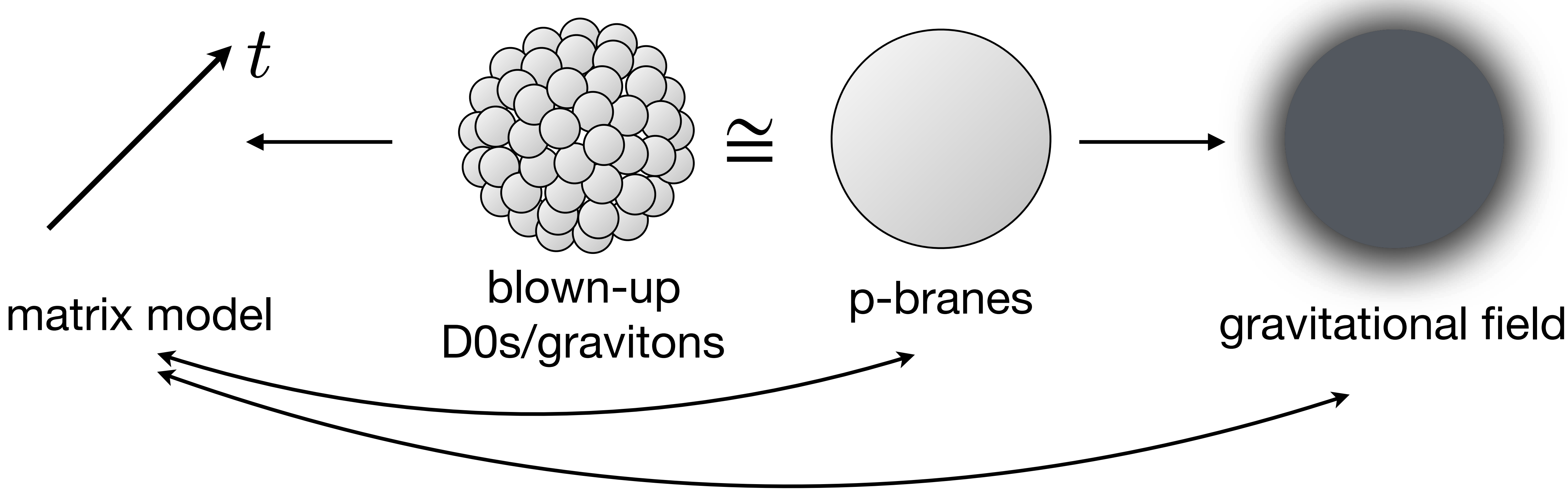
($N_5/q_m \to 0$ and $+\infty$)

Localisation

Applications of the localisation result

- The eigenvalue dist. of ϕ reproduces the spherical geometry of stacks of M5-branes.
  It also reproduces the spherical geometry of stacks of M2-branes at large N_2. [Y.A., Ishiki, Shimasaki, Terashima '17]

- The eigenvalue dist. of ϕ satisfies exactly the same equations as the Lin-Maldacena geometry does, i.e., ϕ constructs part of Einstein's eq. [Y.A., Ishiki, Okada, Shimasaki '14; Y.A., Ishiki, Shimasaki '14]



Localisation

- The double scaling limit for NS5-branes
  (It was obtained with help of numerical computation)
  [Y.A., Ishiki, Okada, Shimasaki '14;
  Y.A., Ishiki, Matsumoto, Shimasaki, Watanabe '22]

  (Purely analytic derivation of the DSL is work in progress)
  [Y.A., Ishiki, Shimasaki]

What hasn't done yet

- Complete reconstruction of the Lin-Maldacena geometry in the matrix model, and the real meaning of ϕ
- $1/N$ correction and coupling expansion (α' correction)
- Reproducing the theory on M-branes in the 1/4-BPS sector
- Relationship to the ABJM/BLG model, in the limit for M2-branes
- Using the exact result as an input for the bootstrapping
- Application of the localisation technique to other BPS sectors, and other matrix models

M-brane geometry

M-brane geometry

The vacuum corresponding to a single-stack M5 ($\Lambda = 1$):

$$\hat{X}^a = \frac{\mu}{3}\mathbf{1}_{N_2} \otimes L_a^{[N_5]} \qquad \text{with } N_2 \to \infty \text{ and } \lambda = g^2 N_2 \to \infty$$

$\cdots$ corresponds to $N_5/q_m \to 0$

$$\rho(q) - \frac{1}{\pi}\int_{-q_m}^{q_m} dq' \frac{2N_5}{(2N_5)^2 + (q-q')^2}\rho(q') = \frac{\beta}{\pi} - \frac{2N_5}{\pi g^2} q^2$$

$$= \frac{1}{2\pi i}\left(\omega(q - 2N_5 i) - \omega(q + 2N_5 i)\right)$$

$$\omega(z) = \int dq \frac{\rho(q)}{z-q}$$

$$\Longrightarrow \quad N_5\left(\omega'(q - i0) + \omega'(q + i0)\right) = \beta - \frac{2N_5}{g^2}q^2$$

~ one-matrix model with quartic interaction

q_m is determined by the least action:

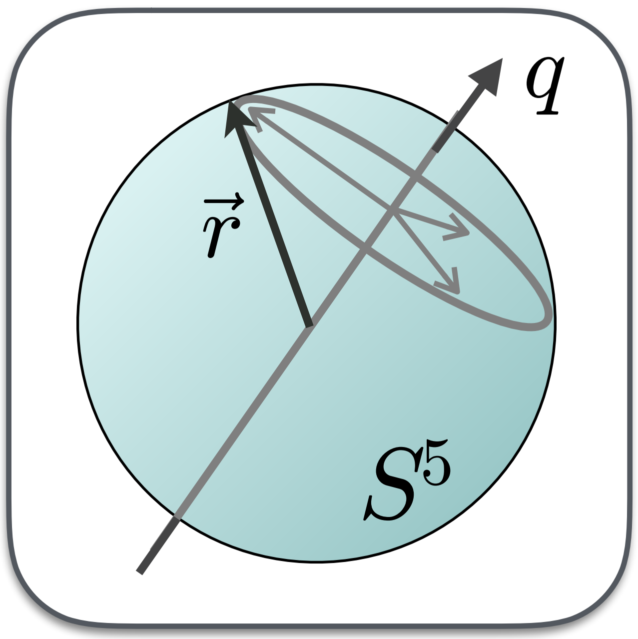
$$\rho(q) = \frac{8}{3\pi q_m}\left[1 - \left(\frac{q}{q_m}\right)^2\right]^{3/2} \qquad q_m = (8g^2 N_2)^{\frac{1}{4}}$$

M-brane geometry

We now have the eigenvalue distribution in the M5 limit.

$$\rho(q) = \frac{8}{3\pi q_m}\left[1 - \left(\frac{q}{q_m}\right)^2\right]^{3/2} \longrightarrow \hat{\rho}(\vec{r}) = \frac{1}{\pi^3 q_m^5}\delta\left(|\vec{r}| - q_m\right)$$

S^5 radius



uplift to 6D

$$\int_{-q_m}^{q_m} dq\, \rho(q) q^{2n} = \int d^6\vec{r}\, \hat{\rho}(\vec{r}) x_9^{2n}$$

$$\vec{r} = \begin{pmatrix} x_4 \\ \vdots \\ x_9 \end{pmatrix}$$

[Filev, O'Connor '14;
Y.A., Ishiki, Shimasaki, Terashima '17]

$$q_m = (8g^2N_2)^{\frac{1}{4}}$$

$\cdots$ exactly the same as the radius computed by the light-cone Hamiltonian for a 5-brane after the appropriate rescaling

$$H = \mathrm{tr}\left(\frac{N}{2p^+}P^iP^i - \frac{\pi^2 T_{\mathrm{M2}}^2 N}{p^+}[X^i, X^j][X_i, X_j] + \cdots\right) \to \mathrm{tr}\left(\frac{g^2}{2}P^iP^i - \frac{1}{4g^2}[X^i, X^j][X_i, X_j] + \cdots\right)$$

[Y.A., Ishiki, Shimasaki, Terashima '17]

M-brane geometry

[Y.A., Ishiki, Shimasaki, Terashima '17]

- M5-brane realisation:

locus: $\phi \sim iM$ at large N_2 and $g^2 N_2 \gg 1$

$SO(6)$ symmetric uplift ⟶ $\hat{\rho}(\vec{r}) = \frac{1}{\pi^3 q_m^5} \delta\left(|\vec{r}| - q_m \right)$

S^5 radius

- M2-brane realisation (at large N_2):

locus: $\phi \sim 2L_3$ at large N_5 and $g^2 / N_5 \gg 1$

$SO(3)$ symmetric uplift ⟶ $\hat{\rho}(\vec{r}) = \frac{1}{4\pi(\frac{3\pi g^2 N_2}{8N_5})^{\frac{2}{3}}} \delta\left(|\vec{r}| - (\frac{3\pi g^2 N_2}{8N_5})^{\frac{1}{3}} \right)$

S^2 radius

We reproduce the spherical thin shell distribution with the correct radius

N.B. The same goes for multiple-stack M2- or M5-branes.

Theory on 5-branes

Theory on 5-branes

We should reproduce the theory on the spherical 5-branes.
But there's a subtlety for NS5-branes.

To reproduce the spherical brane geometry, we took

M5-brane limit: $N_2 \to \infty$ and $\lambda = g^2 N_2 \to \infty$ $\cdots$ OK

The theory on NS5-branes is considered to be Little String Theory.
LST should be reproduced in the BMN model by taking a limit.

NS5-brane limit: $N_2 \to \infty$ and $\underline{\lambda = g^2 N_2}$: fixed (?)

NS5 coupling (?)

q_m doesn't approach ∞

→ The spherical NS5 geometry is not reproduced

The above naive identification of the NS5 coupling may not be correct.

→ This would be answered by the gauge/gravity duality

Theory on 5-branes

Vacuum
BMN matrix model

↔

Gravity solution
IIA/11D SUGRA

Type-IIA Lin-Maldacena geometry: [Lin, Maldacena '05]

$$ds^2_{10} = \left(\frac{\ddot{V} - 2\dot{V}}{-V''}\right)^{1/2}\left\{-4\frac{\ddot{V}}{\ddot{V}-2\dot{V}}dt^2 - 2\frac{V''}{\dot{V}}(dr^2 + dz^2) + 4\underbrace{d\Omega_5^2}_{S^5} + 2\frac{V''\dot{V}}{\Delta}\underbrace{d\Omega_2^2}_{S^2}\right\},$$

$$C_3 = -4\frac{\dot{V}^2 V''}{\Delta}dt \wedge d\Omega_2, \quad B_2 = \left(\frac{(\dot{V}^2)'}{\Delta} + 2z\right)d\Omega_2, \cdots$$

$$\left(\Delta = (\ddot{V} - 2\dot{V})V'' - (\dot{V}')^2\right)$$

(dot: $r\partial_r$, prime: ∂_z)

D2- and NS5-charges reside on (r, z) w/ $y = \dot{V} = 0$, and each config. of the charges corresponds to a BMN vacuum.

It depends only on $V(r, z)$, which satisfies the Laplace eq.

$$\frac{1}{r^2}\ddot{V} + V'' = 0$$

Theory on 5-branes

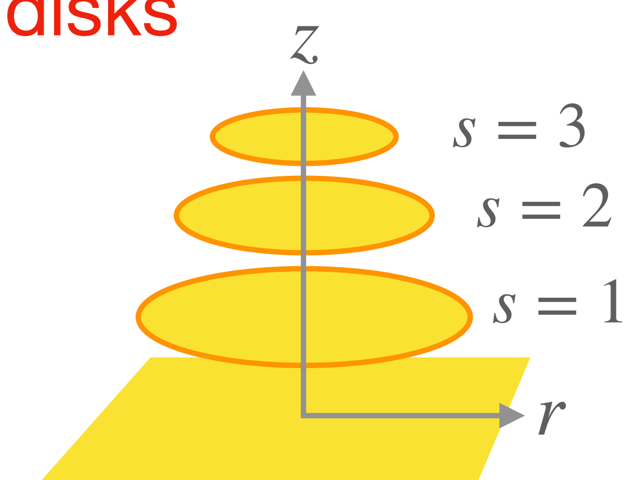
$V(r,z)$ can be regarded as an axisymmetric electrostatic potential.

- Laplace eq.: $\frac{1}{r^2}\ddot{V} + V'' = 0$
- Positivity of the metric: background potential $V_{\text{b.g.}}(r,z) = V_0\left(r^2 z - \frac{2}{3}z^3\right)$
- Regularity at $y = \dot{V} = 0$: configuration of conducting disks

height of the disk

$$V(r,z) = V_{\text{b.g.}}(r,z) + \sum_{s=1}^{\Lambda} \phi_s[f_s(u); N_5^s, r, z]$$

"electric density"



The Laplace eq. is rewritten as

$$f_s(u) - \frac{1}{\pi}\sum_{t=1}^{\Lambda}\int_{-R_t}^{R_t} du' \left[\frac{\frac{\pi}{2}(N_5^s + N_5^t)}{\frac{\pi^2}{4}(N_5^s + N_5^t)^2 + (u-u')^2} - \frac{\frac{\pi}{2}|N_5^s - N_5^t|}{\frac{\pi^2}{4}(N_5^s - N_5^t)^2 + (u-u')^2}\right] f_t(u') = C_s - V_0 N_5^s u^2$$

$$\int_{-R_s}^{R_s} du\, f_s(u) = \frac{\pi^2}{8} N_2^s$$

C_s: constant value of the potential on the sth disk

Theory on 5-branes

On the gauge theory side

$$\rho^{(s)}(q) - \frac{1}{\pi}\sum_{t=1}^{\Lambda}\int_{-q_m^{(t)}}^{q_m^{(t)}} dq' \left[\frac{N_5^s + N_5^t}{(N_5^s + N_5^t)^2 + (q-q')^2} - \frac{|N_5^s - N_5^t|}{(N_5^s - N_5^t)^2 + (q-q')^2}\right]\rho^{(t)}(q') = \frac{\beta_s}{\pi} - \frac{2N_5^s}{\pi g^2}q^2$$

On the gravity side

$$f_s(u) - \frac{1}{\pi}\sum_{t=1}^{\Lambda}\int_{-R_t}^{R_t} du' \left[\frac{\frac{\pi}{2}(N_5^s + N_5^t)}{\frac{\pi^2}{4}(N_5^s + N_5^t)^2 + (u-u')^2} - \frac{\frac{\pi}{2}|N_5^s - N_5^t|}{\frac{\pi^2}{4}(N_5^s - N_5^t)^2 + (u-u')^2}\right] f_t(u') = C_s - V_0 N_5^s u^2$$

They are completely the same equations!

Identification:

$$f_s(u) = \frac{\pi}{4}\rho^{(s)}\left(\frac{2}{\pi}u\right)$$
$$R_s = \frac{\pi}{2}q_m^{(s)}$$
$$V_0 = \frac{2}{\pi^2 g^2}$$

[Y.A., Ishiki, Okada, Shimasaki '14, Y.A., Ishiki, Shimasaki '14]

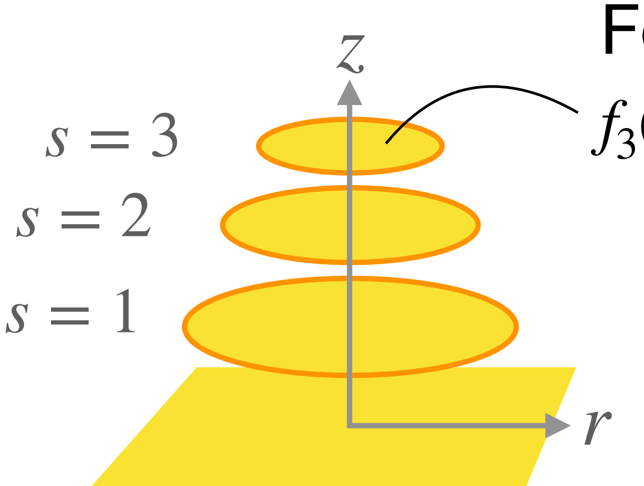
Eigenvalues construct a geometry.

Theory on 5-branes

There's a double scaling limit to obtain the gravity solution for NS5-branes on the gravity side:

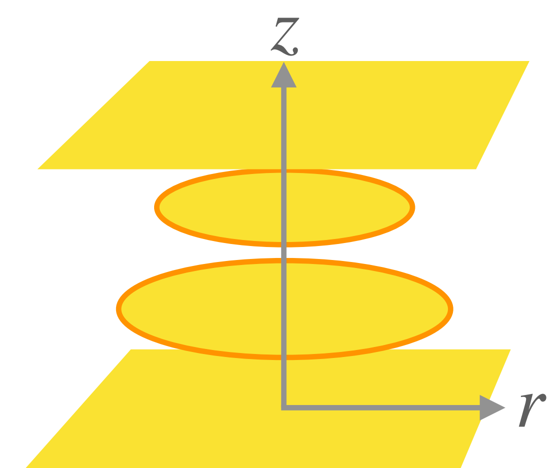
For general Λ,

$f_3(u) = f(u) + g(u)$



infinite conducting plate

sol. for NS5-branes



For $\Lambda = 1$,

$$V = V_{\text{b.g.}} + \phi_1[f(u); N_5, r, z] \longrightarrow V = \frac{1}{g_0} \sin\frac{2z}{N_5} I_0\left(\frac{2r}{N_5}\right) = V_{\text{NS5 b.g.}}$$

$$f(u) - \frac{1}{\pi}\int_{-R}^{R} du' \frac{\frac{\pi}{2}(2N_5)}{\frac{\pi^2}{4}(2N_5)^2 + (u-u')^2} f(u') = C - V_0 N_5 u^2$$

DSL: $R \to \infty,\ V_0 \to \infty$ with $g_0 \propto \dfrac{\exp[\frac{2R}{N_5}]}{V_0(RN_5)^{3/2}}$: fixed

[Ling, Mohazab, Shieh, Anders, Raamsdonk '06]

Theory on 5-branes

According to the gauge/gravity duality,
the double scaling limit on the gauge theory side is

$$N_2 \to \infty,\ g^2 \to 0 \ \text{ with } \ \underline{\frac{(g^2 N_2)^{5/8}}{N_2} \exp\left[\pi \frac{(8g^2 N_2)^{1/4}}{N_5}\right]} : \text{fixed}$$

coupling of the theory on NS5-branes

$$\Rightarrow \lambda = g^2 N_2 \sim {N_5}^4 (\ln N_2)^4 \to \infty$$

For general Λ, the DSL to obtain a theory on NS5-branes around a vacuum with D2 charges is to take the same limit only for the irrep. with the largest dim. N_5^Λ.

$$\rho^{(s)}(q) - \sum_{t=1}^{\Lambda} (\text{interactions with } \rho^{(t)}) = \frac{\beta_s}{\pi} - \frac{2N_5^s}{\pi g^2} q^2$$

$$\longrightarrow \rho^{(s)}(q) - \sum_{t=1}^{\Lambda-1} (\text{more interactions with } \rho^{(t)}) = \frac{\tilde{\beta}_s}{\pi} - \frac{1}{g_0} \sin \frac{2z}{N_5^s} \cosh \frac{2r}{N_5}$$

[Y.A., Ishiki, Matsumoto, Shimasaki, Watanabe '22]

Theory on 5-branes

$$\tilde{g}_s := \frac{(g^2 N_2)^{5/8}}{N_2} \exp\left[\pi \frac{(8g^2 N_2)^{1/4}}{N_5}\right] : \text{ fixed}$$

So far, the DSL was checked at the planar sector, but usually, a DSL is expected to keep all orders of $1/N$ expansion.

The expectation value of a function of ϕ is

$$\langle \mathcal{O}(\phi) \rangle = \sum_{h=0}^{\infty} d_h(g^2 N_2) N_2^{-h}$$

If $\tilde{g}_s$ is the coupling of the theory on NS5-branes (LST),

$$\langle \mathcal{O}(\phi) \rangle = C(g^2 N_2) \sum_{h=0}^{\infty} c_h \tilde{g}_s^h \quad \Rightarrow \quad \frac{d_{h+1}}{d_h} = \frac{c_{n+1}}{c_n} (g^2 N_2)^{5/8} \exp\left[\pi \frac{(8g^2 N_2)^{1/4}}{N_5}\right]$$

Finiteness of c_1/c_0 was checked numerically

[Y.A., Ishiki, Matsumoto, Shimasaki, Watanabe '22]

DSL beyond the planar sector

Summary

- Branes in the matrix model get to be understood better.
  We see emergent geometry in the brane picture and
  the gauge/gravity picture; both pictures are consistent so far.

- Unfortunately, the emergence of the geometry we've seen is incomplete. We highly relied on symmetry of the geometry.

- The susy localisation technique is powerful.
  In this talk, the localisation technique was applied to a theory on the infinite real line. This type of localisation computation would be useful for a test of the matrix-model conjecture and the gauge/gravity conjecture.